Introduction to Helicopter Aerodynamics and Dynamics

Prof. Dr. C. Venkatesan
Department of Aerospace Engineering
Indian Institute of Technology, Kanpur

Lecture No. # 20

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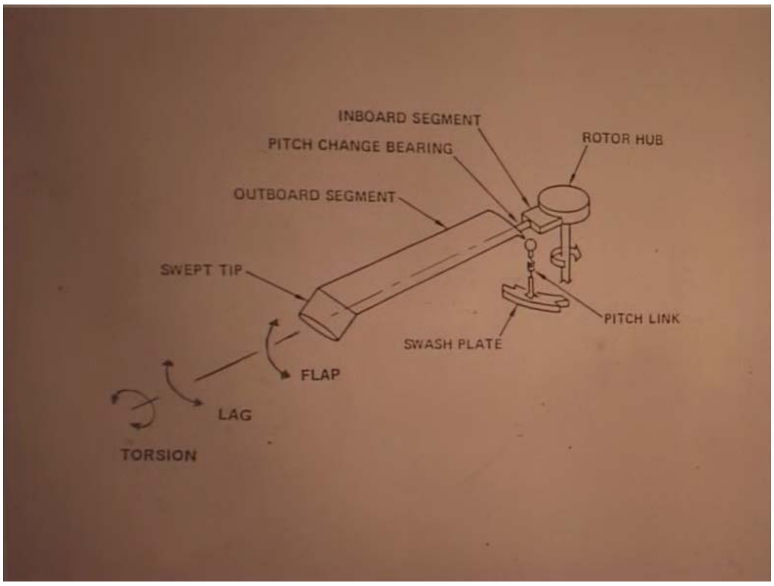

Model and how they are analyzed? This is a typical, you may say, diagram of a helicopter rotor blade. So, is a rotor hub, of course, you have attachment, then outboard segment, then you have pitch bearing, then this is inboard segment, then this is a tip, you can have different tip shapes. Even now, it is not fully clear, which tip shape is best for what; it is all, each company have their own, evolved with a particular tip shape and they use that particular shape; you will find a variety of shapes, alright.

And of course, you say, this is the flap motion, this is a lead like motion and this is a torsional motion. When I say torsion, means it is the elastic torsion, which angle you change at the root of the blade, but the blade also can twist elastically. That is equally important, when you are analyzing the actual blade because your angle of attack changes.

Now, this is a large aspect ratio in the sense, usually it is about 20-25 feet. I just brought one; this is the actual cross-section of a rotor blade. This is the cross section and you see the trailing edge is just a honey comb; this is a metallic, of course; there are composite blades. The trailing edge is honey comb, it just gives only shape, there is nothing else. All the load carrying part is done by only this section, the T-section you can call it and you will see the leading edge is a little thicker.

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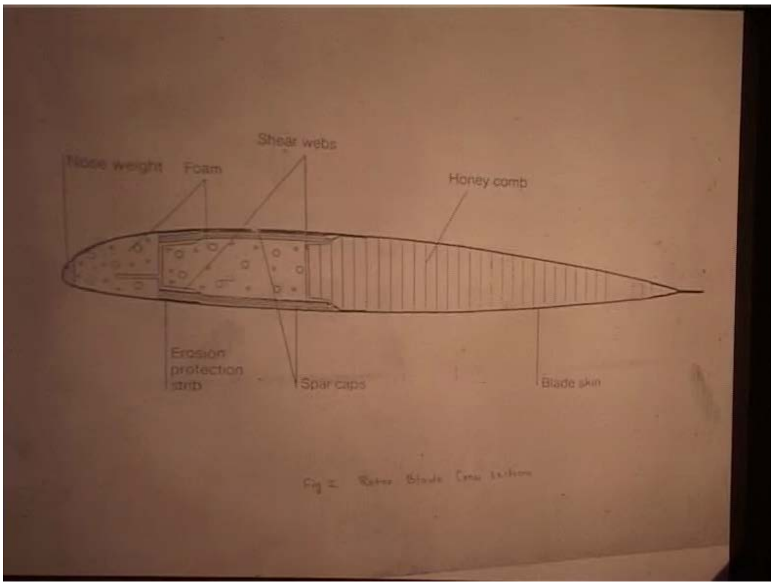

So, this whole design is made such a way, that the, I will show a typical section of a, this is a rotor blade typical cross-section, just similar to this, only thing is each blade has its own, this is the real life actual blade cross-section. You always put a leading edge weight that is to shift the center of mass to 25 percent card from aero-elastic consideration; you will learn that, I do not think in this course you will be able to learn. And then, of course, your aerodynamic center is 25 percent, take it as 25 percent because subsonic. Then, you also make sure, that shear center is at 25 percent. That means the design of your cross-section is very important where you put, that this vertical number is basically this portion, so that this is your load carrying. You can see where the shear center will be and that you try to keep it at 25 percent, but not all designs have to be like that. Here we have given some, see this is the spar, this is just honey comb or sometimes they fill up with foam, very light foam, that is just to give the shape, that is all. And main leading edge weight, this section and this is the foam, sometimes here, there is foam here; in case there is no foam, it is just a hollow.

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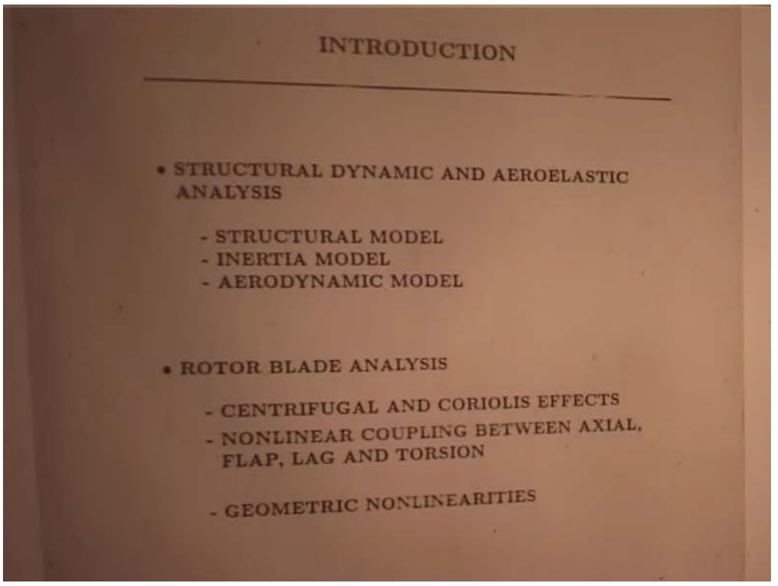

Now, if we try to write that, this beam is going to undergo, I will put it introduction just to say, when you want to study the blade dynamics you need to have the structural model. Structural model means essentially, the how the elastic properties vary along the blade. You say beam, beam type structure because helicopter rotor blade, always it is analyzed as a beam. Then, of course, you have to have an inertia model, then the aerodynamic model, these 3 if you have you can study structural dynamics and aero-elastic analysis of rotor blades. People have spent their entire research career only on this, just sometimes only modeling the structure.

Now, way, the key difference between the conventional non-rotating beam and a rotating beam if you consider, one is the, we introduce centrifugal and Coriolis effects, these are because of rotation and this you cannot neglect it. So, this is the difference between a beam, which is not rotating, which you study in your art of success one. But here is the beam, which is a rotating beam, so you have a centrifugal and Coriolis, both will come because of the deformation. And then, of course, you will have, I put non-linear because it comes because of axial deformation; you have to take because of the centrifugal force. You need to, how do you account for it, there is, earlier days there were talking in a approximate manner, but if you systematically derive the equation, you will find, it is a non-linear equation. What type of nonlinearity is strain to displacements is non-linear. Please note strain to displacement that is one, nonlinearity, which is called the geometric nonlinearity.

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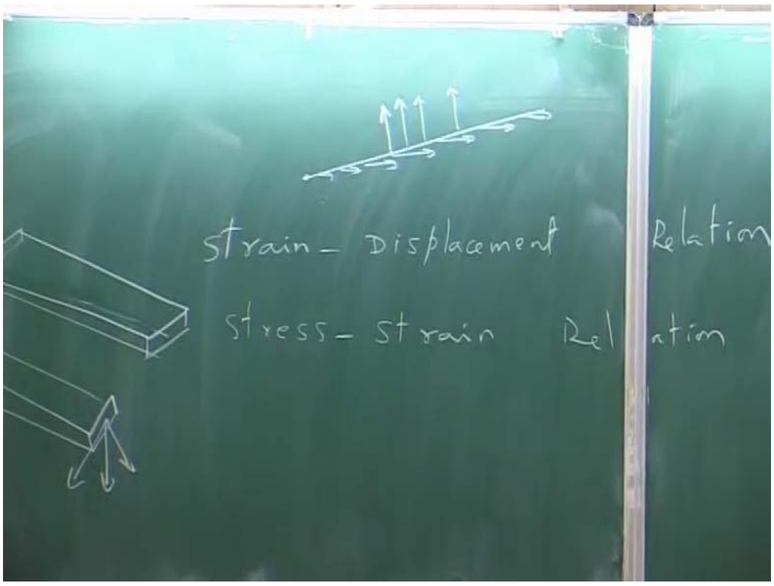

Strain to displacement relation, another one you can have stress-strain relation. These are the 2 in any structural model, there is nothing more. All the problems of what type of material, that comes here, where it is composite, isotropic, anything that is only in the strain displacement relationship, but stress-strain, sorry, strain, sorry, strain-stress relation, strain displacement relation is purely kinematics, that is the deformation. How the strain at any point is, what is the value of the strain in terms of the deformation, and you can... This is what, is the non-linear expression.

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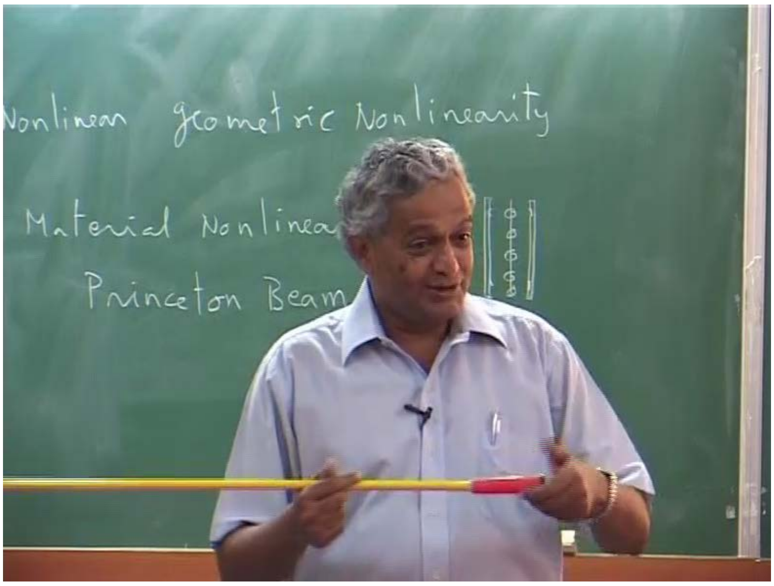

If you take only linear term, you will find, that you will not be able to get the proper representation of the rotating beam, that is the key, you have to have the non-linear relation. So, this particular type of nonlinearity, you call it geometric non-linear, geometric nonlinearity. Whereas, this is stress-strain relation; non-linear means, that is the material nonlinearity, so you call this material nonlinearity. These are the basic things, but we do not go to material nonlinearity. The key is whenever you design a blade, you will make sure, that the strains are small, actually infinitesimally small, strain is small, but strain to displacement is not small because the displacements are, they do not say large, they say moderate deformation. It is not a small deformation, that is why this relationship is non-linear and that is why, I put geometric nonlinearities have to be considered if you want to have a proper treatment of this blade analysis.

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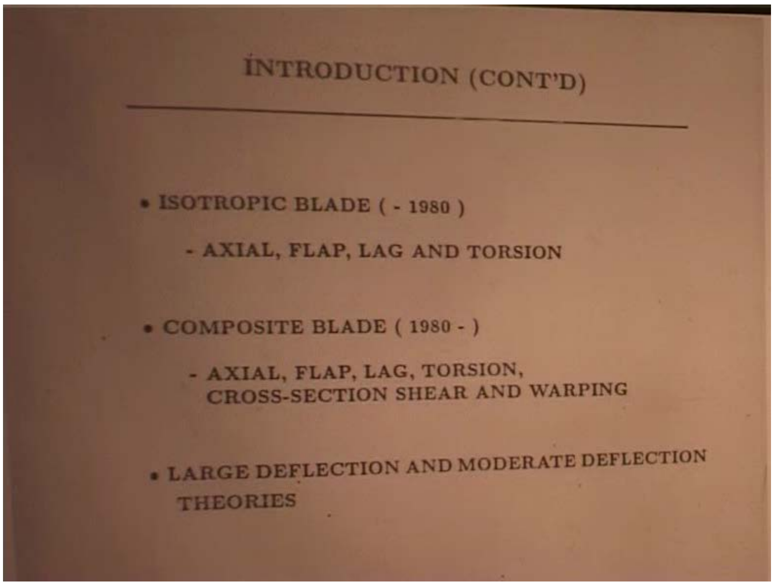

Now, just to give you a brief history, see, isotropic blade, real part, real formulation started actually in 1950's. (( )), that was the first paper he wrote on axial flap bending, lag bending, torsion, but then those, that was the first paper, there were mistakes, few mistakes. Subsequently, groups have started, actually the groups, you will be surprised. One group is actually, turbine blade type of analysis, another group is a rotor blade, other 2 groups because one group was at NASA Louis Center, other was some universities, so they started deriving the equations for a beam, please understand, beam equations. So, it is all isotropic blade, first in 1980's, but then composites people started using, so again, after 1980, composite blade formulation. Then, what are the various things you have to consider is, there is anything extra, suppose if you have a one general formulation, which fits into everything, then it beautiful. But unfortunately, when you were starting the development, always you make approximations because you need to get something. So, they used the, cross-sectional shear deformation was neglected here, then they said we will add this. Then, of course, warping, bending, warping, various types of warping of course, torsion is, warping of torsion is important, that has to be considered here. Then, of course, people also developed large deflection, moderate deflection, moderate means.

You make some approximation because if you use this type of, for I am not writing any equations, here you will find the strain expression itself will be missing, strain to displacement. You will have lot of terms, one is term, another one is you have to now analyze what is the blade dynamics and then, aero-elastic behavior, that is all, only these 2. But to know, whether your equations are correct or not, so there has to be some experimental study. So, one experiment, that is why, I said certain, sometimes one simple experiment will give a very thorough understanding on the status of your knowledge. So, this experiment it is called Princeton beam because it was done at Princeton, very simple experiment. You can look at web page, you see experiment is, you take a beam, it is not a twisted beam or anything like that, straight beam.

So, we all know, that beam means, you take it like this instead of taking it in this passion, you slightly rotate it and then, when you apply the load like this, it will take a load this way and this way. The beam cross-section, if it is flat, you apply the load, this is a beam building you wanted to bend in both directions, so instead of that, they just took the beam, kept little inclined, apply the load, that mean the tip load, you will give reflection in the blade coordinate, it will give in both. So, this experiment was conducted for slightly moderate reflection, lag reflection you may call it, then the data was given. Initially, people could not get that experimental data, then they went back again, looked at the whole equations, started rederiving, then finally, they got the results. That means, you now know, hey, I am able to solve a bending, bending problem; very simple experiment, that was the time people, now equations are fairly well established. You know, that what is the procedure to develop the equations and then, get the equation of motion for a rotor blade twisted beam. No problem, only thing is the, now of course, there are assumptions made, the complexities come in the aerodynamics.

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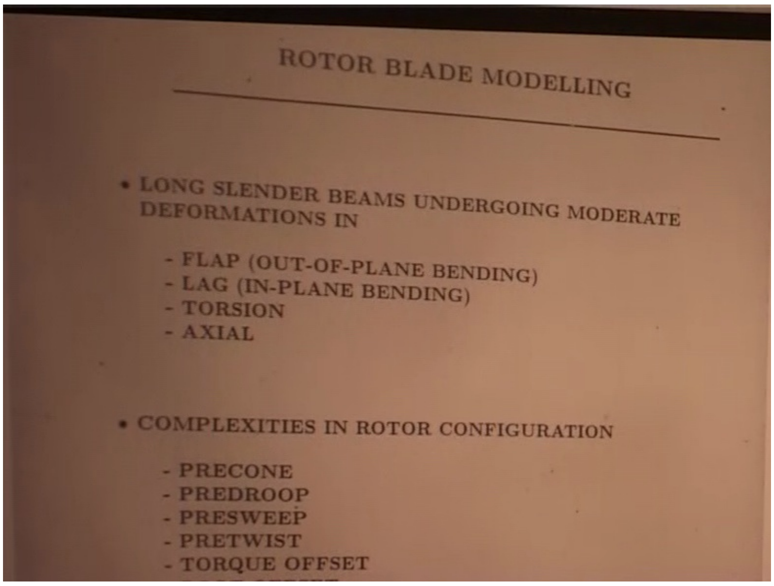

Now, so now when you look at the actual, I have written here the modeling you created as a beam model nothing else, but then rotor configuration, if you look at it, you will have precone, predroop, I will tell you what those things are, then it can have presweep, pretwist, then torque offset, root offset. These all are geometrical quantities, which will be there in actual rotor blade, now you need to take all this.

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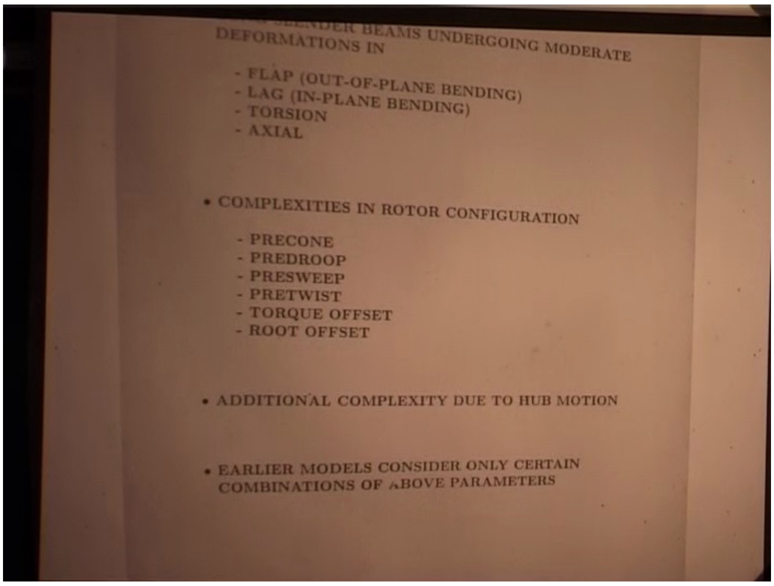

And of course, additional complexity is, if the hub itself is moving, hub is fixed is one. Suppose, hub moves, in addition the blade is moving, then the problem becomes more complex. In the earlier formulation only they will take precone, which we have used beta p, just very simple, but not all the other quantities. Root offset I will show you, we take the key thing, they will take rest of them, but then when you want to take all of them, then the derivation becomes messier. But now, it is, you can take all this quantities in your blade model, I will show just a diagram, both very exaggerated views; this is an exaggerated view.

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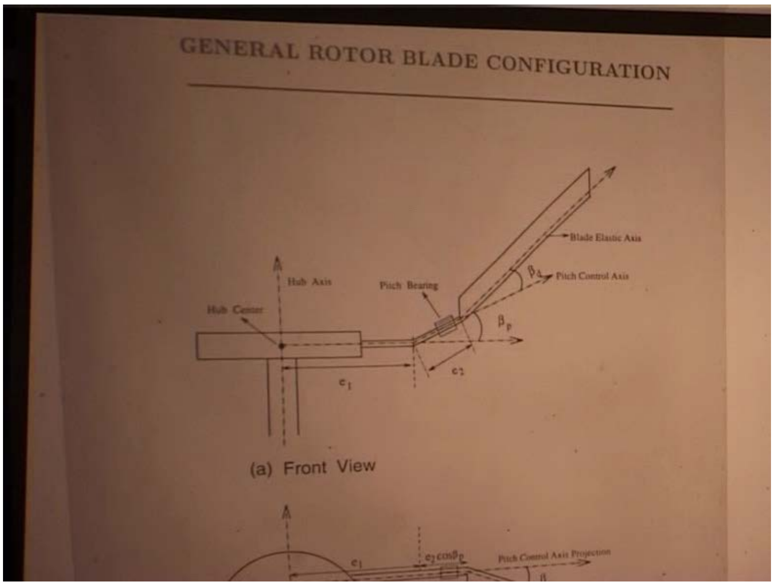

If you look at the, this is the rotor hub, this is one offset, we can say hinge offset, from there you can have, this is the pitch bearing and pitch bearing is inclined with respect to the hub plane through an angle, that is precone, that is called precone; pitch bearing is inclined with respective to the hub plane. Now, the blade can be inclined with respect to the pitch bearing, then that is called pretroop. So, precone, pretroop, these are 2 terminologies. Why this difference is, you change the pitch angle by rotating the blade about this axis. When you rotate about this axis, if troop is 0, the pitch angle will change like this, but if the troop is there, if I rotate about this axis, the blade will go like this. Not only the pitch will change, it will also move back and forth, you understand, because the axis of rotation is not along the blade, it is at some other direction. So, this will go like this. These effects are very critical naturally. The pretroop is called the preflap that is the HAL terminology.

Now, precone, precone I said, no, no, no, ALH because this is used by Germans, but the westerned, westerned means, I mean in U S, they call it troop and then, cone ALH blade has pretroop because the pitch is at that, the blade is like this. So, if you rotate, the blade will go like this and this is a geometrical, these are all, if you see, what if a (( )) 2 degrees. You know, what is the effect of frequent, you say leave the root moment, but it has aero-elastic coupling, tremendous influence in terms of done, already conning I have shown here. What is the difference between these 2? It is, see I am not sure whether one blade has both, I am not sure there are blades with only pretroop, that is the ALH blade. There are blades only with precone, but there is a difference geometrically, they are different, they are not same because they will introduce different effects in the aero-elastic analysis, but you can design a blade with both this end here, hinge…

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No, the earlier, they had actually bearings, now they have (( )), this is actually soft. Suppose, if you want to see our blade, our blade has a, they have the, what do you call, there is a centrifugal, what is that, trust bearing. Trust bearing is, you, you hold this, I am pulling it, if I hold it here I am pulling it, centrifugal force will pull the blade, I have to somehow hold it, but then when I pull the blade it should not come out. So, I have to put a stopper, but at the same time I should be able to rotate. So, the kind of bearing is called the trust bearing, it can take the axial load, but at the same time you can rotate it freely. So, it is like, if you want to see we have removed our thing, you will be able to, we removed, we assembled it, you want we can again easily remove the blade and then, show it to you how it is assembled, you can have, very simplistically you can have a, I have shown a, this is one plate, this is another plate, in between there is plate with lot of ball bearings and there is groove in this, it will come and sit in the groove, you close it and the blade will be tightened from this side, you can rotate freely, this is attached to one, this is attached to the blade. So, you can easily change the pitch angle of the blade without any difficult centrifugal, you do not know how to overcome the centrifugal force.

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If it is a pretroop, yeah, it will go like this, it will go like this, but it, all you give only 5 degrees, 4 degrees, that is all, not much, 10 degrees at the most. The blade will go like this, obviously, see what will happen is, here suppose, I say the blade, it comes here and then it takes a shape. That means, this portion is where I am attaching. Now, this is where the blade is, blade means, this is the aerodynamic, everything, this section is a part of the blade, blade can be manufactured like this; you can manufacture a blade like this.

I can have a bearing composite blade that is why composite blades are easy to manufacture with that shape. Pretroop, this is the troop, because you see metallic blade is tough, but composite blades are easy; ALH blades are composite blades, so it is there. Advantage, that is, I only say word, it, see one is, whether it is a precone or a pretroop in the equilibrium level, equilibrium means whatever we obtained as the loads, it reduces the root load, that is equilibrium root movement basically. But if you take the effect of that on stability, stability means not blade stability, flap lag torsion. In that problem, pretroop gives some tapping, but it will increase the vibratory load in lead lag, but precone will not give the damping, but it reduces the load. So, you have to see that trade-off, what you want, whether you want higher load, vibratory load, not mean load and this will have lot of effect on torsion. So, you see, flap, if you look at it, both configuration, there will not be major change, lead lag there will be change, torsion there will lot more change, which will go to pitching load, then you say, which one is better. This debate goes on because changing configuration from to the other is not an easy task in the actual design. Finally, you live with it, whatever load that comes and you try to minimize something here and there so, but each has its own plus and minus.

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No, no, precone means, precone or, yeah, see it is the root, root will always get the root share force bending movement. You remember, I derived that ideal precone; that means, what is the route bending movement?

(( ))

Yeah, does not matter. See what will happen is, aerodynamic load tries to take it up, centrifugal loads tries to bring it down and there is equilibrium. Suppose, if your rotor blade is already kept equilibrium, manual there is no root stress; you follow what I am saying? That is why, that is, given that precone relieving root moment in the blade, otherwise you have to make root section little thicker.

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It will come, no, no, but elastic deformation, see you are always think, this is another problem, problem not… We always think, if you draw the bending movement shear diagram, please understand, cantilever beam. We always say, root has maximum, you say bending moment, it is not. So, here, that is what you have to know because if you take this beam, your lift force is here and C.F. is here, actually the maximum bending moment is not at the root, you can have 0 bending moment, that is why you can have a hinge and you still have an equilibrium, you understand.

So, you got it; (( )); what; (( )); yeah (( )) here (( )). No, no, that is no hinge, there is no hinge in this things, this is a hinge blade configuration I have shown. Oh, if you have a specific hinge, that you will come to that, that is why I said, this is just like a configuration, I have not shown actual geometry, this is a hingeless blade, I can design like this, I am just showing a schematic. If you take an actual blade, you can have the hinge at the, near the hub, pitch can be outside, no problem, or pitch can be inside; hinge can be outside.

(( )), yes, when it flaps, yes, yeah, yeah, yeah, it will do. I will show you, that, that geometrical thing, how it happens, that is why this was always questioned, where did you put the hinge? But most of the hingeless blades, actually I am pretty much, you know ALH blade, the pitch bearing is inbound and there is, there is no flap hinge per se. But there is an equivalent flap hinge, some soft, stiffness drops, some section where that becomes kind of a hinge, it is a virtual hinge, it is not a physical hinge, but some point is soft. Therefore, it can flex about that point, but the pitch bearing is inbound, most of the hingeless blade pitch bearing is inbound, you do not keep it outside, most of them, you see, that goes into construction.

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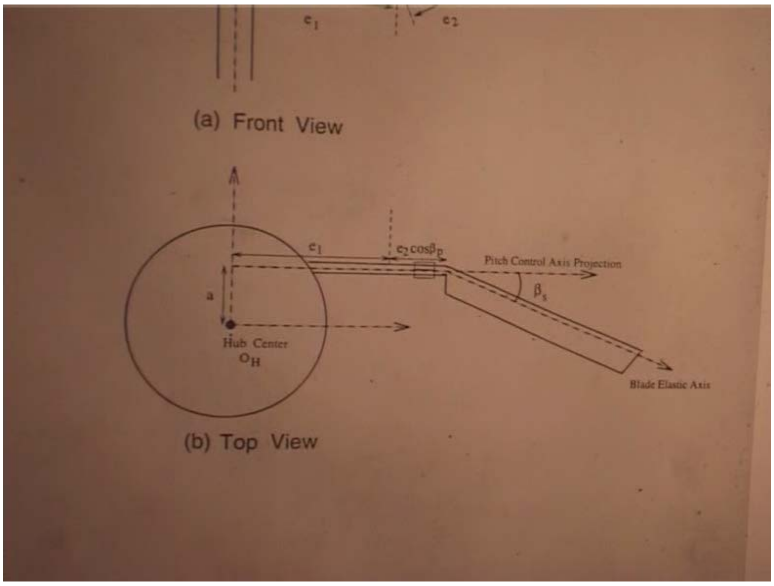

Now, the next, from top if you view, of course, this is just to, I have said, that presweep, you see the axis of the blade need not pass through the center of the hub, it is not that the blade is attached here, it can be attached, but it can also be offset by some small distance, this is called torque offset; torque offset, this is. If you really see what is that distance? That will be about 10 mm, ok, that is all, is not a lot. Usually, it is a very small distance and of course, I have exaggerated this.

Because saying, that suppose if you have a tip, this, blade can be straight and tip element can come here, only tip, 1, 1 small section of the blade can be swept back, that is a tip shape. So, the entire blade is swept, but of course, no blade is made like this. Now, this is the torque offset, these are hinge offset and pre-grown, preflap or pretroop and then presweep, these are the geometrical complexities in a rotor hub, but it is not necessary, that all the rotor blades should have everything; that depends on the experience of the manufacturer.

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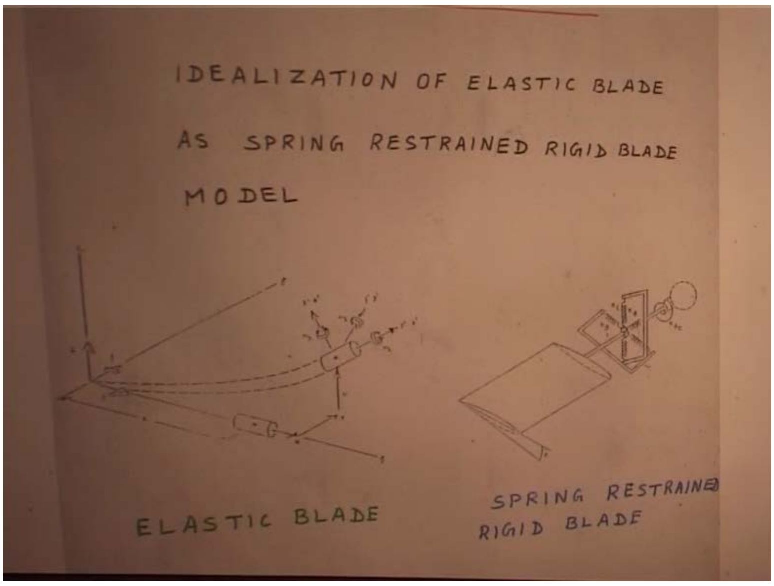

How do we analyze this problem? I will just show idealization of an elastic blade as spring restrained rigid blade model. This is your idealizing; this is the elastic blade I have shown here just for example. If you have a small element because this is how we analyze structure, you take a small element of the blade.

How this small element deforms during operation? This element can have an axial deformation u, a lead lag deformation v, a flap deformation w, and then you can have 3 rotations. Now, this is the model of an elastic blade, now if you want to really follow systematically, because based on that, it is a basically displacement formulation. We, do you assume a displacement and then get the strain, then once you get the strain, then go to stress and then write your equilibrium equation from where the standard technique starts. But of course, you have to take the centrifugal load into account.

This modeling was started only in the, I would say, fifties, 1st paper, then of course, eighties people were really, seventies to eighties lot of books were, lot means, about 3, 4 books, that is all, not many.

But simultaneously, they were trying to get a simple model, which is a rigid blade, but represent the entire elastic properties by a root spring restrained rigid blade. So, I have 3 springs I have put, this is the torsional spring, this is the lead lag spring, this is the flap spring; this is an arrangement. This is my own idealization. Now, I idealized my rotor blade like this and then I go ahead and solve all the problems of rotor blade by this model, you will not bother about this, but only thing is the question will be, what should be my spring constant?

You try to match the fundamental natural frequency of the blade, only fundamental, 1st mode, because elastic blade, it will have infinite natural frequencies, 1st mode, 2nd mode, 3rd mode, 4th mode, etcetera; if you study vibrations, then you will know.

So, otherwise, you know spring mass system, if you have 1 spring mass, single degree of freedom, 1 natural frequency; if you have 2 degrees of freedom, you have 2 natural frequencies; 3 degrees of freedom, 3 infinite degrees of freedom, infinite natural frequencies.

Rotor blade has infinite because a continuous beam, but for practical analysis, usually they take a word, that is from design point of view. Flap, you take about 3 or 4 lead lag, 2 or 3 torsion, 1 or 2 flap modes, 3 you take, 3 or 4. But usually, 3 flaps, 2 lag, 1 torsion. Some people may say, ok, I will take 4 flap, 3 lag, 2 torsion and 1 axial, we take an axial mode because that has to bring in some effect, but you look at the frequencies.

So, in the rotor blade, in the dynamics group in a helicopter, they analyze some, of course, couple of guys there, the, we use to run every day, what is the natural frequency mode, shed angular frequency mode, shed in each mode ?

We have developed the code now; that is given to HAL, we can analyze for an actual blade. Of course, there are lots of small, small details, which we have to take into account, a real blade. Once you have the code ready, everyday you will run. You may not know how it is developed, but that is a finite element code, but this is an idealized model.

Most of the rotor analysis packages in the earlier days, they can use only this model because it is easy, you can understand the physics. But it is a gross idealization of this, you are representing only 1 flap frequency because there is a flap spring, then there is a lead lag spring, 1 torsion. So, you will have 3 frequencies, that is all and the blade is rigid straight blade; that is it, it is a rigid blade.

So, you can get certain phenomena correctly, reasonably good, but you cannot get vibration and the other things because this is a, this is not a true representation of this. So, you will not get all the other loads, it will be in error, but you leave it that.

Today of course, people want to have elastic blade model. Of course, there are several research groups who have developed it and they are also working on further developing the code. Now, we take first the flap because we have been studying flap, I will briefly describe the flap model.

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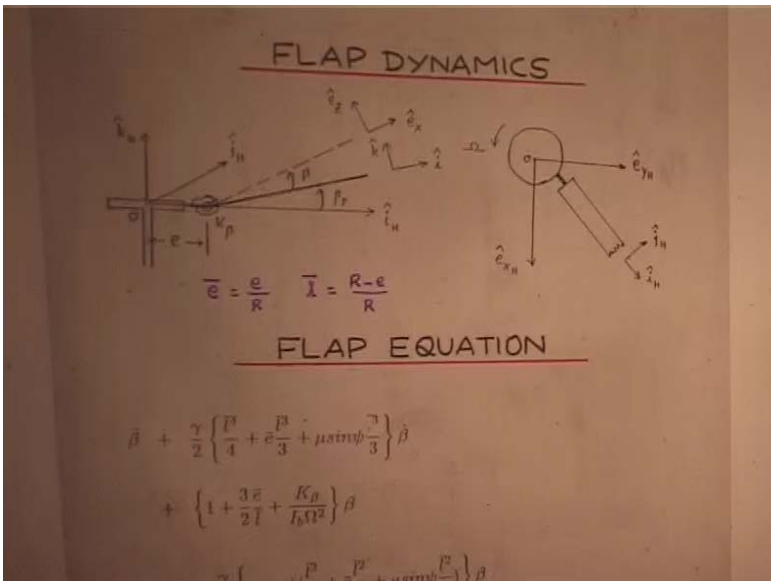

First isolated flap, isolated means only flap motion I am considering, I am not considering lead lag torsion combined because the moment I combined, I cannot show the equations. Now, you see, I have slowly, I am complicating my blade model. The earlier we had a centrally hinged and we put a spring restrained. Now, I am having an offset and a spring, this is what a real life blade is, you can represent it, equivalently offset hinged spring restrained blade model; even our blade is also similar.

Now, you have to write the flap dynamic equations for this model. Of course, the procedure is identical, you take any, I have given a precone, this is the undeformed axis, from there the blade deforms. So, take any point, find out its velocity, find out its acceleration, then take the inertia force, then balance the root moments about this points because here is the route spring. You balance the route moment, take the aerodynamic load and you will have, but now the length of the blade is different from the radius of the disk because the radius of the disk is e plus the length of the blade.

So, your, you have to be R minus e, R is the rotor radius, e is the hinge offset and this is non-dimensionalize with respect to rotor radius, I call it l bar. Similarly, e bar is the non-dimensional hinge offset.

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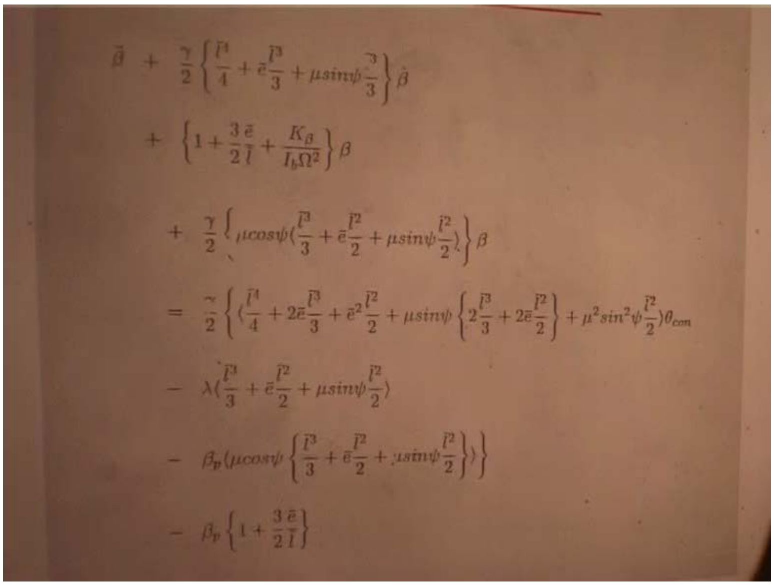

Now, I have to have this and then I derive my equation, equation I have showed it here, I am not deriving, you can have a look at it and the notes which I send it to you. But we will discuss this part a little bit in detail because of, just the hinge offset, nothing else. The, I have not done anything more between the earlier development to current development, only thing is, I added a hinge offset, hinge offset will add lot of additional terms.

When you put that e bar 0 and l bar as 1, you will get back the old equation what you had; I have sent it for you. Otherwise, everything thing is same. Now, you see, every small, small geometrical parameter you add, your equation becomes little bit more lengthier and that is why, otherwise one can really develop the full equation, then start reducing it.

But that is, only when you understand fully you can go, otherwise you start from basics, then slowly proceed further. And I gave you as an assignment, which you have all have solved, only this problem, flap equation with e bar 0, l bar as 1 and I knocked out all the, except the, yeah, not, there is a dot here, knocked out all the right hand side, you solve, given an initial condition, how the response looks, that is what the problem you solved.

I would like to discuss this particular thing a little bit carefully because of the introduction of the hinge offset. What happens to the natural frequency of the blade in flap mode rotating natural frequency?

If I take the hover case, very simple, I can reduce this for hover; hover means just put mu is 0, it will become the hover. So, let us take the hover case, we will have and I have done 1 more approximation there, that approximation I will mention to you, so that I can write the equation here. All the derivatives are with respect to non-dimensional time.

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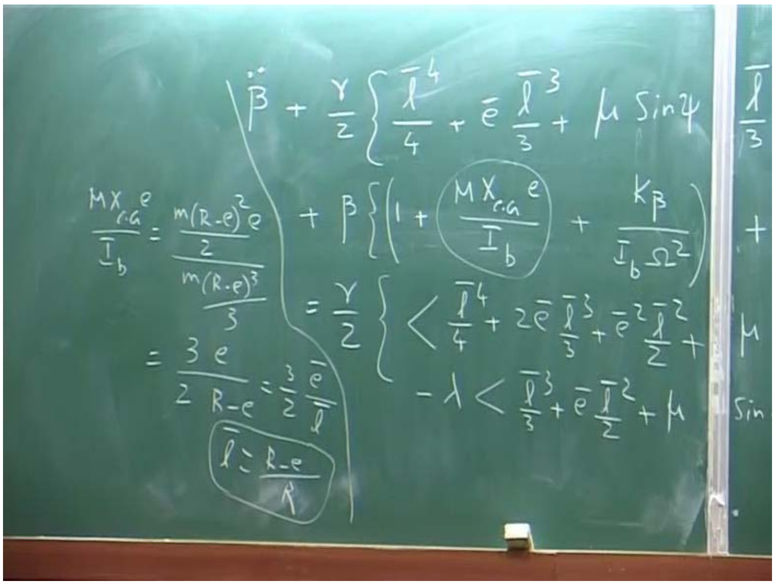

Beta double dot, this is gamma, (( )) number I am just writing it for, mu sine psi beta dot, then you add beta term, maybe I will put it here, plus beta into 1 plus, I am writing here slightly differently, please note, later I will show you what this particular term is. This is the root spring I b omega square, this is one term, plus gamma over 2 mu cosine psi l bar cube by 3 e bar 2 plus mu sine psi l bar square over 2, sorry, this bracket I close it, this I close it, this I close it. This, this is my stiffness term, please understand.

I have 1 plus, this I will explain to you, what this term is root spring divided by blade mass moment of inertia non-dimensional omega square, this is due to the aerodynamics, gamma and this is a time, varying this is only with respect to mu forward speed. If forward speed is 0, this term is 0. And then, the right hand side, you will have equals gamma over 2, you will have the control input 2 e bar l bar cube over 3 e bar square, anyway. This mu sine psi 2 2 e bar plus mu square sine square psi l bar square over 2, this is into theta control input, that is theta pitch input, what the pilot gives, theta naught plus theta 1 c cosine psi plus theta 1 s sine psi, that is this term. And then, I will have minus inflow term lambda l bar cube over 3 e bar l bar square over 2 plus mu sine psi l bar square over 2 and then I will have precone terms beta b 1 plus M X CG e over I b plus mu cosine psi l bar cube over 3 e bar l bar square over 2 mu sine psi l bar square over 2, bracket. This is my entire equation. Now, I will tell you what those I b as a definition because that you have to know, what is M X CG, what is I b, etcetera.

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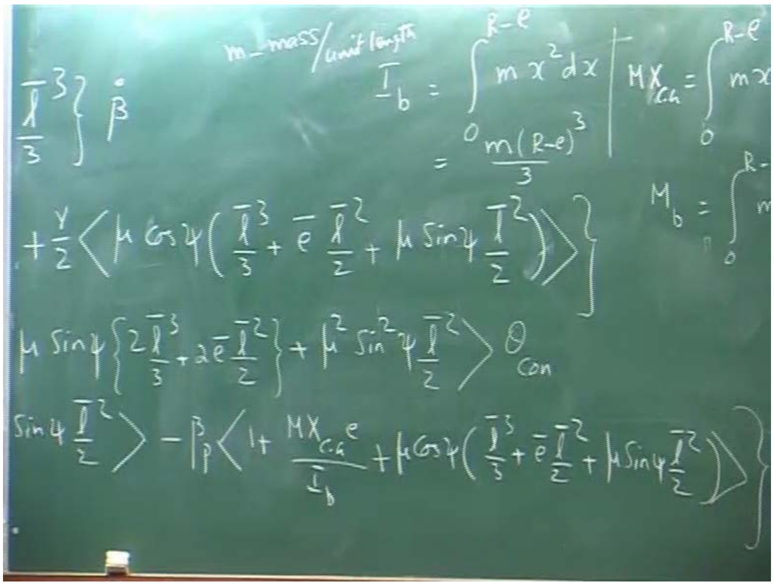

Because I b integral 0 to R minus e m x starts from the hinge offset, from here basically moment of inertia. Then, m is mass per unit length and then, mass of the blade M b is m d x.

Why I am taking? See, there I assume, that the mass is uniform, whereas rotor blade need not have uniform mass. Of course, the major aerodynamic section is uniform, but the blade mass can vary little bit along the span. If you take this term, if it is a uniform mass, this will become, if m is constant, this will be m R minus e whole cube over 3; R minus e cube over 3.

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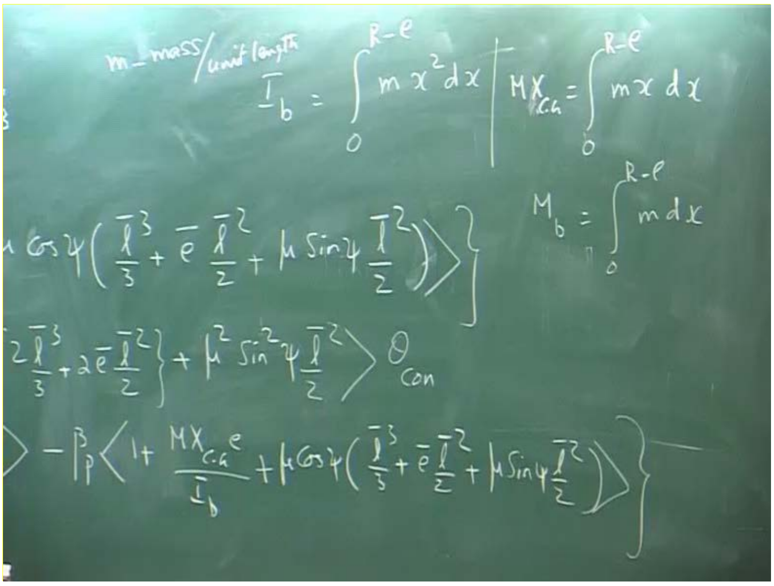

This will become m R minus e square over 2, if m is constant. Now, if I substitute in this place that will be M X CG e over I b becomes your m R minus e whole square e over 2 divided by m R minus e whole cube power 3.

So, this will become 3 e over 2 R minus e, which you can write it as non-dimensional. If you divide by R everywhere, this will be 3 over 2 e bar over l bar, where l bar is R minus e over R, if m is constant and that is what I have written here, 3 over 2 e bar by l bar this place. I have assumed that the blade is uniform, all right.

Now, let us take this equation. What is the frequency? Because this is the non-dimensional thing, if I reduce to hover case, automatically what happens? You know, that this is x double dot plus some x dot plus some x equals some right hand side equations, only thing is the coefficients are of course, time varying coefficients, which you have seen. We do not have any closed form solution for this.

This is the equation; this is the equation, now you can analyze steady state response. That means, if I prescribe my input, find out the steady state value. What is the response of the flap motion? You know, that immediately you have because theta, this one you have theta naught 1 c cosine psi plus 1 s sine psi.

You have already 2nd harmonic, this gets multiplied, 3rd harmonic, you will have, all the harmonics can come, so what you do? You neglect everything, I take only 1st harmonic. But that is not correct because as you go to high speed, you will find, slowly 2nd harmonic will dominate than the 1st harmonic, but this is an assumption we make, we neglect and our results will be wrong.

But it is, you take it, I will write something, I am getting some at random, this is what, otherwise you have to take the full thing. So, for the preliminary study, there are 2 things you can analyze, one is stability of the system, that is, you take a right hand side, you put some initial condition, you have analyzed that problem, which I give it as a assignment problem. But that was simplify, please understand, because you need to have this term, you did not have this term, you had only 1 plus and these are all 1 by 3, 1 by 2, you had only this. If you look back your equations, here I am having a little bit more complicated. So, this particular term 1 plus M X CG e over I b plus k beta over I b omega square, this is my rotating flap natural frequency in hover because this is not that.

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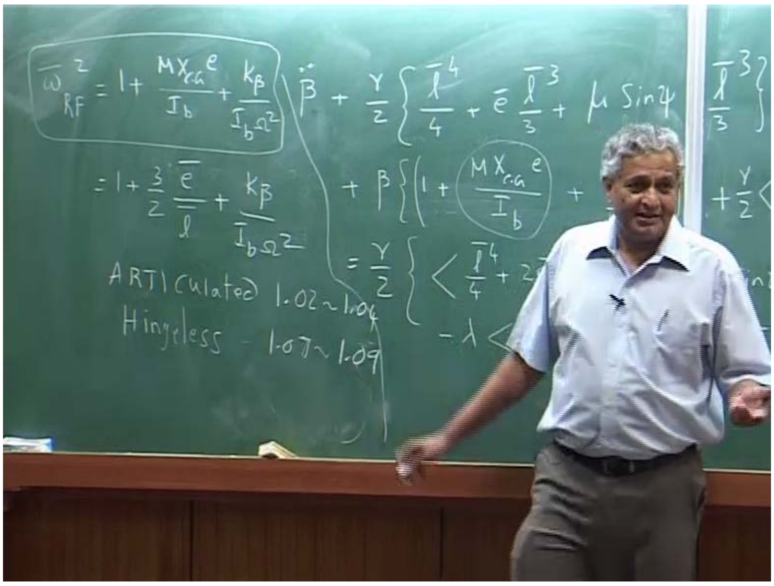

The moment, I said mu 0, my rotating flap frequency, which is omega bar R F square is 1 plus M X CG e over I b plus k, this is my rotating flap frequency. And if my mass of the blade is uniform, then I will have the same expression simplified as 1 plus 3 by 2 e bar over l bar plus, if m is constant.

Now, you see, even if I do not have the spring, if I have an offset root, offset, that is root hinge, offset my frequency is more than 1, that is, my flap frequency will be more than 1 for a rotor blade always, but if I put a spring, I add a one more stiffness term.

So, you just see, the root hinge offset gives a stiffness basically, your flap frequency is more than 1, now how much? So, this is where the hingeless blade, articulated blade, that key differ articulated blade, you do not have any spring, that is not...

So, you usually have in the range of articulated rotor, it is around 1.0, I think, 2 to 1.04. Whereas, hingeless, you go to 09, 1.1 sometimes because physically, you need to have some because you cannot put the blade right at the center, there is a physical dimension. So, articulated blades have flap, natural frequency is in this range, whereas hingeless blade slightly higher. Now, if you want to find the equivalent hinge offset without the spring hingeless blade, you do not know that, maybe you can always say this is having a hinge offset, which is larger than articulated more hinge offset because you have a more flap frequency; more flap frequency. You remember, earlier I said hub moments; you will have a higher hub moment.

Omega bar R F square minus 1 over gamma by 8, remember. That means if you have a higher flap frequency, you will get the higher harmonics; that means, pilot will have, he can generate more. Now, this is where, how much higher you can go, can you go further, then they will say it becomes too sensitive. So, they try to keep it only maximum in this range; do not go too high otherwise higher flap frequency, your higher moment, higher moment, higher vibration.

So, you will find the problems will be if it is a hingeless, your vibrating loads are also higher. So, you get a control moment high, that is why, my vibration is also high. Now, what is the choice? So, this is where every parameter, everything, there is a plus point there is a minus point, plus…

(( ))

Flap frequency, you remember, c m x, c m y, that expression was what? Omega bar R F square minus 1 divided by gamma by 8. Omega bar R F square, if it is higher you get a higher moment, higher control moment you get; higher control moment means pilot's per a unit stick motion, he gets a large moment, he generates more moment. So, that is why, the flap frequency is very, very important in actual helicopter design, but you cannot have anything you feel like because finally, the pilot's opinion matters. If he feels it is too sensitive, then you have to go back and then redo the entire design.

But this is the range, now onwards the course will be like this, I will deviate (( )), yeah, yes. Control respond I do not, I cannot immediately say, if there is a very tricky question you are asking me. Usually, I do not think, that is analyzed from that point of view, precone and pretroop, because I will tell you just flap equation alone, if you take only flap equation, there is no difference between these 2, except for the root loads.

Whereas, if you look at the damping, damping means I am talking about aero-elastic damping, not the vehicle damping, aero-elastic damping of flap lag stability, then lag damping will be influenced by whether it is a precone or a pretroop. Pretroop actually, improves the damping and if you look at the loads, pitch link load will be higher in case of pretroop because of the torsional coupling, torsional coupling it will be there, you will get a higher vibratory load.

Now, from the point of view of vehicle stability, I would not make any statement now because I do not know, its, it is unless we solve the problem you will not know because if I make assumptions, that its only flap motion, then I will, I may not be able to see the difference and most of the vehicle dynamics is analyzed only with flap because they do not take lead lag, because the flap motion controls vehicle dynamics to a large extent.

That is why, if you see the flight dynamic equations, they will take flap, but later somebody said, that damping is not properly predicted in the vehicle. So, it will take lead lag, but then problem becomes messier, your understanding also goes down; may be, when we solve the full problem, hopefully we will be able to answer your question.

Today I do not know, but if, it you consider only flap equation, you will not see any difference, you will not see any difference, you will see both are same. But if you consider flap and lag, then you will start seeing the difference between precone, pretroop; you will see in the stability, you will see the difference in the results, alright, now, this much for the frequency.

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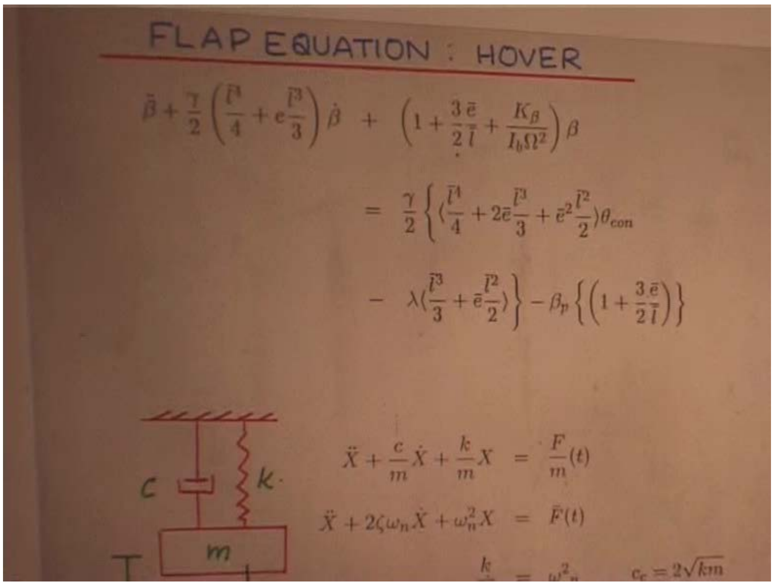

So, I have given here, essentially what I have written there, I have reduced it to the hover case, of course, with the offset e etcetera. So, this is just for comparison, the frequency I said because you know, that the spring mass damper system, you will have x double dot c over m x dot k over m x and this term is c over m is 2 zeta omega n zeta is the damping ratio and this is omega n square. So, this is the natural frequency, natural frequency is this, that is the rotating flap natural frequency.

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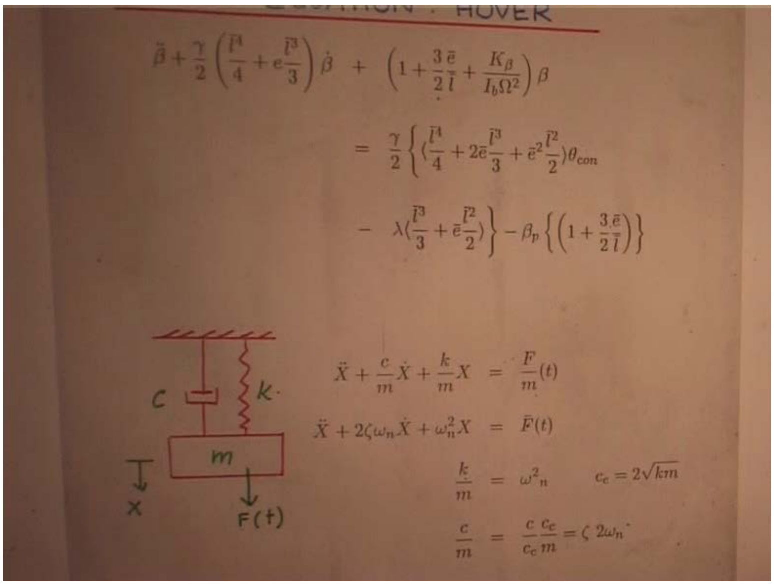

Now, you know what hinge offset affects? Hinge offset influences the flap, natural frequency; actually, it increases the flap natural frequency. Now, what is the value of the damping ratio? You can take a crude approximation here and then, you have to, because this term, 2 zeta omega n is essentially this term gamma over 2 l bar over 4 by 4 e l bar e bar; actually, e bar l bar cube by 3, that is the damping term.

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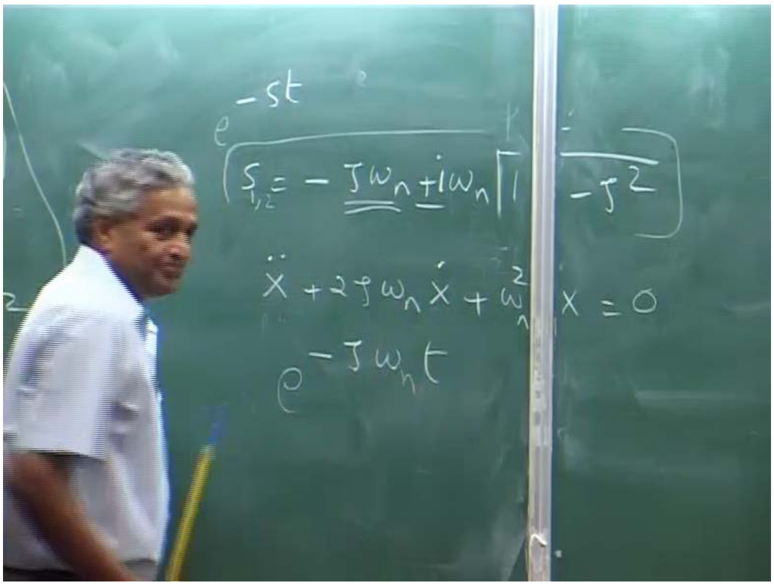

Now, if you look at our, you all have studied the single degree of freedom system and what is the Eigen values, and then the natural, sorry, the roots of the equation, you will write e power minus s t and you will write s as minus zeta omega n plus, it may be, I omega n root of 1 minus zeta square.

You have, you, you are familiar with this because this is the equation omega n square x is 0, this is the homogenous part equation root because you say, that zeta is actually less than 1. So, you analyze the roots, you will get the 2 roots, s 1 comma 2, then you will substitute e power, this you will get a decay, that is sinusoidal. This is a simple single degree of freedom damp system solution. So, the damping, here if I substitute, that is zeta omega n, so e to the power minus zeta omega n t.

This is how my amplitude will decay, this is the decay term, you correspondingly take it to this side.

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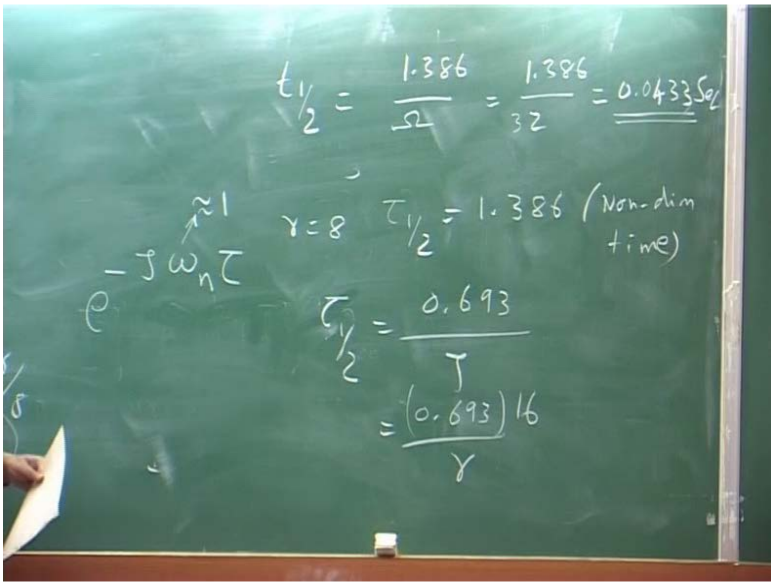

You will see e to the power minus zeta omega n is our natural frequency, of course. Omega n, but of course, since we are dealing with the non-dimensional, everything, this is also non-dimensional, I will put a term tau, zeta omega n tau.

Usually, what they do is, how much time it takes to d k to half the amplitude, that is, what people think. The time it takes, that is the time constant, the time constant (( )) say, how much time it takes to d k to half its amplitude means.

What is the time it takes from 1? Initially 0, it will be 1, how much time will take to get it 0.5, that is the decay and that value is I think 0.693 or something like that. I will just briefly mention here, that is this tau for half, you take this value as approximately 1. Do not try to take it because you know, that is 1.09 or 0, you take it approximately as 1, then you take this. So, the time for this is half, sorry, not, tau time for half amplitude will be 0.693 divided by the real part. The negative real part of this is zeta omega n omega n because 1, so zeta, what is our zeta? Omega n from here gamma by because you look at our equation 2 zeta omega n is gamma over 8 approximately. That means this is taken as 1, so zeta is gamma over 16.

So, here it will be zeta, this is 0.693, that is, gamma is, gamma over 16, you take 16 here, you take gamma here. If you take gamma as 8 gamma equals 8, the value becomes non-dimensional time; the time for half, half amplitude becomes 1.386, this is non-dimensional time.

Now, if you want dimensional time, tau is omega t; so, you will get t is 1.386 over omega. If you take omega as 32 radians per second because which is reasonably a good number, if omega is 32, you will find, I am just writing it, why I put all these numbers is just to give you an idea 0.0433 second 0.043 seconds.

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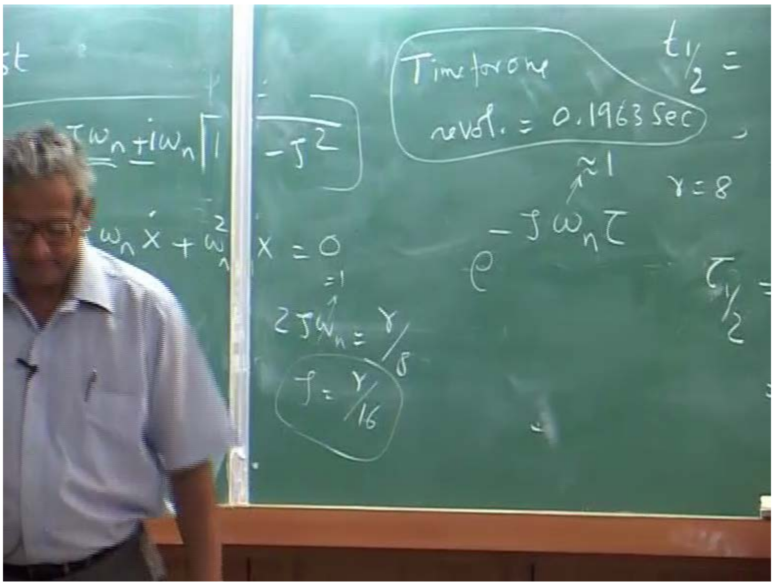

For this omega, that is 32 radians per second, the time it takes for 1 revolution time, for 1 revolution is 0.1963 second. So, for the blade to go around once, it is around 0.2 seconds. Whereas, the amplitude decay, it happens in 0.4, which means the time constant is really very small in the sense, it quickly reaches its value. That is why, when the pilot gives any input, the response of the flap is quick, he would not see, he would not recognize or feel a delay in the response of the blade.

You can take it as, though input output is instantaneous even though there is a slight delay, but that delay you neglect. Why this is important is, this particular number, what I used to indicate, I can now make approximation, which is called the cosine static approximation, I really do not bother about.

Now, I am coming to the your flight dynamics, flight dynamics because pilot gives an input to the rotor blade, blade immediately flaps, then it gives the hub moments and the helicopter response the delay between pilot input to the response of the blade, I am neglecting it.

I say that it responds instantaneously; that means, the flap dynamics I neglect, I look at the steady state value, I give this input, I neglect the dynamics, only steady state. The moment I look at the steady state value, the dynamics is gone, give input, there is a change in force that is all.

How the change in force is happened is because of dynamics of a flap motion. The change is happening over a time, but I neglect that. So, when I solve flight dynamics, they really neglect even the flap dynamics in the sense, they assume this cosine state not that flap dynamic equation is thrown off, but I say, I give an input instantaneously, I change my loads, that is, because of this, in 1 revolution it takes about 0.2 seconds.

Whereas the flap decay, that is the response time constant, it comes to that small value, 04, which is about 20 percent of this, it means very quickly, it responds this is the beauty of the flap and this number is the one, which makes further analysis. (( )) say I can make it simpler (( )) and you start analyzing flight dynamics based on this.

Now, you see, when you really solve flight dynamic equation, you will not have flap dynamics in your model, you will only have fuselage equations, and where is the blade dynamics gone? It is taken as change in force, instantaneous change in force that is because of this quickness.