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Module No. # 01

Lecture No. # 12
Introduction to Helicopter Aerodynamics and Dynamics

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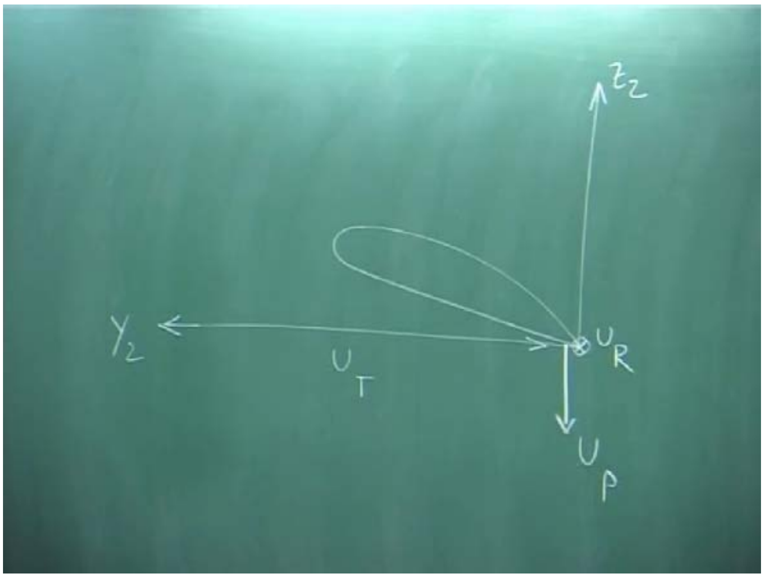

All right. Now, we were discussing about the velocities in a given cross section in the deformed state. So, I will draw the section of the aerofoil. I will write first. Please note this is your Y 2 axis, this is your Z 2 axis, X 2 is going in. And we had the velocity components, U T later I will write what it is, that is the velocity along Y 2 direction. But since it is coming towards it, minus sign is there that is why I put the U T this way. And then there is an U P, which is the perpendicular that is along the Z 2 direction which is again a negative quantity minus sign, that minus sign I have taken here. Then there is a U R, there is a radial which you may I will put a circle here, it is U R. Velocity going into the port, all are air velocity.

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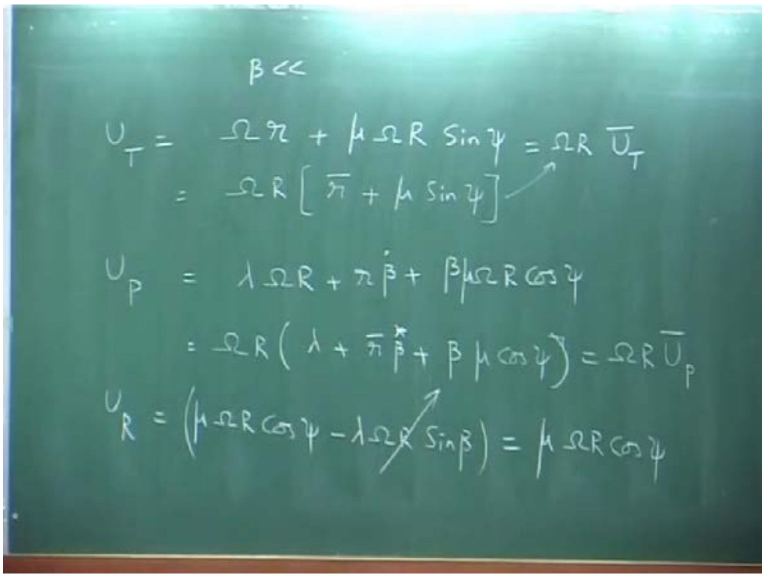

Now, your U T we wrote last time, we make an assumption omega R. Please note beta is small. So, cosine beta is 1, sin beta is beta and lambda is also small. Based on that, we will write the assumption. This is omega r plus mu omega capital R. We will write it as… So, that I take out this omega R outside and write it as R bar plus mu. So, this is basically this symbolically because non-dimensional U T, because later I will use U bar T everywhere. Similarly, you go to U P. U P is actually minus, that is why I have taken that. U P becomes lambda omega R plus… and which you write it as again omega R lambda plus… There is a mu, beta which you call it as omega R U P bar. So, that it becomes easy for later representation, U P bar U T bar. Hut he saying is that this derivative, since you brought it that is good. It is a time derivative we take, but here I non-dimensionalize this. What you said is correct.

So, there is a omega divided. And if I write it in this fashion what will happen is, even though this is that, if you want I can write this as a star just to denote this is a derivative with respect to omega t, then what will I do? So, you can write it like this, d over d t is omega.

Now, this becomes non-dimensional time which is our symbol. Is it clear? Therefore, in all our later things the derivatives are with respect to non-dimensional time. But I do not put star, I use the same symbol as dot. So, take it that the time derivative is with respect to this omega t because I want to take out omega outside. Because in our operation omega is a constant, all these derivations. Is it clear? And then you will have the U R term. Basically we neglect one particular term, which is we know that it is mu omega R cosine psi. But I neglect the beta small, so this becomes lambda beta. So, product of lambda and beta, I am neglecting the term please understand. So, this essentially becomes mu and your omega R is there. Now, you got all the expressions for your velocities. You have to go back and write your lift and drag and if you have aerodynamic movement you have to use the moment. But for our calculation we assume that the aerodynamic moment is 0. So, we neglect the term.

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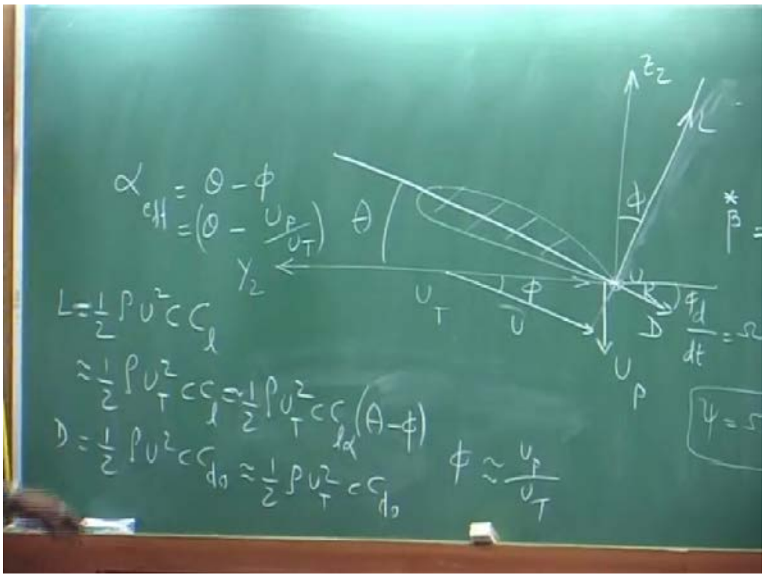

Now, this is our and this is the theta, which is the pitch input given by the pilot. Now, you have U T, U P. So, there is an induced because this is your, resultant velocity resultant velocity. Resultant velocity is square root of… And this angle you call it phi, which is tan phi is…

If you want to compute numerically because please understand these are the expressions. Every section this is changing. And all these quantities are, we assume lambda is the constant that is the most important thing. If you want to put the lambda also varies along the radial, along the azimuth, then you should have a model for that. That is your (( )) model. But right now we assume lambda is constant over the entire disk. You need to calculate this that means, with psi, every time instant things are changing. So, what we do I,s we further make assumptions here. The further assumption is, I say my U P which is this term. I assign some order, lambda is small and the flap is also small and this is a flap angle is small. Therefore, you see all these three terms they are of the same order, because mu is forward speed that is v cosine alpha by omega R. And the that is a little higher term and that is sitting here.

So, I always say my U tangent is much larger than U P, much larger I am just make this statement. It is true as you go R far away, but when you come near the root that assumption may not be valid, but still I make that assumption. That means, I am going to write, that is all. And this is phi is… Because U P is small, therefore U P over U T is small and tan phi I call it as phi. Now, my angle of attack, alpha effective, that becomes theta minus phi, which is theta minus U P over U T, very simple. Now, I have this alpha, now I can get the lift expression, drag expression. Lift is perpendicular to the resultant flow, there I will take this. So, my lift is, this is my lift, and the drag is, this is my drag.

Actually perpendicular means, well it looks like that, but this is perpendicular to not to the chord, but the resultant velocity. So, this angle is phi, this angle is phi. Now, it should not look like. So, this is lift and drag.

So, your lift expression is half, I am writing approximately. Because I already make this assumption u square, I may take this, but I am going to put it half rho. Even though exactly it is U square, sorry U square chord C l, it is lift per unit square. But I am writing this as… which I can simplify as. Let me erase this part. Half rho U T square C C l alpha, that is lift curve slope which we call it a, into theta minus phi. This is my lift expression now. And drag is half rho u square C C d 0, drag coefficient. But hear again I make that assumption that U is U T. So, this also becomes half rho U T square C C d 0. Now, I have lift expression, drag expression. I need to get the forces now. Forces I get it along f z 2. Now, I erase this part. So, I will use only U P U T expression fully.

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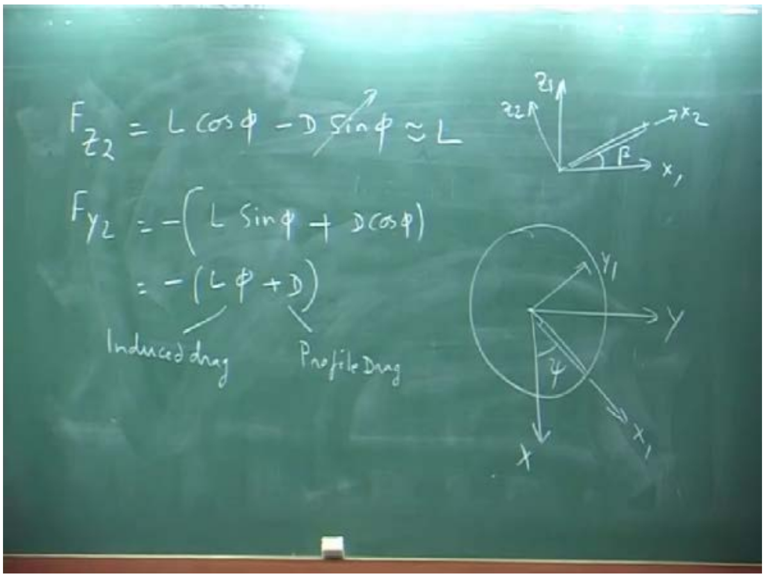

So, my F z 0 is l cosine phi d sin phi and F y 2 is minus of, because Y 2 is this way, so that will be minus l sin phi plus… These are forces in Z 2 and Y 2. I have to convert them into X 1 Y 1 Z 1. Finally, it transfer them back to X Y Z, which are basically the hub coordinates. Because if you remember last class, we said this is hub and this is the projection, this we said this is Y 1 and this is my… And then of course, these two, x 2, y 2. So, we had x 1m this is z 1 and we put this is our blade really, this is x 2 and this is z 2, this is beta.

You have this transformation. So, what we have to do is, you get the force in x 2 y 2 z 2 coordinate, transfer the forces back to x 1 y 1, finally back to x and y. That means, you are doing blade sectional load, this is sectional load per unit length. If you want to get that total load acting at the root of the blade, you will integrate this along the span of the blade, then you will say this is my root load on the blade. You can give it in x 1 y 2 direction, that is root load in the undeformed rotating coordinate system.

But if you want hub load because x 1 y 1 you keeps on rotating. So, you have to get the hub load means hub load along fixed direction x and y and z. But if you want to all our expressions. So, in the design when you go, you need to know what is the blade load, then you also need to know what is the hub load. Because blade load you require further the design of the blade, but when you translate into hub, this is not the only blade you will have four or five blades, every blade you have to add. So, when you go to the hub load, you have to put a summation of all the blades. So, each blade contribution you will take, transform all of them along x and y, then you will say this is my hub load. Otherwise, each blade will have its own load. Now, we make certain assumptions, we are going to do one by one. And then finally, we will write the hub load expression.

Now, let us take the, we make now approximations. This is what we are doing now, lot of approximations which are done here so that you can get something like a closed form solution. Otherwise, if you do not make any assumption, that is all it will be left like this. Then I will say you will take it from some aerodynamic section where it starts, from some root of z to tip of the blade and you get the loads, transform it, put it. Otherwise, you cant get a nice little expression. This is the reality. If you want to really do practical blades they stop right here and you do not even have to make. If you are doing numerically, you do not make even this.

But please remember I am making here. c l is c l alpha this fine. But how c l alpha varies ?c l alpha can be a function of mark number. So, as you go along the span, mark number is changing. That means, your c l alpha will change and c d that will also change. So, you need to take actual aerofoil characteristic. How it varies with mark number? How it varies with angle of attack for the given aerofoil? This is what is done because this they call it polar, that is all, that is the aerodynamic polar. But it is a static data that means, you put an aerofoil, whatever aerofoil which you have chosen for your rotor blade cross section, put it in a wind tunnel, get that data. And in the earlier days they were using not 0 0 1 2 etcetera, symmetric aerofoil. But nowadays nobody uses that. Each company has its own aerofoil. They have some name they give and the rotor blade aerofoil they design and they use that and they will know the aerodynamic data of that aerofoil, please understand, it is a static data only.

Now, if the angle of attack exceeds the stall, so blade will stall. So, you have to take the stalled value. That is why I am saying actual blade calculation to highly idealized situation, what we are doing? We throw away c l alpha is constant, c d naught is going to be constant, I make U P as small comparison to U T, then I do all these assumptions.

Now, I get this. Here also I make further assumptions. The further is, since phi is small, I will write this as l. I am making and throwing this out because d phi, d is anyway small, phi is also small, so I neglecting. When I go here this will be minus l phi plus d. Now, you see l phi. This is d is drag which is due to the profile drag, c d naught this term comes from lift, that is why you call this as the induced drag. So, you have in the (( )), you have induced drag and you have profile drag, two expressions. This is per section, so keep doing and integrate the whole thing. But integrate, before integration you have to transfer this to f y 1 and z 1 and now that also made lot of assumptions again. Now, we will come to one by one. What is f? Can I erase this part? This is not necessary now. Because you know the transformation which I wrote last time.

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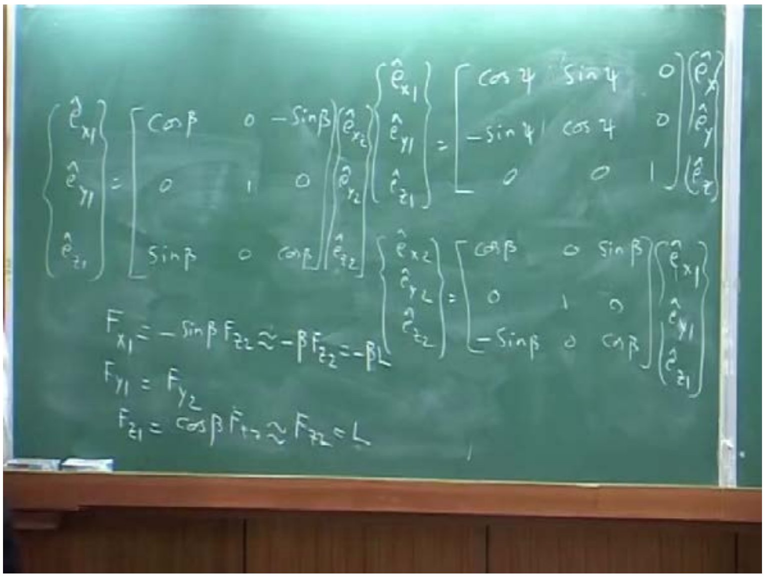

That is e x, I am actually I make the transformation. This is what I wrote last time. This is what was written last time. And another expression was e x 2 y 2 z 2, you wrote cosine beta 0… e x 1 e y 1 e z 1.

Now, I have the forces in this direction, z 2 y 2, they are here. z 2 y 2 x 2, actually x 2 will you have a force? Well you can have a force, that is the drag because it is along the span. And how do you get that expression? That I will tell you later. Right now we say there is no x 2. So, you can get e x 2 because this is all orthogonal transformation. So, you will have just cosine beta 0, just transpose, e x 2 e y 2 e z 2.

Now, I have f z 2 f y 2. So, f y 1 and e x 1 that means, I will have f x 1 is minus sin beta force along f z 2. So, minus beta, in the sense, sin beta I am writing it as, sorry let me put it clearly, minus sin beta f z 2. I am writing it as minus beta f z 2, which is minus beta l from there. And I know the lift expression. And then f y 1 is nothing, but f y 2. There is no change because that is only one. And your f z 2, f z 1 is beta into e x 2, there is no force. So, cosine beta f z 2, that is just f z 2 which is just simply l. That means, I making beta is very small. So, I get this. Now, you go back with these three. Because you know that lift is nothing, but f z 2. That is all. You go back and get the transformation in the, let me erase this part this is not now required.

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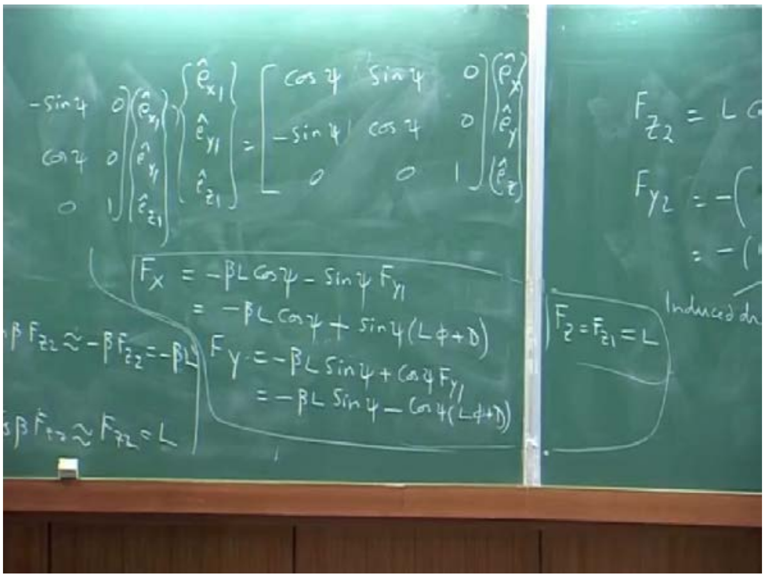

Along x y z, this is hub fixed. So, you will again make the transformation cosine psi minus sin psi 0. Here you will have what? x 1 y 1 z 1. Now, you know f x 1, f y 1, f z 1. Put them here, you get the f x y z. So, you will write f x cosine psi f x 1. So, minus beta l cosine psi. And then sin psi y 1, so minus sin psi f y 1. f y 1 is nothing, but f y 2. Which if you write everything in terms of minus, what is this? This is sin psi, f y 1 is f y 2 and minus and minus will become plus, this will become l phi plus d.

You got it? Now, then f y sin psi e x 1 f x 1 that is minus beta l sin psi. And then cosine psi f y 1. So, you will have plus cosine psi f y 1. And f y 1 you know minus of that, so this will become minus beta l sin psi minus cosine psi l phi plus d. And then f z, that is nothing, but f z is f z 1. f z 1 is f z 2, which is l. So, you will write your f z is f z 1, which is f z 2, which is basically l. So, you see this is my full expression of transformed load along the hub fixed non-rotating coordinate system per section. Basically, all these things you have to now get. Because this is the hub load, so we need to integrate over the span of the blade and then you must also add the value from each blade. Because when you get the hub load, every blade will give some load. That blade will depends on where its location is. Now, you see my hub loads even if you look at the simple expression, these loads are functions of psi, that means with position. Psi is omega t, that means my hub load is the function of time. It is not a constant, you understand. So, as the blade goes round my load is not fixed, it is varying with time. So, how do we really proceed in the entire formulation?

What we make in this course is, let me look at what is the mean value of the load. Mean value of my hub load, that is what. Mean value means I take average. One revolution the blade will go, you will have some load. It will be time varying, then I take the mean value, that is all. What about the time variation? I throw it out. I do not even write that because the time variation quantity essentially represents the vibrating part. So, the helicopter, the moment it starts moving forward, you will have a mean value and you will have a vibrations value. And the vibratory part I neglect. That is why helicopters have to vibrate. If it is not vibrating, it is not a helicopter, in the sense it is very crazy vibration, that is where other problems starts, how do I reduce vibration etcetera. So, that is a different part which will not be part of this course at all. Vibration is a, please note, a major problem in helicopters that comes purely because of this time variation in the load. But then you are flying. How you are flying is you use the mean value of the load. So, you say what is my mean f z, mean lift. If that value is equal to the weight, I support. The mean value is equal to the weight, it is flying, but then it will be shaking and flying. And same thing happens f x, fy everywhere. But this is only sectional load, please understand. You have to get the momentum because aerofoil theory gives you only lift, drag and pitching moment. Pitching moment I said take it as zero. That is for a section, but when I take the load at a particular section, I am transferring this load to the hub.

So, that will give you hub moments also. Because in the route, bending will come. So, essentially you have a lift force, you have a drag force. The lift into this that will give you flap bending moment for the blade. Each blade will give its own flap bending moment. You have to sum up all and get the hub moment. We will come to that moment part a little later. And then the drag force will give you lead lag, which is the torque. So, what we will do is, we will write the torque expression, later we will write the bending moment expression.

So, you see at the hub if I integrate all of them, I get the force and r cross f. Because moment is…, vector is… and r is you take it as whatever location. So, into f, f you put all these f and take the cross product, r is r into e x 2, because r e x 2 is what you have. And if you want to convert e x 1, you can transfer it and then take that x 1 y 1 cross product and then transfer it to hub loads. So, you will have at the hub three forces, three moments. And these expressions you need to get. They are pretty messy long expressions. Finally, I will give only very simplistic stuff with all assumptions made, all integration done everything.

Now, let us go back to writing first expression which is lift. Because lift is my thrust. I am going to write that expression first. So, one by one we will write the expressions. So, I will write that and then we will non-dimensionalize all those loads. Let me erase this part completely, I think this is not required. So, I erase this. Your lift f z 1 which is nothing, but f z 2, which is lift..

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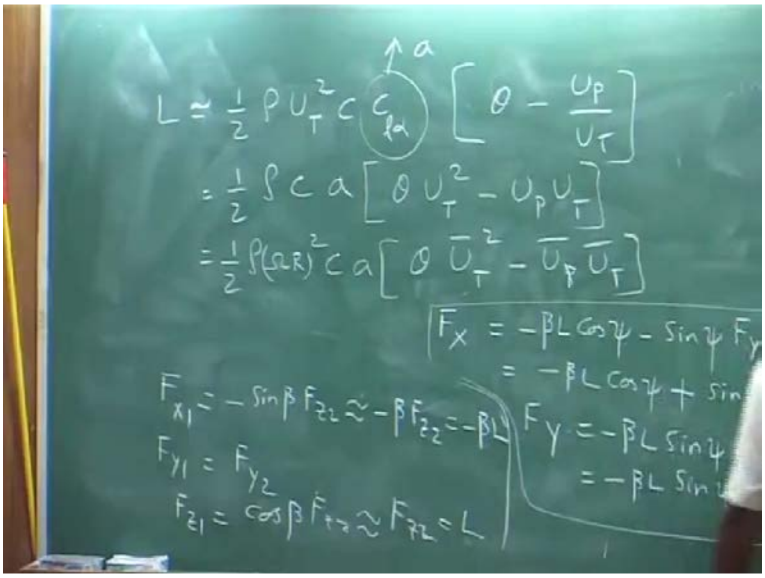

So, I will take it as this is half rho. We made an approximation that is u t square, please understand. I am putting like this. chord c l alpha, c l alpha you may use or c l alpha you replace by the symbol a. So, this is what we use a sometimes. And then multiplied by theta minus. Now, take the U T square inside, you will get half rho c a, I am putting this c l alpha as symbol a, theta U T square minus U P U T. And you know U T you have non-dimensionalized with the omega r outside. So, take out that omega r, you will write half rho omega capital R square c a, it will be theta U T bar square minus U P bar U T bar. I am taking omega r outside because finally, I will non-dimensionalize the whole stuff. Let me erase this part now. Now, I just replace the expressions here. This becomes, please note, I am taking half rho c a omega capital r whole square, substitute for U T bar.

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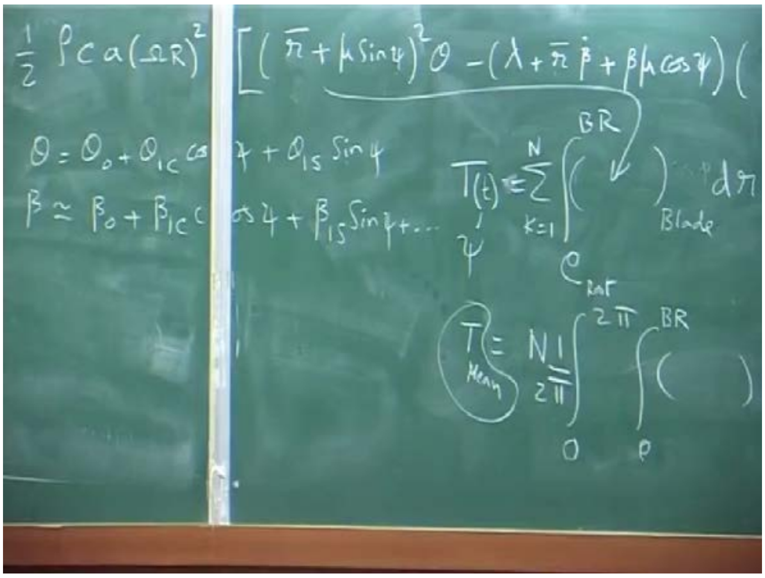

U T bar is r bar plus theta lambda plus r bar theta dot. Please understand this dot is with respect to non-dimensional time derivative, I do not put a star, plus beta mu psi multiplied by… So, this is my sectional lift. Please understand sectional lift. If I want my total thrust. Thrust is number of blades I have to take. I assume now each blade I must add, it is essentially a summation, it is not a integration. Because at a particular time, one blade will be in azimuth, another blade will be in another azimuth. It will be here, another one can be there, third one can be here, and if you have four bladed, it may be like this.

At every given instant, these psi will get replace by psi k. And I must put summation k running from 1 to number of blades. That is what I have to do. 1 to n, where I must replace here all the psi by every blade. And this blade what? I must integrate from 0 0 or some root offset to b r. It will be d r, you understand, is it clear? But this is non-dimensionalize. So, I will actually take a r bar, this will become b, this will become e and 1 r will come out. That r I will take it here. Is it okay? And then this is a summation at every instant. Now, please I am just slowly look, just with the simple expression. You still do not know what is theta. Because theta is theta naught plus theta 1 c cosine psi plus theta 1 s sin psi. That is the pilot input. What about beta? You do not know beta. So, you are assuming now this is the you, assume I know all these quantities. That means, I am essentially saying theta is cosine psi plus theta 1 s sin psi. And beta is I am writing like this cosine psi plus beta 1 s sin psi plus I neglect all the other terms. I am throwing away everything, I assume only this motion. Even though technically it is not correct, you have to take all the terms, but up to what harmonics you will go? These are all real practical problems. Let us take up to one harmonic. Now, you see this I have to put it here, here, here, here. And then I will have a long expression, integrate from this to this, then sum it up. Instead up doing now, I make one more assumption that all the blades as it goes around the azimuth, everything does the same thing, same motion. That means I assume all blades are identical. And the response is also identical. Identical means you take one blade, it goes around the azimuth, that means it you will some flap response. That response is same for all the blades. And this is what is in the practical thing they call it, all the blade first identical in terms of mass distribution both, please understand. Mass of the blade you can keep, then mass distribution how the mass per unit length is distributed. So, when you really manufacture, all the blades are not identically. There will always be some difference in weight.

Aerodynamic shape has to be identical. That we take it mass, but what they do it they measure the blade weight. If the blade mass is not same, they have to add weight. So, they will have boxes where they will put some extra weight, some 200 grams something like that. Make the blade mass is same as well as first movement of the blade mass with respect to the root, that is all they can do. Because you can have, mass is what? Mass of the blade integral m d r, some 0 to r you can take it. First movement I call it m x c g, this is 1 integral m r d r, 0 to capital R, then I blade, 0 to capital R m r square d r. All of them must be same, then only you say they are identical. Otherwise, there will be some difference, but in reality there will be difference. Now, these differences will appear when you really flight, that mean no to blade is same. But they have some criteria how much I allow. That is why I am just deviating now, there is something called a tracking balancing.

Balancing is balance the mass of all the blades because otherwise, if one blade is less mass or different then the other one, then you know that from basic vibration you will have a shaking because the mass is c g f the blade system is not right at the center. So, what it will do? It will start shaking. You will have vibration. So, this is balancing the blade. Tracking is, as the blade goes around you keep every blade some pitch angle, every blade should when at a particular azimuth angle, they should come to the same response. But what will happen is, if you look at that if all the fans are rotating. If one blade is going up, another one is going down at the same location that means, they are not doing the same response, but this is called tracking. So, how do they do is, they adjust some pitch angle of the blade, individual blade. Some blade goes up a little higher, then they will say reduce the pitch of that particular blade or they have a trailing edge tab. Usually most of the blades if you see, they will have something like this. There will be a fixed the tab, small tab will be there. They adjust the angle. Fixed, it is set. It is really reverted and they just adjust it a little bit, so that all the blades almost track. And they give some allowance, this much is there. Some engineers, I will tell you, their specialization is only this keep tracking balancing. Because if the track and balance is not proper, when they fly it will have lot of vibration. So, you balance it, then again fly. Of course, you will always have vibration, even if there all balanced you will have vibration. If they are not balance, you will have more vibration. This is a real practical problems.

Now, for us easy everything is identical. So, all blades perform the same response. Identical blades, identical motion. Now, what I will do is, the movement I say everything is identical. This is per at a particular instant this is the load, I am going to get mean values of the loads. Mean values… some hinge offset b r. Mean value is 0 to 2 pi 1 by 2 pi n. So, this is the mean value of one blade. If I have n number of blades, I will simply multiplied by n. You got it? So, what I am doing is, I am assuming all blades to same motion therefore, I am just multiplying by one blade effect, multiplied by n. So, this is what I finally get. And this I have to do for all the loads. Please understand, I am writing using only thrust. Because I do not want to write everything again and again. If I explain for one, the same thing is valid for f x, same thing is valid for f y, same is valid for movement and torque etcetera everything. Now, this is what I will do. Now, in the non-dimentionalization. So, now let me erase all of that, I knock out everything because I hope you have all these expressions, that is all. This thrust is a function of time, agreed or not? Because this is what is coming here. This entire expression comes here, agreed? And it is the function of psi. Psi is omega is t. k, I have to put wherever psi is there, I must put a psi k. Replace psi by psi k, where psi k is 2 pi over n k minus 1, n is the number of blades.

So, at a particular instant, each blade will have different lift value depending on where it is. Even though their r is fixed, assuming. Because this is a location of the position from the root. You take the same r bar in all the blades. At a particular instant, they will have different lift values. What you are doing is, you were integrating for one blade fully. After integration you are summing it up, each blade value you integrate separately, then add at that time, that is a particular time. You got it? Then again next instant again you do it. That means, your this t is basically a function of time. Time is non-dimensional it psi, agreed? Now, I say I am going to have oscillatory force, I am going to make this is d mean, that is 1 over 2 pi, 0 to 2 pi. Is it clear? That is for one blade I have done. Now, I make assumtions that all the blades are same, identical. Therefore, I simply multiply. I do not go and then add every time each blade and then calculate it, I wont do. In this case, I will simply calculate for one blade, take the mean value multiply by n. It is not that I will go and do for. Actually in my code what we have developed here, aeroelastic code, we do each blade separately. Then you do, but it is technically same, it is basically same. You take this t. t mean what will you do? You integrate 1 over 2 pi to 2 pi, that is basically the same thing, that is what I am saying. But if you really want to capture the vibratory load also, then you have to do this. You will have a t, every t. That is why the blade you keep moving it, every 4 degrees or 5 degrees like that and then keep integrating. When one blade moves, other blade also moves and other blade aerodynamics is different.

When it comes to on the retreating side, this blade may be stalling some sections. That means, you have to take that stalled value of that lift into account. So, when we write the big aeroelastic code, which we have developed of course, the h a l is interested done hence now its a collaboratively we are adding it. It is a very comprehensive aeroelastic analysis. There we do every section of all the blades and we do not assume all blades are same. Even though for computation we do. Otherwise, even if you want change the blade properties the little bit, you can do it. The flexibility is there in the program, but industrial code they assume all blades are same, nobody does that. And after that it is all minor adjustments they do. Because see practical problems sometimes may not be easy to translate into theoretical modeling. Because sometimes it is very complicated realistic problems are. So, we make lot assumption. Because you need to get some values, if you want to fly. See, if you really see the history of, you all heard a history helicopter development, all these things came much later. All the mathematical modeling as all much later, after the helicopter started flying.

But now you try to understand the problems. What is the reason for this problem, how do you fix it, how do you improve it. But improvements have happened substantially. Is this clear? But vibratory load you cannot get it, mean value fine. And that is what is done in practical things. So, if the mean values you capture correctly, you can go ahead and design. Vibratory values always problem because the real aerodynamics you do not know, you understand because the flow is so complicated. Your inflow you have to get properly because if your inflow is wrong, your angle of attack is wrong. If the angle of attack is wrong, everything is wrong. You understand? And it is a time varying. It is an oscillating aerofoil. Unsteady aerodynamics you have to use, but usually industrial use just steady values. But we have a model for unsteady aerodynamics. So, we have incorporated a stall model. But again that is empirical. We try to fit a curve for the dynamic stall and then take that. It is a differential equation.

So, I get my c l, c d, c m from a differential equation. If my blade is oscillating in a particular fashion, at a particular time what will be the value of my c l? What will be my drag? What will be my moment at a instant? So, usually the lift, drag, moment are not just functions of angle of attack. They also functions of time variation in angle of attack. You understand? Time variation in angle of attack also will come and second derivative also may be there. So, these are more, I would say, advanced modeling approaches. But even then that is not precise, but you make assumption which is a better model than assuming everything is constant. But in the course for the basic level, you make all these assumptions. And then I will write the expression for the mean. I will not use the t mean every time, please understand. Because you take it as it is the mean value, it is already integrated over azimuth, 1 over 2 pi etcetera. So, let us write the expression for the thrust.

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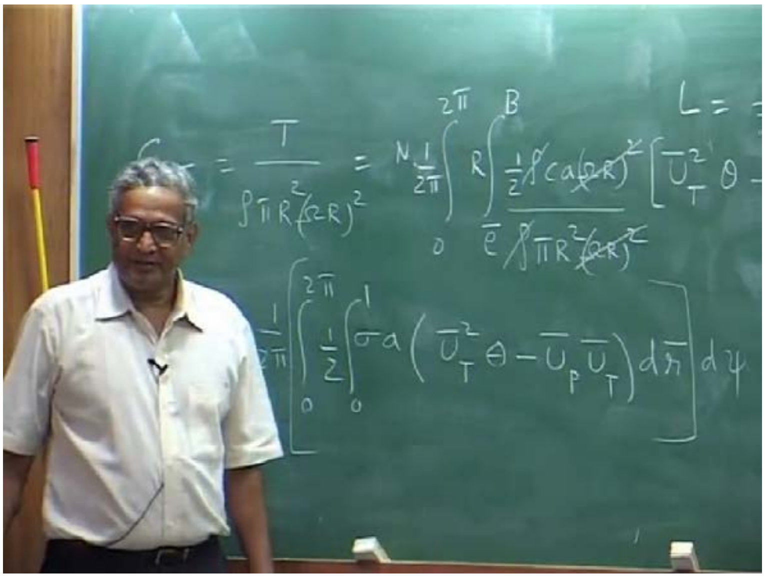

So, your thrust I take it and I will non-dimensionalize the thrust expression also. So, I will get the mean value of the thrust, which is c of t is thrust divided by rho. This is number of blades, putting that, there will be a 0 to 2 pi and one over sorry there is the n, 1 over 2 pi is also there. Then I write this expression. You may say e, I will put a e bar and b. Take out r outside and I will have this half rho c a omega capital R whole square. I am just simple because for simplicity I am writing like this… into… But now I make further assumption. I will not take bar and b. 0 to 1 it is much easier because then you can get a neat closed form expression.

Now, I non-dimensionalize. So, I have to divide by, sorry I forgot to rho pi r square. So, you look at the term omega r, this will go up, rho goes off, n c r pi r square that is sigma. So, you will have 1 over 2 pi, I will have n r c. So, I will put a half this is sigma, this is a. Times your U T bar square theta U P bar U T bar d r bar d psi. This is my c t expression. This integral I have to do. Oh I am sorry 0 to 1 integral. Yeah take it 0 to 1, yeah I put it 0 to 1. Otherwise, you put it e to the tip, whatever you may correct at, tip correction factor 0.97, this root of set. So, this is my expression. Here I made root is also, I do not have any correction I am making it 0, tip I go to integrate till 1. Is it clear? Now, I write this U T, I have to substitute r bar plus mu sin psi whole square. U P bar U T bar I am write these two expression and then I have to theta.

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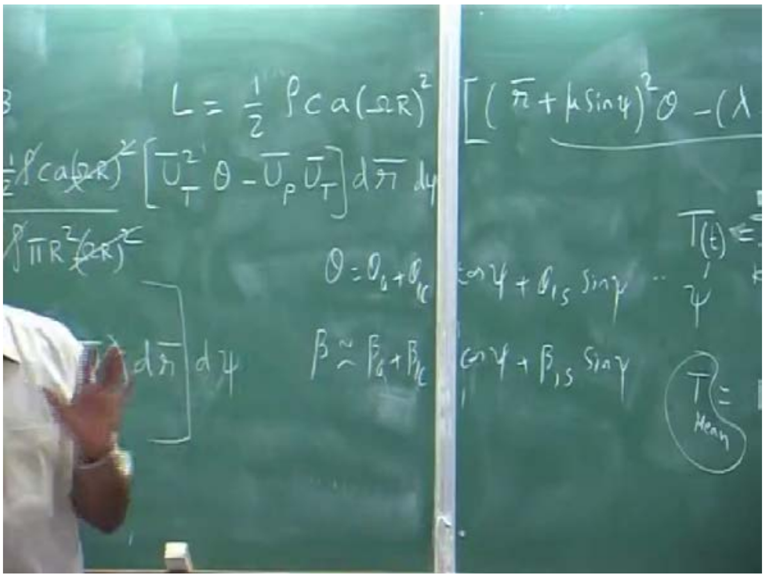

I write that, theta naught plus theta 1 c cosine psi plus theta 1 s sin psi. And similarly I will write beta beta naught plus beta 1 c cosine psi plus beta 1 s sin psi. I write all these expressions. When I take a derivative, beta naught is constant that goes off. Beta 1 c, please understand in this expression beta naught 1 c 1 s they are all constants. Because it is steady state value. You are writing the flapping motion of the blade like a harmonic series, Fourier series. And this is what it is. These are constants. Later I will introduce something these are not constants also, they are time varying. That will come when you have to do stability, then they are not constants, they also can varying. Right now take it as, now you substitute these expressions here. Theta you put it here, beta naught you put it here and here, beta. And then integrate the whole thing. And you write the expression. Now, I will give, this is I will go back there and show you what is the thrust expression. That is all. You do not have do all these things. What I am saying is you will be given the sheet how the thrust is obtained.

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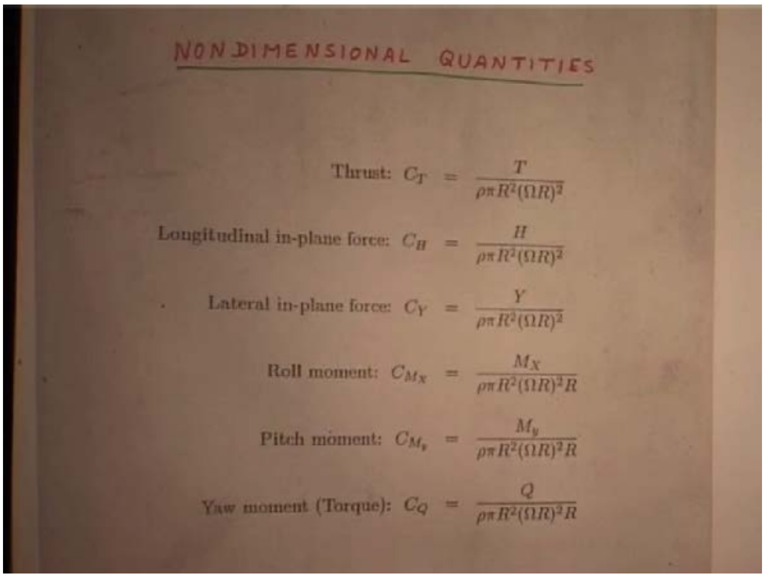

We basically non-Dimensionalize the quantities. So, we non-Dimensionalize thrust, in-plane force that is why I s c sub h, c sub y, roll moment, pitch moment, torque all of them are non-Dimensionalized. And then I give the final expression which is. Here I have just given only thrust coefficient. What I have written there is this expression.

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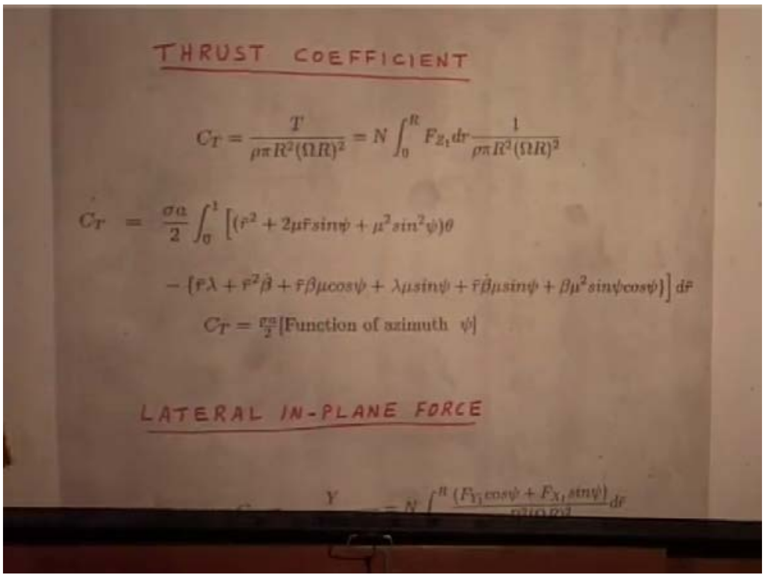

And c t is a function of azimuth. That is what you asked. I did not integrate 1 over 2 pi, 0 to 2 pi, that I have not done. But I have taken all the blades also. So, it is like very, that is why sigma a over 2 some function of psi. That is where all the 0 to 1 everything is sitting there.

Now, like this you have to do for in-plane force, you follow? Side force, longitudinal force, torque everything. Because this you do not have to copy, because this is something which is. I will give you the final expression. Only thrust expression, do not bother about the rest of the things, only thrust you look at the thrust part.

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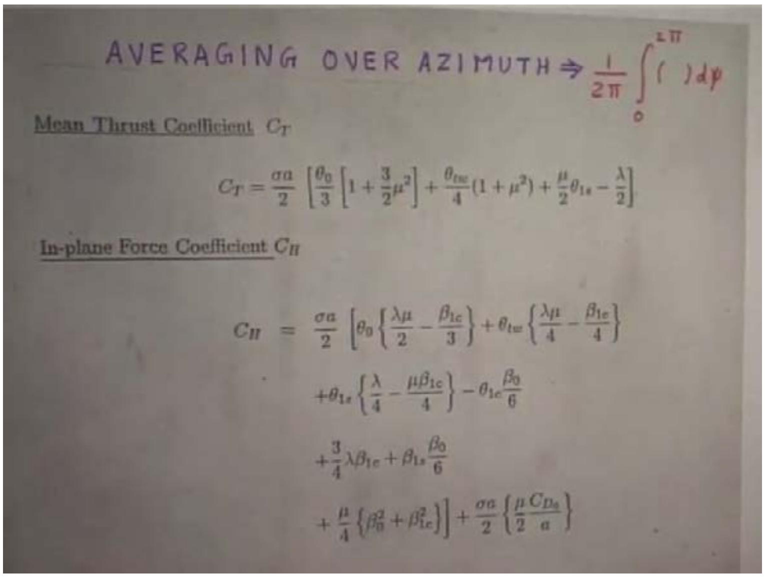

Averaging over this is what I have done. And then my c t, mean thrust coefficient is this expression. And I have included theta twist also. So, I put this plus theta twist r over r. I have added that. Now, this is my c t expression, After doing all, it comes out very simple. But you will find there is no flap term sitting here. Flap term will not be there, but you will get all the flap terms will come in the side force and other things. That will worry later. I want to just look at it only this part. If you set mu, that is the forward speed. So, please understand my c t, thrust coefficient is a function of forward speed, which is advanced ratio mu, mu square. And it is a function of theta 1 s. Theta 1 s is we call it that is why cyclic pitch, whether you call longitudinal cyclic, pitch or lateral cyclic pitch because always that is a confusion will arise.

Theta 1 you say it is the longitudinal cyclic pitch, but the problem is it will be given at 90 degrees. Later you will understand the relation between this and this. Because you need to know how flap. Now, if theta 1 s is positive, that means what? Pitch angle will increase at 90 degree and it will degrees at 270. So, usually you would want on the advancing side, velocity is more, pitch angle is less. Retracing side, velocity is less, pitch angle is more. So, your theta 1 s is always negative. Please understand. When you do actual calculations you will find theta 1 s is always negative quantity.

Now, you see the moment pilot flies forward, if you gives a theta 1 s. Because pilot only will give theta 1 s because he will move the stick forward. When he moves the stick forward, pitch angle when the blade comes at 90 degrees, it will decrease. So, what will happen? When the blade pitch angle theta 1 s is mu is starting forward speed at that quantity becomes negative. That means, your thrust will decrease. So, always pilot will find, if you are starting or you give a cyclic input forward, he will always go down also, he will go down. So, they will always increase the collective. So, that collective you increase it, so that maintain that. So, if you give one input something the vehicle will. So, this is how. These are all very small terms they look like, but then they contribute substantially in understanding how the vehicle will behave in flight. When simplistically what people will explain is, it is like this. The very simple if you want to explain you will say this is my rotor disk, thrust is like this. If I tilt it forward, then what? Because of the tilt in angle the vertical component gets reduced. Therefore, you have to increase the vertical.

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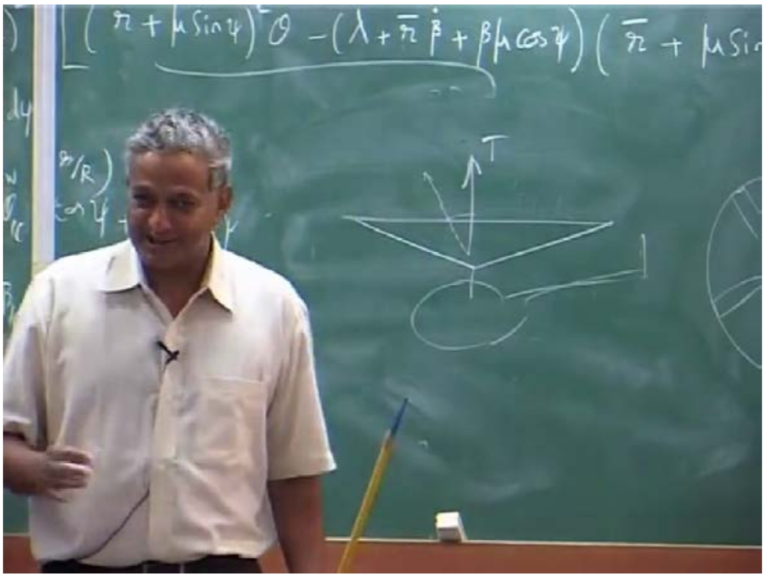

So, whenever you want to tilt the disk, there is a reduction in the thrust. It is a simple explanation you can give. Because only when you tilt forward, you will go forward. So, if this is the thrust you want to tilt it, so your vertical component decreases. But actually mathematically if you want so, this is what is really happening, that is the term. And then of course, lambda I have used constant inflow. Now, you imagine your inflow can be a function of azimuth. Now, imagine you put all of them, these expressions will become more and more miss here.