Introduction to Helicopter Aerodynamics and Dynamics
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Lecture No. # 15

Evaluate the thrust torque for the given data at the operating condition steady state values both starting and finalizing and compare with your experimentally data, the change in values, because the absolute values at (( )) there are some differences. And I want to know what is the reason? You give, think about it give the reasons, why? What? Is clear? 900, 1300; I have given the blade data and geometry data, everything is there, pitch angle, calculate thrust torque with what model you want in that you use it. What inflow model you will use etcetera. And then, see what is the difference between this, and how much is the theoretical difference compared to the experimental difference.

The experimental also cannot jump from one value to another value as it will thrust jump steady state torque jump, this is one set of calculations. In the second one is the time variation. You write your own model, theoretical model. I am not saying you should match with this; theoretical model you formulate, how a time variation will be modeled. I am sure that if your equations predict the similar trend. That is it. Is it clear? So, one is for steady state, and another one for time variation. These are the two and this is going to be, I will evaluate it as in exam.

Sir, we submit the assignment and…

You submit the assignment that is all, what what else.

It is not like we will have to sit for an hour, but how can we?

No, no no no no there is nothing like (( )), you have to submit it on that day whatever date I specified, because I made see I will tell you what are the, there are some still some questions, because the load cells gives only the change, because steady state there is nothing like, if you keep a weight the output of that will keep on drifting. So, you will have one volt at this time then after 20 seconds, you will find the output voltage will be different for the same weight. So, you will not be able to see what happens; the weight has changed as some whereas, whenever there is a jump, a jump is collective.

So, that is why I said calculate for steady state values take the difference; the difference how does compare with experimental data. Whatever you get, that is the result, because then you give…

Reason.

The reason, that is I want you to think and then give your own reasons. What could be the likelihood for this difference, if there is a difference. And the other part is the time response.

Now, is that I I given you the sheet which is the expression for the thrust, side force, later, longitudinal force, torque and also the moment.

Sir, time response need not correspond to this (( )).

No, time response (( )), but you can take the data.

Even, the steady statement value.

You say what changes, because some initial value to final value that is all. How it changes that is enough, but you can take this data is same.

Data (( )).

Same, but then you will find out how much varies, because time response is much more complicated.

Now…

Sir, (( ))

Yes theory.

(( ))

No, no no no no no matching of experiment, theory that is from theory. I want the theoretical equations, formulations of the equations which describe the physics of time variation, not curve fitting, please understand. Is it clear?

Yes.

Now, I have given you the sheet containing the mean thrust coefficient, cost coefficient etcetera. and there are two additional terms which I have added pitching moment, (( )) that that we will cover as we go along the lecture, because I have introduce some new terms.

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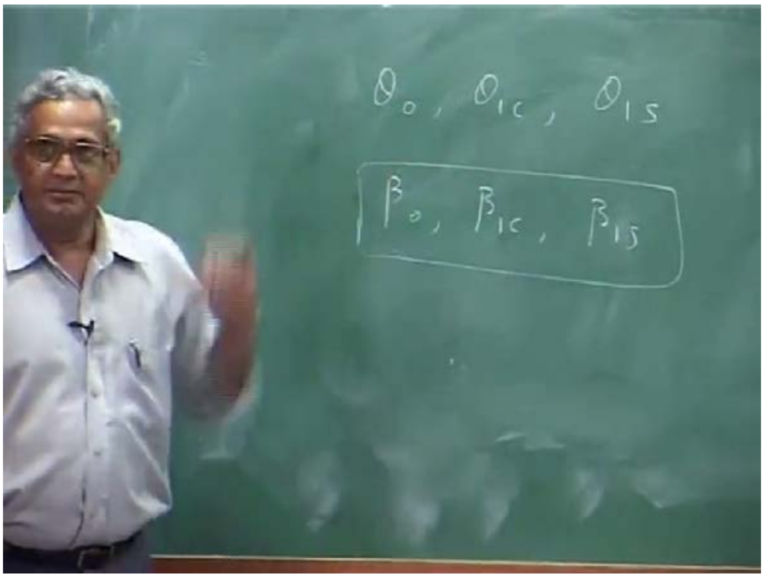

See, in all these loads we had theta 0, theta 1 c, theta 1 s, and then we had beta naught, beta 1 s. Of course, sigma solidity ratio is there lift curve slope is there, everything is there, but these six quantities are given in the loads expression.

Now, you have to know how do I get the flap response for a given flight condition with an input of pitch angle.

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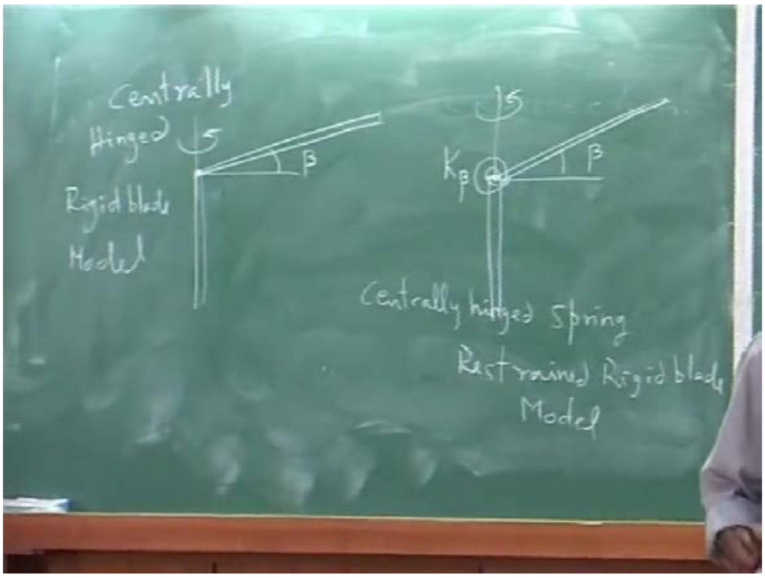

So, for that we need flap equation. So, that is why now I am going to say rotor flapping model or you may call it flap dynamics, only flap motion. That means, we are going to derive the equation of motion for the flapping of the blade.

But before you go and derive, we have to make a idealization, what type of idealization you will have, because you know that the blade is attached to the hub. It can be with the (( )), and it need not be with the hinge, because it can be border there is a hinge less blade.

So, what we will do is we will have a hierarchy of models, the very first one is you put this, this is called centrally hinged rigid blade, and this is rotating about this. Centrally hinged this, it is the hub center again pendulum, because this is pendulum you hold it in some place that is just oscillates.

Same thing the hinge point is here. This is one type of model, this is actually easy first to write the equations subsequently we will, because this is the way you are a code model. Of course, teetering rotors - teetering means where the hinge is right at the center, this model is valid, but only thing is this is attached to the other blade. So, when one blade goes up the other blade go down. So, that is why that you have to derive separately, here we are taking one blade only. Then we slightly more, I would say complicated; that is also centrally hinged, but this is this is also a centrally hinged, spring restrained spring restrained rigid blade model.

And the here, see this is the hinge, but I replace the hinge, it is still there, but I took a spring, it is possible they put some (( )) in certain blades, and what is the effect of that that we will see later, as we go along with this.

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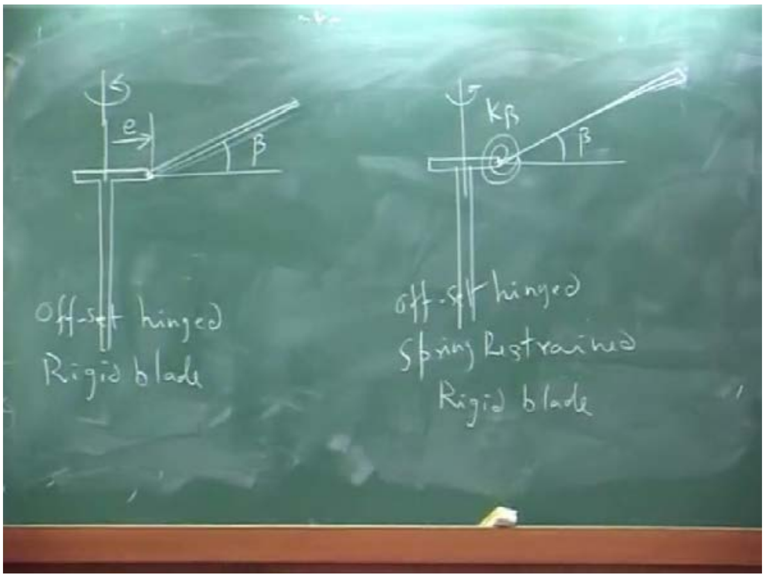

But slightly more advanced model is, you can take.

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This is the this is the articulated blade; that means…

Off-set.

Off-set hinged, off-set rigid blade, and then the final model is this. We put a spring also. This is a off-set hinged, spring restrained, rigid blade model, always I say rigid blade, because the blade is rigid, in real life. The blade is not rigid but it is flexible, but then we rotate it. So, how do we really match this model? What happens this? The flexible blade it should have more number of vibratory modes.

The first mode, because if you have done operation continuous systems, you have to assume that you have done then; a natural frequency of continuous system you have infinite number of natural frequencies, you are working at the fundamental frequency. And then the fundamental law of whether it is a hinged blade, articulated blade, hinged blade, they all different match. So, this model all this represent only first frequency of the blade in flap is the idealization, and this is good enough for most of the preliminary calculation, and more in fact some of the industrial codes use only this, because today of course, people have added more things.

But (( )) earlier days, what was developed in the sixties, they were all using spring restrained off-set hinged rigid blade, because that gives the changes clear; of course, it has its own limitations not in this good as a model. Now, we have to derive the equation of motion for the blade flapping as it goes around the azimuth, so blade dynamic. First we will take this, and then we will add this, later we will come to these two, because I would like to proceed in a systematic manner.

I will first write this then we will have, then we will go further, because each one has that interesting characteristics. So that you will slowly understand this is what you really means, when you do the blade analysis, because please understand this is we are taking only about the flap motion. Similarly, you can have lead lag motion, and then torque motion, and then all of them can be coupled, but we will restrict our self to first this type of model.

Write down the equation of motion. So, how do we write the equation of motion for a this we have the basic dynamics, and of course aerodynamics is that you have to combine both. The external loading is aerodynamic, and because it is moving you have an inertia; and there is no restrained here. So, the moment must be balanced.

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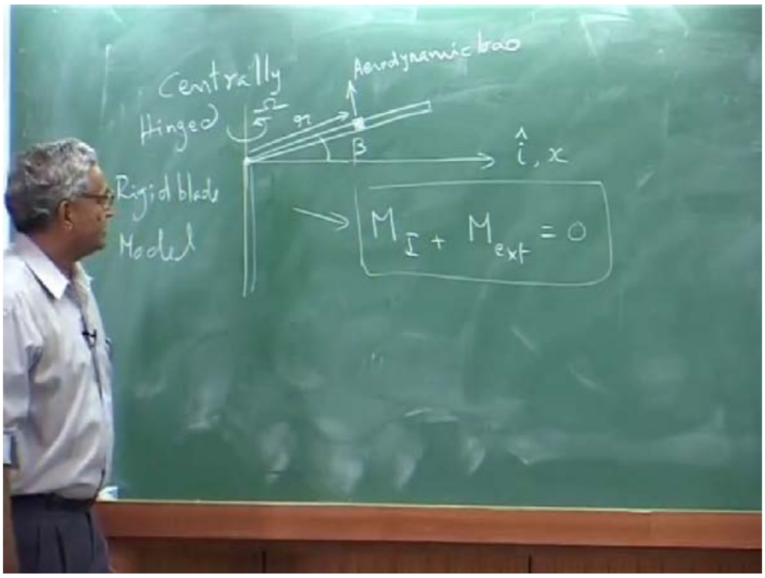

So, the equation, first you write inertia M I plus M external which is actually aerodynamic, this is 0 for this model, please understand. That is for this model, well when you put a spring you will get a spring force also. So, that you have to add it. Here there is no root spring, it is a free like a pendulum, it is free to only thing is you have to model it properly this is rotating you have to take this into to do, load into account and we make when we derive, we make lot of approximations.

That is why what I felt was first I derive for this, make all the approximations then you will know that this is what is being them. And finally, I will just directly use the same equation add the spring, when I add the spring I will get the natural. Of course, these two models are slightly different, because you have an off-set, and off-set will come into the equation that part we will do later.

Now, we take the equation of how you develop the equation, because this is basically dynamic of a rotating beam. So, we will go very systematically. So, this is my omega, and please understand now this is in the rotating frame.

So, this is my i x, and if you take any section, this is like a beam that is all a line, we have ideal it like a line. You will have an external which is the aerodynamic load, this is the distance r.

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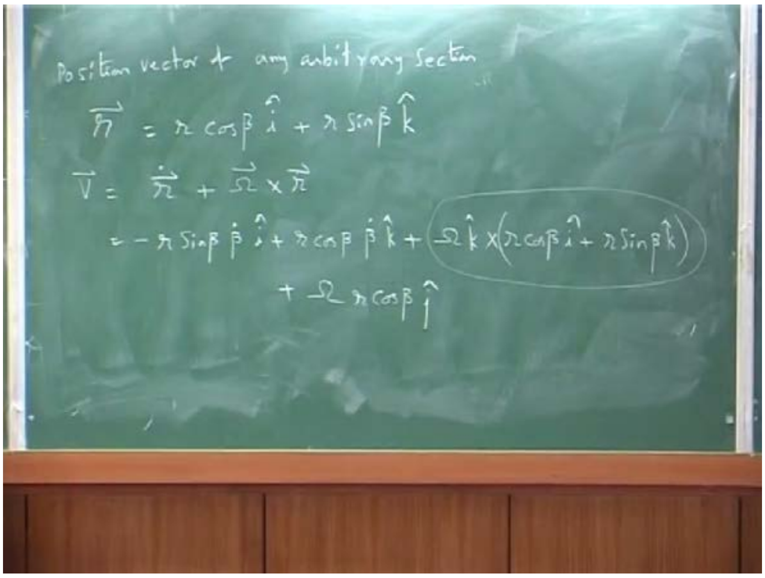

So, the position vector of this r p, position vector of any arbitrary point arbitrary section, actually this is (No audio from 18:08 to 18:23) because this is k, this is k, this is my j axis. The rotation is about the j axis; j, this is k.

(( ))

Flap flap flap flap motion, flapping is alone, that is the rotor r p m. This is capital omega is about k flap motion is about j.

So, now you know the position vector, you can find out the velocity; velocity is r dot plus omega cross r, because this is the rotating coordinate system alright. So, this will become, r dot will be minus r sin beta beta dot i plus r cosine beta beta dot k plus omega is omega k cross your r cosine beta I plus r sin beta k.

So, you now (( )) i is j. So, that other term will drop out. So, you will have this can be replaced as plus omega r cosine beta j.

This is my velocity, then I go and get my acceleration of the same point, acceleration is if you want directly acceleration, because if you know see when I say acceleration, it is the absolute acceleration of the mass point which is attached to the which is in the blade which is rotating. You can directly use that expression, acceleration will have you have the relative acceleration, you have the corialis acceleration, you will have the centripetal acceleration, you will have the angular acceleration.

So, you will have four terms alright, and then if your hub is moving that acceleration also will come alright. So, that is how you have to develop the acceleration of any point. Here I am directly writing every step I am going through, because this particular term comes, because we would we would differentiated this term, r is constant whether question (( )) is this distance is fixed.

Beta is varying, i angle of the changing. So, you will have d i over d t, and k is not changing. So, d i over d j becomes actually omega (( )), this term that is how because in the rotating coordinate system that the factor omega is actually the angular velocity with which the coordinate system is rotating, because this is a basic dynamic, I hope you all of studied some couple of years back. You do not forget it, because dynamics is very very important, here I cannot go and then start teaching that part. You take it and though this term we go back and read brush up your fundamentals in deriving the equation.

Now, you write the acceleration – acceleration will be derivative of each one of them plus again omega cross, because i dot k dot is 0, j dot also will be there.

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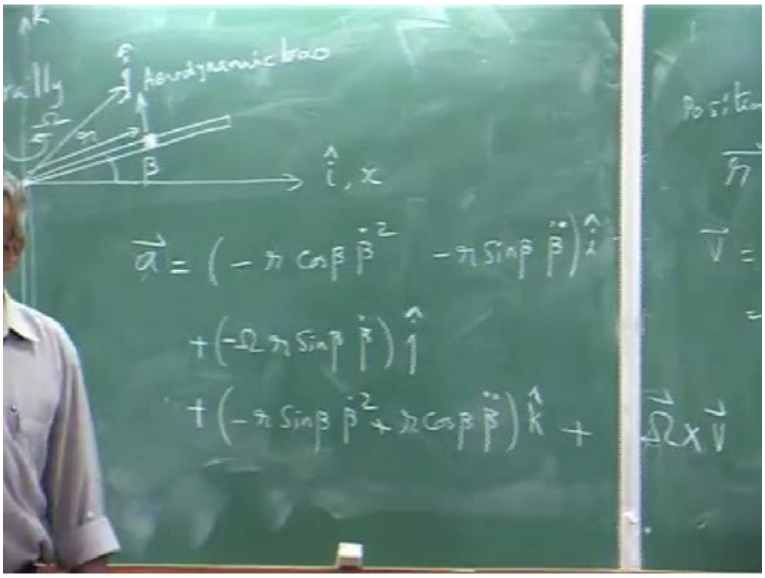

So, omega cross this expression again come. That is why I will write that acceleration is first take this derivative that will be minus r cosine beta beta dot square i, then we will have another term minus r sin beta beta double dot may be I will put the i outside. Then j term, plus omega r; so, I give the minus sign sin beta beta dot j plus minus r sin beta beta dot square plus r cosine beta beta double dot k plus we have be, because of the rotation you will have omega cross omega cross, this expression; it is a v omega cross v.

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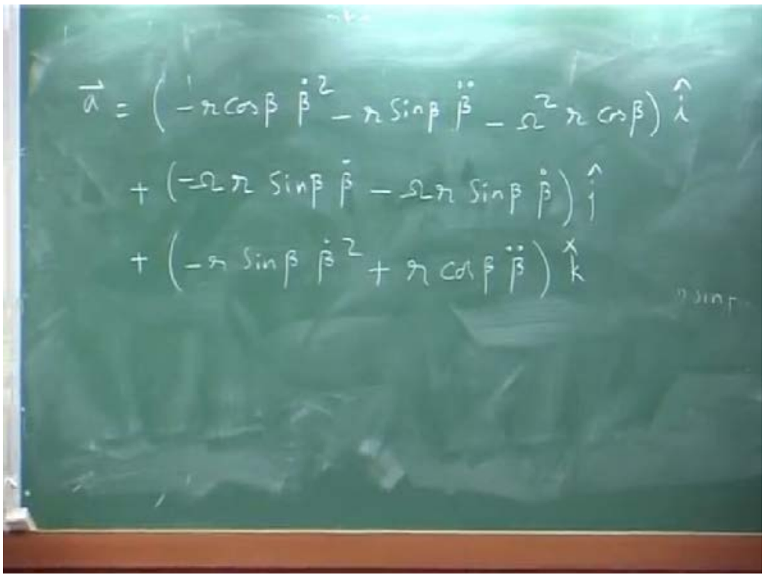

Now, when you or I would say when you add those things I erase this part, your acceleration expression will be minus r cosine beta square r sin beta beta double dot, and then minus we will have omega. What is the term omega square r cosine beta i. Then plus omega r sin sorry beta minus sin beta beta dot omega r sin beta beta dot this is j, then plus minus sin beta beta dot square plus r cosine beta beta double dot double dot .

So, this is my full acceleration expression, you will find that now I erase this part, how do we get that inertia force? Inertia force is basically, take the negative of the mass times the acceleration, that is inertia force.

So, you will get inertia force is minus mass of this element I call it d m. That is the elemental mass, this is my inertia force into the entire expression acceleration. Now, I have to get the moment, because my equation about this point all the motions are 0, because there is a fixed point - external moment, inertia moment everything is 0. If you want to write in terms of Newton second law, you can get external moment is equal to mass times the time load accelerating that part you can write it.

But otherwise what normally we follow is because if you have a very complicated motion, write the complete acceleration expression take the inertia moment, force, and then take the moment, then you will get external moment, and inertia moment. You add (( )). If you do not have a spring it is 0, if you have a spring add the load to the spring also.

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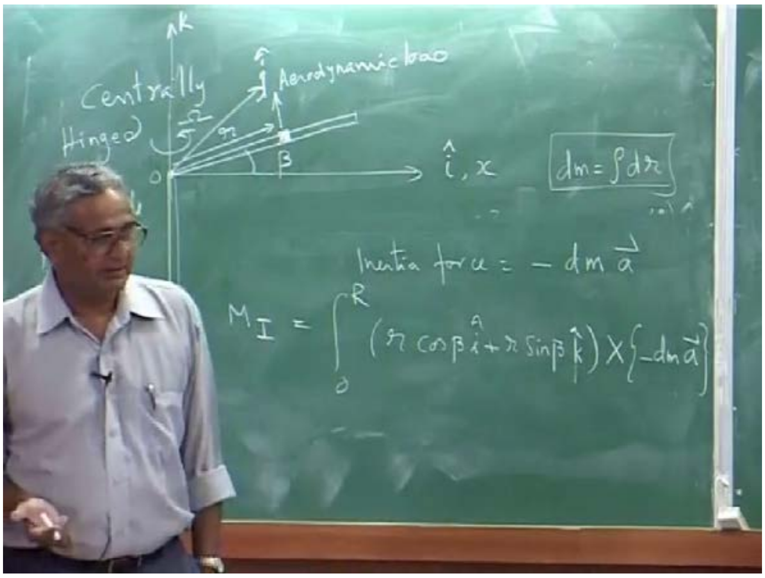

So, here your M I, inertia moment about the roots that is about o this is 0 to r, you have to take… This is the inertia force acting at this point. So, we want to get the moment with r plus f.

So, your r is r cosine beta I plus r sin beta k cross minus d M a, again d M a alright; d M a you can write it as you can write rho d r, you can write it rho is the mass per unit length, d r is the element. That means, what you have taken this cross product. So, you substitute all the things, take the entire process, and that is dot product we will give you this (( )), may be I erase this part.

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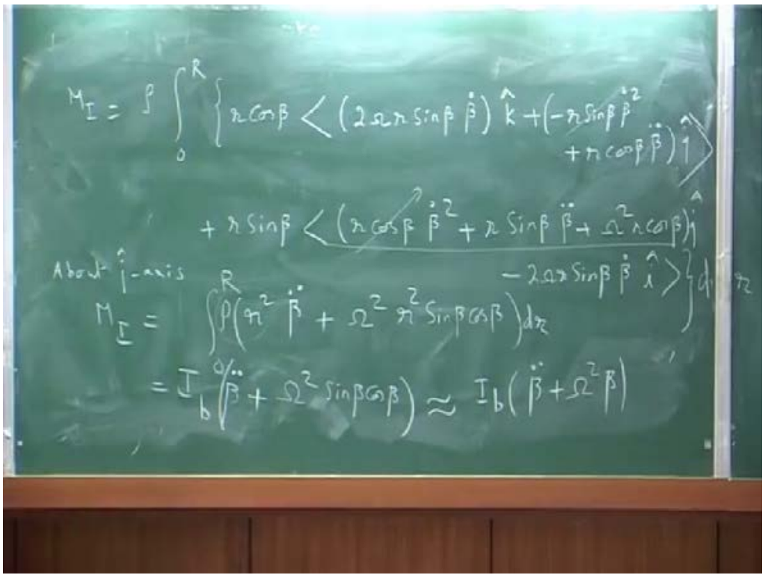

You will have M I is rho 0 to capital R r cosine beta 2 omega r sin beta beta dot k plus minus r sin beta beta dot square plus r cosine beta beta double dot j, because this algebra you please check it out, plus r sum of beta r sin beta beta double dot plus omega square r beta beta dot i d r, this is the full expression. You have all i, j, k all components are there.

Sir, why is it minus d M a?

See, because I am adding see f equals to M a right, I am taking the M a to the left side. So, external force plus inertia force is 0, basically (( )) that is why external moment I am taking it as external moment plus inertia moment is 0.

Now, this is my full expression. You find, I have if you simplify, but I need the equation of motion only about the, because j axis, because even though I may have moment about the k axis alright or I, I do not take that into account, because I am writing the equation only for out of plane motion even though inertia will have other moment coming in please understand, because you will have other terms come in the picture.

But we basically say if you are writing the full equation flap lack torsion, then you take every term. Right now, in our formulation we are consulting only the motion about j axis, but the j axis, please note it is also changing. It is not fixed j axis is changing, and calculating what is the moment about the j axis, because that is the one which goes to flapping.

So, moment about j axis, if I look at it the component of j is one term is here, and here all the term. If we add both that is about j axis M I, because M I is a vector, now I am taking only about the j we will have 0 to r r cos beta minus r sin beta beta dot square; here you have r sin beta r cos beta dot. So, they will cancel out. This term will go out, then here you have r cosine square beta double dot. Here you have r sin square beta double dot, sorry r square sin square. So, you will have sin square cos square one, you will have r square beta double dot.

And then, you will have this term plus omega square r sin beta cosine beta d r which you can now write it as density I have taken it out whether it can be inside or outside it does not matter, we can put it inside also, because if the mass is varying, along the blade rho d r is the mass per unit length into r square; that is basically mass moment of inertia about the hinge point.

I will call it I b beta double dot, no here also one r square is there, plus omega square sin beta cosine beta. Here the double dot there are derivative with respect to time. This I am approximating now, when my angle is small cosine beta is one, sin beta is beta. So, I will write this as I b beta double dot plus omega square beta, that is all. So, my inertia moment in the clock direction about the j axis is this. Now, you see there is an acceleration term, and this term omega square is due to centrifugal force. That is the centrifugal stiffness, because you know when you are looking at an equation whether you later.

Now, it is only an expression when we get the equation, if we all know M x double dot plus k x equals some external force. This will be in the similar form, and you find this particular term is due to rotor load r p m, that is the you call it centrifugal stiffening. Because if if if omega is 0 then that term is not there, then the blade will simply come down like this.

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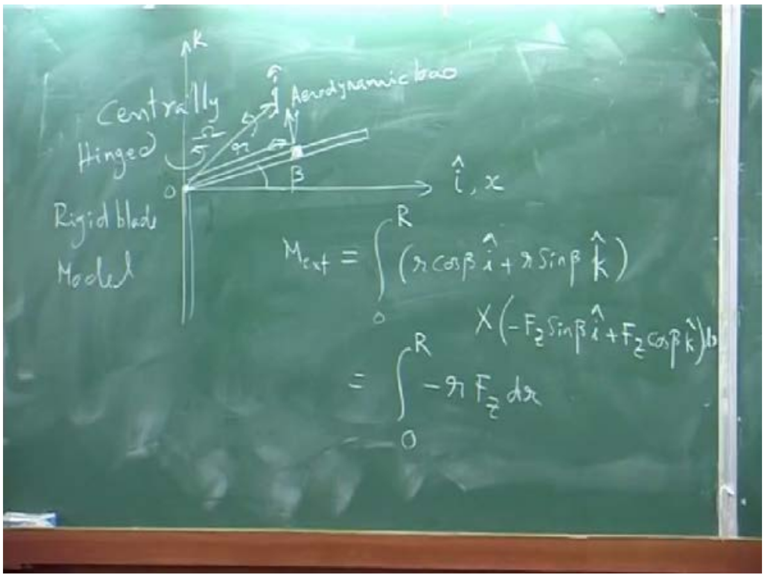

Now, you look at the external force - external load is only the aerodynamic load, we will write that that is the badly simple expression, because your aerodynamic load is again you can take 0, capital R.

This will be because this is the f z we are taking lift force, because the drag force is along this direction, and the drag is not going to give any contribution to…

Flap.

Flap moment. So, we neglect that we take only the lift force, when you take lift is f z, f z into r. So, you will have r cross. So, I will write r cosine beta I plus r sin beta k cross your force, force is you will have minus f z sin beta I hat plus f z cosine beta k b r, because this force is along. So, I take a component like this, and like this. Now, you can take I, because I is 0; I cross k is j. And similarly, I cross that is minus j this k cross I is j. So, this moment will be only along the j axis with minus, because this is cosine square, this is sin square.

You will have one, we get minus r f z d r; that is all. Now your flap equation is here as you write the flap equation.

(( ))

Now, this is not j only j, j is an only moment - only one component along the j. So, I am just taking moment.

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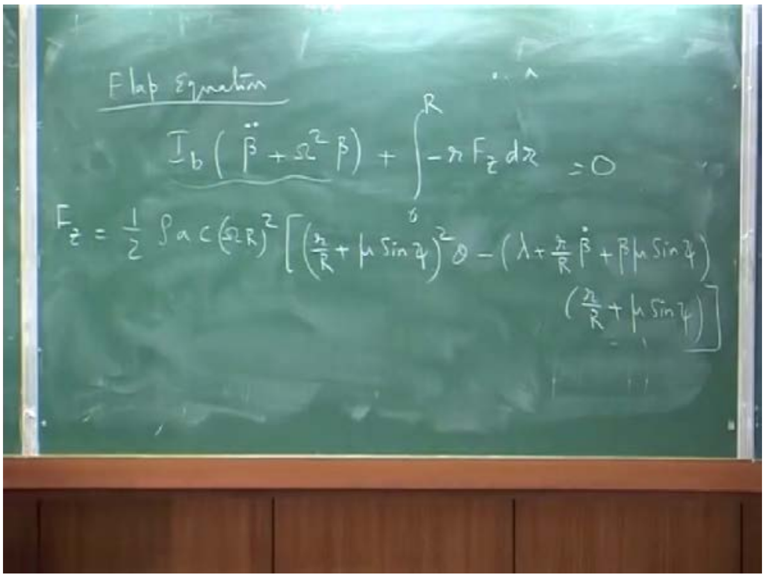

Now, I add both inertia, and aerodynamics and write my flap equation which is flap equation. This is I b beta double dot omega square beta plus f z d r 0 to R. This is the equation. Now, we have to go back and substitute what is f z, because f z is nothing but our lift, and we wrote I will write f z expression just for you look at your notes half rho a c omega; this is an mu sin chi whole square theta minus lambda plus r over r beta dot plus beta mu sin chi into r over r plus mu sin chi.

Now, what I am going to do is, here these dots are derivative with respect to time; here I have written this dot also this is derivative with respect to time please understand. This is non-dimensional whereas, this is dimensional.

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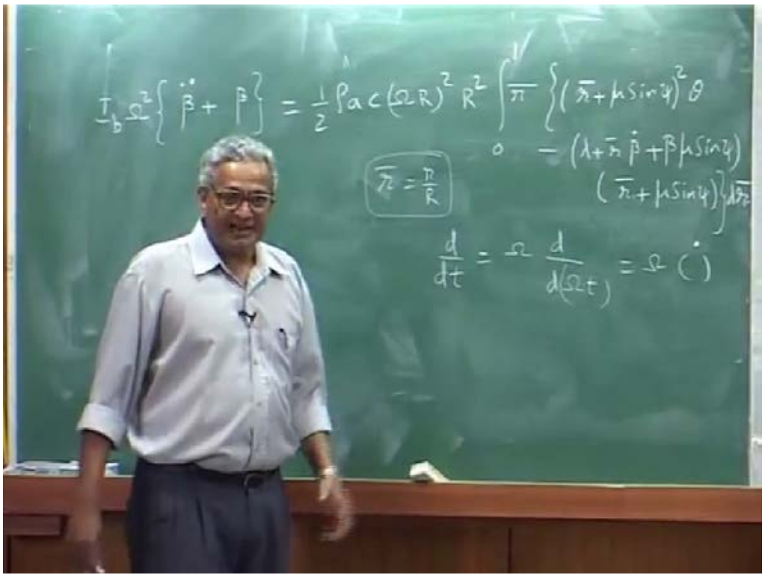

So, we will replace this also we put it d over d t, we can put it as omega d over d omega t. This we call it as omega dot. So, that means this term if I non-dimensionalize, there will be an omega square coming out. So, I can write that part, and then put all the equations in non-dimensional expression, because you know that this is already r over r everything.

So, when I put this d r over r, I will put a capital r make it 0 to 1. And here also I put a r over R; that means, I will have a r square getting multiplied; this just a non-dimensionalize in the above equation. I erase this one, I will write the non-dimensional equation then we start discussing the whole thing.

You will have I b omega square beta double dot, please understand here double dot is non-dimensional time derivative, that is d by d omega t this term, that is why I taken a here we have beta equals, I will take half rho a c omega r whole square r square 0 to 1 r bar, where r bar is r over capital R; r bar into r bar plus mu sin chi square theta plus lambda sorry not plus minus lambda plus r bar beta dot, this dot is as it is, because this is the non-dimensional time that dot plus beta mu sin chi multiplied by r bar plus mu sin chi closed d r. This is my full flap equation.

Now, you find within this, this is an integral over d r bar sorry d r bar, now I can actually multiply everything, because mu sin beta beta dot lambda assuming lambda is constant over the span. Then I can integrate the entire thing bar r bar separately, like that moment equation. In the relatively simpler form, and even theta - theta is not this is the theta what pilot gives, that is theta psi.

You can have a, if you have a twist in built in that, then twist will come here; that will have a theta twist r bar that term also will come here.

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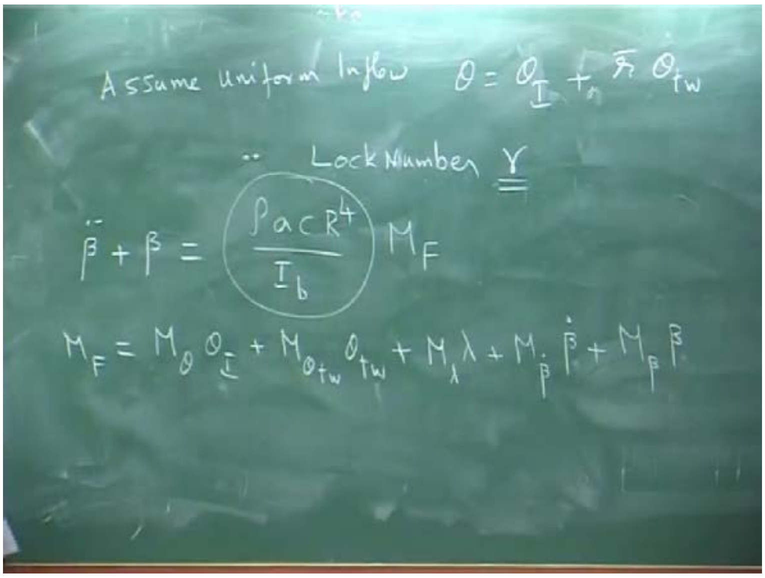

Now, if you write assume uniform inflow inflow, then theta equals theta I plus r bar theta twist. We assume that linear twist, put it back here integrate.

You will get your equation like this, (( )) finally I am going to write it in this form which is relatively simpler. This is beta, and double dot plus beta equals. Now, you see I take this I b omega square divide here, when I do that rho a c, one of the omega square will go off. And then r power 4 over I b. So, you have rho a c R 4 I b, and I am putting some M flap; just I will write this expression what is M F? M F is - it consists of several terms, you will have M theta theta I M theta twist theta twist plus M lambda lambda M beta dot beta dot plus M beta beta. Please understand M F as theta I theta twist lambda I am taking to the constant beta beta.

Now, each one of the term I am just writing it, and then complete my flap equation. And I am going to count this number, this is a non-dimensional number. This is lock number, and the symbol is gamma. That is this is the aerodynamics, because rho a is the lift of slope, the density of air. So, this is aerodynamics, this is inertia. So, this term lock number represents the ratio of aerodynamics force to inertia of the blade.

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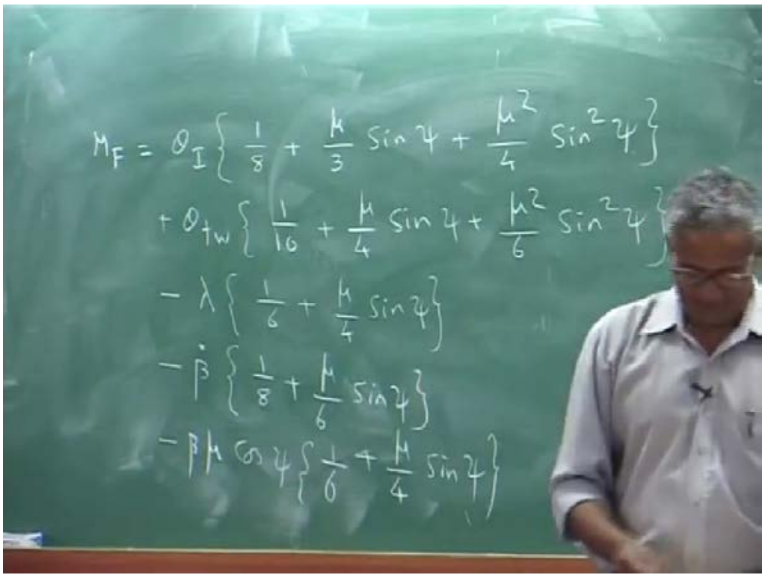

Now, let me write M theta you can actually write M F itself. You can write theta I 1 over 8 plus I will show later, (( )) not that, we now plus theta twist one over 10 plus mu over 4 r mu square over 6 sin square chi minus lambda 1 over 6 plus mu over 4 sin chi minus beta dot 1 over 8 plus mu over 6 sin chi minus beta mu cosine chi bracket 1 over 6 plus mu over 4 sin chi. This is my full equation.

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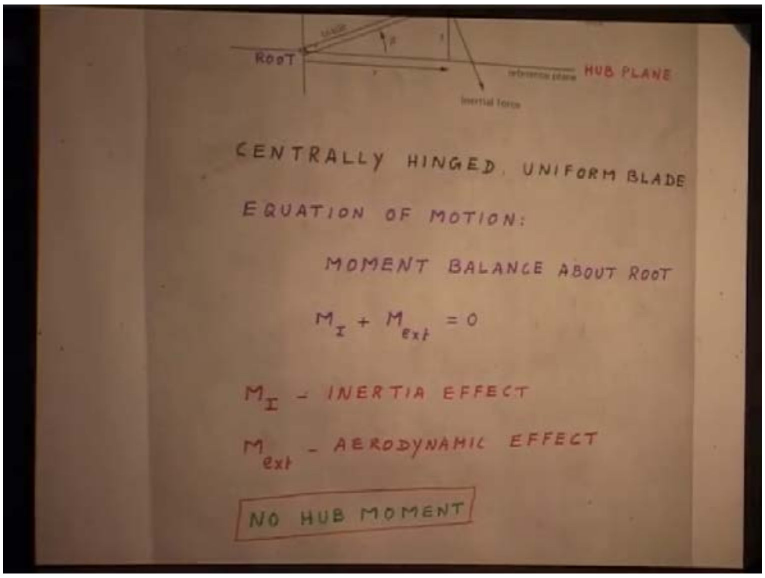

Now, I will go and then show, you do not have to worry I will give you that equation. This is the model whatever I derived the just showing it, because I will get back to the final equation then we will start discussing the equation. We have the, because uniform blade, because I integrate equation of motion moment balance about the roots, and I get inertia effect to external aerodynamics, no hub moment, I am not putting another spring here.

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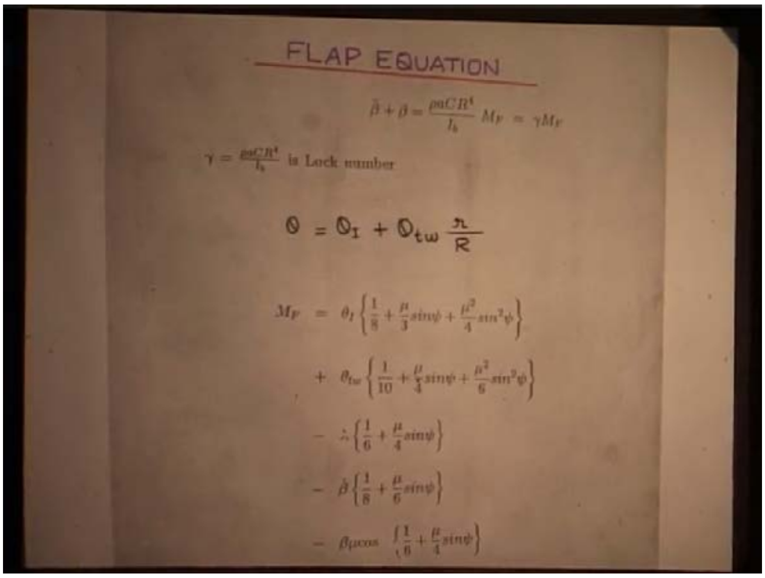

So, put a spring I will add that term here, that part will come later. Now, when I get the full equation - the equation is, this is what I have written there; beta double dot plus beta is gamma M F gamma is the lock number, and M F is given here.

Please note M F is this. Now, if you look at this equation, we will theta I what I use theta which is actually theta I. Theta I is given by this is what the pilot gives theta naught 1, 1 s sin chi cos psi; that is the input pilot is given.

So, you have to multiply that term here, theta naught theta 1 c cos chi theta 1 s chi everything will come in this place input. These two terms there is a dot beta dot, which is actually like a cos derivative clock motion like a damping term. And there is another term which is beta cosine psi; that means, this is a spring term.

But the spring term associated with the forward speed mu. Now, if you take these two terms to the left hand side, you will have a inertia, you will have a dot angular theta, and you will have the spring term. And then rest of the quantities are directly input and twist play twist if it is there it is there otherwise it is 0, 0, and your inflow whatever constant if you have, that is the constant (( )).

Now, you are having the flap equation with coefficients which are of sin (( )) let me, it is alright with the sin chi cosine chi within the coefficients of beta dot and beta; you follow, because beta dot hat the sin chi here beta mu cos chi 1 over 6, and then another mu over sin chi. so this will be going mu square over 4 etcetera. That means, your spring and damper, whatever damper means there is not damper, there is actually is derivative term - damping term. They are functions of the sin, and chi is your…

Independent variable.

Derivative is respect to chi. So, your coefficients, please understand left side is purely inertia, and getting the damping term coming from aerodynamics, please understand; in the flap motion the damping comes from aerodynamics. So, I have a spring term which is looking (( )) like an aerodynamics term, when you put everything on one side your equation will be beta double dot may be I will write, then you will be able to see here how it looks, because you can take this.

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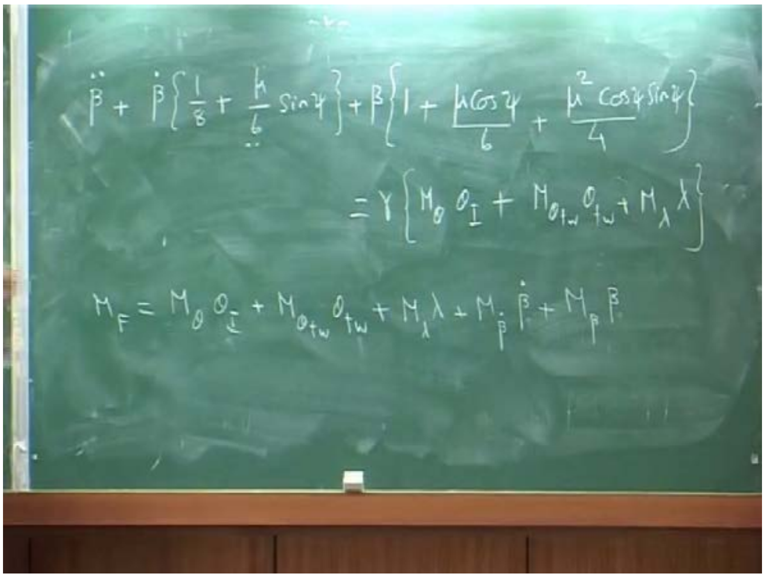

Beta double dot plus beta dot term is here, beta dot 1 over 8 plus mu over 6 sin chi plus beta 1. 1 is already there plus you will have mu cosine chi and over 6 plus…

Mu.

Mu square cosine chi sin chi over 4, equals rest of the terms; that is gamma times M theta theta I plus M theta twist theta twist and plus M lambda, this is what your equation. Now, you are having a time varying equation. It is the parameter excitation type of problem, yeah you have gamma, then gamma here, and gamma there. Now, what is that we are looking for is solution, because solution is we said lambda is a constant, we have to substitute for theta I.

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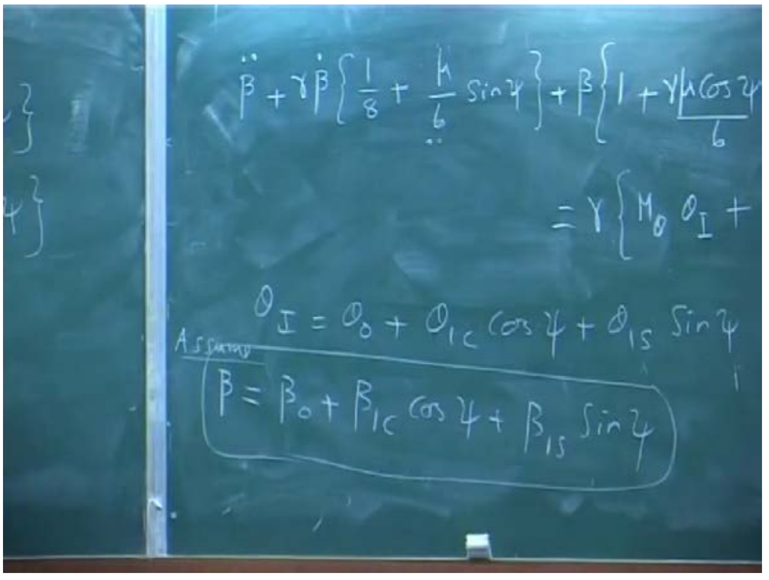

So, I erase this part, I have to substitute, theta I is theta naught 1 c cos chi plus theta 1 s sin chi. and you substitute this. Then essentially I need to know what is the solution? Please understand lambda is assuming lambda is I am not to get this what it is. So, I am assuming only first half (( )), I am not considering other harmonics. That means, assume this is my solution assume; that means, what you do you do? You have this equation, substitute this term on the left hand side, substitute this on the right hand side, and collect all the constant terms sin chi terms, cosine chi terms, and then you will have three equations; and those three equations you will get, there will be algebraic equation.

Because all the derivatives are known algebraic equation from there you can get a relation between this, this, this; so this, this, this to lambda, and theta twist. And of course, mu is always there, because every where mu is sitting there.

Now, this is called the harmonic balances, because you are looking for steady state responds. Steady state means these are 6 tangents, there is no change. How as the blade goes round and round, what will be its harmonics of flap motion. It is like a steady state solution, there is no is only the particular, there is no transient part - transient part is 0, but if you take this, I am not going to write the procedure. I am only saying you take this substitute here. There are two ways of doing this problem.

One is called to substitute collect all the terms put down together this is an algebra, another one is an operator form - operator form is you integrate that equation first, and then again multiplied by sin chi. and then again multiplied by cosine chi.Like that, that is like a Fourier coefficient how do you get?

Same.

Same thing, that is an operation form. Another form is put this harmonic balance, this is another thing. Now, this is the algebra which I will not be going it, and only directly write the final answer.

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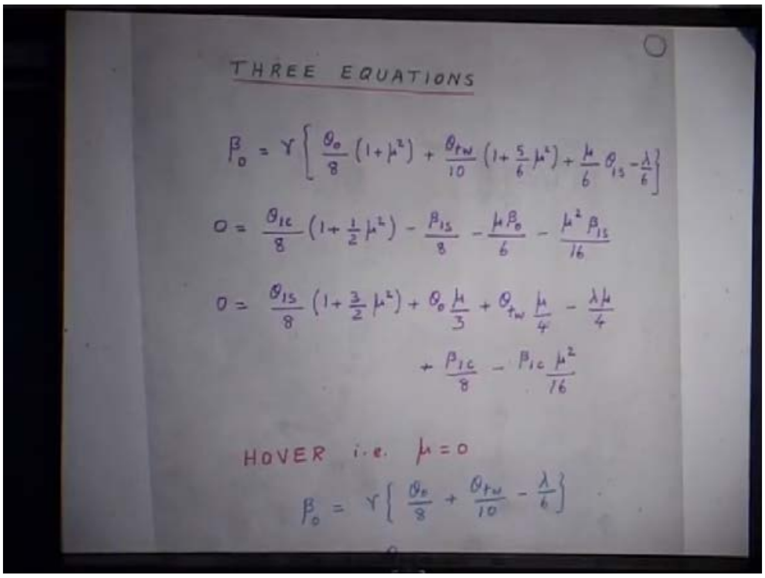

The final answer is this, which I will give you. The final answer is you will get three equations, other we will just only discuss.

Now, because there is no need to take, because this if not keep on writing into thing, now onwards you find that it becomes messier, messiser, messier with too many things.

We have three equations, this is the constant term part, and actually this is the sin cosine - sin cosine (( )) there. You have to solve beta naught beta 1 c and beta 1 s, from these three equations, but beta not is given here directly, and that is the function of beta 1 s, as well as lambda theta 1 s mu theta naught. If you know this values, it is beta naught. From here, you have to relate, because this is beta 1 s beta 1 s. So, you can get in terms of theta 1 c, and beta naught; that means, you have to get this value put it back here, then you can get beta 1 s.

Similarly, when you go here, you can get beta 1 c, you take all the terms on that side you get beta 1 c. You have you have to solve this equation. This is for centrally hinge, please understand. Now, let us defect two parts - one part is just this equation alone only that equation, assume that right hand side it is 0.

Assume the stability of that equation, stability in the sense for a given initial condition. You give the small flap motion, this is just a purely academic interest, please understand. It is not for an actual blade, it does not happen; that academic interest. You have a differential equation. Now, treat it no doubt bother about it is a flap equation or anything, it is a differential equation which is given like this. You know the value of mu which you can vary, gamma is fixed; lock number of a rotor blade which is a aerodynamic by inertia, usually that is in the basis 5, 2, 8 is a lock number- 5, 2, 8, 5 is derivative no.

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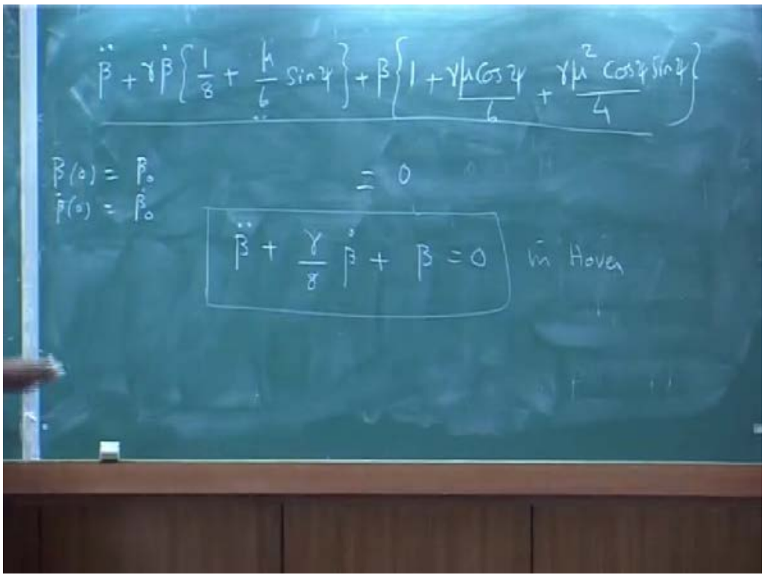

You get these values, and then you give an initially input; that means, initial condition is you can have beta at 0 some beta 0, and beta dot at 0 (( )). Please understand this is none, no longer the helicopter problem, I am not solving a helicopter problem, I am only looking at a differential equation of this form. So, this is going to be I will send you a home work, which you have to solve. This also part of your term, same day I will give. This is the equation depending on the value of mu, what will happen is give it, suppose mu is 0, assuming mu is 0; that means, it is in hover. What is the condition? Your equation is beta double dot plus gamma over 8 plus right. This is my flap equation in hover; there is no (( )). Of course, gamma over 8 is the damping term.

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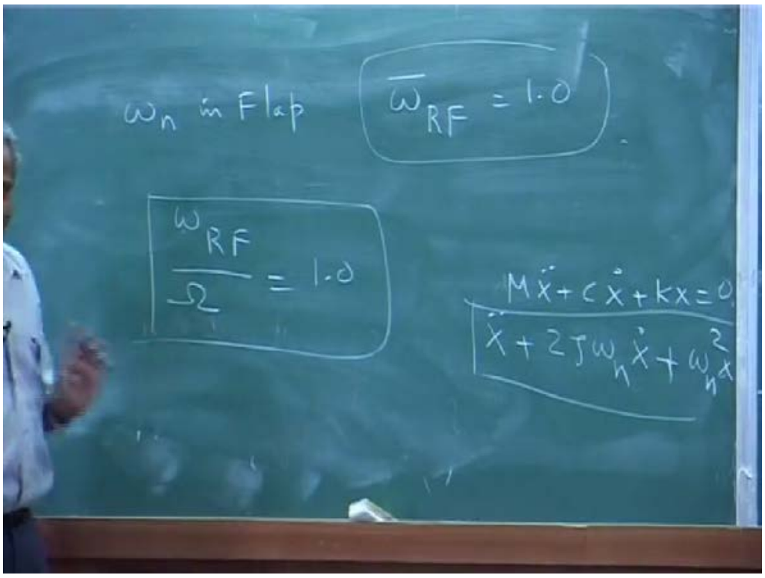

This is very similar to, you have your M x double dot c x dot plus k x equal f of t, I am taking the 0. You divide by n, we will have x double dot plus c over M you will write it as 2 zeta omega n x dot right plus k over M is omega n square x is 0.

Omega n is the natural frequency of the…

Spring.

Spring. Here the natural frequency is one; one means here it is a non-dimensional. So, when the blade if it is rotating into tapping it, it will up lift the time for way to go up and down, one oscillation is exactly equal to the revolution time.

So, the rotor r p m is actually flap frequency, only in this case please understand. That is why we call it 1 per (( )) 1 per (( )), 1 oscillation per revolution; that means, if you tap it up; it will go down, and then up down then it when it comes to one oscillation blade would have gone through one revolution.

Now, you look at the napping part. So, your omega n in flap which I am going to call it rather than omega n, the notation I am going to use is omega bar, bar is because it is non-dimensionalize with respect to omega. So, you may call it omega R F over capital omega (( )). This is the natural frequency of the plane in flap, non-dimensionalize with respect to, and R F to denote that rotating flap.

Please understand R F I am using that substitute only to denote rotating flap, if it is not rotating there is nothing then it will just come down that is all. So, the centrifugal force, actually use the Stephanie, and the natural frequency non-dimensional is basically one, then what about the damping? Gamma over 8 it is exactly 2 zeta of omega n.

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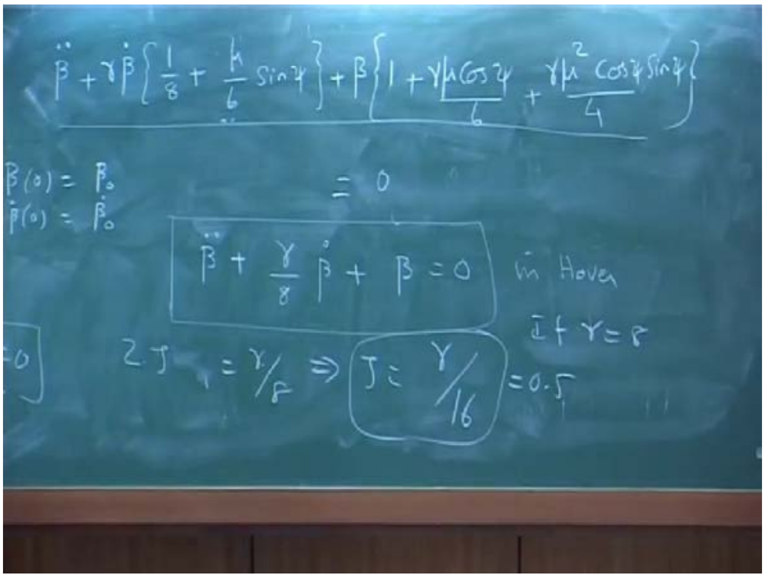

So, you will have 2 zeta omega n is and omega n is 1. So, you will have 2 zeta is gamma over 8 or in other words this will give you zeta as gamma by 16. That is that, what is zeta (( )), that is damping ratio. If you have gamma of the order of 0.8 sorry not 0.8, now as 8. Now is the order of 8 not 0.8 sorry gamma is the order of 8 zeta is 0.5; if gamma equals to 8 zeta is 0.5; 0.5 percent this is the damping ratio is 0.5. It is high damping, very high damping.

So, basically the flap motion is heavily damped, and it takes very short time to reach its steady state. If it is damp it will take later we will calculate how much time it will take if (( )) how much time it takes to reach its steady state value, that what we will from where we will analyze it clearly. So, so 0.5 flap motion is heavily damped, and the damping comes from gamma, gamma is aerodynamics suppose you rotating vacuum there is nothing, because there it is the rho that is k of a lift curve slope, they contribute, density (( )).

Now, this is a heavily damped system in hover; in the sense when mu is 0, but when you mu is present. That means, you have these terms forward flight. I cannot write like this, because I do not have an equation where I take this term. So, that my damping is a time varying term, stiffness is also time varying. You will find just for theoretical academic interest for some values of mu, even if you give a distance. In this you know that if you give a initial condition, it will damp out, because you have a damping between d k.

Whereas for the value of mu beyond 1.3 or something, you will find that it will never damp up. You give a disturbance it will keep one oscillating, but you do not go to that value of mu in helicopter blades, because helicopter blades operate below 0.4 maximum, whereas that kind of that tendency comes with the high value of mu.

So, as I will give one problem which you are going to do numerically, this is a simple this is an ordinary differential equation only with time varying coefficients, how the solution looks for a different values of mu for given initial I will send you with assignment today, you analyzing you plot it, you will find. It is a very interesting things, that is why a helicopter blade, if you want to analyze the stability.

Just for how the system will be stable or not that part will cover later. Right now, as a numerical problem you cannot write the solution like this. Draw numerically, get the solution, and then just plot it look at for a different values of mu. How the solution really looks, for small values of mu solution will directly converse immediately, whereas when you give high values of mu, it will not you could start them oscillating then it may start diverging also.

Sir, what if we take sin beta into instead of initial equations sin x starts.

No, (( )) the see, then you have to take the full large amplitude equation, but we use small values of beta only, even then it will (( )); see that is the non-linear equation you were going. This is still the linear equation only, linear equation with time varying coefficients, but why we do not take sin beta as it is, because there in most of the rotor blade analysis, beta is order a couple of degrees 2, 3, 4 degrees.

Then we at most 5 degrees; 5 degrees is what? 0.1 radian sin beta is almost beta. So, that is why that approximation is still (( )) that is the reason, but this is as an academic exercise, because you are getting exposed to a different kinds of equation which is normally not in regular math, because this is a time varying coefficient, but this will create problem later, not in flap, but if we have flap like passion everything coupled. How will you analyze the stability of the system; that means, you have to learn a new technique. So, that is a (( )) theorem, then we have to do that; there is a transition method that part I will cover which is a fully math towards the end of the course, because that is just for information.

So, right now you take graph equation with this just for exercise. Otherwise with all the right side, you put the whole thing you will get this, we will discuss this part next class.