Introduction to Helicopter Aerodynamics and Dynamics

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Lecture No. # 19

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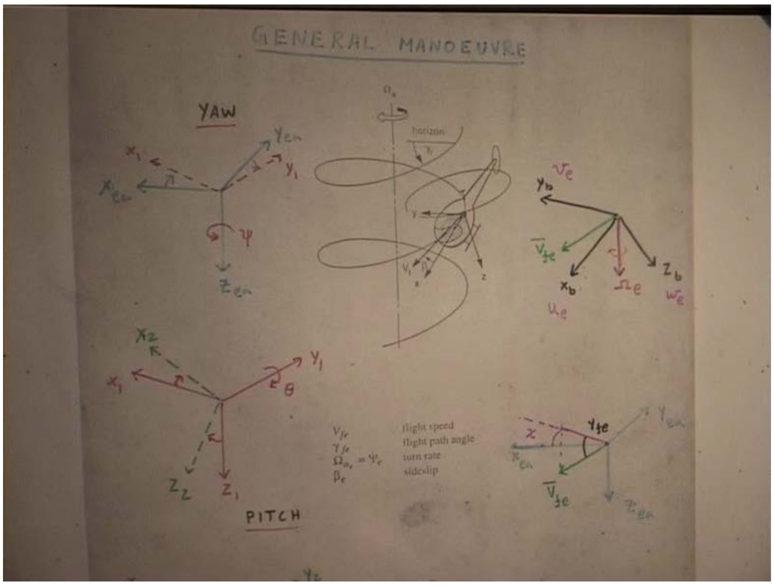

The, this figure is taken from Patfield book. What the kind of maneuver we are talking about is, see, the helicopter goes in a spiraling path, but it can have a sideslip also. Now, if this is the kind of motion, how do you trim the helicopter? Because it is a very complicated maneuver, not like our what we solved earlier, level flight. But if we solve or if we formulate equations for this problem, then you will be able to solve any problem, which will be a simplified version of this. So, essentially, the helicopter attitude can be anything and the velocity vector V f is oriented in space and that vector is also turning.

Now, the flight speed is given by velocity vector, you may say, that is a constant, the magnitude is a constant. And you will specify a flight path angle, that is, you have to define what is that flight path angle because this vector is oriented in space in some direction and then, you also define one turn rate, the rate that, which the vertical from the helicopter, other helicopter is spinning. You can say turn rate, spin rate, anything, but that is about a vertical axis.

And of course, one more quantity you specify, the sideslip. You will define what sideslip is, the only, these quantity are specified, they have nothing to do with that type of equation they are looking at, but if you see overall for this type of a maneuver, you have to treat nine equations.

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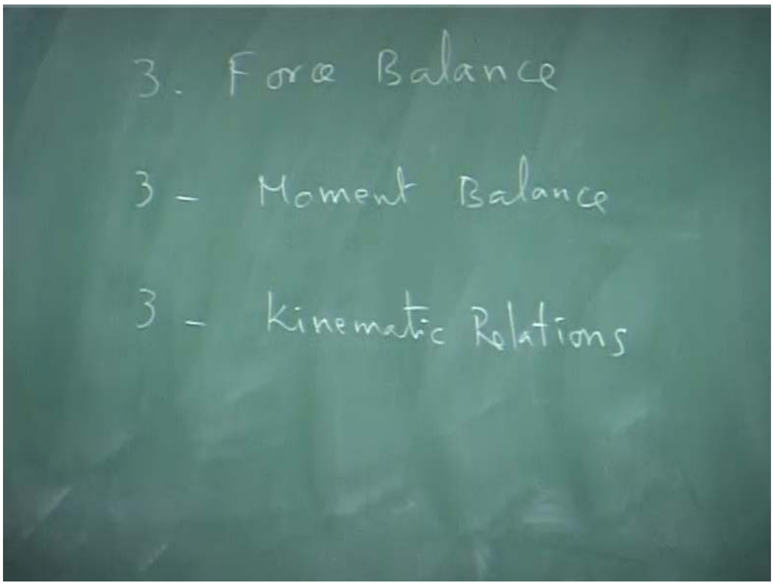

Those nine equations are - three force balance, three force balance equations and three moment balance and then another, 3 kinematic relations. Now, force balance, moment balance, you know for a rigid body because you say f is equal to a method of Newton’s law and the moment balance is moment related to angular acceleration, etcetera.

But the kinematic relation represents, essentially, the instantaneous angular acceleration or, sorry, angular velocity instantaneous because at every instant the aircraft is turning. So, it will have an angular velocity, which is a vector, but how that angular velocity vector is related to the orientation rate of the helicopter with respect to some earth fixed coordinate system.

Now, this is, like, orientation of the helicopter is one, angular velocity is another, how do we relate this? This is where you have to have, now, coordinate systems very precisely marked.

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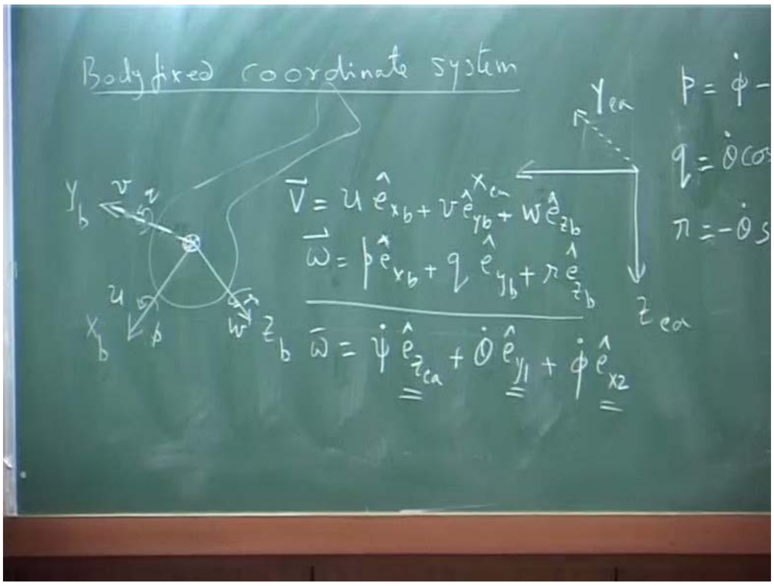

First, you write the body fixed system; body fixed coordinate system. This is, it will, you take the helicopter, you, and please note, I am using slightly different coordinate system for this case because this is what, generally, most of the flight dynamic people use, x body and then, this is y body and then, this is z body. This is the center of mass of the helicopter; this is body fixed coordinate system. That means, when the body turns, this axis system also turns and this is attached to the centre of mass.

No, no, z is going down, you can take this as the dash, dash, dash; this is going in; this is into the board. So, you can say, x b, z b is in the plane of the board, this is the standard thing. Now, how do I get this? This means you have to first define an earth coordinate system, earth coordinate system. You can say this is my x earth, z earth and then, this is y earth; you can have this any direction, local.

Now, you see the sequence of rotation from this to this is actually, you follow yaw, then pitch, then roll. And yaw is psi, I think, then it is, theta is pitch and then, phi is roll; yaw, pitch, roll, this is the sequence, all counterclockwise. So, that is what I have shown here. If you look at it, first this is the z earth, x earth, y earth; I give a psi rotation counterclockwise.

I get the coordinates system, x 1, y 1 and of course, z 1 is same as z ea, I am not writing the transformation matrix fully, then what you do is you take y 1, x 1, z 1, give a pitch rotation, then you will get x 2, z 2 and y 2; y2 and y 1 will be same.

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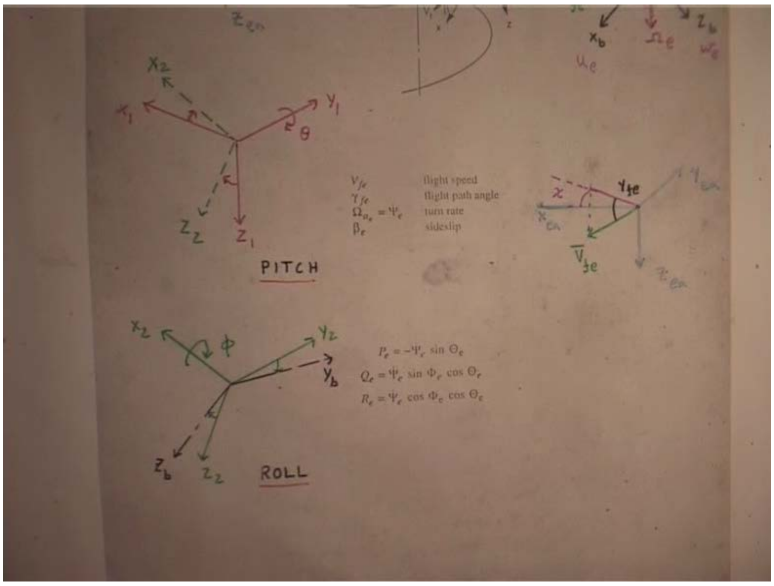

Next, you go to the roll, about x 2. Now, this is the sequence of transformation. If you follow this sequence, this is the very well set orthogonal transformation.

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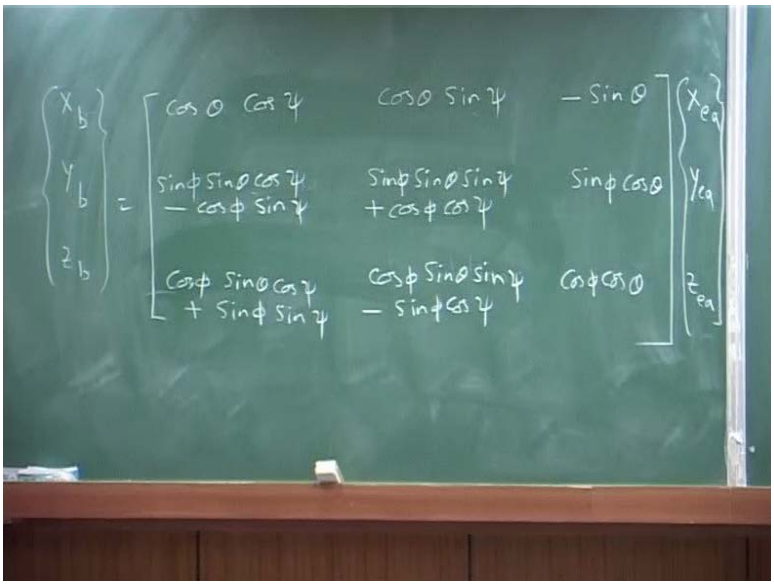

I will just finally, write the transformation between, this is the x b, y b, z b, this is cosine theta, cosine psi, then cosine theta sine psi sine theta, then sine phi sine theta cosine psi minus cosine phi sine psi, then sine phi sine theta sine psi plus cosine phi cosine psi and then, this is sine phi cosine theta. And here, cosine phi sine theta cosine psi plus sine phi sine psi, this is cosine phi sine theta sine psi minus sine phi cosine psi and then, cosine phi cosine theta. And this is x earth, y earth, z earth, this is the full because this you may find in text books also this transformation, because this is the product of 3 matrices, that is all. But follow this sequence, that is the most important thing; yaw, pitch, roll sequence.

Now, what is the angular velocity of the helicopter? At any instant, you define by angular velocity of the helicopter as pe xb qe yb plus re zb, the, that e is the unit vector. Now, this is the instantaneous angular velocity vector defined with respect to body axis.

Now, you will see, if you want to know what is the angular velocity vector, but defined in terms of time, rate of change of these angles because when the body is rotating, which means, they are also rotating with respect to the earth axis. But you will find the same omega, you can write it as, I am putting it, psi dot e z earth because this is with respect to the earth and then, plus theta dot e y1 because look at this, theta dot is y 1 plus phi dot e x. These are time rate of change of orientation of the vehicle, as defined by my yaw, pitch, roll sequence, because yaw is about the z ea, this is about the rotator y 1.

Now, what you do is you make a transformation of each one of them to body axis because you will find each one is the intermediate position, you can get the transformation from z a to x b, y b, z b; y 1 also, you can transform, but this is actually the body, phi dot is along the body because that is x 2, which will be the body axis.

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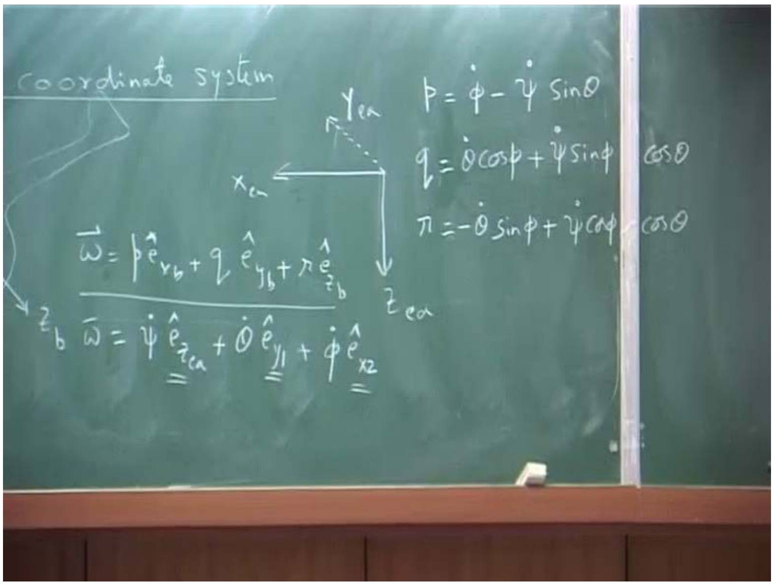

If you do the transformation, you will get this relationship, here is this part here. I will write the relationship is finally, you convert everything into x b, y b, z b; you will get p equals phi dot minus psi dot sine theta and q equals theta dot cosine phi plus psi dot sine phi sine, sorry, cosine theta and then, r is minus theta dot sine phi plus psi dot cosine phi cosine theta.

So, this is the relationship between instantaneous angular velocity to rate of change of the orientation angle. But please remember, this relation is not that, they also depend on what is the theta and phi. That means, you have to solve for the orientation angle, this is a non-linear relationship because theta dot is there, then there is also theta, so this is where, solving this problem, this is a, this is what navigation, even the satellite communication navigation, everything this is what is used.

So, now, you have a relationship between instantaneous angular velocity to orientation of the vehicle because orientation of the vehicle can be with respect to earth axis. In any orientation, you always want to know what orientation it is there, so particularly, if you are designing autonomous, any vehicle, you have to know all this, yeah, and the estimation of this.

Now, you can always solve for this in terms of p, q, r also, that is like you take because psi dot, phi dot, theta dot, you can get it in terms of p, q, r, but again, theta will be sitting, theta and phi. So, this is like a just transformation of one to the other coordinate, we will not use that.

Now, what I want is the body, what I specify is my velocity vector, I will specify velocity vector of the helicopter, which is actually, I define my velocity vector as u e xb plus v e yb plus w e zb. That means, this is my u, v, w and you have p, q and r.

So, at, at a time, I have a velocity of the body and the angular velocity of the body defined with respect to body fixed system. Now, if I want, yeah

(( ))

Yes, as we have defined here, that is why, that sequence you follow. What sequence, that is the most important thing; this is the sequence we are following. Suppose, if you change the sequence, you will find the different thing, usually, that is why I said, yaw, pitch, roll sequence is followed, so that the convention everybody understand, and the coordinate system also, they try to use the same thing.

Now, the question is, I have the velocity and the angular velocity, translational angular, all, now I can go and calculate, what is my acceleration of the centre mass. So, maybe, I erase this part because this transformation, I use it as a transformation from this coordinate to this coordinate, that is all.

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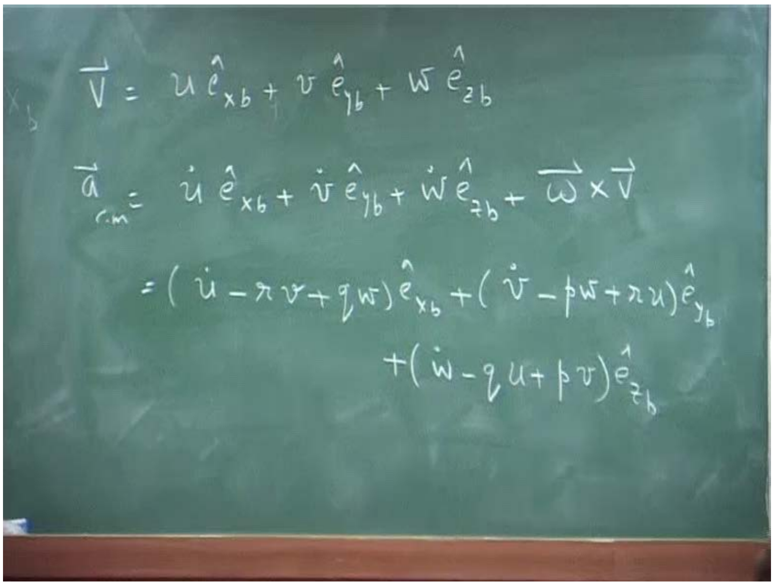

Now, my velocity is, velocity of the centre of mass is u e xb v e yb plus w e zb. Now, what is my acceleration of the centre of mass, time derivative of this and then, these vectors are also changing their orientation, but they change with the angular velocity. So, omega cross V will go. So, you will have, u dot e xb plus v dot e yb plus w dot plus omega cross V. Now, you put, the omega is there; p, q, r, you use p, q, r; do not use this omega because that we will keep it as separate relationship, always use p, q, r. Take a cross product, you will have the acceleration of the vehicle, vehicle. I will write it here, this is u dot minus rv plus qw e xb plus v dot minus pw plus ru e yb plus w dot minus qu plus pv e zb. So, I have my acceleration, only thing is, I have to now apply Newton's, this is the absolute acceleration of the (( )).

So, what are the loads that act on the helicopter? You have a main rotor load, tail rotor load, fuselage aerodynamics and then, any external bodies, aerodynamic surfaces, then gravity, all. So, what you do is you put, take all the loads, transfer them to the CG. So, you will write, f x equals mass of the helicopter times, this f y is mass of the helicopter, f z is this; that means, you have 3 force equations.

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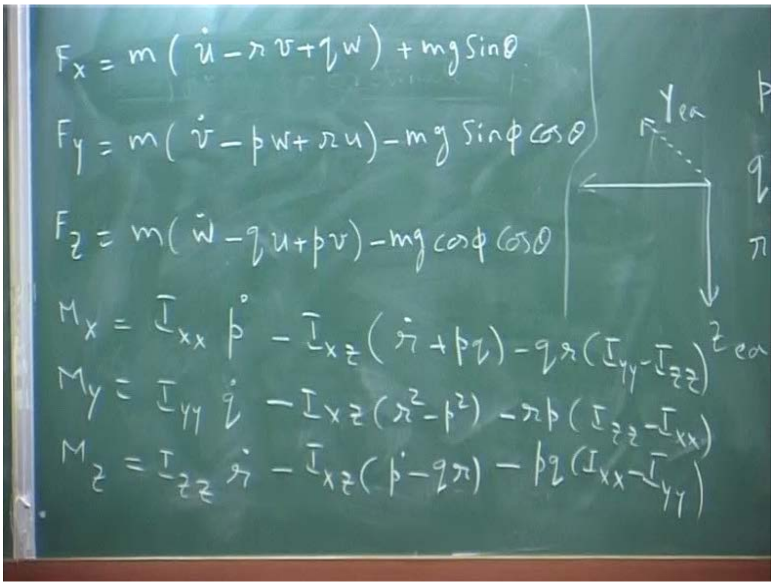

So, you have F x is u dot minus r v plus q w, but what normally done is, the gravity load you write it separately, so you will get, I am putting it, because please note, that gravity load acts along on z earth. So, you transform the z earth to body and that will come.

And then, F y because this is the standard way of writing, v dot minus pw plus ru minus mg sine phi cosine theta. And then F z is m w dot qu plus pv minus mg cosine phi cosine theta. So, these are my three force equations. Can you look to the transformation relation, you will find, that this is very obvious, all right.

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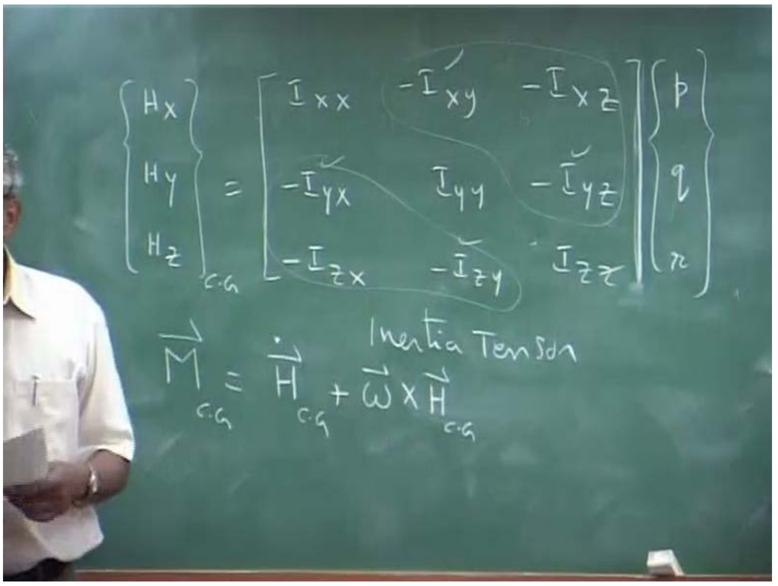

Now, you have to get the rotation equations; rotation equations are a bit messy because you know, if you have done rigid body dynamics, you know, that angular momentum, angular momentum is given by I omega. So, I am writing here H x, H y, H z because angular momentum is a vector, this is symmetric matrix, I yz minus I zx I zy I zz, sorry, the p, q, r, this is my angular momentum of the body H vector. So, H x, this is along the body axis. Please note that because this can be derived, I am not going to the derivation of this because this you must have done in the rigid body dynamics.

Angular momentum and the axis system is the body axis system, that is why, in any aircraft, anything, you first decide your axis system and then I xx, I yy, all these quantities are defined with respect to that axis system. Now, imagine, if you put a load, if you put some equipment, your initial change, that is, the reason for using body fixed axis system is only to simplify the moment equation, otherwise it will become really missing because these quantities, they do not change, you say with the body fixed system. Of course, if fuel is expanding, it is changing, but you do not bother about that.

So, this inertia, this is the inertia tensor, I hope you know, that this is a 3 by 3, it is known as inertia tensor because this follows the tensor transformation, that is all. Now, what I want is that is the moment, this is defined with respect to my axis system in the body p, q, r. Now, I want moment, moment is essentially H dot, this is about the C.G., this is about the C.G. If you want, I can write it the C.G. H dot moment, about the C.G. plus omega cross H because these vectors, they are also changing with time. Therefore, when I take a derivative of that, I will get omega cross H, so this is my moment equation. So, I may put C.G., that is why, always you take moment equation about the centre of mass, it makes simpler. Now, what you do is, this is the moment, due to all the external loads, anyway, the gravity passes through the C.G., so it will not create any moment. You have to take a dot product of this, which means, you expand the whole thing, take a dot, dot means p dot, q dot, r dot, that will come.

Then omega cross H, omega is p, q, r; H is also a function of p, q, r, so you have to take the cross product, write the full equation, that is the correct exact way. But usually, what happens is they make certain assumptions. The assumption is, if you write the full stuff it becomes a little messy, so I leave that part, I will make the simplified assumption, that cross product of inertia, that is, these quantities.

If it is a symmetric body, if it is a sphere, perfect sphere, then the cross product is 0, you will have I xx, I yy, I zz, but if it is a symmetric body about 1 plane, which is the longitudinal plane, if you take then what will have? For every plus y, you will have a minus y, so wherever y is there, they will go off. That means, these quantities, they will go to 0, but this will stay I xz, because they say, that number for help is for aircraft or anything is substantial. Even though, this may not be 0, I xy, I yz, you neglect it, but if you do not want to neglect, carry the entire term, it is not very difficult. If you have computational aspect, you can carry it, it is not a problem.

But for ease of understanding, they try to throw away lot of things and then try to get a simplified version. Now, I will write generally all cross products, except I xz. That means, these are neglected, that is because you say, but in the aircraft case, yes, there is symmetry, but in the helicopter it is not exactly symmetric. Because you have tail rotors sticking around one side, but you say, that it is all right. Let me make that assumption, then it becomes, I can write it in small, so I will have my M x. M x is moment of the all the loads, that is, the rotor loads, tail rotor, horizontal surface, every aerodynamic load, whatever. You get inertia, aerodynamic load, everything, transfer it to the C.G., that is the along the x direction is the M x.

That is why we get c m x, c m y, c m z, all those things. The rotor, they transfer to fuselage centre of mass, this becomes p dot minus I xz because I have made the assumption r dot plus pq minus qr into I yy minus I zz and then, M y is I yy q dot minus I xz r square minus p square minus rp I zz minus I xx. Then, M z I zz r dot minus I xz p dot minus q r minus p q I xx minus I yy, these are my, normally you see, I xz is comparable to I xx magnitude wise.

So, this has to be, that is why weights and C.G. group in any aircraft industry or aircraft or aerospace, any company, they have to get these values. Before the aircraft is build, it is like, you make estimate from the drawing and then you try to calculate the complete I xx, I yy, I zz, all these numbers for a given coordinate system because if you make a mistake, there your flight dynamics is all wrong. So, this is done, that is why some time it is a routine job, but one has to do it in an aircraft industry. As a matter of fact, all your dynamics is only this, but of course, I made an approximation. If I do not make approximation, that is, all the full dynamics of a rigid body, there is nothing more beyond all problems, whatever. You have learnt in physics, one on one, or dynamics you can solve with this, with this set only, only thing is you must be very clear.

Now, you see, how do I solve, I have nine equations, how do I solve for my problem because what are my unknowns? (( )) problems, first you say, hey I am not, I am going to fly a steady maneuver and subsequently, I will say, if there is a perturbation about the steady state, what is the stability? Only these two are solved, using these equations otherwise, this is a fully non-linear equation. Because you see p square, r square, p q, everything is non-linear, so solving it is not that straight forward. So, how do they attempt to solve this problem? In the case of helicopters, I will go to the helicopter thing, see what is done is, first I say I am doing a particular flight condition. That means, I am not accelerating; my u, v, w are fixed; p, q, r are also fixed

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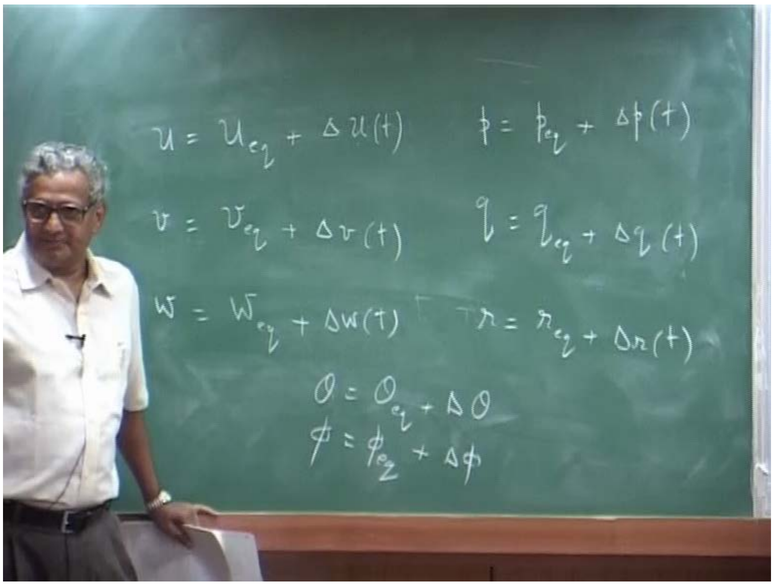

Then, I will write my u is, u equilibrium plus delta u, which is a, and v, which is the v equilibrium and w, w equilibrium delta w time. And then, p, p equilibrium plus delta p; q and r, r equilibrium plus delta r time t.

Then, you need to have also your theta equilibrium, phi equilibrium because theta and phi, that is the orientation of the vehicle. So, I will have, theta is theta equilibrium plus maybe, delta theta. And phi, which is the phi equilibrium plus delta phi except psi because there is no psi equilibrium because the orientation can change; as the helicopter turns, my angle psi, you can keep on turning, there is no equilibrium for that. Please remember, that is why, that turned allow because it is like a yaw rate, pretty much it can keep on turning. But the vehicle orientation is theta equilibrium, phi equilibrium there, so what you do is you substitute these quantities now in this equation.

And then, you say, my deltas are small, product of deltas I neglect, and I will collect all the terms with the equilibrium quantity separately, and collect all the terms with the delta separately. Now, I will have one set of equations, I call it as the steady state equilibrium equation and another is the perturbation equation. Perturbation equation I need to use it for my stability, whereas equilibrium I use it for my trim. This is how it is done, otherwise you will go nuts. This is highly complicated formulation, you just cannot get anything; first you split, when you split you will write the equilibrium equation. So, I will just write that equilibrium force equation.

So, you, you understand this, you substitute here, then take. So, when I take u dot, that means, it is only delta u dot; u equilibrium is constant value.

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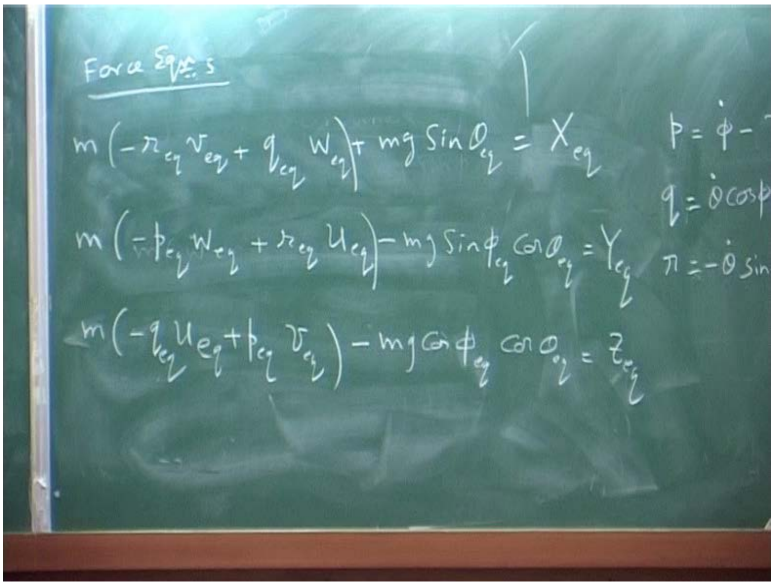

Now, I can write my equations. This is force equation and moment m minus r equilibrium v equilibrium plus q equilibrium w equilibrium plus m g sine theta equilibrium is X equilibrium; that is the force along X directions equilibrium force; that means, the steady state force, that is what it means. m minus p equilibrium w equilibrium r equilibrium u equilibrium minus, sorry, I should put a minus m g sine phi equilibrium cosine theta equilibrium is Y… Then the Z equation, minus q e u e plus p equilibrium v equilibrium minus m g cosine phi equilibrium cosine theta equilibrium is Z…

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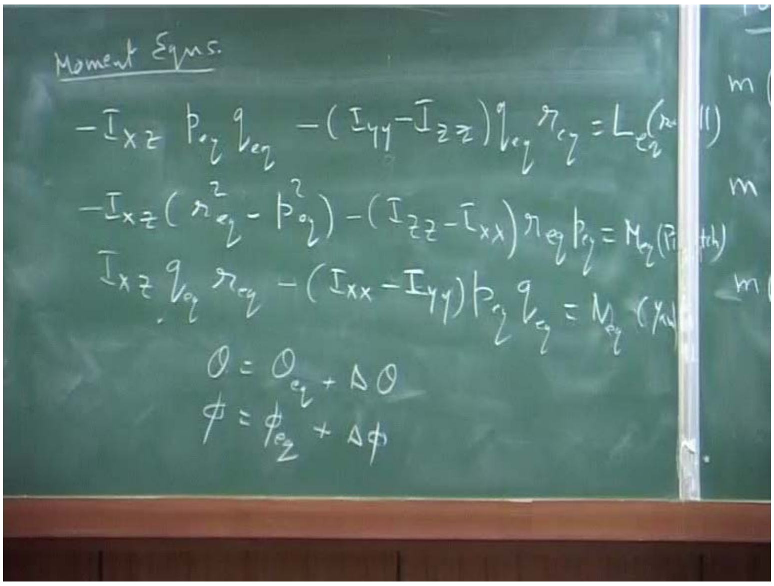

Then, you write the moment equation also, 3 moment equations, that is, moment, you will have minus I xz p equilibrium q equilibrium I yy minus I zz q equilibrium r equilibrium and this the standard notation L e roll, this is roll equation. And then, minus I xz r square minus p square equilibrium minus I zz minus I xx r e equilibrium p equilibrium is M, we may say equilibrium, this is pitch. And then, the yaw equation is I xz q equilibrium r equilibrium minus I xx I yy p equilibrium q equilibrium, which is N equilibrium, which is the yaw. And then, you have to change this equation also, in this equation because you know that these are dots. So, dot means that is the time derivative.

So, what will happen is, p equilibrium is minus psi dot equilibrium theta equilibrium; q equilibrium will be sine phi eq cosine theta eq; and then, r eq becomes psi dot equilibrium cosine phi equilibrium cosine theta equilibrium.

Now, you see, this is my full equation, nine equations here, here, here. Now, this I xz, no minus 1. Now, you see these 9 equations you have to solve. What are the quantities, which I have to know? Now, I will show this because I would like to come here, which will be the nine, sorry, not nine.

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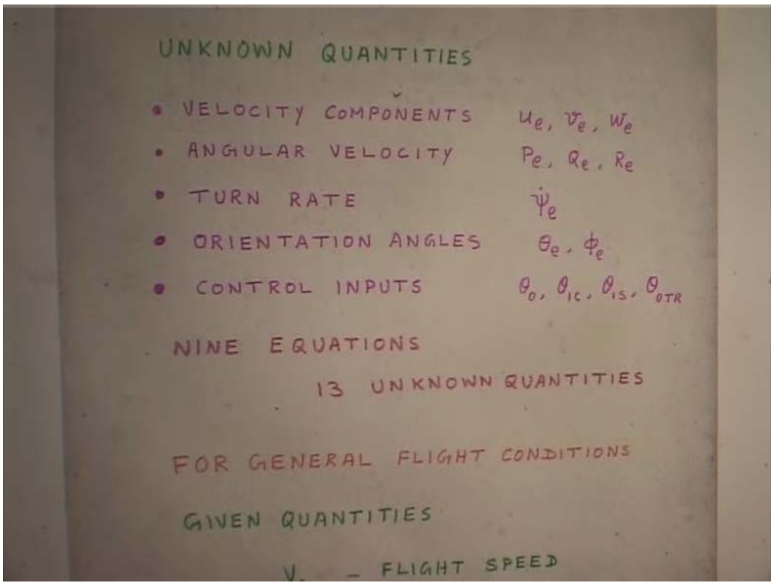

The unknown quantities of the problem, what are the unknown quantities, because when I want, these are 9 equations, what are things I do not know? You say, mass of the helicopter you know, please note that, that is, is very important, I know the mass of the helicopter; I know the inertia properties the tenser. External loads, these are my rotor loads and fuselage aerodynamic loads, any horizontal surface, any lifting surface, everything, they provide those loads here.

I do not know what is u, u e, v e, w e. I do not know, that angular velocity P e, Q e, R e, I put a capital because for equilibrium I just used a capital here, that is all. Otherwise, if the p equilibrium, q equilibrium, all the e is equilibrium, then this is the turn rate, psi dot and orientation angle, theta equilibrium, phi equilibrium and then control input 4.

So, I have thirteen quantities, which need to be noted, but I have nine equations. Therefore, you have to prescribe 4 quantities before you really can solve the problem. Now, what are those four quantities you will specify? You will not specify four from here that is the problem.

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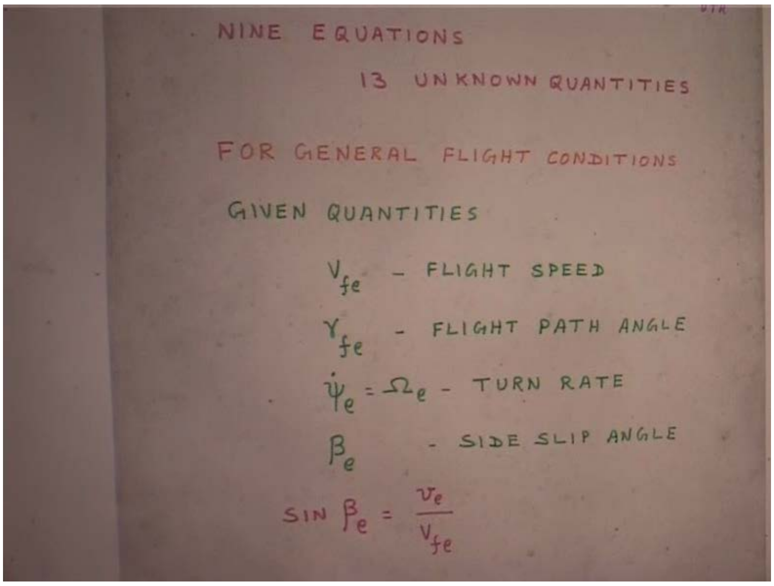

The four quantities, which you specify are flight speed and flight path angle. I will tell you what the flight path angle is, then turn rate (( )) and then, you will say what is the side slip angle.

The definition for side slip angle sine beta e is v e by v fe. Please understand, now the quantities, which are mentioned there are different from, except this, this is different, this is different, I do not have any, I do not have v fe defined there, you follow, I do not have gamma f e defined there, I have only u e, v e equilibrium quantities.

But what I specify is in some other four quantities. Now, you have to relate this 4 from here to some of those quantities, that is again a, I will briefly describe that, we will, may be go what, to the first figure. If it is there, I will specify, that what we need is, this is their z earth axis. Body axis is x b, y b, z b and velocity vector of the body is v bar fe. The components of this are u e, v e, w e, they are equilibrium quantities, components of the velocity vector along the body fixed system, please understand, is u equilibrium, v equilibrium, w equilibrium.

Now, the flight path angle is defined, that this is the z earth, x earth, and y earth; the velocity vector can be in any orientation, can be up, down, anywhere. What you do is, you project this velocity vector on to a horizontal plane, on to the horizontal plane; project this vector on to the horizontal plane. And then, the angle between the vector, velocity vector and the horizontal plane is the flight path angle and then you project it and the angle between the projected vector and the x earth, you call it as a chi; just put some because that will come later.

Now, these are your quantities, I have the velocity of the vehicle, I do not know what is the chi is, but flight path angle I specify, velocity I know; that means, I know this and I know this, but I do not know this. But from here, if you look at it, I know omega e, that is the turn rate, but I do not know u e, v e, w e, none of these quantities I know.

Now, what you do is you take the velocity vector, transform them along the earth fixed system first. That means, I have the velocity vector, I am going to get components along the earth fixed system because this is a little interesting. Finally, you will not use that.

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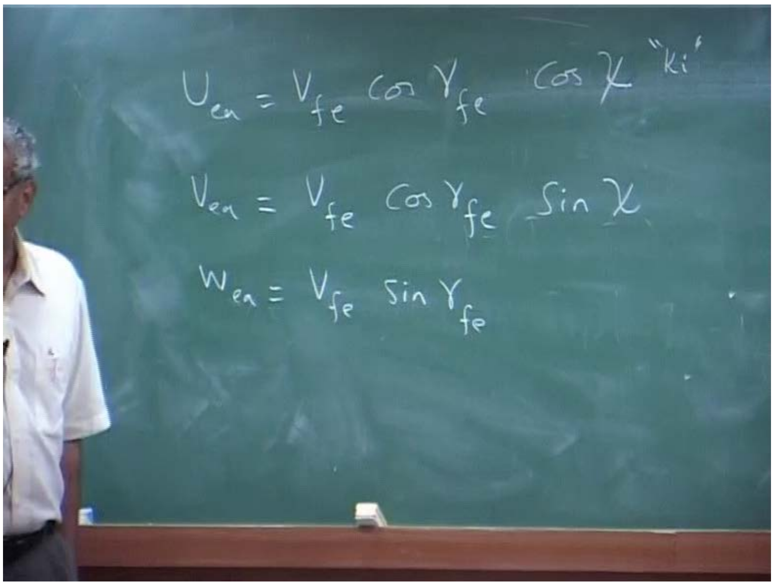

The velocity vector in the earth bound system is U earth phase V fe cosine gamma fe, this is the flight path, and cosine chi, this is not psi, this is chi. And then, V ea, V fe cosine gamma fe sine chi and then, w earth is V fe sine gamma fe.

This you can get, only thing you do not know, these quantities are mathematically, I can write it, but I know this, given this sine gamma fe, I know w ea. Now, what you do is you go back, transform this in the body fixed because you know the transformation relation between this and the body.

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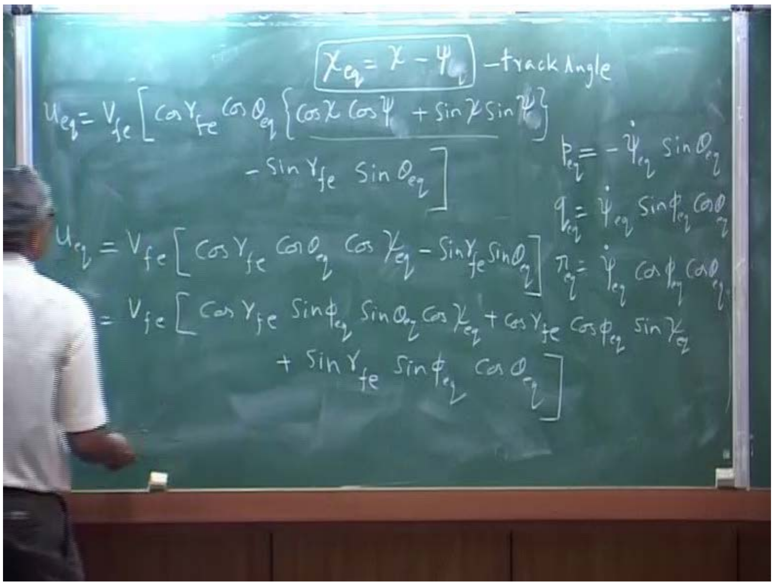

When you do that, you will get, now I am, say I am writing for, I will write it for one quantity, U e is v fe cosine gamma fe cosine theta equilibrium cosine chi cosine psi equilibrium plus sine chi sine psi, this is psi equilibrium minus sine gamma fe sine theta equilibrium. You will find this is the exact transformation in u e or u equilibrium, sorry, I, we are using equilibrium. What you will find is you will have a term cosine, cosine plus sine, sine. This is nothing but you call it by a chi e is chi minus… This you call it track angle.

This is something new, track angle, that is why you do not define it, but later you will get, the track angle will come, you have to solve for the track angle. Now, if I write in this my u equilibrium in the body because this u is in the body access system, I will have it in this form v fe cosine gamma fe cosine theta equilibrium cosine chi. Maybe, I will put, if you want equilibrium, you can put equilibrium, equilibrium minus sine gamma fe sine theta equilibrium.

In a similar fashion, you can write v eq, I will later tell you what is, why I am using because in the equilibrium state you say, that my velocity is constant, flight path angle remains constant, orientation angles are constant and this is also constant, they do not vary with time. But you will find chi and psi, what is psi equilibrium, then nothing like psi equilibrium; it is also changing with time.

So, I cannot put a psi because you said, psi dot has an equilibrium; that means, psi cannot have equilibrium because it is turning. So, usually, what is done is you do not try to put an equilibrium here, you say these two quantities are varying with time, but there difference, it is a constant.

Both of them are varying in a same amount, the difference is a constant. And now, you have to get that, now flight mechanics possibly they may, because the track angle is…, Now, what you do is, you will have your v e is v fe cosine gamma fe sine phi equilibrium sine theta equilibrium cosine chi equilibrium plus cosine gamma fe because these are all long expression phi equilibrium sine chi equilibrium plus sin gamma fe sine phi equilibrium cosine theta equilibrium.

Similarly, you will have w because the combination, it comes such a way, here this sine chi, you will have; if you substitute this, you will have sine, sine chi cosine psi minus something. So, that is how the expression will come, that is why they will come on to be sine chi and cosine chi.

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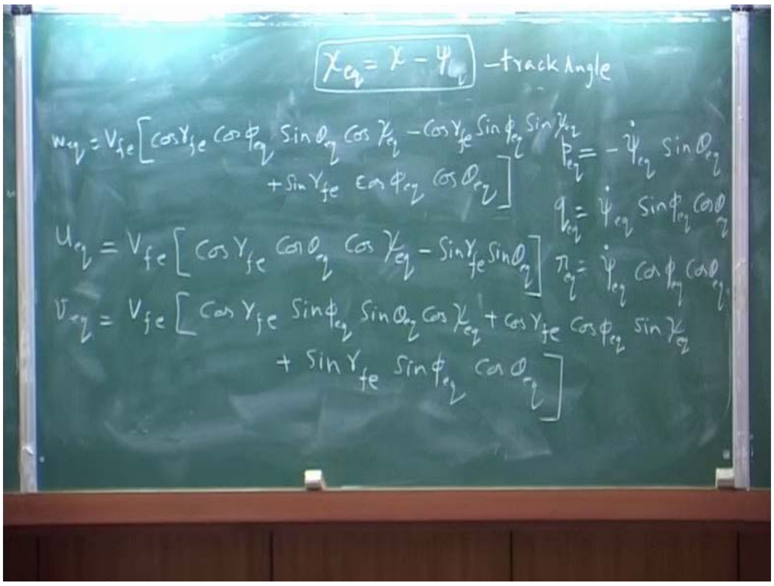

Now, you will have w and your w equilibrium will be again, v fe cosine gamma fe cosine phi equilibrium sine theta equilibrium cosine chi equilibrium minus cosine gamma fe sine phi equilibrium and then, sine chi equilibrium, then you will have one more term, plus sine gamma fe cosine phi equilibrium cosine theta equilibrium.

Now, you see, for me to get u e, v e, w equilibrium quantities, I know flight path, the velocity I know, the flight path angle is given. What I do not know in this is, I do not know chi, I do not know theta, I do not know phi, these 3 equilibrium quantities I do not know; you follow what I am saying? That means, if I want to get the velocity along the body axis, given the quantities of these four, I still need to know three, chi equilibrium, theta equilibrium, phi equilibrium.

So, what do I do is, I first go and solve for, how I define my chi equilibrium. What is given here is sine beta e because side slip angle is specified, you have to specify what is side slip; side slip angle is what? This is v equilibrium in the body system, whether you are moving along the y body, along the y axis of the body system, if you have a velocity there, the ratio of that velocity to the total velocity resultant; that is my side slip. That means, side slip angle is specified; that is a given quantity. Now, you know from here v equilibrium, v by v fe, this is nothing but sine beta, which is the cyclic.

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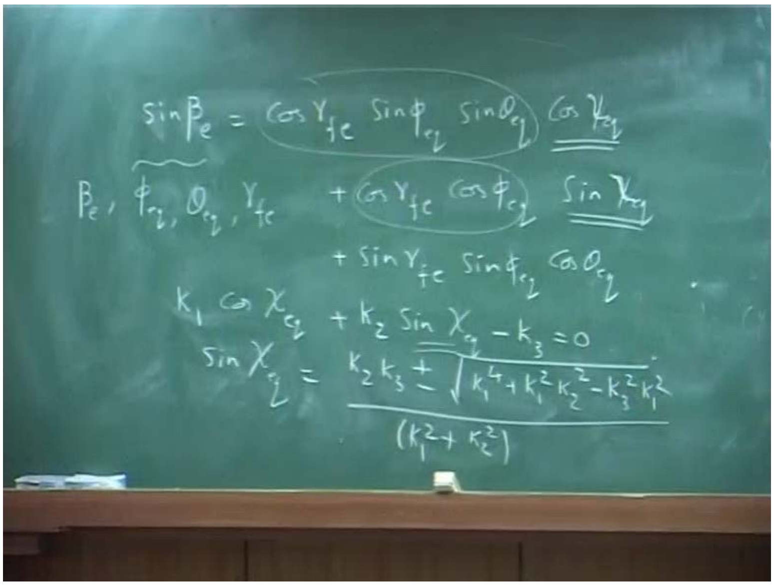

So, I write sine beta e, this entire expression, that is, sine beta e is cosine gamma fe sine phi equilibrium sine theta equilibrium cosine chi equilibrium, then plus cosine gamma fe cosine phi equilibrium sine chi equilibrium, maybe I should put it, plus sine gamma fe sine phi equilibrium cosine theta equilibrium.

Please note, that here, what I will do is, I know this quantity, this quantity I know, I can form a quadratic equation in sine of them, call this as k 1, k 2, k 3 and then, take this term sine square cos square, something chi, you solve that quadratic equation, you can write it as k 1 cosine chi e equilibrium plus k 2 sine chi equilibrium minus k 3 equal 0. Because you bring this term here, you can say this is k 1, this is k 2, and k 3 is, you bring this term and then put a minus sign. Now, what you do is, you convert to either sine to cos or cos to sine and then take this term on this side, this term on the other side, square both, you will get cosine square. Then, this is sine square, sine square can be or cosine square can be written as 1 minus sine square. So, you will have an equation in sine chi. So, finally, you will write sine chi equilibrium is given by k 2 k 3 plus minus root of k 1 power 4 k 1 square k 2 square minus k 3 square k 1 square over k 1 square plus k 2 square. This is what you will get.

That means, there are two routes, you always pick the meaningful route, that is what, in that, what you know in this is, knowing beta e side slip angle and you have to assume phi equilibrium, theta equilibrium and then, these are assumed, gamma fe is given. That means, when you are starting, you specify those 4 quantities; first thing is I will go and get the track angle.

How do I get track angle? First I solve, I assume the orientation of the helicopter, you assume, then you get that track angle. Once you get the track angle, you have already assumed these two, track angle you know, gamma fe you know, you can immediately get u e, v e, w e. That means, you have all the three quantities and given psi dot, that is the input equilibrium turn rate. You have already assumed theta eq, v eq; that means, I know p e, q e, r e, you go back the six equations and then solve. I will just briefly mention because this procedure is a quite complicated, that is where the convergence, all the, see here, I briefly describe the procedure, that is done.

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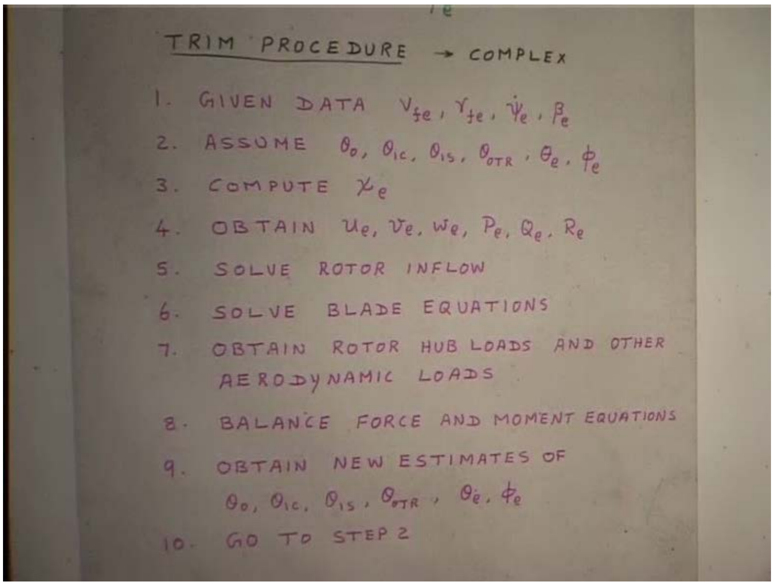

Trim procedure, I just put it complex. Given the data, velocity flight path and then turned rate, side slip. Once I have this, I go back and solve first the track angle, but assuming theta eq and phi eq, so that is why, assume. I say, here these 4 quantities are pilot control input, I assume these two quantities, which are eq, compute the track angle. Once you get the track angle, obtain u equilibrium, velocity equilibrium, w equilibrium, then P, Q, R. There, I use capital, here P, Q, R. Then, you solve rotor inflow, that is, the, you may assume uniform inflow or any other inflow, first assume that, go and solve rotor inflow. Once you get the rotor inflow, then you solve the blade equations, even if you have only flap equation just solve them. But you should write the flap equation based on a hub, which is having all these motions because you cannot use that level flight flap equation. So, your flap equation must contain the hub motion. That means, all the equations, you have to develop, then solve them, blade flap, assuming only flap. If you have lead lag torsion everything, then you have to solve the full, get the rotor hub loads and other aerodynamic loads, transfer them to the fuselage, get the force and moment equilibrium. You will find, that they do not match, but you take the average load. Please understand, you do not hub loads only, you get, obtain the rotor hub loads. That means, you take 1 revolution, get the mean values, then transfer the mean values for balancing the force and moments, it will not balance this. You have six unknowns, which are these pilot input and orientation of the vehicle.

So, six equations, you use these six. Now, once you get this, you go back here because for getting the blade nodes, you need the pitch input, otherwise you cannot get the blade load. That is why, you assume this once, you get a new update, go back here again, compute the track angle again, calculate all the u e, v e, w e and then, you do this. This is quite a complicated procedure, sometimes the program will not converge; it is very, very tricky problem and this is the complicated trim procedure.

Pardon.

(( ))

No, it is not how long, it is actually the convergence becomes complicated problem. See, we are going to do this because we have developed the equations now we plan to do, but initially we will do a steady turn.

Do not go and complicate all sides of (( )), just give a, we have done for a flap, we need a steady turn and we are getting results, which are quite similar to what are obtained by some other people.

Now, we want to do more complicated motion, ultimately the problem is the industry required. Hey, if I give a control input how my vehicle will perform, that one is you can go and solve my linearized, that delta u, delta v, delta w, that equation, that is a perturbation only, but that is, will give you only at the instant what this happening, it will not tell you over a long time because long time. However, you would have done the equilibrium about a hovering condition and a perturbation about the hovering condition.

But you go to forward flight at some other speed, then you will have equilibrium under perturbation, for that, like that you will have equilibrium, perturbation equilibrium, perturbation, you cannot say this perturbation will lead to that perturbation. If you want to solve the full problem, you have to solve complete, non-linear problem and this is where in between you have to get the rotor inflow. Please understand this is also equally complicated problem because rotor inflow is related to rotor thrust, so you can have, you know, various types of inflow models, dynamic wake model and we have incorporated something, which is (( )) model, you can put, but that is again a constant, even though it varies along the radial and azimuth, but it does not vary with time. So, you can have different types of inflow models.

So, usually, these problems, first you try to solve in a simplified way. Take only flap, check whether a program is converging because you will be able to get the mistakes from that. Once it is done, then you can say, these are my loads and if you want to include stall model, please understand, when I get the aerodynamic load I should have stall model for the blade, all those things. I cannot have a closed form expression, everything has to be integrated in these this part, so that is why the aero-elastic problem is part of flight mechanics problem. It is not that this in independent problem in helicopters and if you are doing this kind of a maneuver, this is, this is a real challenge problem. I think with this we close.