

## Introduction to Helicopter Aerodynamics and Dynamics.

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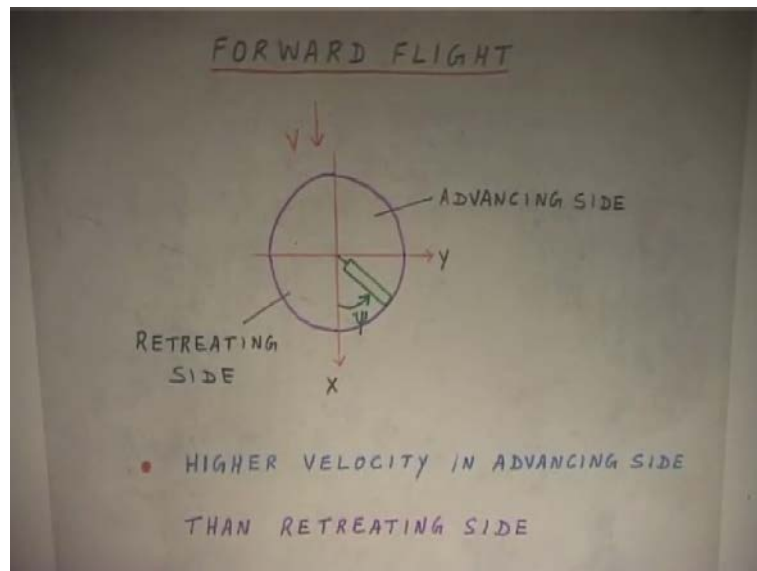
Department of Aerospace Engineering

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Module No. # 01

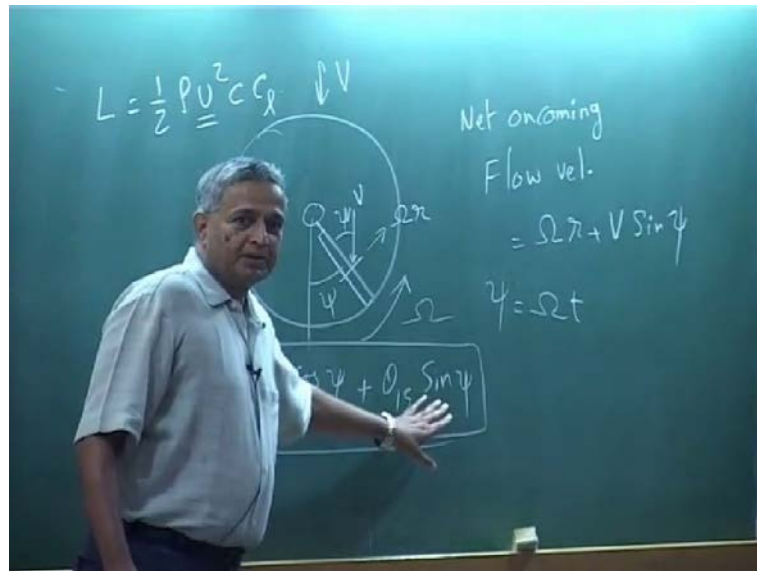
Lecture No. # 10

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We will directly start. This is what I was mentioning to you last class, that from now on we will be dealing with forward flight. See the forward flight has more complexities particularly with regard to the aerodynamics, and because of that, the structural motions are the blade motion occurs, and that again influences the aerodynamics load. So, as the result this problem becomes as you can say that aero-elastic problem. So, you cannot treat forward flight. So, we have essentially divided the disc into 2 regions; 1 region we call it advancing side, that is because the oncoming flow and the rotational velocity add together.

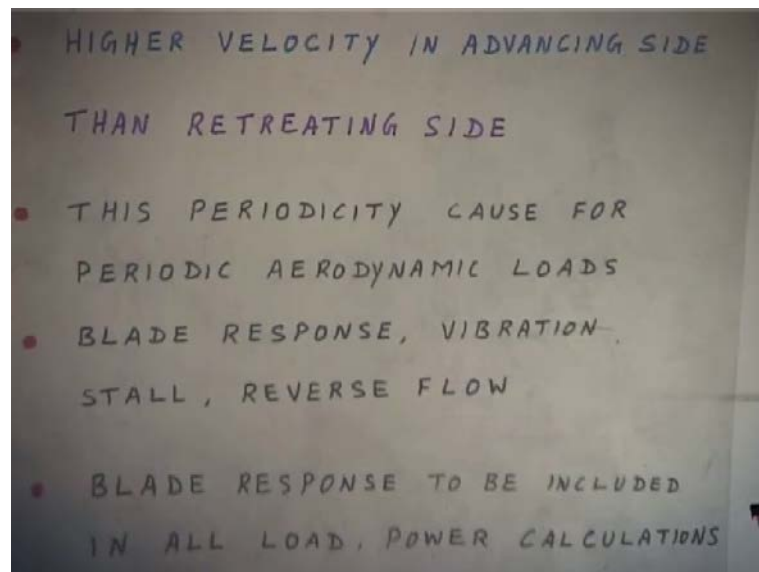
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So, if you look at **the** just a simple expression for the velocity this is omega and this is my oncoming; that means helicopter forward flight. And please remember this is on the rotor disk. So, slowly you will add more and more complications. Now this angle is psi always it is measured from the rear this is a standard from the rear of the disk. And at any section you will have omega r this is the rotational speed but, this velocity is here this is psi this is also psi. So, net on coming flow velocity is omega r plus V sin psi omega r plus v sin psi.

Now, you know that psi is varying because psi is omega t. So, immediately it tells me my oncoming velocity is time varying. But, what is the frequency it is basically the rotor angular velocity omega t. So, we always say the variation this is a terminology once per revolution that is called 1 per rev because the variation is once in a revolution this is the 1 per rev; that means, it represent the rotor angular rotor velocity. Now, this if you use the simple aerodynamics like last time, I told you lift per unit section you take half rho u square card c l lift per unit area. This q is this, of course there are other components immediately you will see this is a square. So, you will have second harmonic sin square psi which will become cosine. Now, you see slowly you are adding time varying quantities in the system the moment you have time varying quantities you will start having....

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That is why, this periodicity cause periodic aerodynamic loads and that will cause blade response, because you have a time varying, load time varying, load acting on a beam it will make it vibrate.

So, blade response is the cause of that and this will cause vibration. Now there are other two aspects which I have added stall and reverse flow why the blade should stall. Blade means not the entire blade will stall, some sections will stall. Why they have to stall is because you see in this disk advancing side my velocity is additive; that means oncoming velocity is more.

So, if the velocity is more I am going to get more lift when I get more lift one side is having a higher lift, the retreating side is having a less lift you have to balance that balancing is done by changing the pitch angle of the blade.

So, theta you remember if we started earlier, collective  $1 - c \cos \psi + \theta$   $1 - s \sin \psi$ . Please note this is my blade pitch now you see what is your  $c_l$  lift curve slope into angle of attack the angle of attack has this as the component; that means,  $c_l$  is also time varying you see now  $c_l$  will have if you simplistically say this is the 1 harmonic.

Now,  $u^2$  you multiply you'll have higher harmonic more and more. So, you'll have more and more components coming in. now that is why you see the swash plate mechanism which is introduced in the blade is essentially to change the blade pitch angle

once in a revolution please understand that is why 1 per rev you do not change 2 per rev 3 per rev.

Later we will see there is something called a higher harmonic control that will come may be towards the end of the course just for a description that part now you see this is varying now by in the advancing side I make this angle small usually it becomes negative .

But do not bother this is small, and on the retreating side I increase this in the sense increase means this component when I do that collective is always fixed suppose if this angle is negative that is  $\theta_1$  as is minus what will happen advancing side it will subtract retreating side  $\sin \psi$  negative. So, that'll add. So, you find I increase my pitch angle on the retreating side.

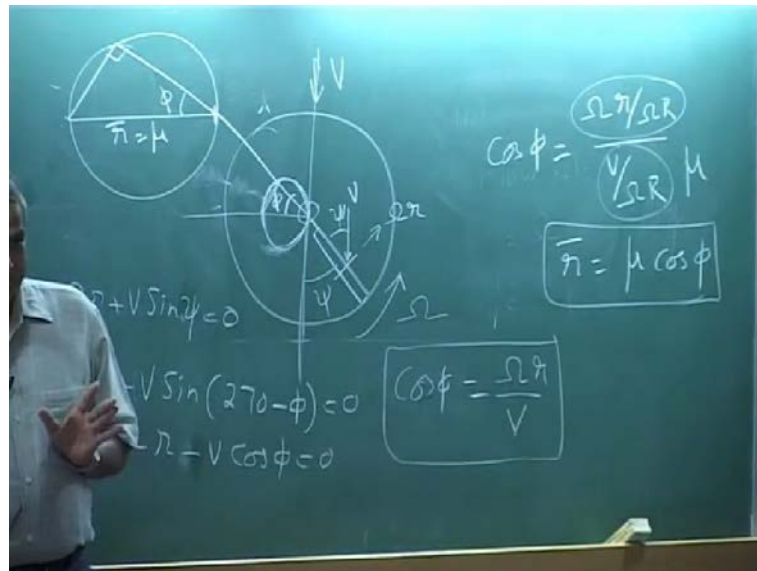
That is again in once in a revolution you are deliberately changing the pitch angle once in a revolution and you will find at some particular speed you may start having because the pitch angle required may be little bit more.

So, you find some sections will start, but, that stall is not that it will suddenly drop the stall will happen and again when it comes the flow will get attached. So, it is like flow detachment attachment detached flow attached flow this is what will happen in that and that is why it is called not static stall this is actually a dynamic stall condition.

Then you have reverse flow reverse flow I will just mathematically show it here that is this  $\omega r v \sin \psi$  on the retreating side  $\omega r$  is coming this way  $v$  is coming this way. So, they try to subtract because  $\sin \psi$  is greater than 1 eighty degrees.

So, it'll have a minus sign. So, some regions the flow will be coming this is  $\omega r$  please understand this is because of the motion this is  $\omega r$  this is  $v \sin \psi$ . So, if  $\omega r$  is small the flow will come from the trailing edge towards the leading edge and that is called the reverse flow region.

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Now, that region can be shown mathematically to be a nice circle that will be like this is a perfect circle this is reverse flow how the circle is made is we have to the boundary of the circle this is  $\omega r + v \sin \psi = 0$  that is the boundary beyond this  $\omega r$  is large below this  $\omega r$  is small now you take this angle to be  $\phi$ . Now  $\psi$  is two hundred and seventy minus  $\phi$ .

So, you will have  $\omega r + v \sin$  please understand I use the same symbol  $\phi$  do not get confused this is just for denoting then this is nothing, but, because  $\sin$  two hundred and seventy is minus 1 and cosine  $\phi$ .

So, this will become  $\omega r + v \cos \phi = 0$  and cosine  $\phi$  is  $\omega r$  over  $v$  right. So, you will write no is there a plus sign or minus sign, sorry I am sorry. So, cosine  $\phi$  is right is it correct is it right or wrong check it.

Now, in a right angle triangle you will find cosine  $\phi$  is nothing, but, this divided by this if we take a circle from 1 end because this is the diameter if you take if you know that this is always ninety degrees and this is my  $\phi$ .

So, this gives me the condition for the boundary and that boundary is nothing, but, when  $\phi$  is 0  $\omega r$  is  $v$ ; that means, this point that is the radial location where  $\omega r$  is equal to the oncoming that is the helicopter velocity that becomes your diameter and then this is when it is that another angle this is the radial distance.

So,  $\frac{\omega r}{\phi}$  represents actually  $\frac{\omega r}{v}$  represents this is nothing, but, a right angle triangle therefore, this becomes a perfect circle and the reverse flow region if you want to non-dimensionalize that is normally done because it is never written in this fashion I will just show you divide by capital  $\omega r$  then what will happen  $\omega$ .

This is  $\bar{r}$  you call it approximately I am saying approximately this is a new symbol that is used which is  $\bar{\mu} r$  is  $\mu$ ; that means, you immediately know the reverse flow this is  $\bar{r}$  this ... this which is nothing, but, non-dimensional velocity

Now, you see the reverse flow region actually will increase with forward speed because if  $v$  increases  $\mu$  is increasing; that means, this region as you keep on increasing your helicopter velocity that will become bigger and bigger and bigger on the on this side retreating side.

So, you know that you will get the lift only when the oncoming flow is towards the leading edge not when the flow is from the trailing edge now that region is getting reduced as you keep on increasing your speed of the helicopter the reverse flow region will become bigger and bigger... bigger because it'll occupy large.

That is this is one of the reason I will not say this is the only reason this is 1 of the reason why helicopters cannot fly very high speed because you cannot make the entire retreating side non-generate lifts because please remember the flow is highly complex.

In this zone the flow is coming like this here it is coming like this the net little flow net air velocity. So, you have stall problem you have reverse flow problem. So, what will happen is if this region becomes bigger and bigger you need to generate lift here

This means you have to increase the pitch angle; that means, if you keep on increasing the pitch angle you may stall. So, you see there, these are a restriction that is why helicopters cannot fly even if you put a powerful engine it just cannot fly at any speed this is only one of the reasons I am not saying this is the only reason because there are several reasons.

Because what will happen if you keep increasing your helicopter velocity advancing side velocity is  $\omega r + v \sin \psi$  and that velocity is also increasing; that means you are increasing the mark number in the advancing side at various sections.

So, as we keep on increasing Mach number you may get into transonic zone you may start having higher drag and there are some drag divergence everything because the drag as it will be up to some Mach number it will be it'll not change much then suddenly it will start going up; that means, you will start having drag divergence increased drag; that means, you need to have more power.

So, these are all the problems which you start facing in forward flight now we have expressed the basic complexities I would say the basic complexities we have not gone to inflow yet we are only looking at the flow in the rotor disk inflow comes next because inflow is a very important.

That is why usually it is mentioned these are complex I will show one diagram then later next. So, because of all these, one of the critical things which are blade response to be included in all load and power calculations.

These are important because blade response you cannot avoid we have not talked about blade response here till now I am only purely from what we know  $\omega$ ,  $r$  and  $v$  that is all what do I mean by blade response.

A blade is a beam the beam can bend up and down beam can bend this way also and then it can have elastic twist only these motions of course, centrifugal force is pulling please understand that is always there.

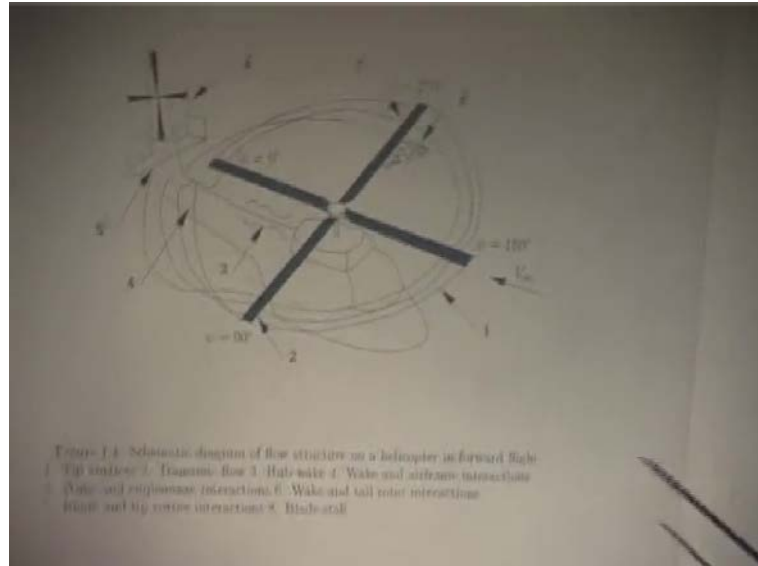
So, you have these motions are blade motions. So, you have to have a model for the blade that is I would call it a structural model very simplistic. So, we will start that whole thing as we go along the course I will introduce each aspect.

But simplistically please understand because it is still a very complex problem. So, how do people treat this at the preliminary level now we have to have a model for the blade number 1 please understand

Now, how do we idealize? So, far the course we will idealize in some particular fashion highly simplify please understand, but, meaningful it is not that you simplify to a large extent that it becomes meaningless you simplify which is reasonably meaningful and then we need to get inflow.

Inflow is another complication.

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So, I will show the next thing what is the general flow is the picture is right I will just put what all I said as a the blade schematic diagram of flow structure of helicopter in forward flight the blade will start giving vortices.

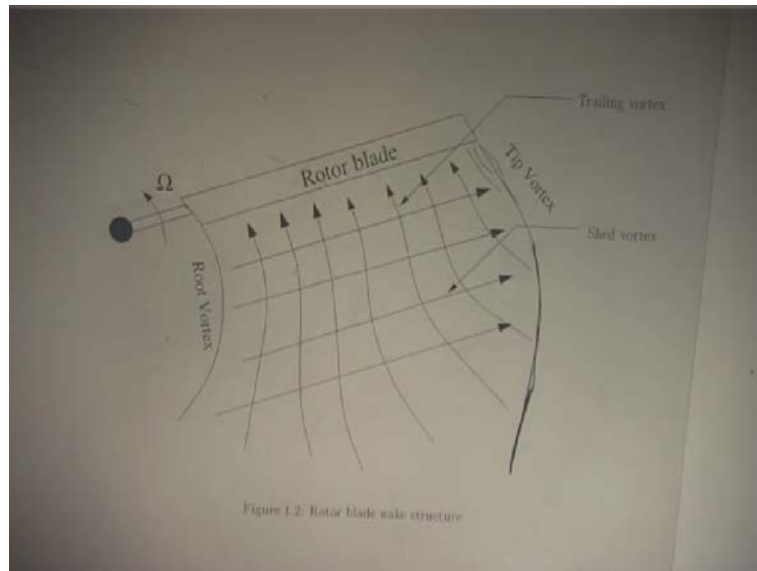
Because vortices is basically disturbance you know lift you have vortex coming up and then they will come they will hit the blade which is coming behind the in the sense they will all come interact this is they call it blade vortex interaction, and then at the tip you will have a transonic flow then the hub is there hub and then where the engine deck and in forward plate that will start giving its own disturbance to the flow and this flow will all go into the tail and the wake means basically the disturbance from the rotor and the fuselage they hit the airframe and then wake and then tail it will interact.

So, you will have all sorts of aerodynamic complexities please understand now you cannot treat everything even today. So, what if you could do in industry of course, the work is still going on people will keep on doing it is a very complex structural dynamic aerodynamic interaction problem.

We will make it highly simplify how do we make it simple is I have first some more complexities I will show this is just to tell you the wake which some of the models which people use in their studies.



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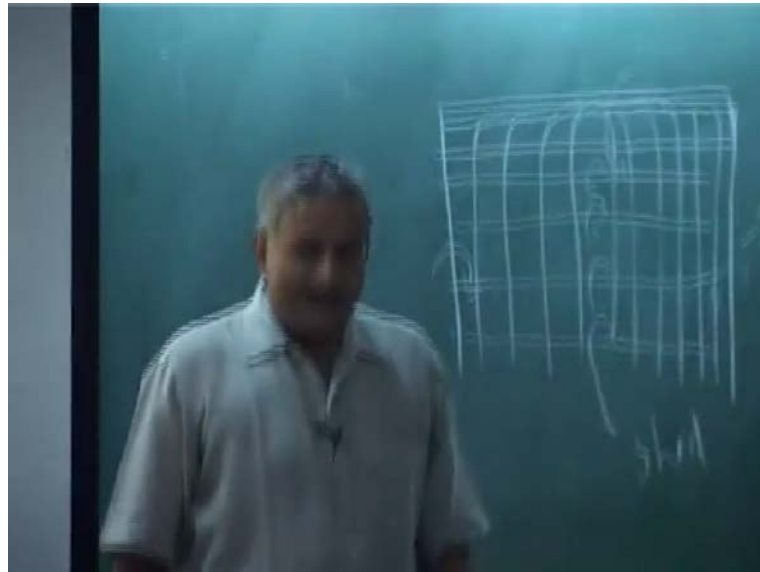


This is the rotor blade and you will have tip vortex you will have shed vortex shed. So, there are two words please understand trailing and shed these are two different vortices trailing vortex shed vortex if the circulation is constant then there is no span wise variation of the circulation on the blade.

So, you have your horseshoe vortex which you have learnt in your aerodynamics course that will come out from the tip that is your lifting line theory it says that my circulation is constant comes out that is called the trailing because it trails and that vortex will be like this.

In the sense usually I will draw from the point of view of clarity suppose you take a simple wing just a wing.

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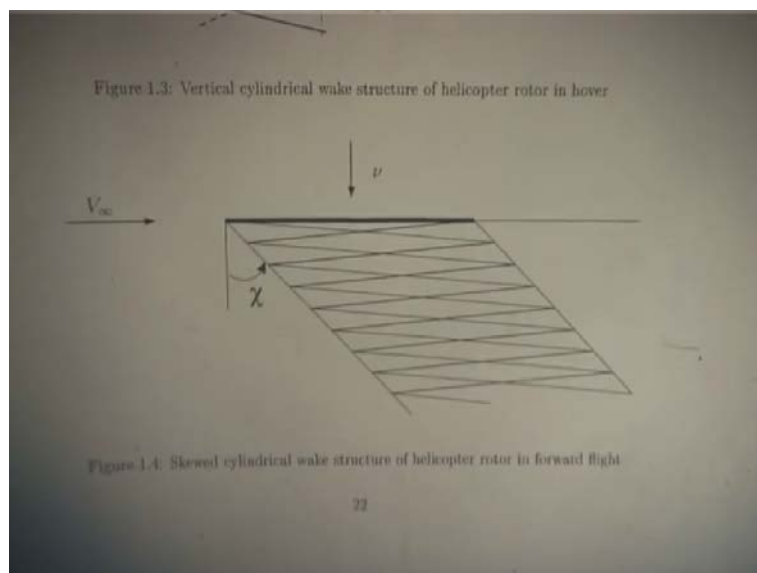
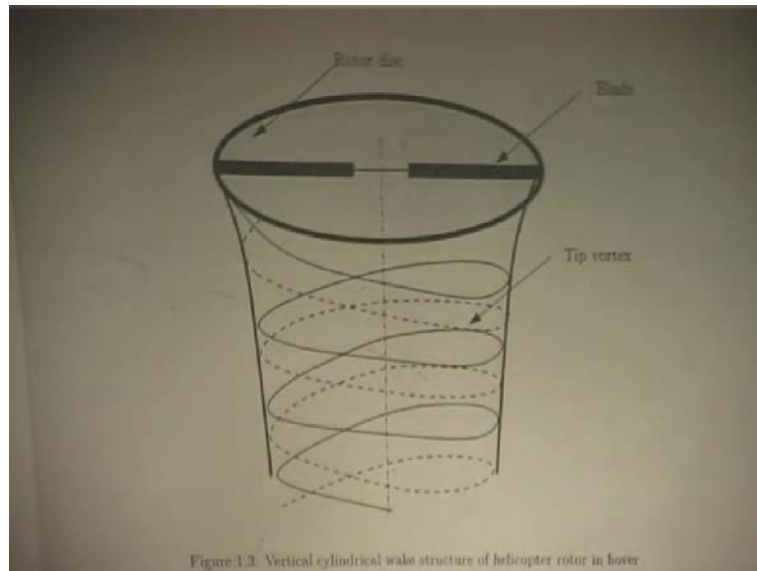
This is this is trailing, the trailing will be in this direction, this is trailing.

Then what is shed is suppose if this circulation varies with time please understand if it is a span wise variation is there you will get the please understand this is constant gamma you will get only here suppose if the gamma is the function of span you will get what like this you will get all of them are trailing this is span wise variation of circulation suppose if circulation is the function of time then you will have like this these are shed.

So, you have shed vortex you have trailing vortex, but, usually thee are lot of assumptions made for the simple first I will say I do not bother about the shed vortex good I will only worry about as though the load is uniform only tip is coming out it is the uniform loading case.

This is how assumptions are made and based on that assumptions they try to get the effect of that wake on the inflow which is very approximate, but, then people will update their models to include complexities one at a time.

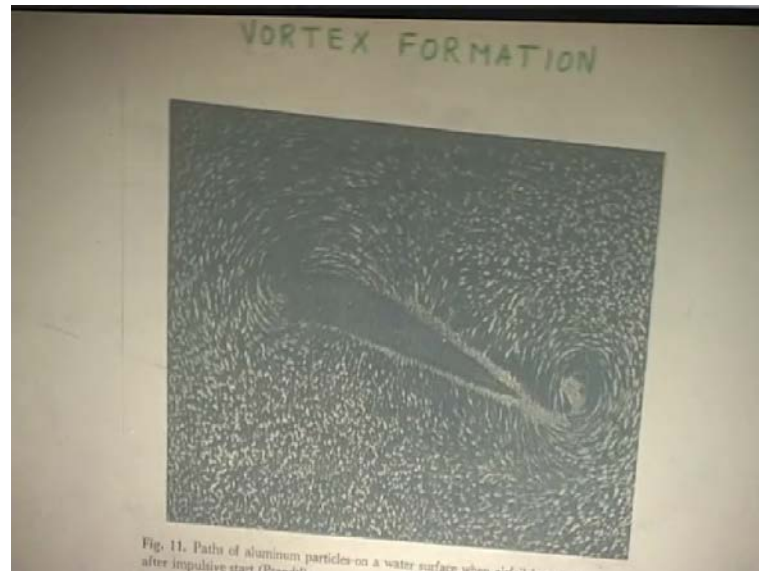
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So, I will show some pictures of the approximations that is made is a (( )) circulation is constant it does not vary with span please understand this is what you called uniform loading; that means, what you will have only the tip vortex.

So, the tip vortex in the case of hover it'll come below because it is push down only tip vortex I do not consider anything else, but, when you go to forward flight please understand what will happen is the rotor disk is not stationary it is moving when it starts moving whatever it leaves that wake is going back and that will go we will take it like this although this is pushed backwards and I have indicated  $\chi$  angle which is phi.

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That is actually wake skew angle that is just to indicate this how my wake is there. So, this is called the wake skew angle do not worry too much about this because this will come later when we try to describe certain things.

This is wake just and the I am taking although this is a cylinder skewed cylinder and on that face I have the vortex now I may use actuator disk theory please understand actuator disk says what I have infinite number of blades.

That means I will have a normal number of blades my wake in hover it just a cylinder in the case of forward flight it becomes a skewed cylinder now the skewed cylinder gives its own problems people have derived this with a skewed cylinder how the inflow is calculated we will not get into that part now I am just wanted to give you the complexity involved in this.

Now, I will show a few pictures from purely aerodynamic thing whatever I said because this is interesting to have a look at this pictures because then you will understand the wake this is a 1 picture see this is a when you start an aerofoil because it is there is no lift when you start means what your velocity is increasing that is why the vortex theory whatever we've that a lifting line theory which you call it is a very powerful technique.

Because as an observation when the aerofoil starts from rest there is no circulation that is why we called it irrotational field those of you were familiar with fluid mechanics and

this irrotational field then what happens a vortex starts from the trailing edge and this is the starting vortex.

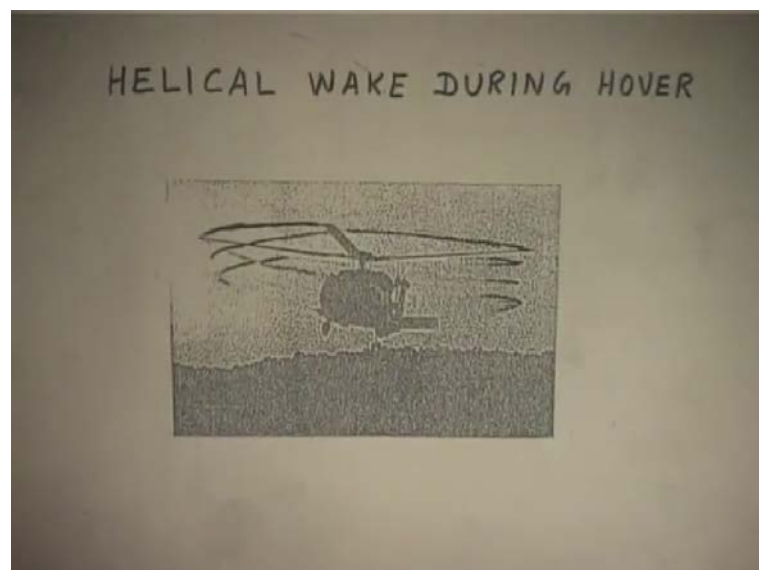
You see the direction is like this and there it should be another bound circulation which you will come over the aerofoil 1 is the shed now this is what is shed you understand.

Because initially there is nothing. So, it is shed when you shedded what will happen because of the oncoming velocity in aerodynamic thing this gets taken back and the effect of that vortex on the aerofoil is you say zero. So, I have only a bound circulation that is why you say gamma the lifting line theory you would draw what simply take gamma rho mu gamma only thing is you have to this is.

The effect of all you study this only in getting a induced angle induced angle this gamma that is why I am saying when you start you will have 1 when you stop you see 1 is this another 1 is this.

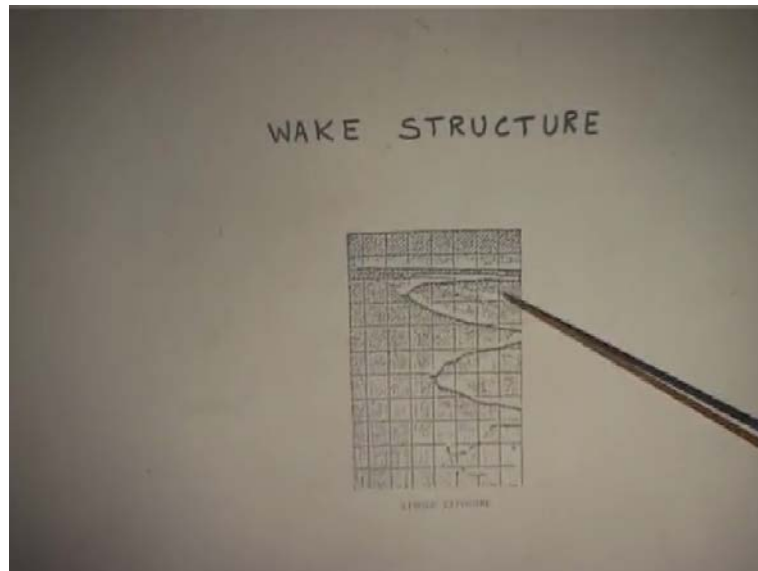
Now, there is no more that gets shed. So, when you start and stop this is the picture. So, when the lift is also varying you will have all these things coming out, but, this is attached flow please understand once you have detach flow things are much more complicated now if the show few more pictures see when a helicopter is hovering this is taken an I think in morning dew or something like that.

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So, you can see distinctly they tip vortex in a hovering helicopter see there coming now these also there are some pictures which are actually made in flow that is how wake structure looks this is on a lab

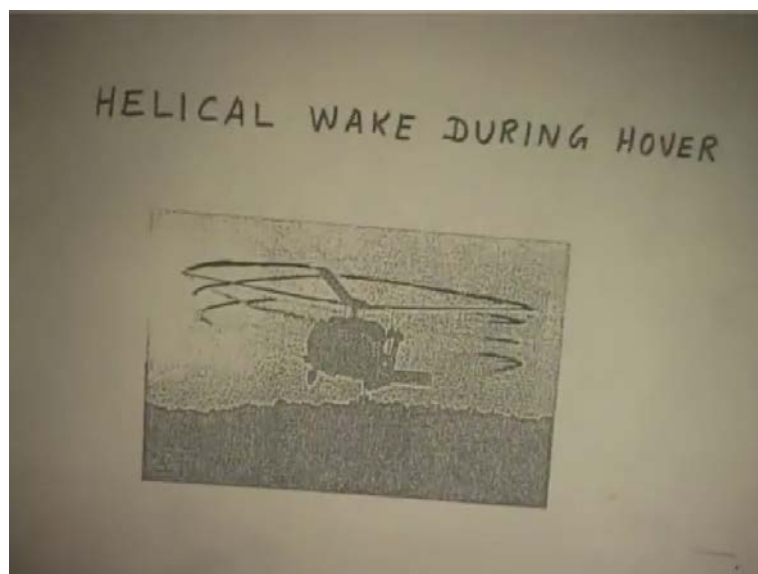
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See this is a blade how it sheds and how the.

Wake contraction these are all flow I would say visualization after that you use a theoretical model now the same wake structure which I showed here if I put it.

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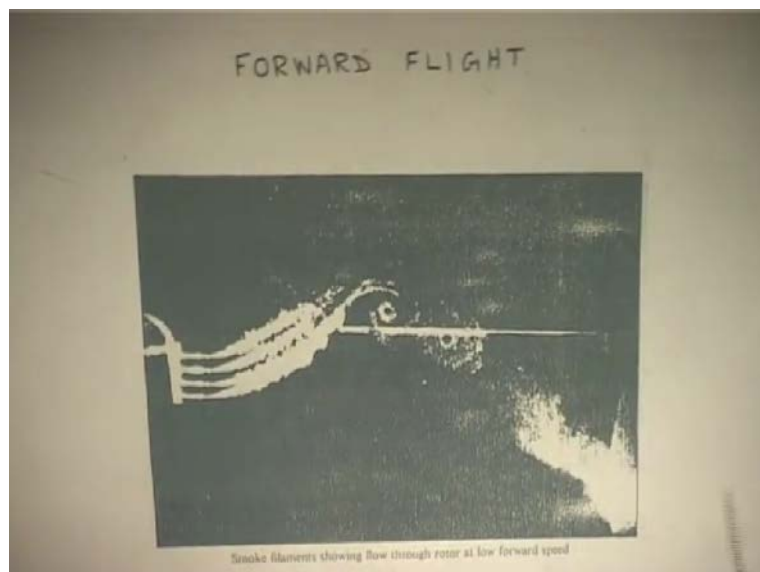
This is in the hover case you may start moving in forward flight. So, they again took some pictures that is a very interesting picture I will show that diagram it was actually a flight test is a flight picture the bottom 1 you see the

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flow because something was push some smoke how the smoke goes in and then goes back it is not push to down and the similar picture which is shown on a small scale thing the.

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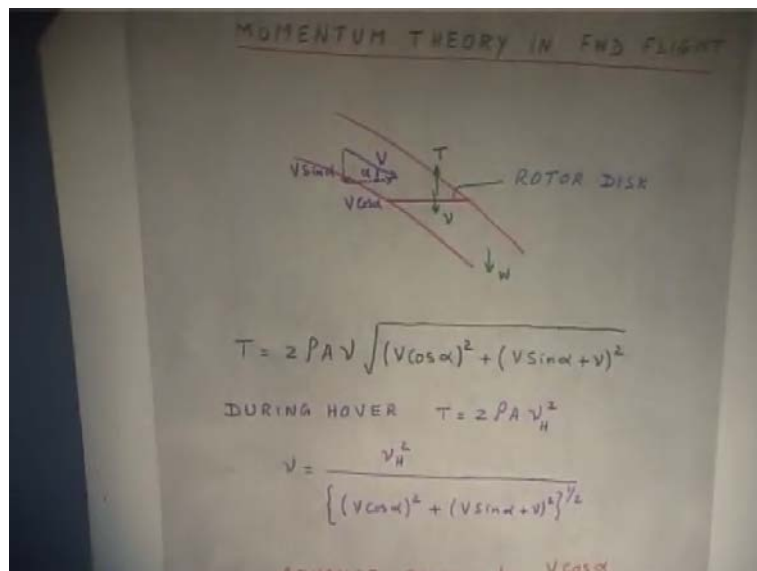


Smoke is given the flow is in this direction please understand and.

This is rotating now how the vortex which gets released from the tip in the front how does it move if you see it moves up and then comes inside you follow now all these are complex phenomena, but, you see the reason I wanted to point out the top 1 is that is that is 1 of the reasons till 8 is there results between theory and experiment the quite far away this 1 this 1 this is the rotor is white stick white line see the vortex the flow goes like this it is not it goes what we assumed is a cylinder like this you assume you saw that, but, actually it goes up and then down now you need to capture all these things which is still the difficult task.

So, there are models which are use to approximately capture please understand these effects. So, in this course what we will do is I will introduce briefly some of these approximations which are used and the gross approximation is what we will use in our calculation and. So, now, with this a small background on the flow in forward flight because forward flight is a highly complex problem very simple I want to get the inflow at the rotor.

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I have very complex flow, but, I am going to say hey that is not worry about those things I want like you said in hover you applied the momentum theory and you predicted the inflow through the rotor disk similarly, you apply the momentum theory to forward flight, but, please understand, but, I will not write all those various things this is all

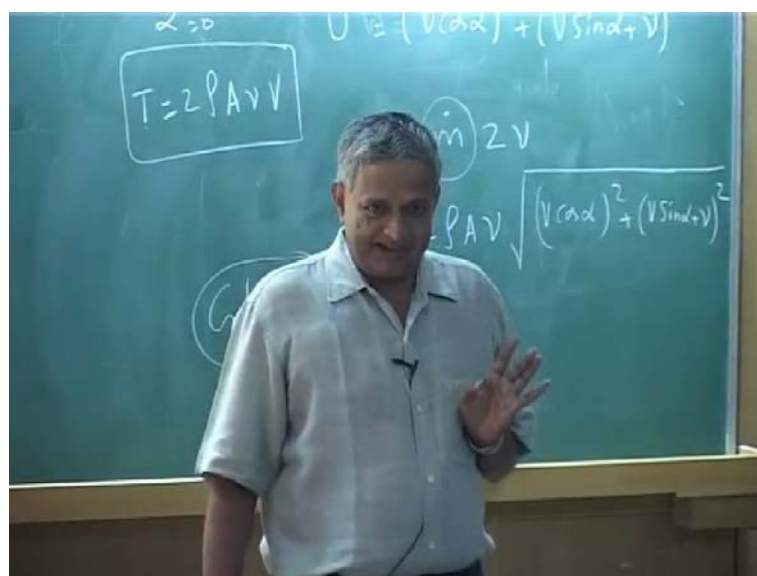


simple extrapolation of what was there in the hover momentum theory because some proof here it is only from analogy to fixed wing this is where the ingenuity or whatever it may call it. So, there is a lot of approximation even in this now what is that approximation how the inflow.

Simple as you this is my rotor disk my rotor disk for the sake of a general the disk is moving forward and up please understand it not just going only horizontally it is also flying up. So, you will have that is the reason why I gave the this is the velocity of the oncoming flow please understand the velocity of the oncoming flow to the rotor disk the rotor disk is not moving like this rotor disk is moving like this up now this is you divide this into component 1 parallel to the rotor disk another 1 perpendicular to the rotor disk.

So,  $v \cos \alpha$   $v \sin \alpha$  and then I put thrust because the helicopter is holding this is a steady velocity there is no acceleration it is moving steady thrust is up because you cannot support the weight of the helicopter. So, therefore, for supporting that that should be an inflow, but, that inflow is actually normal to the rotor disk normal to the rotor disk because I am pushing they are down. So, I am going up again simple far field you take it as  $w$  and that  $w$  you take it momentum theory tells you  $w$  is  $2 \mu$ . So, whatever happens, the inflow at the rotor disk the induced velocity at the rotor disk  $w$  is 2 times that value that is all. So, these are all from I am just please understand.

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Extrapolating from hover momentum theory now what is my thrust is momentum theory tells we  $\dot{m}$  mass flow rate into change in the flow  $2\nu$  the change is  $2\nu$  initially 0 here it is change to  $2\nu$  now what is  $\dot{m}$  dot  $\dot{m}$  dot is flow through the rotor disk mass flow rate, but, this is where the approximation.

$u$  where  $u$  is because you always normally you think that the velocity because this component  $v \cos \alpha$  is parallel to the disk what is flowing through the disk is  $v \sin \alpha + \nu$ , but, you take in defining mass flow rate this is based on glauert he said that use  $u^2$  as.

now you may say why question will come the why how what comes from the limits of this expression that is whatever I showed here the limit of that expression when  $\alpha$  is 0 and  $\alpha$  is ninety when  $\alpha$  is ninety what happens it is a climb  $\alpha$  is 0 is high speed flight.

High speed flight of some disk because these are all actuator disk mind you this is not a rotor system because the momentum theory assume it is an actuator disk in the high speed flight I have to get an expression please understand this is how the whole logic goes for the lift which should look because you know from the lifting line theory for a elliptic wing and horizontal flight you will get the lift you get the drag induced drag expressions please understand that is how it was bought in I will explain to you later how that whole thing came now you see you are getting an expression if you.

Use let me put it that way. So, I will write it a  $\rho a$  sorry  $u \dot{m}$  is all right. So, thrust is I will write it thrust is  $2 \rho a \mu v \cos \alpha$  whole square plus  $v \sin \alpha$  plus  $\nu$  square  $a$  is a area of the disk if  $\alpha$  is 0 right 0 means I am flying like this then this will become limit  $\alpha$  is 0.

Which a very high speed flight and you can represent this entire expression because  $\nu$  is small because  $v$  is very large  $\nu$  is small. So, this whole thing will be just  $v$  and the root of  $v$  and the root of  $v^2$ . So, that becomes  $v$  high speed flight  $\alpha$  0 thrust is  $2 \rho a \mu v$  high speed we will try to get that from fixed wing theory then you will see wow this is the reason it is put it suppose if you say I neglect this  $v \cos \alpha$  from this whole expression.

High speed flight what will happen this goes up right alpha is 0 this is done this is only nu square that will be nu. So, you will have 2 rho a mu square this is same as hover when alpha is ninety or a hover is v is 0 you find that that comes like a hover theory it does not make sense that is the reason glauert that is why he wrote you take this expression please understand now you see my inflow is constant over the rotor disk it is a constant it is not varying along any radial location.

Is it clear now in the straight vertical flight yes it matches when alpha is ninety this term goes this is v plus nu you put it here that is nothing, but, what you got for hover. So, the 2 extremes in hover it matches in high speed forward flight how does it match. So, that I will just briefly before I go I thought I will non-dimensionalize these quantity these are the new expressions of the same velocities we define please understand we define v cosine alpha over omega r as mu which is called advance ratio.

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The image shows handwritten mathematical derivations for rotor performance metrics. At the top, the thrust equation is given as  $T = 2 \rho A v \sqrt{(V \cos \alpha)^2 + (V \sin \alpha + v)^2}$ . Below this, the condition for hover is noted as  $T = 2 \rho A v_H^2$ . This leads to the equation  $v = \frac{v_H^2}{\{(V \cos \alpha)^2 + (V \sin \alpha + v)^2\}^{1/2}}$ . The advance ratio is defined as  $\mu = \frac{V \cos \alpha}{\Omega R}$ . The total induced velocity is defined as  $\lambda = \frac{V \sin \alpha + v}{\Omega R} = \mu \tan \alpha + \lambda_i$ . Finally, the induced velocity  $\lambda_i$  is expressed as  $\lambda_i = \frac{C_T}{2 \sqrt{\mu^2 + (\mu \tan \alpha + \lambda_i)^2}}$ .

and then the other term which you call it as the total induced velocity because I would like to use it total induced velocity because it has 2 components 1 is due to v s sin alpha that is due to the flight itself another 1 is the induced component. So, this we will call it lambda which is a sometimes you loosely use in flow ratio. So, you can call that as lambda is v sin alpha plus nu over omega r you may take this term itself.

You divide by coos alpha divide by sin alpha then this sorry divide by coos alpha then v coos alpha over omega r will become mu. So, you will have mu tan alpha plus lambda I.

So,  $\mu \tan \alpha \lambda I$  now directly go back to the original expression thrust is you know  $c t$  and you will see how do I get inflow because in the hover case it is square root of  $c t$  by 2 here you will have you use this expression non-dimensionalize divide by  $\rho$  you will get that expression of  $\lambda I$  will just briefly write that because you can non-dimensionalize  $\lambda I$  becomes  $\lambda I$  is nothing, but,  $\frac{v}{\omega r}$  this you obtain from the this expression here from here you divide everything. So, this is  $\frac{t}{2 \rho \text{ area}}$  right because you have to divide another  $\omega r$  also. So, you will have  $v$ , but,  $\frac{v \cos^2 \alpha}{\omega r}$  you will have a  $\omega r$  if you want to non-dimensionalize what you will do you will divide by.

$\rho \pi r^2$  is the area of the disk and  $\omega r$  whole square. So, you will also divide this by same  $\rho \omega r$  whole square. So,  $\rho \omega r$  will go up 1 of the  $\omega r$  will go inside another will cancel out with this leaving behind now your  $\lambda I$  becomes  $c t$  over 2 what is that there is a  $2 \sqrt{v \cos^2 \alpha}$  is  $\mu$  square this is  $\lambda I$  square or you may write it as.

$\mu \tan \alpha$  plus  $\lambda I$  whole square now you see this is my inflow now this is not that easy to calculate because if you want to calculate first of all you must know  $\mu$  is advance ratio now you understand this is what we use as a terminology in this course advance ratio advance ratio means loosely forward speed by tip speed please understand loosely forward speed by tip speed because you were neglecting that cosine alpha, but, precise definition requires  $v \cos \alpha$  over, but, normally alpha is very small angle. So, it does not matter you take it as forward speed by  $\omega r$  now.

This is the  $\mu$  which I mention earlier in getting the reverse flow diagram it is a same  $\mu$ . So, if you increase your now the question is if you want to get this you need to know  $\mu$  you need to know alpha; that means, the orientation of the disk with respect to the oncoming flow that you must know and the next part is this is a iterative procedure  $\lambda I$  is here that is what you were getting here now this is not very easy to obtain you have to write a small code because you have to solve it iteratively whereas, you see in hover it is very straight forward now forward flight the moment you go.

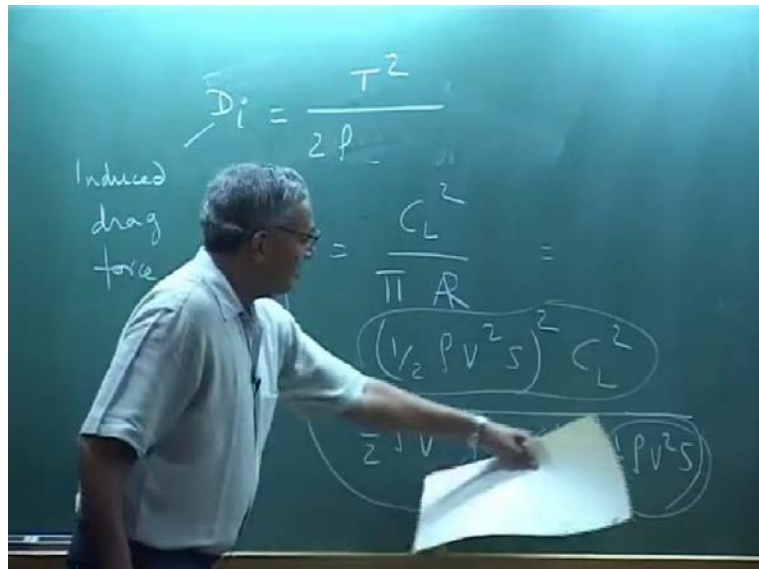
Forward flight all come and this inflow is a gross approximation of what really happens on the rotor disk because you have assumed that it is constant over the entire disk please understand it is constant it is uniform everywhere nothing, but, this is a gross

approximation and this is still used it is not that this is not it is still used even the research level because this gives pretty decent result of course,, we add more complexities to that rho a mu into v right at what speed it is valid is a we will see it is more than I will say non-dimensional point 1 1 5.

Onwards you will find that is fairly all right mu advance ratio point 15 is fairly good point 1 I will show the diagram I will prove it, but, first of all you have to know how you got this I thought I will this is a very interesting thing because 1 is you professor says take this everybody to get that is 1 thing another 1 is a because somebody says you take it as you take it you do not take it you ask questions why then you say 1 end it should be valid any expression you derive it should valid at least at the extremes in between. So, the extreme case of hover it is valid hover and vertical flying, but, in forward flight how. So, that comes from essentially. So, it is not there.

It is in my notes I will show you this is a small which an interesting thing I will go there is it.

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See you know fixed wing theory tells you induced drag co efficient c d I because fixed wing please understand I am directly jumping to this is you all know pi aspect ratio because those of you have done aerodynamics you'll you will know this if you have not done take it because then you can read a aerodynamics.

Book and then you say oh for a elliptic wing loading say minimum because this is the minimum induced drag that is what elliptic loading all those assumption then you get this value now what you do is you take this you multiplied by what is  $c_l c_l$  is nothing, but, lift coefficient of the wing please understand this is a finite wing theory this is a finite wing theory because aspect ratio is there you remember this are now you multiply both side by half  $\rho v^2 s$  whole square  $c_l^2$  square divided by you do that half  $\rho v^2 s$  of the values I am taking.

$\pi$  aspect ratio is you know that if it is a  $b$  is span. So, you will have this is aspect ratio and then another half  $\rho v^2 s$  I am just now this term is nothing, but, the force which is I am going to write it as maybe I erase here because the board is I will take 1 of these term take it here if I take this term and then put here half  $\rho v^2$  area.

$C_d I$  that become the drag force induced drag force. So, I am going to call it as  $d I$  induced drag force this is nothing, but, I am using the helicopter terminology thrust because area into  $c_l$  is a thrust  $t$  square and in the denominator it is going to have what half  $\rho$  this  $s$  will cancel out with this  $s$  you will have what  $\pi b^2$   $b$  is the diameter. So, what you do is you divide by 4 multiplied by 4 when you do  $\pi b^2$  over 4 is area of the because  $b$  is the span that is  $2r$ . So, that is going to be the area of the disk. So, you will have and the 4 this will become  $2\rho$  area  $v^2$  is it clear now my drag force induced drag force is this expression what is my power, Induced power. So, I let me erase this part.

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What is induced power induced power thrust into nu this is also equal to d I into because d I is the induced drag force into the velocity. So, d I into v now you want to get what is nu what is the induced velocity you equate that is nu becomes t sorry d I v over.

This is the induced velocity please understand I am relating rotor wing rotor wing kind of a thing for the rotor this is the induced power into nu for the wing. So, this is the induced power because this is induced drag into the velocity you are dragging it to the forward flight now I want mu is d I v over t now you know here d I over t you can substitute here d I is d I over t will become what t because over 2 rho a this we will cancel with the 1 of the v right.

So, you will have in other words thrust is high speed you follow. So, you see, but, this is nice jugglery, but, what we started off with a large aspect ratio minimum induced drag etcetera, but, the aspect ratio of a wing is what aspect ratio of the wing there you take it the large aspect ratio here if you take aspect ratio of a rotor aspect ratio is what rotor aspect ratio will be just b square that is nothing, but, 2 r.

Over pi r square which is 4 over pi which is 1 point I think 2 7 this is aspect ratio of a rotor. So, you say this is too small that is why your inflow that is inflow is that mu cannot be assume to be you know uniform everywhere there will be lot of variation

because the aspect ratio is too small. So, inflow cannot be same. So, it has to vary quite a bit that is all that is all this statement says, but, glauert got this hey this is a nice relationship. So, at high speed the rotor behaves like a wing.

Just a fixed wing you understand, but, with considerable variation in the inflow, but, it is like a wing of that sides is it clear that is why you will replace I will show you some of the results which we have generated here as we go long that was the PhD work this is a very complex aero-elastic you find you use any aerodynamic model as you go to high speed now I will answer your question later as you go to high speed you will find they all come close you may use different models in gross estimates, but, not in loads vibratory loads please understand when you look at the vibratory load it varies with what theory you use.

Then when it is flying that equilibrium at trim condition you will find does not matter that is the beautiful result that is why I am saying this is all from you relate the fixed wing on one hand, but, you have the momentum theory on the other hand from rotary wing and then you simply say hey have to bridge between hover condition to high speed flight and that is the reason the net that is in defining the mass flow rate you add  $v \cos \alpha$  and  $v \sin \alpha$  that is the reason. Now we will solve because I thought I will take next class which will your, answer will become clear because this is a very neat expression, but, only question is. At what  $v$ , you can start using this? You can start using approximately.