

## Introduction to Helicopter Aerodynamics and Dynamics

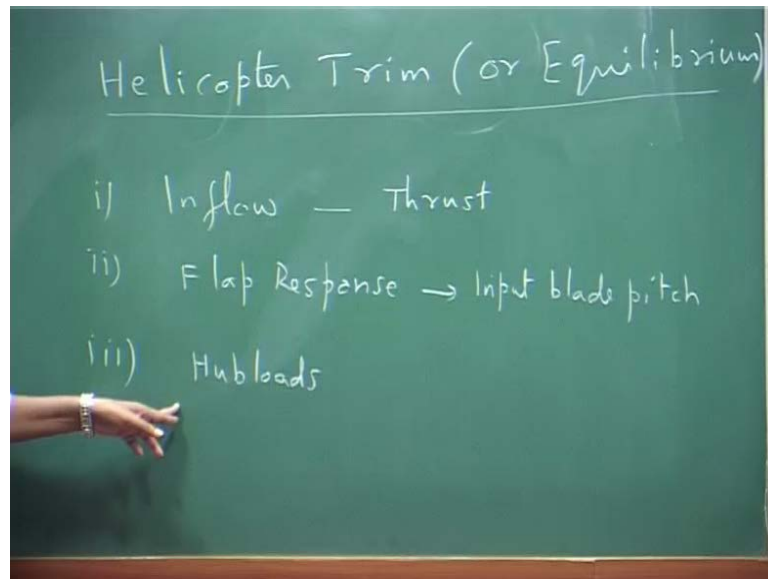
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### Lecture No. # 18

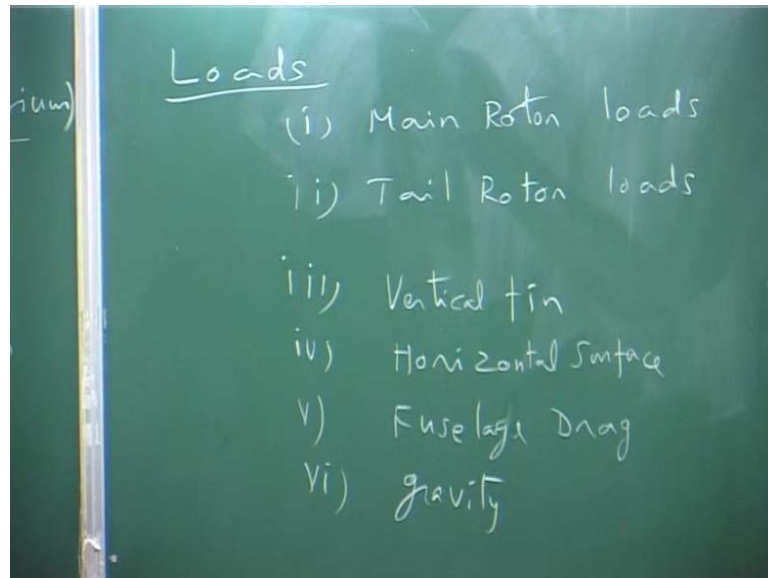
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Today, we will start with the section, which is a helicopter trim or you say, equilibrium, that is, if the helicopter is flying at a particular flight condition, what should be the control input, that is what you call the trimming of the helicopter. Now, for that, pilot input will be evaluated theoretically. How will you get that trim from what we have learnt till now? Because we have learnt inflow, first inflow and then inflow was related to thrust, so this is one equation. Rotor inflow, of course thrust is their key, you have forward flight condition, attitude, everything is there and the 2nd one is flap response. For this your input is pitch angle, blade pitch and the 3rd set, which we obtained are actually the hub loads, rotor hub loads, but the hub loads are in terms of flap, in terms of blade pitch and inflow, everything is there in the hub loads.

Now, if you want to trim the helicopter, that means, you must know what are the all the external loads, that act on the helicopter.

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You will have, in terms of loads if you say, it has several components, one is of course, main rotor loads, main rotor that means, load means, all, all the six components. And then, you have tail rotor and 3rd one is, if you have any vertical fin, so any vertical fin, that load and then, horizontal surface, these are all fixed. Then, fuselage, fuselage aerodynamic, fuselage loads you can say, fuselage means one is aerodynamics. So, you may say fuselage drag because we normally take only drag, but if you want to do, you have to put internal to take a particular point and evaluate the loads at different orientation, different mark number. So, the fuselage loads is 1, aerodynamic loads and then, the last one is the weight also, that is gravity.

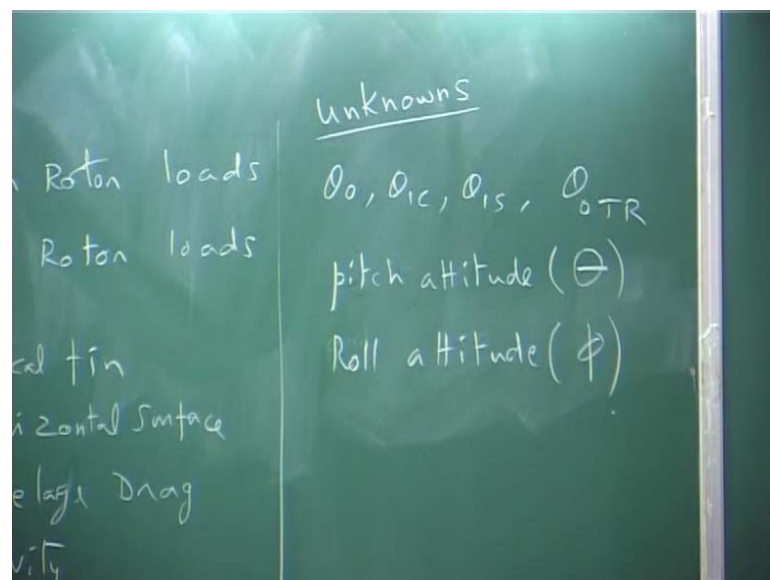
Now, what we obtain is only for main rotor hub loads, we have not; we do not have any of these things, yeah. The shaft tail rotor can be mounted on a vertical fin that means, it is like actual conventional aircraft and the fin can give its own load. So, that is why, but if fin is not there, then you neglect, that horizontal surface, you will find, you know this is a very, some helicopters have full big ones, some will have very small one, some may not have horizontal surface. And of course, fuselage drag you have to do from (()), then gravity, gravity (()) the CG.

Now, once you have the loads you have to transfer all the loads to the center of mass, but that means, you must know the location of center of mass for a particular configuration, means, you have the weight of that, how much fuel, how much, everything. And then,

write the six equations of equilibrium, three force equations, three moment equations, but they will be in particular coordinate systems, whether you want to write the equation in a earth fixed coordinate or it is the body fixed coordinate because you have to be very clear about that. All these loads, they are obtained in the body fixed coordinate system, so it is always preferable to use body fixed coordinate system, transfer all these loads to that and then write the equation. So, you will have essentially six equations, force equations.

Now, the helicopter, you know, that it can be flying in the hub loads, where obtain, then it is flying straight as a rotor, but the rotor can fly in any direction, that means, the component of velocity you must take proper wing. Now, what will you do in this is because my fuselage may be like this, rotor may, helicopter may be going in this direction, lateral, you can have all velocity components, but what we have obtained is only one velocity. So, you will restrict our self to only forward flight steady condition and how these entire equations are solved. Now, the question is what are my unknowns? What are my known quantities? That is very important before you really start off the entire sequence of this problem. You must know what are my unknowns and what are my known quantities.

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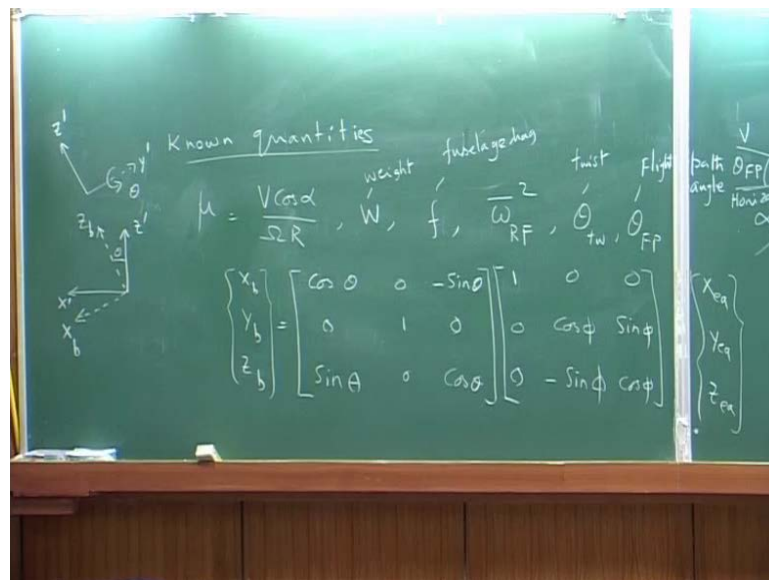


If you look at that, the known quantities if you say because let me put it, the unknowns, I will write unknowns, no one will worry about that later. Unknown is a theta naught, 1c,

1s and theta 0, tail rotor, then pitch attitude, roll attitude. We normally use **the symbol...** Pitch and roll attitude of the helicopter we do not know, these are the six quantities and then of course, power.

Sometimes you can do a particular case where power becomes the equation, but usually that is not done. What you do is, you calculate power after trimming the helicopter, evaluate the power, keep on evaluating, may have power available. It is not, that if you want to see what is my, I want to use all the power what should be in my flight condition, that is a bit tricky, that is why, best is you prescribe the flight condition and then you want this, calculate what is the power required.

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Now, when we prescribe the flight condition, first we take the very simplest case. Simplest case is I will put it known quantities, your mu, which is the advance ratio. This you say, I know it, which is, please note this alpha is attitude of the, basically alpha is that pitch, attitude pitch, theta alpha (( )). We do not know this to start with, that is why you always use advance ratio I am using. Assume, that this quantity is known; then, weight, this is the weight of the helicopter. Of course, CG location must be known for the given weight, you can have forward CG, half CG, etcetera.

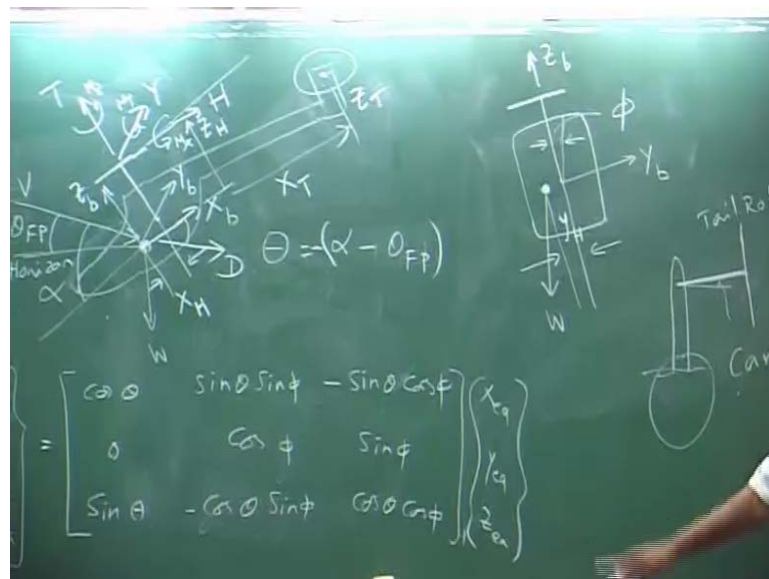
Then, that f, which is the fuselage drag, related to fuselage drag that f area; then, of course, the location. Then, you need to know omega bar R F square flap frequency.

Then, if you have the blade with the twist, you must know theta twist and theta; I will write what are these? This is the twist of the blade, twist and theta F P, which is the flight path, flight path angle, I will call it, I will, later I will describe.

In addition to this you have to know other geometric condition, where is the tail rotor, what is the distance from CG, the tail rotor location and the tail rotor dimension? If you want to take what we derived for main rotor expression, completely you can use it for tail rotor, absolutely no problem, only thing is you have to use the dimensions, operating conditions only for tail rotor. But tail rotor has only collective pitch, there is no cyclic.

We have simplified our problem by considering only flap frequency that is why I put what is the rotating flap frequency. And of course, if you want geometric, what is the precone beta naught, then you must specify that also, geometric values. Now, given these quantities, I will go and get those quantities and what is the power required for that particular flight condition, this is the basic question.

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We will draw one diagram and then I will we will describe the diagram is...

And you may have your CG somewhere here, this is weight and the horizontal, this is your longitudinal. This is the horizon, in the sense, the helicopter is flying in that direction, this angle is theta F P, that is the flight path angle with respect to horizon, that is all; horizon is normally earth bound. So, horizontal, with respect to horizontal, what is

my velocity vector? That is the flight path, this can be 0, means, you say level horizontal flight and this angle, I call it as alpha; that is the angle because this is what is required. This alpha comes here because  $V \cos \alpha$  in the hub.

Now, you may say, this is perpendicular to, if the shaft is perpendicular to the body fixed coordinate system, then it is fine, that this will be 90 degrees because you can have fuselage access system and that will be same as hub access system. On the other hand, if the hub is tilted a little bit with a respect to the fuselage, then you have to take that into account in the transformation, usually in, sometimes it is tilted by 2 degrees, something like that. The shaft itself, it will not be perpendicular, that will be slightly kept. So, these are small, small details, which one has to consider when you are writing the equilibrium equation.

Now, given these quantities and what is my pitch attitude? Pitch attitude is basically, theta is alpha minus theta flight path, but I will use some minus sign because the rotations are counterclockwise, clockwise, positive, etcetera. This is the angle pitch attitude of my helicopter

Now, this reference line, so I will fix two coordinate systems, one is earth coordinate system, which we may have. X, Y and Z these are earth system, you may, all X ea... With respect to this, this is the body, capital X, capital Y and capital Z, you may call it b, b, b. That means body system you have to get the transformation from this, this to this, that is, by rotation. But you assume initial because more complicated motions will take little later, right now we assume, that the helicopter is only flying in, flying in the plane X-Z earth axis system, but you may find it can fly in another direction.

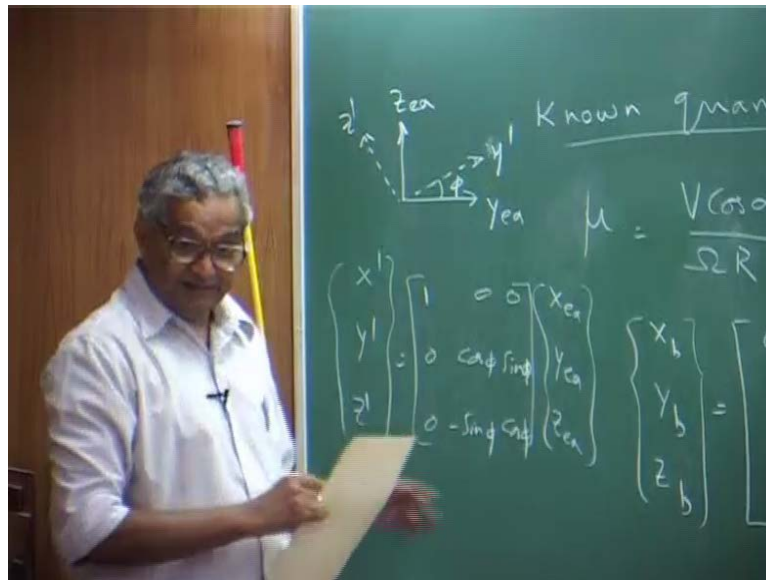
Then, you may have to say what is the  $\psi$  because the heading can be something like this, because if it is making a maneuver, that means, earth is a fixed coordinate system, but the helicopter heading can change, you follow. On the other hand, you can say, my heading of the helicopter is same, but my velocity vector is this way, then you have a side slip, you follow, because you are also moving side. So, all these flight conditions there becomes a little complicated. That part is a most general  $\psi$ , that I will just describe, if time is there at the end of the class I will show that, otherwise because that is the complicated thing, which of course, we are developing it. Right now let us say, this is the motion, the helicopter is flying only in the X-Z plane, but the center of  $\psi$ .

I will draw one more diagram by the side, that is, the view from rear and weight. So, there is a tilt about, because if you say with respect, this is my Y body, Z body and this angle, with respect to this is my roll. And this distance, you may call this distance as Y, Y hub. Here, you may say this is X hub and this is Z hub, then you can take to tail rotor. This can be X tail rotor, this is Z tail rotor; you can have all. That means, essentially, you have to locate every point. I have not taken any horizontal surface, nor the fin. I neglected those things in my simple formulation.

Now, you know that, because these are helicopter maneuvering, anything; these rotations are not small angles. So, you have to follow Euler angle transformation. The Euler angle transformation, I will write between this, you find out the sequence of rotation. First you rotate about x-axis, that means roll and then, after the rotation of x, you rotate about y-axis, so that you get the body access system, you understand. From the earth access you want to go to body access, so the transformation sequence, you first say roll, then pitch. So, this sequence if you follow, it will be like this. First is, what is it, this is x, y, z, this is the body, but the rotations are counterclockwise positive I am taking, that is why, there will be a minus sign I have to introduce there, multiplied by  $1 \ 0 \ 0 \ 0$  cosine phi phi 0 x m, this is the trap. That means, earth access system, you first rotate about phi you get the intermediate access then that intermediate you rotated about the pitch and then, but these data are counter clockwise rotations. So, phi is a counterclockwise rotation positive, then if you use that, this data because this is here as per diagram, I put to the access system clockwise rotation theta; I put it as alpha minus theta f p. So, actually, I must put a minus sign, that you look at, you, you look at, if you want I will describe it.



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See, this is Y earth, Z earth, X earth is back, so you rotate about phi. So, I call it X, sorry, Y some intermediate prime, Z prime, this angle is phi. So, Y earth to Y prime cosine minus sine; this is this matrix if you put x 1, y 1, z 1, you follow. Here, x prime, y prime, z prime. Now, you take here, here is X ea, Y ea, 1, 0, 0, 0. Here, you will have cosine phi, the same thing it will give you, minus sine phi. And here, this is sine phi, in the Z ea sine phi cosine phi, this is fine. Now, you got x 1, y 1; you got x prime, y prime. Now, you are in the prime system, you are giving a rotation about y prime.

So, you will have, this is my Z prime and X prime is here. This is actually my Y prime, X prime is coming out of the board, you are giving a rotation about Y prime. Now, what it will be? This is Z prime, this is X prime, this is the Z body, X body. Now, you will see, that you will get that, do not just blindly, that is why, in these transformations all of them, how they are given. Because now, you see what is the transformation, because here you, what X prime, Y prime is 1, Z prime, X prime cosine theta and X prime sine theta. Whereas, if you go to Z, Z prime cosine theta, Z prime, it will become minus sine theta. So, that is why, the minus sign there.

Now, with this you have the relation between earth to body. Why we have to use this is, because please note, that I still do not know this and this angle, but I use, I would not make any small angle approximation because that is why, please understand. The moment you have large angle, they are not vectors, you have to follow this



transformation. The weight is, acts along the earth access system, that is why you have to transfer the weight into body system and then, you can write all your equilibrium equations in the body system. That is the reason you need to have this transformation properly done.

Now, you got this, only thing is theta, you have to put up minus sign because as per the diagram, that is the key. Now, please understand, that is, the phi is rotated about earth access, whereas, theta is not rotated about earth access, theta is rotated about the rotated access; it is not about the fixed access systems you are doing the rotation. So, every rotation is about the rotated access after you get a new position about which you rotate, this is the sequence you follow. Now, if you want, there is another transformation, which is there, we have used all those things in our work, you rotate only about a fixed access system. What you said is, because you say, should I rotate theta with respect to Y prime?

(( ))

These are (( )), if you have a heading, that also will come.

(( ))

Not exactly, these are Euler angles; do not say earth coordinate systems.

(( ))

Because **jairo** will give you rate, that I will come to later. **Jairo** will give you angular velocity; angular velocity is a vector because instantaneous angular velocity, it will give you about three axis. If you have a three axis jairo, you will give rotation about the three axis. Now, the rate of change of attitude, please understand, there is a lot of difference between angular velocity vector of the body at an instant to rate of change of attitude of the body, there is a relationship. If the angles are small, you say angular velocity is same as rate of change of angle; if not, see because in the airlines, you have to do one more, that is kinematic relation, that data we will do it later, if it is interesting, if need it to know from flight dynamics point of view, because if you really want to solve that problem you have, know little bit more, that is a real life, real life problem, you have to handle that, I will describe that part.

This angle is the rotation about earth, I can find out how much roll rate is happening, roll rate with respect to earth,  $\dot{\phi}$ . How much  $\dot{\theta}$  is happening here, but  $\dot{\theta}$  is about the  $y$  prime axis, yeah, already tilted, well,  $\dot{\phi}$  is about  $X_{ea}$ ,  $\dot{\theta}$  is about  $y$  prime. So, when I want to get the angular velocity vector, what I will do is, I use the same equation, I will put  $\dot{\phi}$  here and then get this.

What is the body fixed? What is the  $\dot{\phi}$ ? But then, when I come here I throw this off, I will put here  $\dot{\theta}$   $y$  prime axis and then I will calculate what is the effect of that, you follow, because you see  $y$  prime is nothing but  $y$  body. So, basically,  $\dot{\theta}$  becomes basically pitch rate, whereas the  $\dot{\phi}$  is not exactly the roll rate experienced by the **jairo** on the vehicle, that depends on what is the orientation of the aircraft at that instant, that is why, you need to solve even in the **navigation of any...** What is that navigation, satellite navigation, **(( ))** navigation, everything you have to follow this because these angles are not small angles, they can be large angle. So, you solve the angle, you get the attitude, you have to solve for that.

Now, let us take as, it is this one and we will write our, so I thought I will show directly, yeah, you can do, there is nothing wrong. See that is where, what is the sequence I should follow? There is some convention that is it; you can follow another sequence, absolutely no problem. But so long as whatever sequence you follow, the result will correspond to that, only thing is you cannot compare his that number with the result, which is following another sequence. Numbers may not mean, but the orientation will be fine, that is, this problem cropped up when people were deriving the equation for the rotor blade. Whether you use flap, lag, torsion sequence or lag, flap, torsion or these are so many and in the 70s, that was a big debate. Finally, when we do rotor blade, I will explain that part right now, you will go **into the equation for...**

So, you got the, now let us look at what are the equations, which we need to solve.

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1. Inflow Equation

$$\lambda = \mu \tan \alpha + \frac{C_T}{2\sqrt{\mu^2 + (\mu \tan \alpha + \lambda)^2}}$$

$$\lambda = \mu \tan \alpha + \frac{C_T}{2\sqrt{\mu^2 + \lambda}}$$

2. Thrust Equation: Vertical force

$$T - (W \cos(\alpha - \theta_{FP}) \cos \Phi + D \sin \alpha) = 0$$

3. Horizontal Force Equation: Drag

$$H + D \cos \alpha - W \sin(\alpha - \theta_{FP}) \cos \Phi = 0$$

4. Side Force Equation

$$Y + T_T - W \sin \Phi = 0$$

5. Roll Moment Equation

So, I said, first is inflow equation in forward flight, uniform in flow I have taken, please understand, this equation is one, then I have trust equation. When I say vertical force, this is along hub. So, this is, this is the thrust, this is T, this is my capital H, this is my, because T H Y, then you have m, x then you have m y and you have m g, all the loads at the hub.

You transfer these loads to the CG and then of course, tail rotor I have used, only thrust and tail rotor is giving, only thrust in this y direction that is all I have neglected. All other loads, which is, which is reasonably all right for most of the analysis, unless you want to complicate more for a good solution, that is enough. That is why you see the drag force D, which is the fuselage drag, that is, along this, along the flight path. This is the D that is why you see this angle, that horizontal angle, this angle is alpha. So, D cosine alpha, D sine alpha; that is why, weight is acting here. When the earth system, you put weight, put minus w because this is acting in the negative z direction. So, minus w you convert it, find out along what is the force along the z body because this product you can (()). The product is actually and this is a, this is essentially a cosine theta sine theta sine phi minus sine theta cosine phi 0 cosine phi sine phi and sine theta minus cosine theta sine phi and cosine theta cosine phi, x ea, y ea, z ea. Now, you see weight is acting along minus w because that is minus Z-axis z earth. So, that will be along the body, you will put this; that will become minus w cosine theta cosine phi. Theta is minus alpha, so you will put cosine that is why, here I have put cosine alpha minus theta flight path cosine phi. And D

is here,  $D$  acts along the velocity direction and the velocity, we took it although it is in the  $X-Z$  plane. Please understand, that is why, I said velocity is in the  $X-Z$  plane. Suppose, if the velocity you have a side, the drag will not be  $X-Z$  plane, drag also will give a side force, you follow. So, all those, that is why, when you do vectorially, you have to do everything properly.

Now, you have the vertical force equation, similarly horizontal force. Horizontal force, you have the hub load because tail rotor, you say it is not giving any vertical force. Suppose, if the tail rotor is given sometimes in the design, the tail rotor fin may be like this, something like this, you may give a slight tilt of the tail rotor, it is not perfectly the shaft. This is the tail rotor; looking from back you may have, there is the fuselage is going you, see this is given a small angle  $((\ ))$ , maybe in this they can also give it in the front direction, this is called Cant angle, few degrees.  $((\ ))$  take a little bit, they can do, why?

Now, a component of thrust, you can use it in supporting the weight, a component; that is all. But usually, cant is not, you know, arbitrarily you do not do because not that everybody goes and gives the cant the advantage or disadvantage, whatever it may be. The advantage is, they say, that I can use the tail rotor force a component in the vertical direction; that is all.

$((\ ))$

No, you cannot change, fixed, not that helicopter, no way to introduction in the design configuration; that is all. But not all helicopters have, not all the helicopter have no random means, what do you mean by random?

$((\ ))$

Zero; most of the design they will have 0. Some helicopters have some angle and I have to be very honest, I only know the effect of this is, it will give some component because you need to have a tail surface always. If the helicopter has to fly, it is slightly higher speed, we need to have a tail surface, why? It means, if you do not have, you will not get equilibrium, moment equilibrium; we will not be able to achieve as a result, which means you cannot fly the helicopter at that speed. Engine may have power, but pilot

cannot fly it will not be in equilibrium. So, that is why all the fixed surfaces they introduce.

Now, you may say why they have to have a vertical fin because you have vertical fin. What happens is, that vertical fin, you will not be at the 0 degree, it is slightly given 1 or 2 degree to the longitudinal line of the helicopter at, there is at an angle of attack. Now, what it will do is, it will try to relieve the tail rotor load in forward spin a little bit.

That is all nothing more because tail operates in a very complex environment. Industries will have a test wing, they will do some test, otherwise all simple calculation. You take the thrust, whatever we have, for the main. Main rotor is much more complex in terms of pitch angle, etcetera; tail rotor is more complex in the aerodynamic environment.

And, and the distance these are all important. This is mounted because first of all, the vertical fin you need because if you want to mount the tail at a slightly higher position from the  $(C)$ , because you may have why I have to shift it up, that is like a configuration because the main rotor, the wake, you do not want to go and hit the tail rotor. So, you try to keep this center along this height rotor hub height, so that tail rotor is not affected by the flow from the main rotor. If it gets affected, you do not know what is going on, you can say, well I will put it this here, then you put it. Whatever you get you have to take it because you do not know how they will interact with the wake from the main rotor, the tail rotor rotation. So, best is, you put it up even in tandem helicopter, you see the back one will be up, the front one will be a little lower. This is to avoid because you do not know honestly and, but some helicopters you may say, it is there, our model has not that lifted, it is right on that so, but we take it, that is all.

Now, the horizontal force, you have again the hub load, the drag and the component of the weight because the component of the weight multiplied with this will give a horizontal force in the body system. So, you use that term and then side force, you have the  $y$ , you have the tail rotor and you have again the  $w$  in the  $Y \sin \phi$  minus sign because this is a  $w$  becomes, so that side force has again the weight. That is why, you see, the weight comes in all directions in the body fixed system.

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$$H + D \cos\alpha - W \sin(\alpha - \theta_{FF}) \cos\Phi = 0$$

4. Side Force Equation

$$Y + T_T - W \sin\Phi = 0$$

5. Roll Moment Equation

$$M_X - Y Z_h - T_T Z_{ht} + T Y_h = 0$$

6. Pitch Moment Equation

$$M_Y - T X_h + H Z_h = 0$$

7. Yawing moment

$$M_Z + Y X_h - H Y_h + T_T X_{ht} = 0$$

Then, similarly, you do the moment balance, roll moment, pitch moment and yawing moment. So, you have the roll moment  $M_x$  from the main rotor, side force into  $Z_h$ , this is the height; side force. And similarly the tail rotor, tail rotor thrust into  $Z$  height tail rotor. And then, you will have main rotor thrust into  $Y_h$  because this distance, the thrust is this way that will give a moment if the CG is not on the longitudinal line. No, I will send you these things, this just for explanation. And then, you have pitch moment, again rotor hub and this is due to thrust and that is due to again side force, sorry, the longitudinal force will give, and tail rotor I have taken nothing.

Now, you see, in the pitch moment I do not have any horizontal surface. If I put a horizontal surface here, like a small aerodynamic surface flat plate kept at an angle that will give a pitching moment. So, usually, big helicopters they have that, if it is a small weight class, they do not have. But some time you will find some helicopters have only one half of the tail, not on both sides, it will be having only one half; these are all just for equilibrium. And you can have a vertical plate also, like you have horizontal surface you can have a vertical plate, which is inclined at a slight angle like your vertical fin; you can have like this, they will give a force, these are all to relieve little bit.

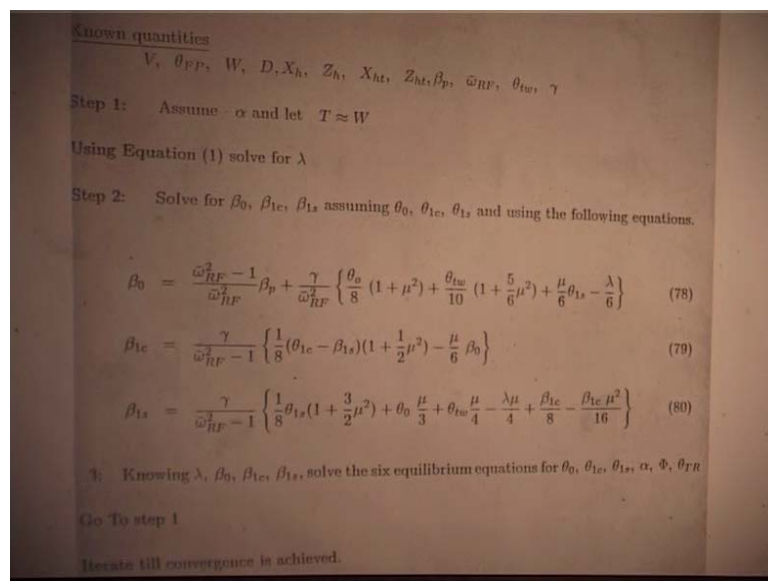
And yawing moment, again the main rotor and the side force will give a yawing moment because of this distance side force. And of course, tail rotor will give and your horizontal

force also will give because of this distance. Horizontal force will give lateral shift in the CG.

So, these are my seven equations, of that inflow is the first equation. So, six load equation, you may call it equilibrium. Now, how do we really solve, that is the question you have to because where is the flap equation coming in between, because flap equation is sitting somewhere because all these thrust H, Y, M x, M y, they all are dependent on flap response.

Now, flap response we approximated it as though first harmonic beta naught, beta 1c, beta 1s, that is all, rest of them we neglected. So, all these expressions require flap. So, you have to now introduce the flap equation in between; that is why, now you see your equilibrium.

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This part we will come to that later, this is an aeroelastic problem, is part of trim problem because you cannot decompose aeroelastic problem in helicopters. Aeroelastic problem is response of the blade to air ((C)), that will be sitting in between because you cannot just like that solve equilibrium equations separately, because you must know blade response. Then, blade response depends on what type of model you are having for the blade. Here, we have included only flap, please understand. In real helicopter you have to have flap,



lag, torsion, all must be included and it will be much more complex, I will show those results a little bit later.

Now, how do you start because it is an iterative procedure, you do not know inflow. So, first is you assume my thrust is equal to weight and always start these analysis from hover case, do not straightaway go and then start solving, I want at  $\mu$  0.3; straightaway go and start means, sometimes it will not converge. So, you start from hover, which is the simplest case. You take, that equilibrium quantities and then slightly change those quantities, then go to next forward speed, which is point 0.5  $\mu$  because you have to assume. First assumption is, assume  $\alpha$ ,  $\alpha$  is  $\theta$  F P, maybe there flight path may be 0, may not be 0, that is a given quantity, please understand, given quantity flight path is there. I may have flight path 0 that means the FP is 0, means,  $\alpha$  I do not know. So, I will assume  $\alpha$ , you may say I assume 0, no problem because hover I may take it 0. That is why, first case hover, whatever result you get in the  $\alpha$ , if it is 0, when you go to first forward flight of point 0.5  $\mu$ , you assume 0. Then, thrust is equal to totally weight, just approximately thrust is weight. Then, you solve for  $\lambda$ , inflow because inflow equation I showed you, that requires only C T and  $\alpha$ ,  $\mu$  you will know, there is no problem,  $\mu$  is given, that is a flight condition. So, you first iterate within this, get a  $\lambda$  that means what you have done? Very first step, you got the inflow, after knowing the inflow, **then you go to...** So, that is why I said, use equation one there is, solve for  $\lambda$ . Once you get  $\lambda$ , I have given you in your class notes the flap frequency not equal to 1; equal to 1 I have both,  $\beta$  naught,  $\beta$  1c,  $\beta$  1s. Solve for  $\beta$  naught,  $\beta$  1c,  $\beta$  1s assuming  $\theta$  naught,  $\theta$  1c,  $\theta$  1s; please understand you have to assume something.

Now, how do we assume? This is from previous flight condition, that is why, you take the hover case first, solve for that because hover is easy to converge. Assume these values, then substitute here because here this requires a  $\lambda$ , please understand.  $\lambda$  and  $\theta$  twist, everything, you will get  $\beta$  naught,  $\beta$  1c,  $\beta$  1s. Now, I have  $\lambda$ , I have all these quantities, then go to the six equilibrium equations. What you do is, what are the unknowns? For that case you say, I know  $\lambda$ ,  $\beta$  naught,  $\beta$  1c,  $\beta$  1s, but I do not know the six quantities of  $\theta$  naught, 1c, 1s,  $\alpha$ ,  $\phi$  and tail rotor. Tail rotor you can take it as a tail rotor thrust itself, later you can, once the

thrust get converged you can actually go and use the simplified path to get the tail rotor pitch angle.

Now, when you solve these six equations, six unknowns, they are all algebra equations; please understand, they are all algebraic equations, but none can be used it may be, but after getting  $\theta_{naught}$ ,  $\theta_{1c}$ ,  $\theta_{1s}$  and  $\alpha$  and  $\phi$ , you again go back to step 1, because now you know  $\alpha$ , you go back, you can again and you may get the thrust also from the problem. You can calculate because all these angles are known, you can get what is the thrust. Know the  $\alpha$ , know the thrust, again get a  $\nu_{\lambda}$ . Once you get the  $\nu_{\lambda}$ , again go here, solve and this whole procedure you iterate, till equations are converged. Now, the convergence itself is a very interesting thing, you converge here, then whatever, you come here, you take it, but the key problem, which is spaced is, you will find the thrust, which is a dimensional, it is like a 4000 kg helicopter or 5000, that is a very big number. Whereas a side force (( )), they are all very small numbers. Then, when you try to take equations, you will find, that the big number will drive the result, but it may not converge properly.

So, always, all of them are non-dimensionalized such that they converge. They are all given equal importance and then, you get the converged solution and once you converge in the pitch angles, then you say your result is over, but you will never get everything equal to 0 beautifully, some small number, that is ok. If it is 4000 kg, if it comes to 3950, it is alright, already 980 because otherwise, it will keep on accelerating the result, you keep on shunting. And these iterations if you go, real complicated thing, it takes about, the problem, which we solved, it takes 14 hours for one flight condition because that is much more complicated, much more complicated. This you can do because you have everything in the closed form solution, you can write a code, that is why, I am saying, you can write. I have given every expression to you in the class derived, this can be actually exercise for the student, you see, how to solve the problem. Now, you see all these equations are there, you write a program, it will not work initially, it will give you garbage. Then, you have to find out what error because even the solution technique has to be proper, it is all algebraic equation.

Now, I am just extrapolating the complexity. We made several assumptions, one is in the inflow. Uniform inflow in it is not uniform, it is variable. The 2nd one is flap, who told you only 1st harmonic, I should have all the harmonics. Then, I should have lead lag

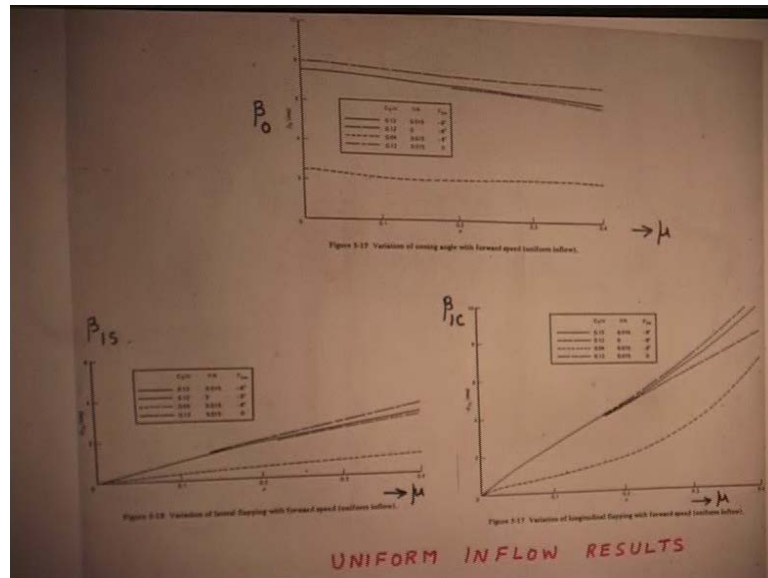
motion, I should have torsion motion. That means, your problem gets complicated and you should have stall, you have reverse flow effects. When you start adding everything the problem becomes sometime intractable, it is a complicated problem. So, what industry was doing, initially they do lot of approximations, get a result first, after that they will go, that is why, here when I use flap, so if you take only flap, you call it flap term.

And another thing is some of the research work, they do not take all the six equations because I do not know tail rotor, I am not interested. So, they will say in most of the research publications, when they are analyzing the aeroelastic response of rotor blade, only blade. Then, they will take side force equation and yaw moment equation, they neglect it that means, you have thrown away two equations that means. What are the two unknowns you will throw? You basically throw roll angle, you throw from that and the tail rotor is thrown, that means, two unknowns are thrown, remaining is theta naught, theta 1c, theta 1s and alpha, that is all. But that is, they have for a research publication, where you want to study more about only blade response, blade characteristic influence of certain geometry and the aeroelastic thing, they do not do full analysis of the helicopter.

Now, another aspect is you can do wind tunnel. In a wind tunnel what happens is, it is kept at a specific shaft angle, that means, alpha is fixed, that is called a wind tunnel trip. There you do not go and balance the thrust because you know, it is held. Whatever force, that comes, it is not, that it is equal to weight, so thrust equation is out. You may directly solve what is the developed force, developed only. You can say, there is no equilibrium equation wind tunnel. What people do is, they give prescribed collective, prescribed cyclic, prescribed theta naught, 1c, 1s, now find the response of the blade. But for that you need to know inflow. So, they will give a shaft angle, that is all, but you do not know thrust, you follow. So, you have to get the thrust and then get the inflow and then get the all the response quantities. So, depending on the situation, you use that particular set of equations and solve them.

I hope you have, I have conveyed the message here. Now, let us look at some of the, how, see once you have converged, you go back and calculate power, what is the power required for this because power equation, you know it earlier I had, we have derived in the class.

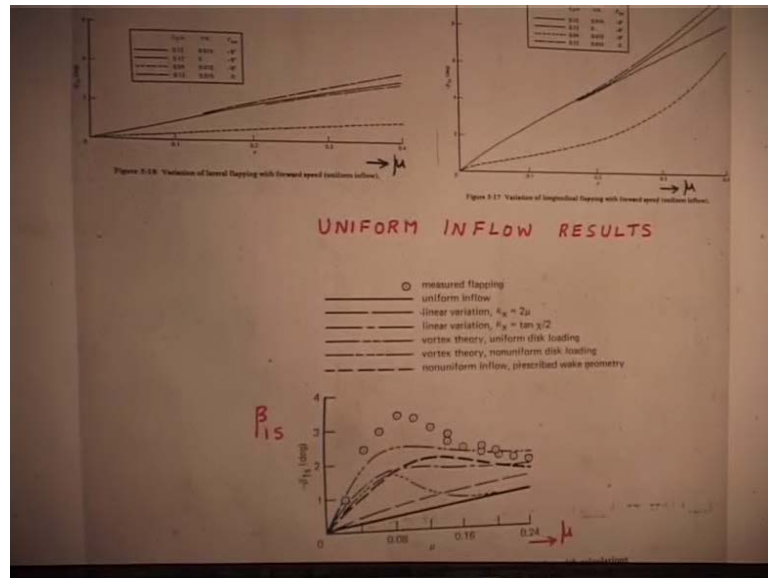
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Now, I am going to show few results, which I have taken from the book and some, which we have generated. You see, in general what happens is,  $\beta_0$  does not vary so much with respect to forward speed. It does not vary a lot up to some 0.3, some 0.3, 0.35. So, very small variation because the thrust vector, it is almost supporting the weight of the helicopter. So, it just, only with change in weight it may change.

Whereas,  $\beta_{1s}$  and  $\beta_{1c}$ , I have shown here like this, but actually this is not the, this is some theoretical calculation.  $\beta_{1c}$  they increased, this is also increased with, but this is not correct. This was a main point of research in 80s, later I will show some other results.

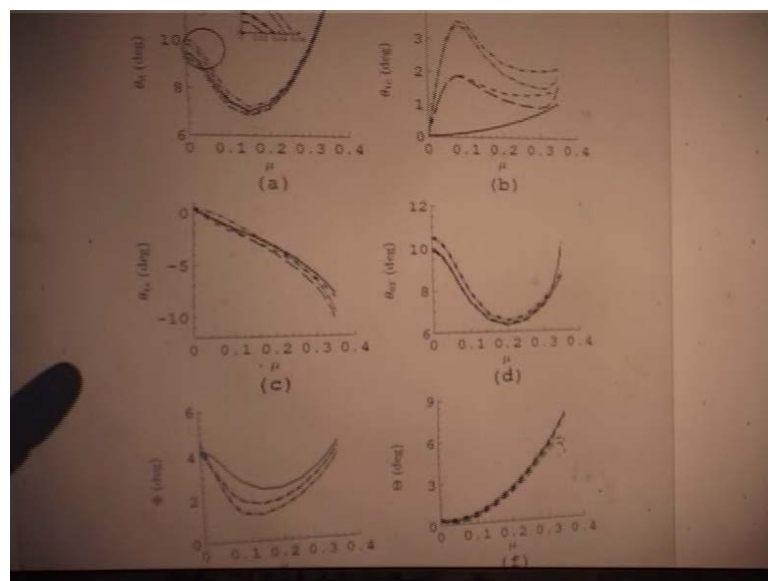
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Some of the experiment show, that at low value of mu, the lateral flap, lateral flap, it is about 2 degrees, 3 degrees that is all; it goes up and then it decreases. These are all various theories, this line, I wanted to say this line is my uniform inflow theory, this is the line.

Now, I will come to our results, which we have, this was the, we published, just last year it came, I think last year, yeah, 2009, yeah, 2009.

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Here is the, we have, this is a complicated problem we have solved, same thing, flap, lag, torsion completely, but full 6 degree of freedom helicopter analysis. This is where it took about 14 hours I was telling you with various aerodynamic models. So, if you look at the collective, collective it will decrease and then it will increase.

But you see, as you go to high speed, that is, around 0.35  $mu$ , you see the slope of this curve, it is very steep, the program will not converge so easily, that means, even if the vehicle has power, enough power. But the helicopter cannot be converged, in the sense, trimmed properly because of, it requires very large, it is not restricted. Now, how do you, why this curve shoots up like this, that is, because of the fuselage drag, if you reduce the drag of the fuselage, but how would you reduce it, that is the problem?

Now, that is why, that  $f$ , which I showed you equivalent flap plate area for the helicopter, that has to be, if it is brought down somehow, then the curve may tilt. You can go further if it is there, you just cannot trim. The drag will shoot very high, that is one of the key things and of course, you see,  $\theta_{1s}$ , that is the longitudinal cyclic. You see, this is a negative and that keeps increasing, these are with different aerodynamic models.

Now, here is the  $\theta_{1c}$ , which is the lateral cyclic. With different aerodynamic models I am getting different, different curves. This is the uniform inflow curve, the lower that continuous line and these are, I have included some dynamic stall model and dynamic wake, some additional more complex models, you see they change drastically just in equilibrium, this was a point of (( )).

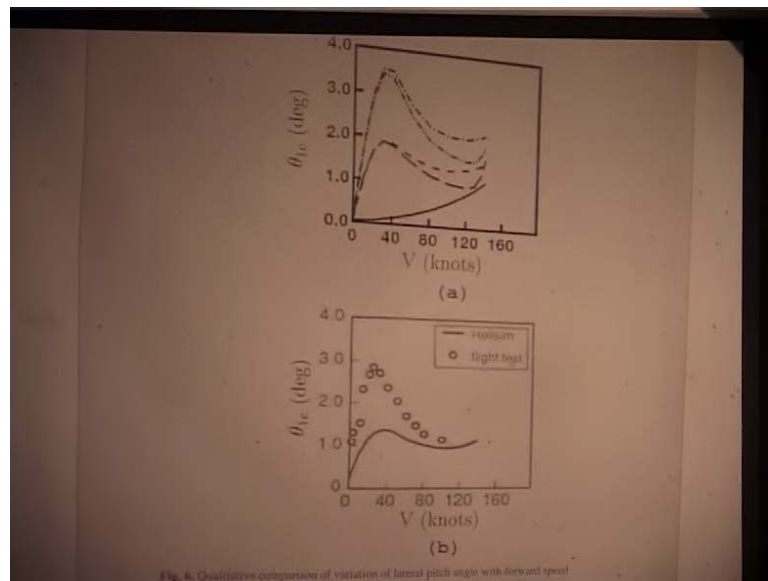
Why the helicopter it is like this? Pilot wants to fly forward, usually the general tendency is, what if you want to go high speed, you give more angle, but here, initially you give more angle, then as you increase this speed, you start decreasing the angle. So, from hover you go to very large and then you start coming down, this is, it defies the, you know, traditional (( )). You want to go high speed, what you do? You increase the throttle. It is not that you go to high throttle for low speed and then come to low throttle for high speed, it does not happen that way.

But here, this was why it happens; this was a question a lot of people... Of course, this is due to inflow modeling, but inflow and stall put together it came, whereas (( )) model, which I have mentioned to you, that gives slightly this result, but we need to have stall

also. And of course, the tail rotor follow similar trend as a main rotor and these are roll and pitch, but roll attitude we cannot, I do not have data to compare with experiment because I do not have data for this with a real helicopter. So, any way HUL, once we develop, they will be using this code for them because this is, you cannot measure it and then you have some other mechanism.

What is a rotor tilt, tilt angle? You should measure it and then plot. Because the tilt angle depends on where you put your tail what happens, etcetera. So, and this is the pitch attitude. So, you see, the angle values, that is, what I am telling you approximately, this is about 10 degrees, it comes to 7 goes up cyclic, that is theta 1s, which is directly related to beta 1c they, go up to minus 10, minus 20. So, the pitch angle can go very high values in the retreating blade.

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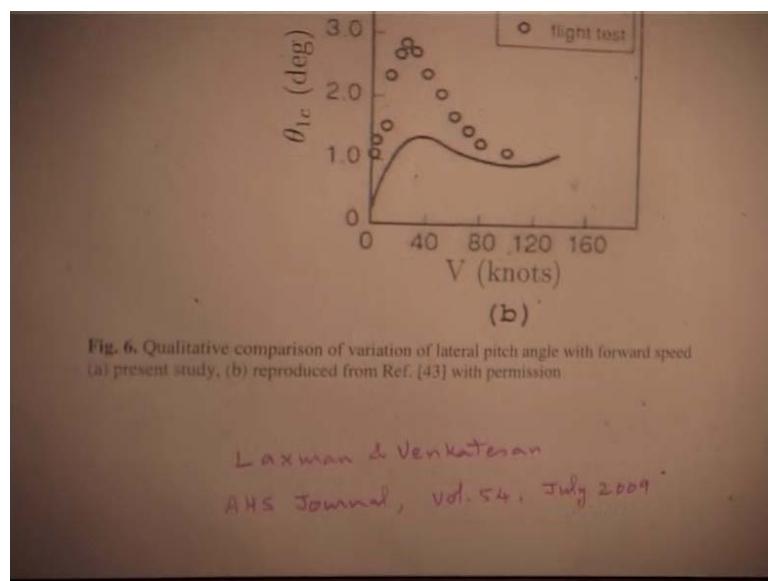
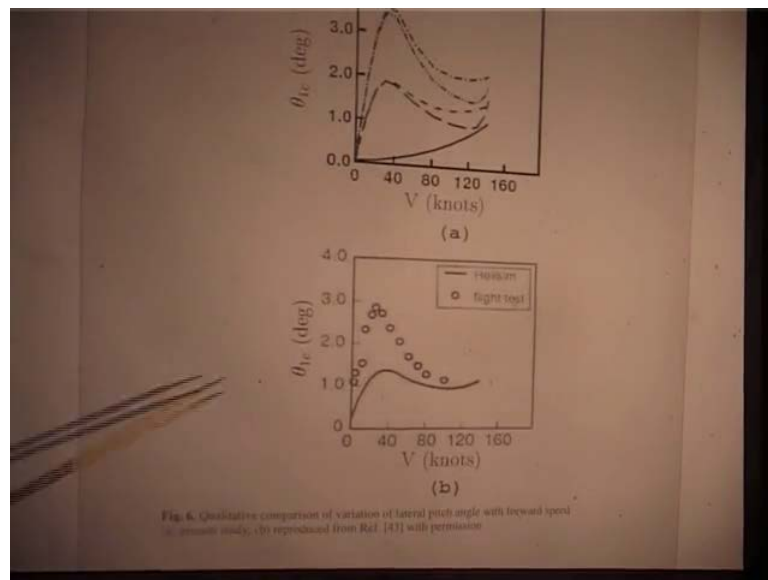


Now, I just want to show this result because this is what this is also we presented it in it is published in the journal see this is the taken from a book this is the flight test this is the code which they have used this is a paper if the pat field book I actually took permission from him to put it in my publication in our publication I would say because the laxman student did I took his permission. So, that I can reproduce this in our journal paper just to say because I do not have practical data because usually even if you have one practical data and then show the quality of the paper will go drastically very high.



So, I just wanted to show to the people look here this is what the experiment or flight test qualitative this is how my **my** data for the given helicopter test. So, different models if I use see this line looks somewhat like this **this** is only lateral cyclic please understand rest of them I do not have any data and this is you see goes up to three point something here also it shoots up; that means, the rise t price in the angle for trimming this is only trim problem please understand we have not gone to stability other things this itself has then we said this is what that is why we put qualitative comparison of variation of lateral pitch angle with forward speed.

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Now, I just want to show this result, because this is what, this is also we presented it in, published in the journal. See, this is taken from a book, this is a flight test, this is the code, which they have used, there is a paper in the Patfield book, I actually took permission from him to put in my publication, in our publication I would say because Laxman student did, I took his permission so that I can reproduce this in our journal, paper, just to say, that I do not have practical data because usually, even if you have one practical data and then show, the quality of the paper will go drastically very high. So, I just wanted to show to the people, look here, this is what the experiment (( )) qualitative, this is how my, my data for the given helicopter does.

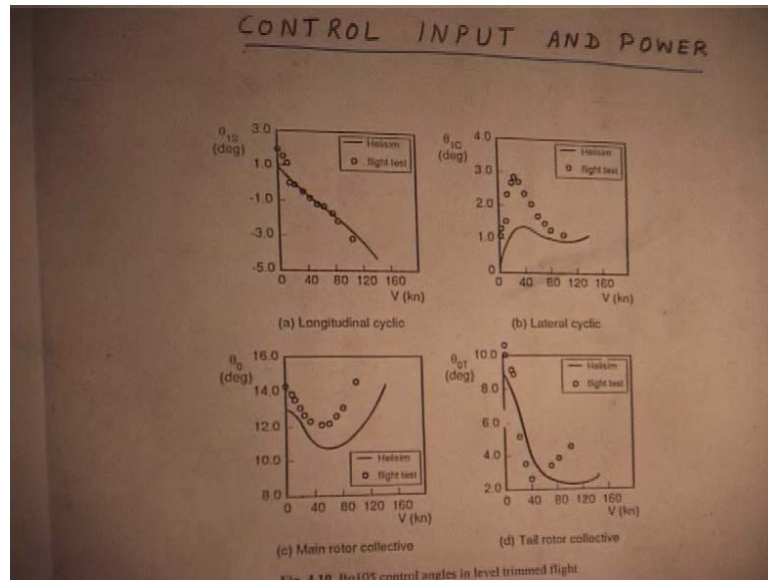
So different models if I use, see, this line looks somewhat like this, this is only lateral cyclic, please understand, rest of them I do not have any data, and this is you see, it goes up to three point something, here also it shoots up. That means, the rise, steep rise in the angle for trimming, this is only trim problem, please understand, we have not one to stability, other things, this itself has, then we share, this is what, that is why we put qualitative comparison of variation of lateral pitch angle with forward speed. So, this is a present study, this is reproduced from some reference, that is, the Patfield book, what I am saying is, this we just published last year AHS, this is the helicopter society journal.

(( ))

This is a flight test; this is a flight test, but flight test for some helicopter, please understand. But this, why I took it is, we all know, everybody knows this is what happens in flight, but how would you get that, what model affects your trim result, simple. So, ours is purely a theoretical study, but we gave various aerodynamic models just to see, hey if you change this model to another model what happens to the trim, you follow and this is only a qualitative comparison. Now, if you want quantitative, then we should take HAL that is all; HAL should supply the data, see they also know that.

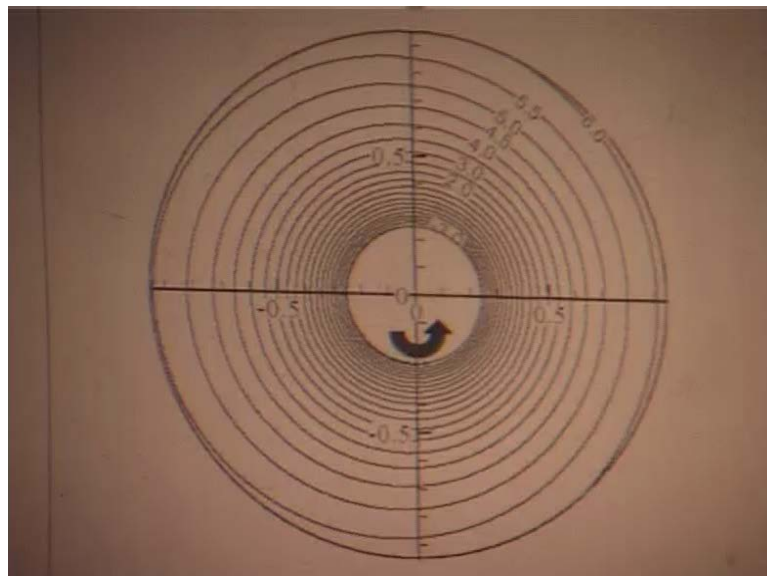
Now, there is a collaboration to make it our own Indian developed code for the analysis of helicopter thing because please understand, these things take 15 years, it is not one day's job and a lot of companies, they are also working in US, they have put lot of millions of dollars for developing analysis code. Today, they are doing maneuver, we are now doing maneuver for the next thesis, after this is the maneuver.

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Now, I will just show you one diagram, it is an interesting, maybe I will show this first, this is what I took it from the paper. See, these are all experimental data, this is the one, which I took because the other things are all, everybody gets the trend, there is nothing special, only this is the key. I just want to show how the angle, pitch angle varies.

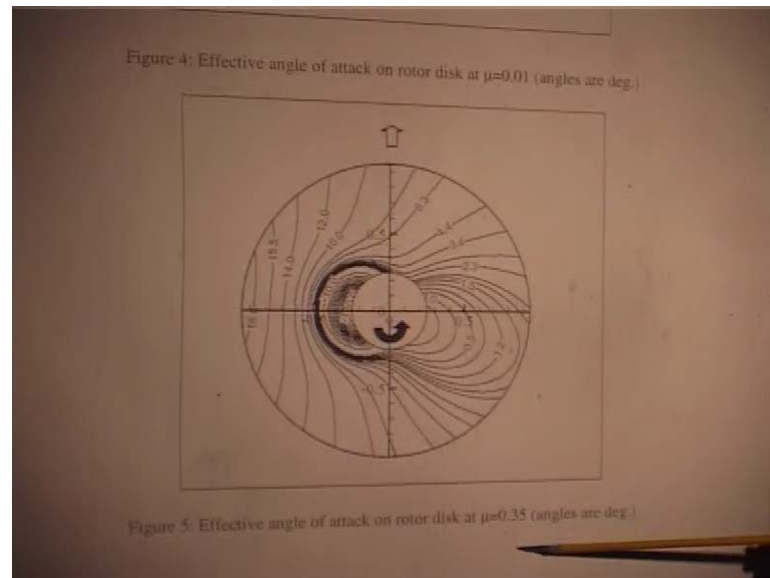
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I have put the numbers here, this is 0.05,05 these are nothing, but the midpoint of the rotor. And these small numbers 2, 3, 4, 4.55, 5.6, these are effective angle of attack on the rotor disk at various radial locations because this we took it as 0.23 as the hub for a

advance speed 0.01, very low speed, everything is within the stall angle, that is, 6 degrees is the maximum near, somewhere near the tip.

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But when you go to high speed, high speed, this is 0.35. You see here, these angles are 16 degrees, 15 degrees; that means the blade starts stalling here and on the advancing side, there about 1.2 here, 3, 4, 5 here, there is no stall, all the stall goes here. And this is the reverse flow region, very high angle, you can even have, I think we have put minus 180 degrees. That means, the aerodynamic data we have for the full 360 degrees, the rotor airfoil, we took the data from a text book only.

Now, that is why, at high speed you start stalling on the other side and that will give you vibrate. We did vibrate, load, etcetera, it is just for you to know the, how do you do trim problem, but trim problem will give you the vibration load as a part of solution. That is why you do not solve trim separately, aero-elastic load separately, you have to get both of them simultaneously, but simultaneously means, at one shot you will not be able to solve it; iteration.