Introduction to Helicopter Aerodynamics and Dynamics.

Prof. Dr. C. Venkatesan
Department of Aerospace Engineering
Indian Institute of Technology, Kanpur

Module No. # 01

Lecture No. # 11

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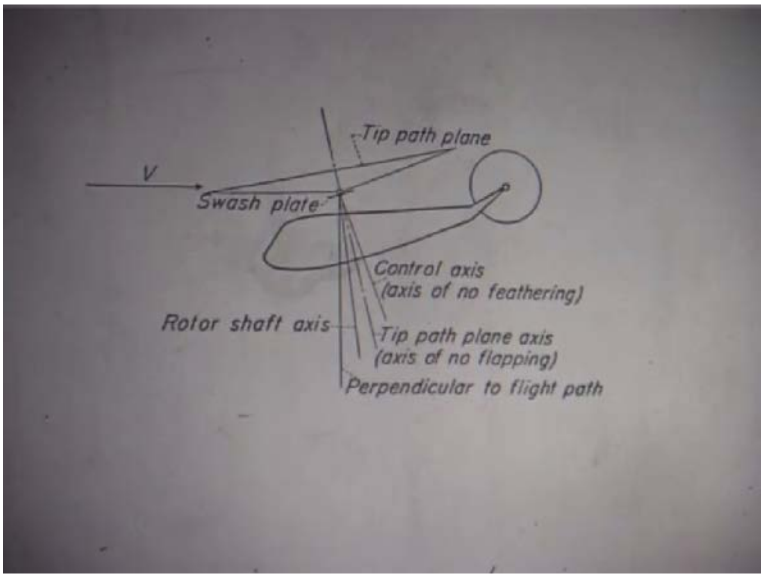

When you start describing the motion of the rotor blade in forward flight, you can refer to several planes, in the sense reference frame. See, this is the rotor shaft axis and this is actually swash plate and there is a tip path plane. And this is perpendicular to the flight path. We can have a flight path and then whatever is the oncoming velocity you can define a coordinate system with respective to that.

Now, you can have several coordinate systems. But for ease of formulation, you have to decide what you are choosing. And if you decide to choose a particular coordinate system, then you have to stick to that throughout the development of forward flight equations. Because the blade is flapping lead-lag torsion about the hub and the blade also is given a pitch angle which is varying as it goes around the azimuth. And more complicated situation, which the fuselage itself is moving up and down, three translation, three rotation. So, this is the actual full dynamics. Now, because the roll or pitch and yaw motion plus up and down, this can be any large angle also. So, you have to describe the motion of the blade, get the loads on that, transfer to the hub and then how the fuselage will move. Unless, you are very systematic in the development, it becomes quite complicated.

So, what normally done. Normally in the sense, it depends on the person who uses that. But it is preferable to choose the hub plane. That means, hub plane is, I am choosing the shaft axis. So, the shaft is there, hub is there. So, I decide that my coordinate system is fixed. And with respect to that coordinate system only I will start describing all the motions. So, coordinate system, we will say, if this is the hub, this is my z axis and I may have my x and my y.

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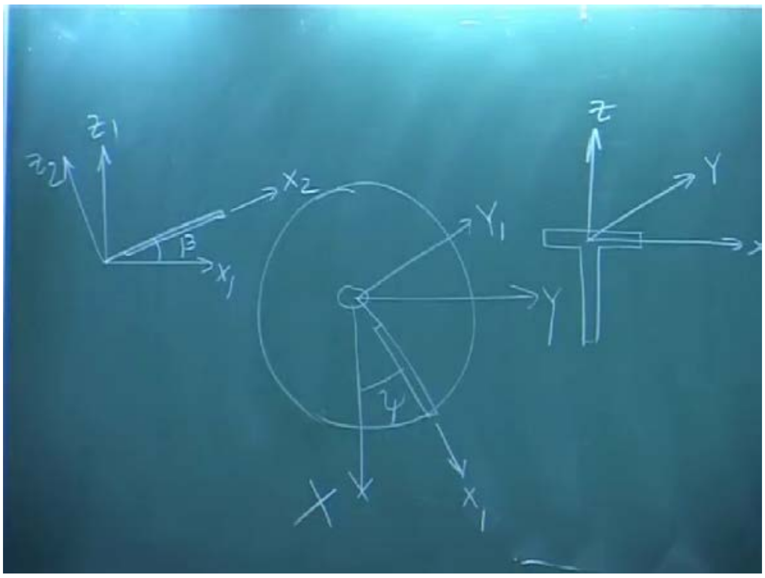

And the blade can be inclined, flapped, it can do anything it wants. But my motion is describe in this plane. This is my reference plane, which is same as what I do in the fuselage, because this is attached to the fuselage, so once you decide this frame, you can say this is parallel to the c g frame also if you want, but it can be little different also.

But when you are calculating the aerodynamic load, what we have learnt in aerodynamics is, you have an aerofoil, there is a oncoming flow and there is a angle of attack may be, and you get the lift and drag. But in the present case, you will have of course, you have a normal flow because this is not the only flow, you are going to have a flow like this, which is due to the inflow plus the motion of the blade, because the blade is flapping up and down. That mean there is a relative velocity due to flap motion. And then this is, all these are in the plane of the aerofoil please understand. That is, if you cut a section of the blade, that section you are describing all these aerodynamic loads. But my blade, when it rotates, so I will write some. This is the hub, this is my capital X and here z is coming, that is the hub plane which I have drawn.

But if I look at, view it from the top my blade may be going like this. I am drawing only the projection of the blade which is in this. But, actually I will call this as may be x 1, y 1. x 1, y 1, z 1 is actually a rotating coordinate system. It rotates with the blade. But the blade can flap. Now, if I look at from y 1, assuming I am viewing from y 1, I will have, this is my x 1, this is my z 1 and my blade can be like this. This is my flap beta. So, I call this as x 2, z 2 and y is into the. Is it clear?

So, you see I am defining several coordinate systems. Because the idea is, if you are very clear about the, this is the most I would call it the simplest problem, hover is the simplest of all the cases. And when you go to the forward flight, taking only flap motion is the simplest. But, if you want to really understand the full, if you understand this then you will learn more and more as you go along, if you are interested in this field, otherwise it stops right here. Because if you have all the other motions, here we are not, in this what we developed now, we assume the blade is only flapping. That means it is coming out of the plane of hub. I define that flap as the angle beta, x 2, y 2. Now, you see which is my, if I look at this is my aerofoil, cross section of the rotor blade, my x 2 is coming out here. Because that is the cross section of the blade. And I know aerodynamic theory is for the section. So, I will simply apply. When I want to get the aerodynamic loads, I use this with what is the angle I am having with respect to, this is the u tangential, this is u perpendicular. And then of course, I get the induced angle of attack. So, I will subtract. Finally, I will get a lift and a drag in this frame in the z 2 x 2 frame, the lift.

And then I again transfer back to, if I want to get the hub loads, I have to transfer back the aerodynamic load which is getting in this axis system, I should transfer back to this system. So, we have a transformation of coordinates which we have to apply. That is systematic, these are all orthogonal system. So, the transformation you just use. So, you define motion in one coordinate system and then you describe a position vector, velocity, then you get the aerodynamic load and again you can transfer back. So, if you are very systematic, then you will find everything follows a very clear cut procedure.

But most of the books, as a matter no books, they do not give any of these things. They will finally write this is the velocity component. But in writing that component they make lot of assumptions. So, what we will do is, we will use this. I will go through math part here up to some point, describe how you really get the lift, drag, moment everything and how they are converted and then what approximation we make. And at the end of it you will get loads here, like your thrust. And you will have a longitudinal force, you will have a lateral force, you will have moment also, yaw moment, pitch moment, role moment everything will come in forward flight. And that is where the key development in forward flight is, first part. And this is like assuming my helicopter is in a steady level flight. Steady, that means it is flying with the constant speed. Now, before you go to describe a little bit, I wanted to show you a just a brief very brief introduction to modeling the blade.

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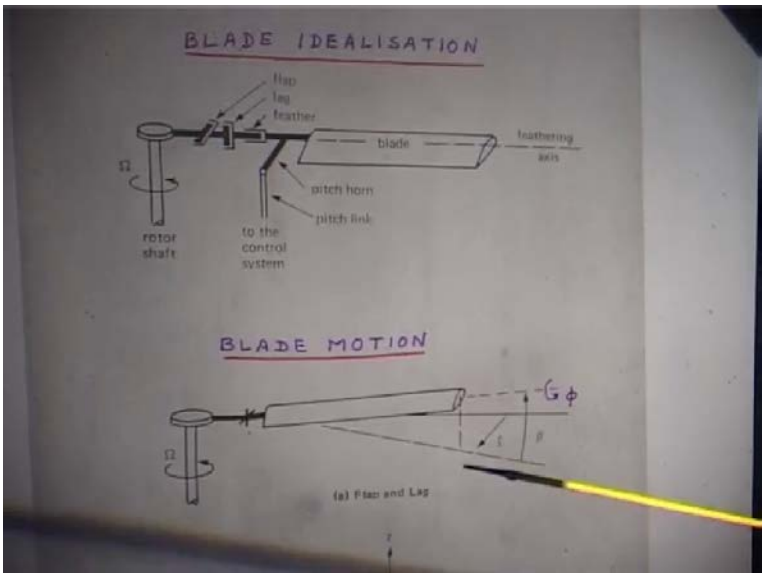

Because if you remember right at the beginning of the course we mentioned the blade is attached to the hub and then a if it is articulated, you have a flap hinge and you have a lead-lag hinge and you have a pitch bearing. And in the case of a hingeless, you do not have any hinge, but you have a pitch bearing, because you need to change the pitch angle of the blade cyclically, as it goes around.

Now, this is a very interesting thing. In this diagram, this is a rotor shaft, blade is attached to the hub. It is very schematic, I am just showing schematic. I put the flap hinge near the root, after that lead-lag hinge, after that the feathering. Feathering is the pitch bearing, Please understand. They are kept in a different particular sequence moving from the root to the blade. And feathering axis is the axis about which you change the angle of attack of the blade, basically the pitch angle of the blade, by moving this control. Because if you move it up and down, you change the pitch angle.

This is a symmetric. And now is this really important that they have to be in this sequence? No. It need not be because if you take hinge less blade, there will no hinge, hinge is somewhere in the design of the blade. When you make the blade you make it such that this portion acts like a hinge. But you will change the pitch angle here by putting some bracket and taking it you rotate it here. That means, your pitch bearing is inboard of flap and lag hinges. In this case, it is inboard of flap and lag hinges. Please understand, inboard, outboard. What is the big deal you can ask.

If it is an outboard, suppose flap hinge is this, it is flapping like this. If my pitch is outboard, I will change the pitch angle to any value, but the flap hinge will not rotate. It will be still in the same. So, the blade will flap like this. On the other hand, if my pitch is here, flap is outboard. If I rotate, the flap will be going, then the blade will flap like this. You follow? Now, these are very critical. So, you will learn about these things slowly later. If you have outboard, if you have inboard, what happens? Because that introduces something called a structural coupling. Because flap is what? Flap is something out of plane motion of the blade. This is the hub plane. But the blade will instead of going like this, it will go like this. That means, you have both up motion, lag motion.

So, you have essentially coupled flap lag motion. You have a coupling. But is beneficial or is it determental? Unfortunately, it is beneficial in some case, it is detrimental in some other case. So, these studies has been performed earlier. And then that is why the choice of the rotor for a particular helicopter, the industry, they go by their experience and what they have learnt in designing one rotor system, they will stick to the same thing. Only thing is when they went from articulated, because articulated rotor was there in all most of the articulated rotors. And I do not know you may correct the pitch barring is outboard. In (( )),is it outboard or inboard? Outboard. Inboard? Outboard. Whereas in the (( )), it goes inboard. Inboard it introduces coupling. So, you change it from articulated to a hinge blade. It is not that I take it and I put it, then let me try what happens. Because if you try what happens, the blade may break because of the aero-elastic coupling. All are very small effects only, please understand. That is why I am just giving you a glimpse of the physics involved in this. But later we will time permits, we will introduce a little bit.

But in the beginning, when we are starting, what you do for this? And then the question is, where is the hinge? Is the hinge right here or here? Here it is moved from the hub. So, as a first approximation, please understand, now this is where I say first approximation I do not bother about any of these things. I will simply say all of them are coincident and sitting there right at the center, at the center of the hub. So, I consider only flapping motion, somehow I can change the pitch angle, that is all. I do not consider my flap axis is tilting, there is no lead lag motion, these are assumptions I am making right at the, that is why I said reality when you see, when you try to model when you see some of the literature or books or anything, there is a tremendous gap between what is their what is modern, because you make assumptions, lots of assumptions. So, the first assumption which we make in modeling the rotor blade is it is centrally hinged and I am changing the pitch angle. This is the flap, centrally hinged it is. Later we will study blade modeling, after we finish the forward flight which will take a few lectures, then we will go to the blade modeling. So, right now for our development we make this approximation that. And here you see I make coincident hinges, flap and lag same. Actually there is a interesting thing, they made a blade because most of the analysis we do coincident that we do not distinguish between this end and that end, lead lag flap hinge they are all the same place. But technically they can be at different place. So, what was done is, they made a blade, designed a hub such that the flap and lag hinge exactly at the same location and then tested it. And then they introduced different things to see the effect of all this on the blade dynamics.

Now, having known made the assumption, we now look at it. This is the feathering. Please understand. What is the axis? Again I go here. When the blade flaps, this axis going up that means, I change my pitch angle along this axis. I do not change the pitch angle about this axis. Because if I change the pitch angle here, what will happen? The blade will go in a conical path. Even though I may rotate here, the blade will do like this. And these are all are very small things, but they have tremendous aero-elastic effects. You have to model them properly. So, the rotation about what axis you are giving. We make very simple assumption now, we rotate the pitch axis only along the blade axis, not about some other axis and this is what I put phi. And phi is of course, elastic torsion plus in this case we take it there is no twist of the blade happening by elastic twist.

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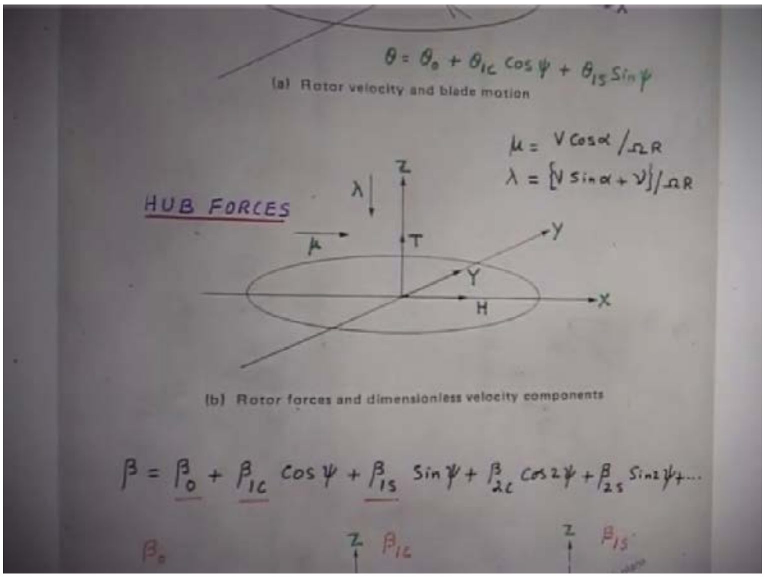

Normally those things happen that is why I put flap, lead lag elastic torsion. All of them are happening on the blade. But I throw everything. I say only flap is happening, there is no phi, but theta is the pilot input. And that is there he gives. And the blade experiences that variation as it goes round and round. So, these are the basic things which you have to know. And now I will go to one more new graph, I have written here the same diagram, little bit more. So, you see this is my hub plane x y z and the rotation vector is along the shaft that is, the hub plane. And the blade centrally hinged and it is flapping beta and the pitch angle is theta. And pitch angle is given by this expression, you know that this is the pilot input, collective two cyclic 1 c 1 s.

Now, my hub system is what I have, what are the forces which I have to calculate. Later I will derive then you will know. This is again my hub access x y z, even though I put a circle the blade is not in this. It is a hub plane. I have a h force which is along the x axis. Please note the convention is helicopter is flying like this, x is behind towards the tail in the hub. And y and this the z. And I defined my mu v cosign alpha over omega r. You know that velocity, because earlier we said only some rotor disk, now I am defining mu in the hub plane. Alpha is the tilt of the helicopter. Because helicopter normally when it flies forward, I have exaggerated.

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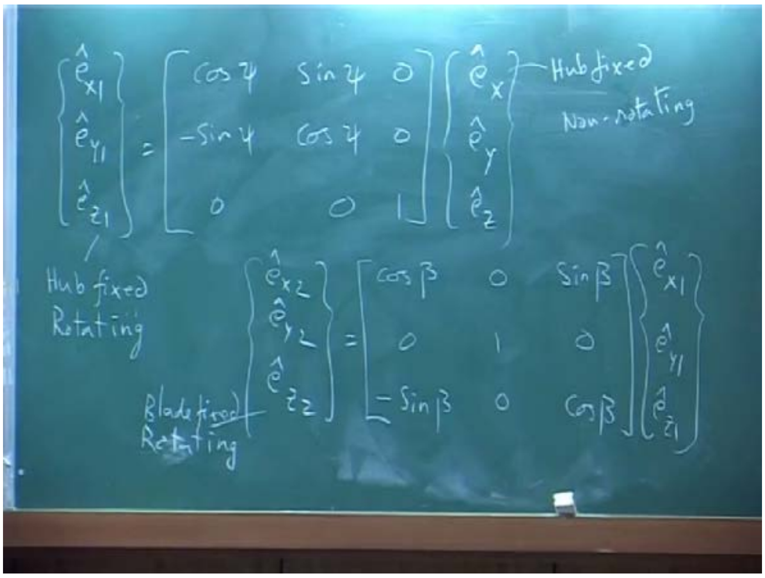

Leave that part. This is my, I have exaggerated. This angle that is, the latitudinal axis to the earth, you can take it or the oncoming flow. You define the oncoming flow as alpha. You are now, your hub plane is what? This is your hub plane, this is your v because helicopter is flying forward, what do you do? In the hub plane, you will have v cosine alpha and normal to the hub plane you will have a v sign alpha. And of course, you will have an induced velocity also. Now, you know induced velocity is defined normal to the hub plane in the formulation. I am not really looking at tip path plane nothing, please understand. Then you can define the tip path plane with respect to the hub plane. It is very easy because tip path plane whatever is the 1 c beta 1 c, that is the tip path plane.

So, you see v cos alpha, v sin alpha and mu. These are my definitions of advanced ratio and inflow, total inflow you may call it. Because I divided by omega r tips phi, but only thing is I do not know what is alpha. When I am hovering you may say alpha is 0. Because I am having straight weight is below, but please understand suppose weight is not right on shaft, weight can be anywhere here there. So, that depends on even in hover you can tilt it. But anyway mu is 0, therefore does not matter in hover case. So, in forward flight you need to know alpha that means, what is the pitch attitude of the helicopter which you will not know beforehand up.

So, in all over formulation what we are going to do next. We will simply say, hey I know mu I know lambda. But please understand this lambda is v sin alpha plus mu. Now, mu is the induced flow. It can be any model. You can assume uniform flow, you can assume grease model, I can consume (( )) square model, you can have various inflow models. Because my mu is changing, it can change with the location.

But for simplicity of this I am assuming it is constant. This is constant uniform which I will get it from basically the momentum theory in forward flight which I mention in the last class or before. Now, you know how do you get that inflow. Assuming that you know this, because I will tell you just for, see the inflow expression was this, either this or this. Because you multiply by omega r you get the mu, but this requires again alpha. So, you see to get the inflow you need to know the alpha. Only when you define the alpha you know mu.

So, it is like one is related to other. So, there will be an iteration in forward flight if you want to solve the flight condition, that is the basically the trim or equilibrium. To do that you need to start with only symbols and using that we will derive all the expressions for this. We will get the expressions for the six t, h, y and then moments torque, roll moment, pitch moment. All of the them we will obtain in forward flight and this is where the lot of complexities there.

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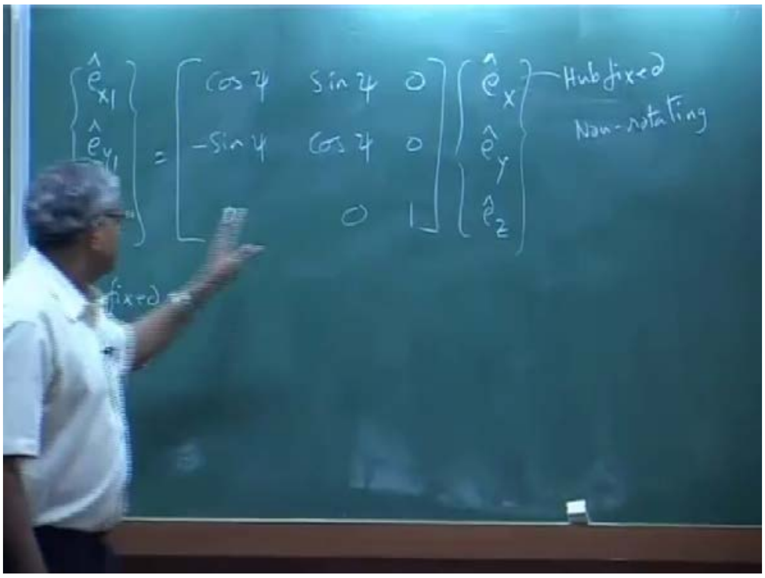

Now, of course, this I mention to you last time. How will I describe my flap motion? I am assuming my flap is periodic that means, because it is going round and round my velocity is varying. So, I make the assumption first that this is going to be my flap motion, but I knock out the higher harmonic terms, I neglect them. And I try to give physical meaning to these three which are nothing, but this is the conning because collective all the blades go up simultaneously, that is we call it conning. Then if they tilt longitudinally, that you call it 1 c and this is lateral tilt, that is 1 s. Of course, the other things all are rest of the perturbations you may call it in the tip path plane.

Now, you see I have essentially kind of define what type of motion, what my axis system is and what my mu and lambda. Using this, how will get my aerodynamic load first? But please understand I am assuming mu, lambda, beta. You may ask how will I get beta naught 1 c 1 s, how will I get it? Because that nobody going to supply it to you. I am only saying it is periodic and you can write it any periodic signal, you can write it as a Fourier series, this is essentially a Fourier series. And your pitch input is given here. So, this is the pitch input or you can call it pilot input which goes to the main rotor. We are talking about only main rotor please understand now. And then these are the velocity components and this is the blade motion, flap motion. That is all. Using this how do we get the aerodynamic load at any cross section. Is it clear?

But this is most primitive I would call it. Because in the helicopter this is simplest one you can have. Because this part is, I will do it on the board for clarity. Now, the symbol now the symbol I use, T for thrust, T is the thrust, H is the longitudinal force and Y is the side force, side force to the helicopter. This is the symbol please understand I am using the same symbol we all know, because this is the standard symbol which people use. T H for longitudinal force, Y for side force, all are uppercase. And then you can have moments. Please understand you can have a moment m x, m y and m z. m z is basically your torque. And if you know the rotor rpm torque, you can get the power. That is all.

Now, you have these defined. We have to go and get expressions for thrust H y, m x, m y torque, these six quantities. That means, I know what are the loads that act on the hub. Knowing this, but still in assuming this, there are unknowns that is why I am again emphasizing the unknown is alpha, I do not know. And here I do not know beta naught ,beta 1 c, beta 1 s I am neglecting all the higher harmonics again, please understand I am neglecting all of them. Because if I do not neglect it is your expressions everything will become massive, very big ones. So, we make assumptions.

Now, let us start with the simple formulation, the transformation. So, you understood this. We will come back to this figure later also. Now, how do we get the, because we will write the transformation matrix. The transformation matrix because e… So, this is hub fixed non-rotating. This is hub fixed rotating because this is the transformation. Cosine psi, this is x minus sine psi. This is the orthogonal transformation. All of them orthogonal. If you want from this to this, you simply transpose of that. Because this is an orthogonal, so inverse is transpose. This is a simple transformation. Then you have to have another transformation between this system and this system, because this is the blade fixed coordinate system rotating. So, I will write, now here y 2… e x 1 e y 1 e z 1. This is blade fixed rotating. And I am assuming my blade is rigid. It is the rigid blade, it can only do like this. It is not a elastic blade. So, please understand. It is a rigid blade assumption only flap motion. Now, you have this transformation. You need to know for getting the aerodynamic loads on the cross section of the blade, you have to know the velocity components. The velocity you have from two sources. One is due to the motion of the blade, another one due to the motion on the vehicle.

So, first you say motion of the blade that means, the velocity at any particular cross section. Because we say it is a rigid blade, cross section is fixed, so you draw a line. That line is you say your pitch axis line or you may call it elastic axis line. Because that is the pitch axis about which you are actually changing your blade pitch angle. I want velocities obtained at that point. Because you may ask, if this is where, velocity at which point I will get it, you may ask.

So, this is the axis of pitch and in the design of the rotor blades, this is a usually for subsonic 25 percent chord is the aerodynamic center. Then you have learnt about shear center in structures and you have learnt mass center for every section. In the design of rotor blades, what you try to do is you try to bring every center here, 25 percent. Every center, aerodynamic center is shaped which is a subsonic. You can take it as 25 percent, but mass center, that is why they put a leading edge mass to shift the c g, onces they design. Leading edge mass they put such that the c g also comes center of mass at 25 percent chord. Elastic axis of the structure is also at 25 percent and pitch axis is also at 25 percent. Please understand everything they try to do. It is a very complex design, it is not that easy, it goes through several iterations. And then finally, they will try to fix it, put it there. The idea is you do not want introduce too many couplings because if the mass center is somewhere there, if it start doing, then it will have its own inertia, there are problems. So, let us say this is our reference line 25 percent chord and this is what my pitch axis is also. Usually the pitch axis for the rotor blade is kept at 25 percent chord. Now, I define the velocity components, u tangent and u propagate. That is all. Because you realize you can find out velocity at any point. This point will have different velocity because if your pitch is changing about this axis, this will have different velocity components.

So, which point you will really take the velocity? So, these are all very basic questions. Now, you start thinking what do I do. How the aerodynamic loads are obtained? Now, you say let me go for CFD or wind tunnel. Wind tunnel, what they do is they put that they will rotate about 25 percent chord pitch, they will do that and then get the loads.

So, these are some of the. When you really start doing only you will start realizing hey there are so many things which we have to know before we really start analyzing or understanding, otherwise books will simply take it. Take oncoming flow because it is not a fixed airfoil. Please understand because this is a moving, moving one wing it is doing something and your pitch angle is changing continuously that means, it is no longer a static problem, it is a dynamic problem. So, we do not have assume, we do not have any dynamic data, we only know static. If you give me what is the angle of attack, I will give you what the lift is.

So, every instant I am assuming this is the angle of attack. So, this is what I call it the quasi static approximation. Because even though my pitch angle is continuously changing that means, I am oscillating my blade. It is a dynamic situation, but do not consider that. So, this is another first, another assumption we make in getting aerodynamic loads. Purely from what you have learned aerodynamics, given the airfoil what is the angle of attack, take it and then put it. When you want to complicate a little more, then you say let me introduce unsteady aerodynamics, but we will not do that.

So, I am going to take this. So, for me when I am trying to get the velocity expression, I will just take that 25 percent which is the reference line. So, I am going to say what is my position vector. This is my origin. Please note origin, all the origins are fixed, same. Position vector, I take a point p on the reference. I will say r is the position vector of, this is r, any point. Because that is in the deformed axis, I have to get the velocity. If you want to get the velocity, one way is you convert x 2 to, using this transformation relation, to x 1 and then again differentiate. Or another way is, I am not going to do I can directly go ahead and differentiate. You can do many ways. That is why I said basic dynamics is very important here.

Because when I differentiate velocity of that point, I will write velocity of point p, r dot, but r is a constant. So, that means, e x 2 is a varying. What is the angular velocity of this coordinate system x 2 with respect to fixed frame fixed, this term is capital X capital Y capital Z. This is what you need to be careful, is it clear? Because basic dynamics is very important. Another way, slightly easier way is, you know that this is my omega. x 1 is rotating with the omega, which is the rotor angular velocity with respect to capital X capital Y capital Z. So, one axis rotation. Whereas, when you go to x 2, it is not one axis rotation because you are also rotating about this. Because the flap y 1, flapping is also a rotation.

So, when you want to define rotation of x 2 y 2 z 2 with respect to fixed frame you have to include capital omega, you have to include flap. So, that will be like this. If you define omega here. This will be omega e z because that is the z axis, counter clockwise rotation positive. But my flap is about what axis? Flap is clockwise about y 1. y 1 y 2 both are same. So, I will have minus y 1 or y 2, both are pretty much same. You can put it actually y 2 is better because you have to convert y 2 only. Because y 1 y 2 both are same

Now, you see this is along z direction, e z. This is along y 2 direction, this is defined along x 2. We have to get the velocity that means, covert all these vectors in x 2 y 2 z 2, then only you can get the velocity. You know that r dot because the position vector is r is fixed. So, that dot is zero. So, you will get omega cross… This is what the basic dynamics, omega is the angular velocity of the coordinate system. And this is the position vector.

Now, you convert this omega. You know e z that means, e z, basically e z 1. Then you put it e z 1 here and then get it in this coordinate system. So, you will have. Can I erase this part? y 1 y 2. See, x 1 y 1 z 1 flapping is coming out of plane, this is flapping. This I call it beta. But onces it comes up, that is x 2. That is why x 2 and z 2. This is z 1. z 1 and z are same. y 1 and y 2 are same. And now this is beta is the angle, time rate of change is the angular velocity, beta dot. Now, I have to convert this into in the coordinate system of, what I will write it as you put it omega, I am writing directly. Because please note I have put omega here. Omega is omega, that omega comes here. Omega sine beta x 2, omega cos beta z 2. Now, I know r p, I know omega. So, I can calculate my velocity. Velocity of the point p is… This is what and here it is.

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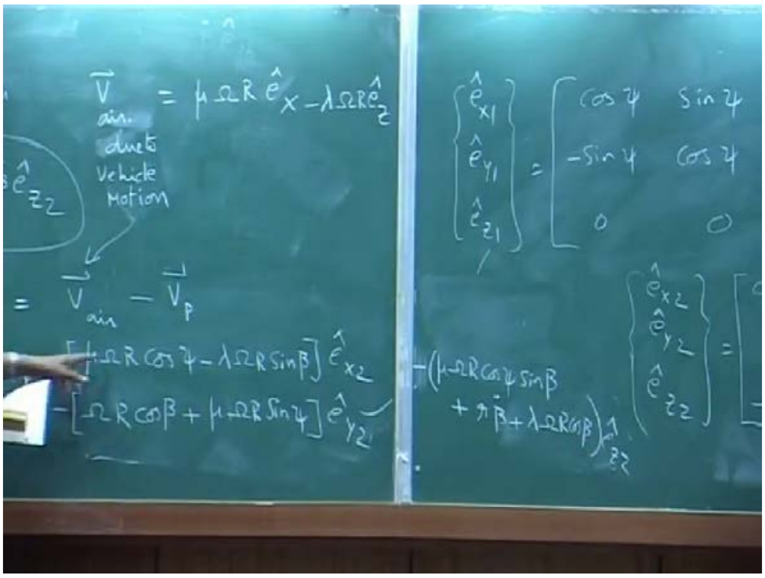

Now, what is v p? Velocity of the point p is x 2 is 0, y 2 omega r cos beta into e y 2. Now, this is the velocity of the body. Now, what is the velocity of air? Minus of this. But you have to add velocity of the helicopters also. See, helicopter is now going forward. So, I will have these are the two components which are due to the vehicle. So, I will write the vehicle velocity.

So, I erase this part now. Now, these are all not necessary. Due to blade. What we are writing now is, it is due to vehicle and inflow. This is (( )) velocity that is all. Now, this is due to vehicle velocity or you may hub velocity actually, the origin. Because velocity of the origin you have to take, because the hub is moving and the blade is rotating. So, you have to add the velocity of the hub also into that. So, that is what we are getting here. But here I am directly writing instead of hub velocity, I am putting it in terms of air. This is body motion, this is air. But if I want total, then I will subtract this from here.

Now, this and then your lambda. Lambda is in the z direction downward. So, I will put minus lambda omega r e z. Now, you see these two are in x and z. So, you substitute that, get it in this, again put it, get it in this, in x 2 y 2. So, you go through that full transformation. So, you will get, I will write the final velocity of relative air at airfoil at p or you may call it point p. This is nothing, but what? You take this velocity and you subtract this. So, that is this expression minus v p. This is what your final velocity expression is. But you can write this whole thing. Now, I will put it here. What is v air? Mu omega r bar in the x direction that will have a cosine psi and then it will have a minus sin psi. Then again you have to substitute here and then get it. So, that whole expression will become mu omega r cos beta. So, I am carrying on this here. Plus, not plus actually it will be a minus sign. Minus mu omega r cosine psi sin beta plus r beta dot plus lambda omega r cos beta e hat z 2. May be I will write it again. So, you see here mu omega r cosine psi minus lambda omega r sine beta e x 2. Then minus omega r cosine beta plus mu omega r sin psi, this is along y 2. And this is a minus sign, mu omega r cosine psi sin beta plus r beta dot plus lambda omega r cos beta, this is along e z 2. Is it clear? You have now three components of velocity. You go ahead and you substitute and get it. You take that as a exercise. Now, I will write that and just briefly mention then we can, onces I write the velocity expressions. I am going to make approximation now. And that is what is the key because you make approximations. What kind of approximation I am going to make? Beta that is the flap angle is small, the first approximation.

So, when I make beta small, sine beta is beta, cosine beta is 1. Then I also make another assumption. So, I will put v relative air. The first expression will be mu omega r cosine psi. I am going to like this, minus lambda beta omega r. Then this is minus omega r, cos beta is 1. Sorry this is not uppercase r, I am sorry, this is a lowercase r, I am sorry about that. Please change that. That is not r, because this is a lowercase, please that location.

Omega r cosine beta is 1 and then plus mu omega r sin psi e y 2. And then minus lambda omega r because I am writing this expression first. Then r beta dot, then this expression plus mu omega r cosine psi into beta. Please understand that is a beta mu omega r cosine psi. Now, I will briefly show the diagram. And here again I make one more assumption. Lambda is also small. So, I say beta is small, lambda is also small. So, the product of, I am neglecting.

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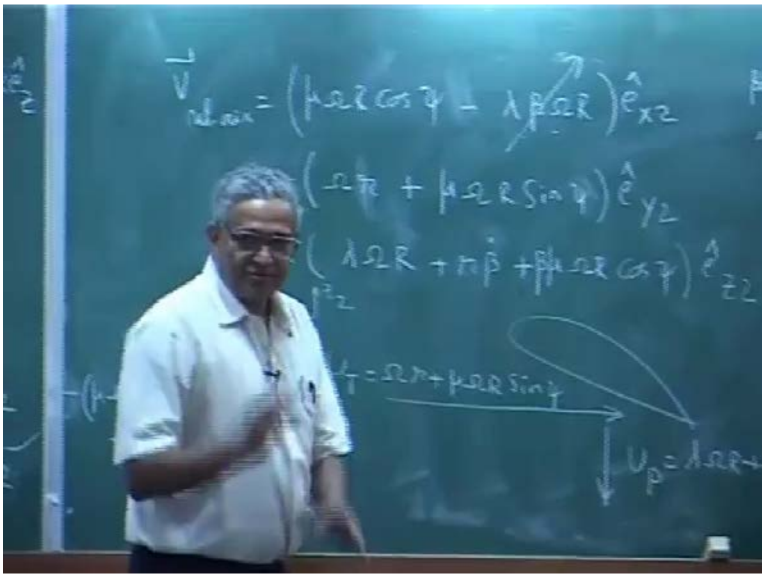

Now, you see I have written the expressions. Please understand this is along x 2. Along x 2 means along the span of the blade it is going, that is this. You call it radial velocity. But y 2 is, it is the velocity which is coming towards the leading edge, minus sign is there that means, it is coming towards. Because y 2 is forward, minus means. So, your airfoil if you take, this is my u t tangential, which is omega r plus mu omega capital R sin psi. This is that velocity and the cross section. And this is z 2 with a minus sign that means, it is coming down, so I will put it this is u p. Because the sign is already taken because z 2 is up. So, u p is lambda omega r plus r beta dot plus beta mu omega r cosine psi.

Now, you have u t, u p, that means, and u r, which is the radial velocity. Because please understand the radial velocity is going into the board. Because you now knew that this is my y 2 axis, z 2 axis, x 2 axis. So, this is y 2, x 2 is going into the board, this is z. You have velocity component and you say my theta because as usual I am defining my theta which is rotation of the blade pitch angle only about its elastic axis, which will happen about 25 percent chord. So, I define this is my theta. Now, you got it. This is what I have shown here. We will continue next class.