Instability and Transition of Fluid Flows

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Lecture No. # 03

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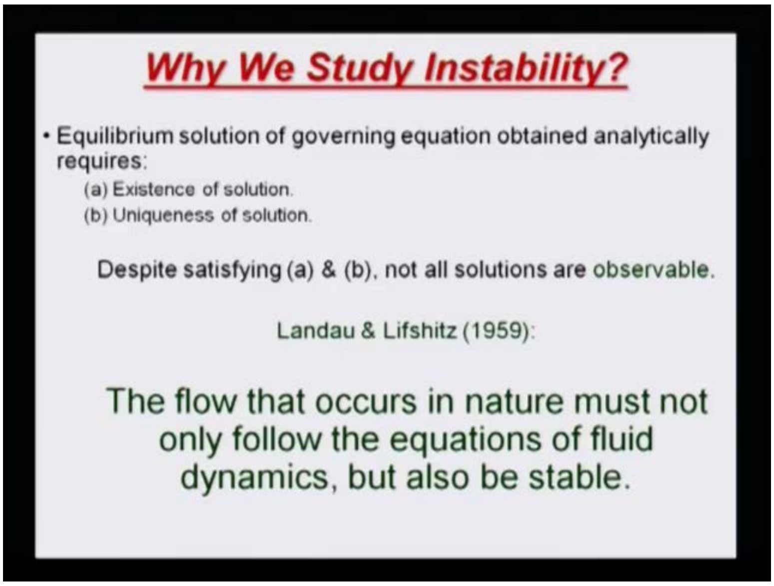

Yesterday's class, we started discussing about instability and receptivites. And what we did was, we focused our attention on Reynolds pipe flow experiments where, he pointed out that the critical Reynolds number could be raised as high as that, by controlling the disturbances. And that made him observe also that this kind of growth of disturbance could be related to non-linear instability, because you need certain magnitude of the disturbance to trigger such instability. So, this was the non-linear aspect which was later on found out when linear stability theory was developed. That pipe flow indeed remains stable to linear growth of disturbance modes. So, that is also true for Couette flow, that Couette flow and pipe flow remains really stable for all Reynolds number.

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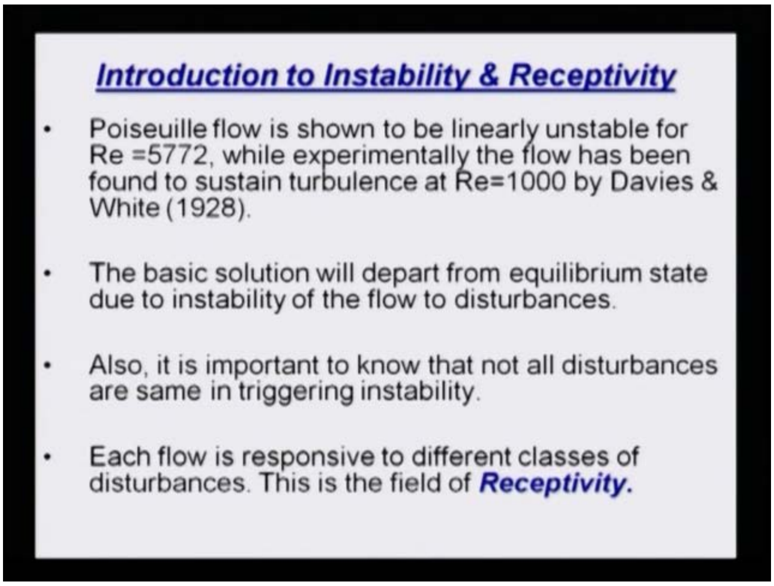

There is somewhat of a difference that you see in a channel flow or what is called as a Poiseuille flow. This shows instability in the linear mode, but the critical Reynolds number that is predicted by linear theory is about 5772, whereas people have done experiment. For example, this paper by Davies and White shows that flow becomes unstable at Reynolds number as low as a 1,000. So, this also should be cited as failure of the linear theory. So, this is one thing that we must understand.

So, we must also point out that the basic solution which we call as the equilibrium solution departs from its equilibrium state to another state because of instability due to disturbance. It is important that, we know what are the disturbances that trigger those instabilities, and that is what is studied in receptivity. So, this is what we studied yesterday.

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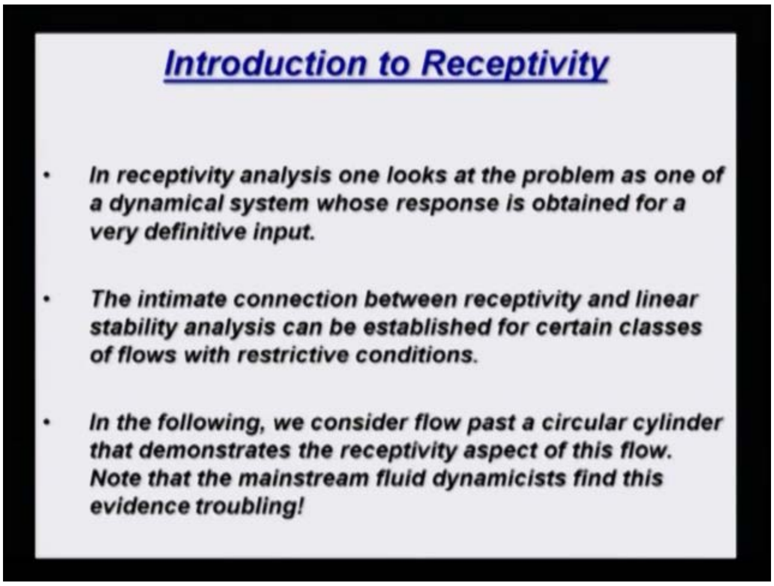

So, let us now talk about receptivity a little bit more. As I mentioned to you that in receptivity studies, you do look at problems as one of a dynamical system and you will find out what are the inputs that triggers the dynamical system to give you those growing disturbance in the output stage. They intimate correction between receptivity and linear stability theory can be established. We will do so provided we restrict our self to some conditions like what we talked about yesterday - being the equilibrium flow has to be parallel, then only you can apply the linear theory, and same thing you can do with receptivity theory, but I must say that receptive theory brings to the table much more things to offer than linear stability theory. That is why we will spend lot of time talking about receptivity rather than stability theory, because once you have the receptivity analysis done, linear stability theory becomes one of the sub cases.

I want to show you the one aspect of receptivity in the context of flow pass to a circular cylinder and this is rather an interesting example. We did this experiment here itself few years ago. We have a very noisy tunnel and we wanted to characterize the noise and what happens? So, what we did was we set up an simple experiment, flow past a cylinder. We kept the Reynolds number free by considering two different cylinders of different diameter so that we change the speed to get the Reynolds number same.

So, if you keep the Reynolds number same and do the experiment in the same tunnel, and then, what do you see? Tou see this and that is what I say that many of the fluid dynamists should find this result troubling, because we keep teaching you in basic fluid mechanics course that similarity parameter if the Reynolds number is kept same, flow should be same and here is an example.

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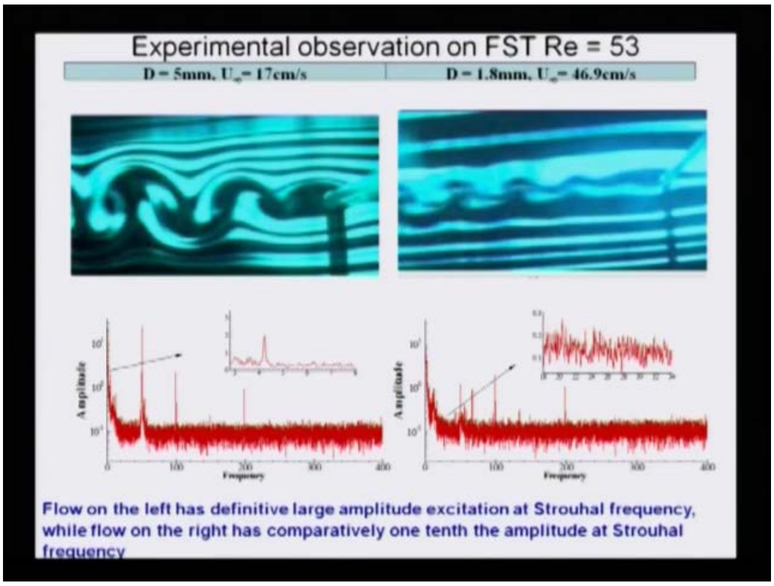

Flow does not remain same. In both the cases, Reynolds number is kept at Re equal to 53 here. For this, we have used a 5 millimeter diameter cylinder and the speed in the wind tunnel is kept at 17 centimeter per second, that gives you a Reynolds number 53, and in this case, we take a thinner cylinder of diameter 1.8 mm and that the speed is raised to about 47 centimeter per second.

So, what happens? Why are these two flows so different? We are working on the same tunnel; we are using the same similarity parameter for Reynolds number. This was at the back of my mind, so, I told our group that let us measure that background disturbance in the tunnel by removing the model. So, for example, if I look at the disturbance level here for 17 centimeters per second in the empty tunnel, this is what I get the spectrum of the noise. So, the spectrum means we plot the amplitude versus the frequency, and what you notice that this inset shows what is the condition near the Strouhal number. We have heard of Strouhal number - Strouhal number is the non-dimensional frequency of vortex shedding.

So, near this Strouhal number, we notice there is a peak, and for the Strouhal number here the other case, we notice also a peak, but if you look at the scale of this corresponding to this, this is ten times more.

So, what happens is if you have the disturbance of the Strouhal number, strong enough to excite the flow. You will of course see a better shedding, that is what you have seen here compared to this. So, you can understand that if I could do an experiment in a tunnel and I can even keep this disturbance level further down at that speed. I could probably not have vortex shedding at all, and to tell you a interesting story, if you look at the book by Bachelor or by (( )), there is a result given by which is sort of attributed to a student of Prandtl.

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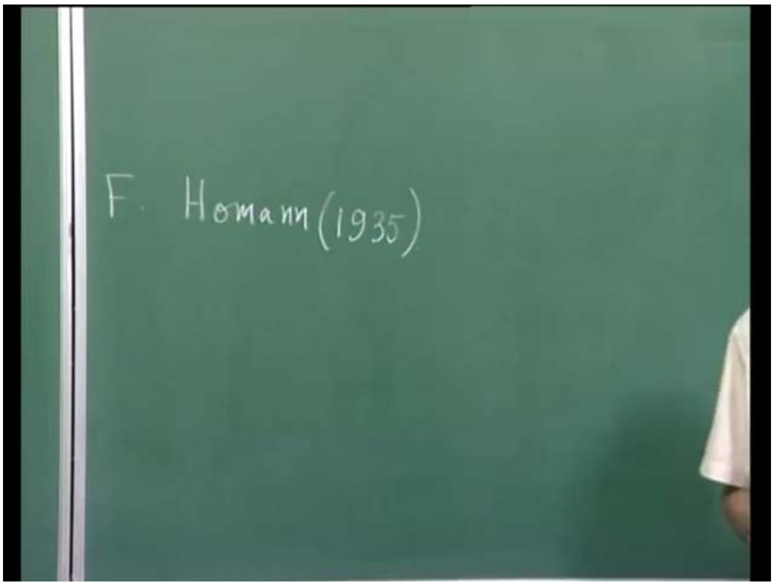

This experiment was done in 1930 and the students name was Fritz Homannn. I think it is 1935, or so, I do not exactly remember, I can give you the reference. He actually set up a very interesting tunnel. Instead of using a wind tunnel or a water tunnel, he constructed a small tunnel which worked on a very high viscous fluid, a liquid which is almost like your lubricating oil, very high coefficient of viscosity, and then, he put the cylinder and he did not see vortex shedding till the Reynolds number was 65, whereas people have done some kind of a linearized stability analysis of flow pass to a cylinder by not making parallel flow assumption, but you linearize the Navier-Stokes equation and you studied stability as Eigen Value problem and people keep telling us everyone that flow should become unstable at a Reynolds number about 45 to 47. However, they are all aware of this Homannn's arc because these are two topmost text books that you can think of in the field but nobody did explain what was happening.

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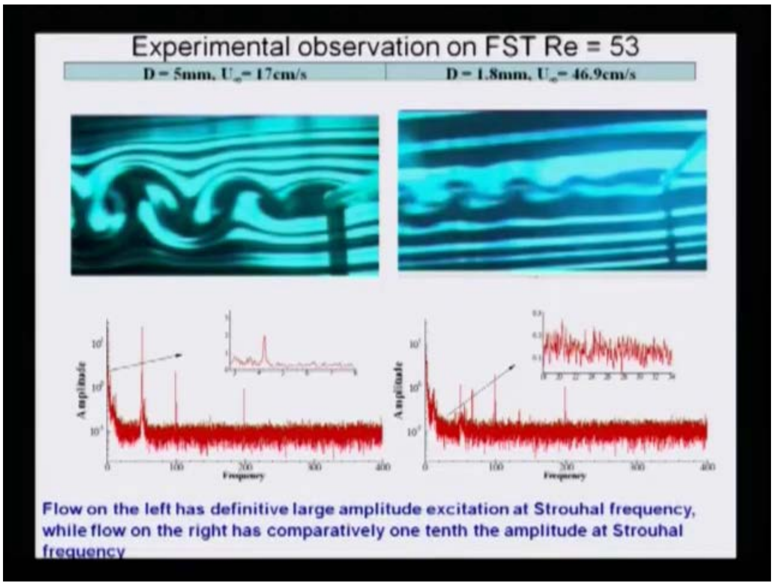

So, in that context, we are also interested in this. Our contention that we put it in a recent paper in journal of fluid mechanics which has just come out. We said that receptivity is the clue if you can stop the background of the disturbance like what Reynolds did for his pipe flow, you can delay in the primary instability and that was what was achieved by Homann.

So, please do understand that what you have studied in your course that earlier course in fluid mechanics that pipe flow becomes unstable at Reynolds number of 2,300 and above, and flow past cylinder becomes critical above 45 to 47. These are kind of myth. We need to study them much more clearly; we need to understand the background disturbance and dynamics is of course important; the transfer function is important. See what that transfer functions tell you that the flow is ready. If you provide the disturbance, it will pick up. See, this is of the order of about 3 on the left hand side. So, this value is roughly about 2 here, while this value is about 0.25 or 0.27 in that range. So, you can now very clearly understand that what you are seeing is a very clear demonstration of a receptivity of flow.

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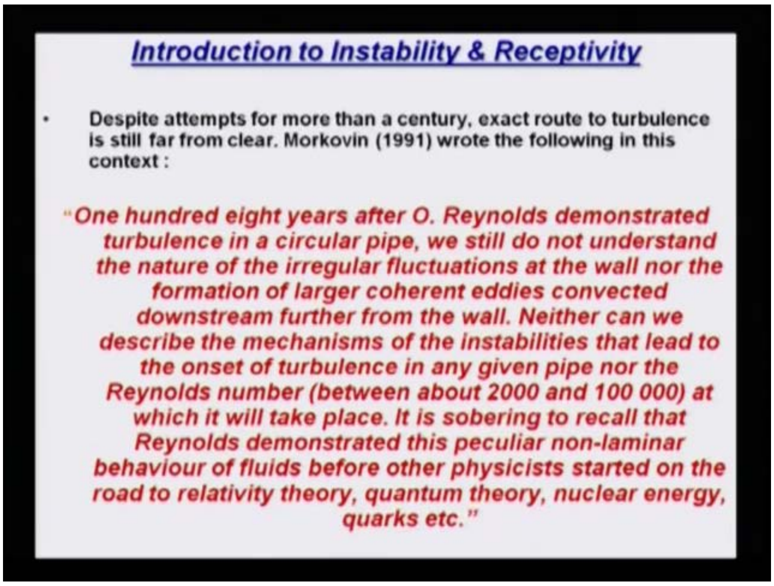

In fact, this led Markovin to the write the following in 1991. That is exactly 108 years after Reynolds famous pipe flow experiments. He said that Reynolds demonstrate turbulence in a circular pipe, and even today, as of that day or I could say even today 2010 twenty years almost down the line, we still do not understand the nature of turbulence that you see in a pipe flow. How flow becomes fluctuating near the wall, and you see those eddies outside the near wall position and those we have no mechanism to explain. So, this is something interesting, because as I told you that Reynolds number can rise from 2,000 to 1,00,000 and you could keep the flow lamina. So, that is what it is, and this is what is a kind of a, sort of a advise that he flayed that it is sobering to recall that Reynolds demonstrated this non-laminar behavior of fluid before other physicist started on the road to relativity theory, quantum theory, etcetera. We have made so much of progress in those areas of physics, but when it comes to explaining turbulence, we are not very much comfortable even today. So, I think we should keep this thing in mind.

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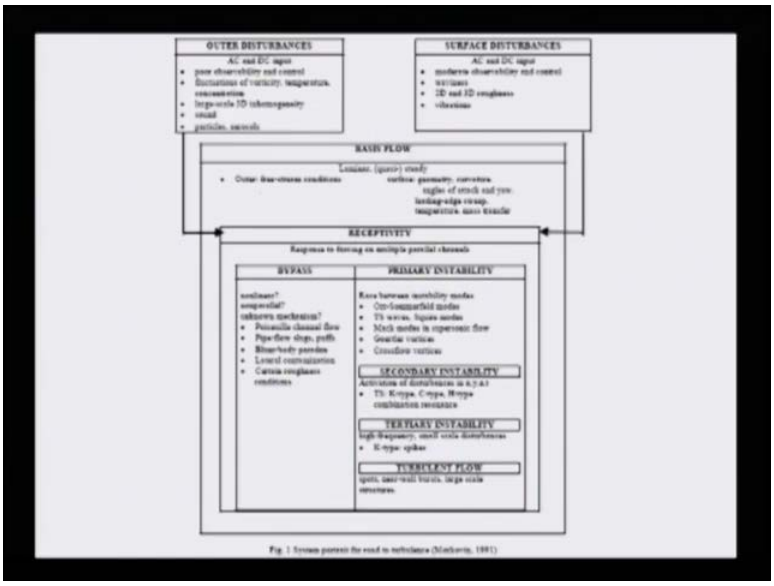

This is basically a portrait as Markovin called; this taken for the Markovin's paper of 1991. He says that this is what you have. You have the basic flow, equilibrium flow, that will determine the transfer function, and then, you could have disturbances; these two blocks are disturbances. Disturbances can be of two kinds - one kind of disturbance could be near the surface. I will send this slide to you. So, you could take a look at it much more clearly.

So, this surface disturbances, that means the disturbances within the shear layer could be time dependent or it could be time independent. So, in this paper, it has been called AC and DC meaning unsteady and steady disturbances. You can have different types of disturbances at inside the shear layer or you could have the disturbance outside the shear layer.

So, that is what is called as the outer disturbance. Those outer disturbances also could be steady or unsteady, that could affect the flow. This aspect is related to the receptivity and kind of scenario that we see. Either we could see those what is given by primary instability theories. Whether you are doing it by linearizing, parallelizing or just simply linearizing the Navier-Stokes equation and solving, that is what you could see or you could get the bypass route. So, these are possible, and if you come from the primary instability side, you could have a secondary instability; you could have tertiary instability and then you end up getting. This block, bypass block has been all written with question mark.

So, in 1991, many of those mechanisms are considered are unknown, but today, twenty years down the line, we did some studies. So, in various parts of the world, and now, we see that we can identify if we have a non-linear mechanism affecting it or nonparallel mechanisms are there, or if there are some unknown mechanisms, I must tell you that in this course, we will show you some of those mechanisms. Today itself I will show you some visual to explain that.

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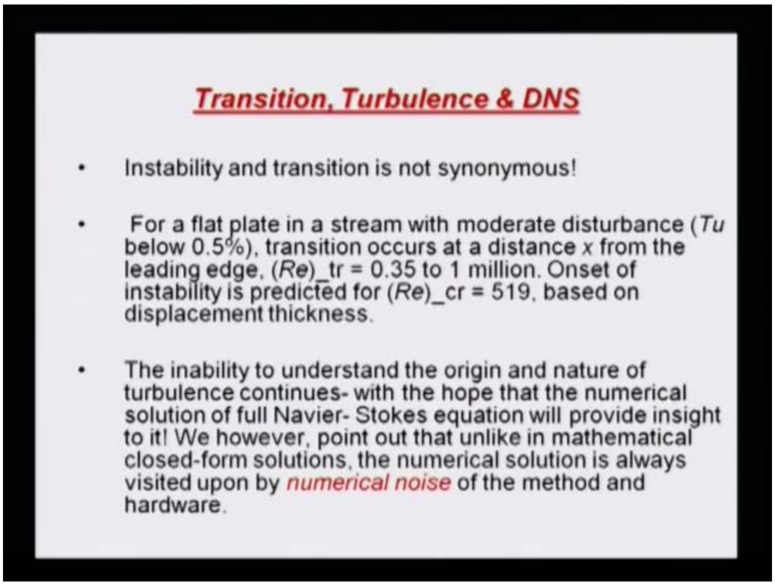

This is one thing that to we discussed earlier also that instability and transition is not synonymous, they do not occur in the same place. You may have a transitional flow. For example, if you look at a flat plate in a flow with moderate disturbance where, that turbulent density is less than 0.5. What is turbulent intensity?

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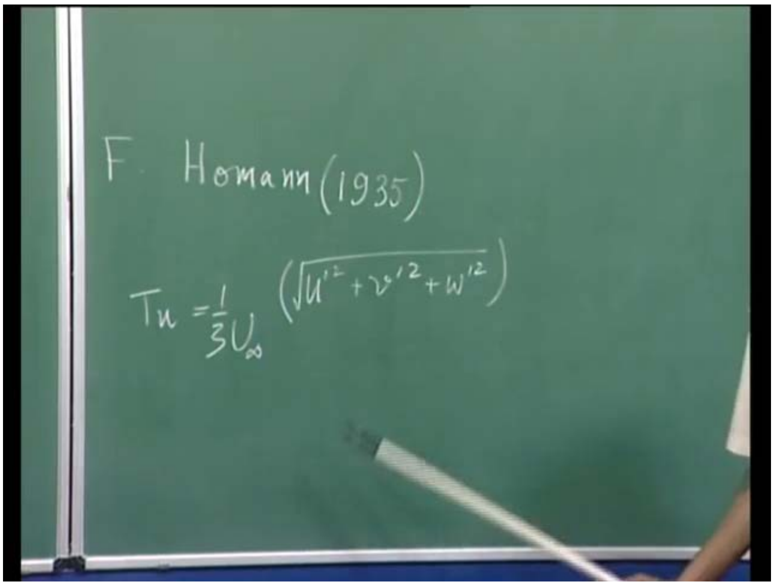

Turbulent intensity is defined as the following that, if we have a uniform flow infinity and then you measure the disturbance quantities, and if these are u prime, b prime, w prime, you look at it and that is your turbulent intensity. So, it is a basically non-dimensional RMS fluctuation of the flow field.

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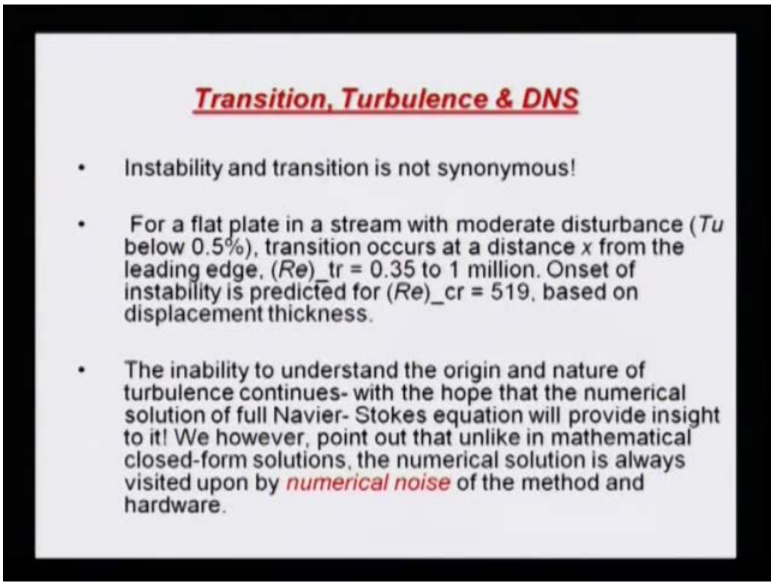

So, if you keep that turbulent intensity less than 0.5 percent, that means 0.005 or lower, then transition occurs at a distance x from the leading edge which is given about 350 thousand to 1 million, whereas on state of instabilities predicted at a Re critical of 519 based on displacement techniques.

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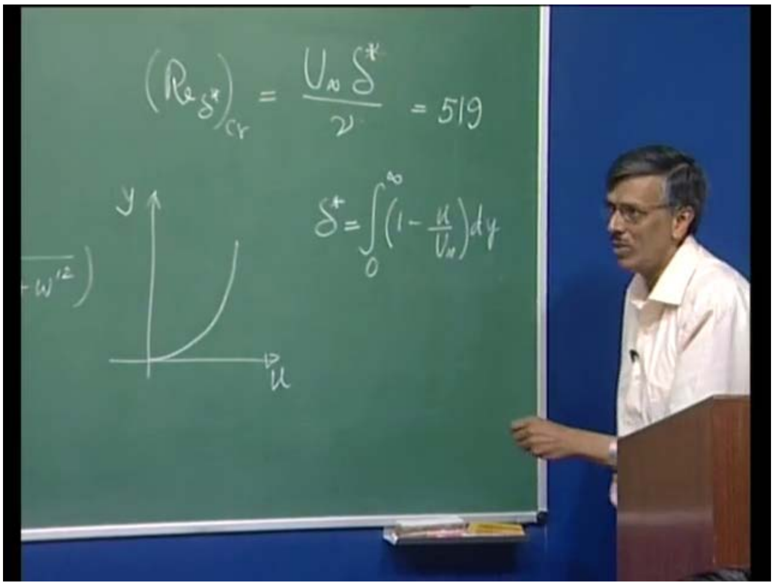

See, this value is rather critical. So, basically Re critical that we are talking about, this is based on displacement thickness. We will define what displacement thickness is later if necessary which is given like this, that is this, and if I have a flow field, if I get the boundary layer growing like this, u as a function of y, then I can get delta star as what? For a incompressible flow, I will integrate 1 minus u y u infinity d y 0 to infinity.

So, that is going to be your displacement thickness. If I use that, that Re critical for a flat plate is actually 519.2 to be precise. So, this is a very well defined value for a flat plate. What do you mean by flat plate flow? This is a flow with zero pressure gradients, like what you get from Blasius profile.

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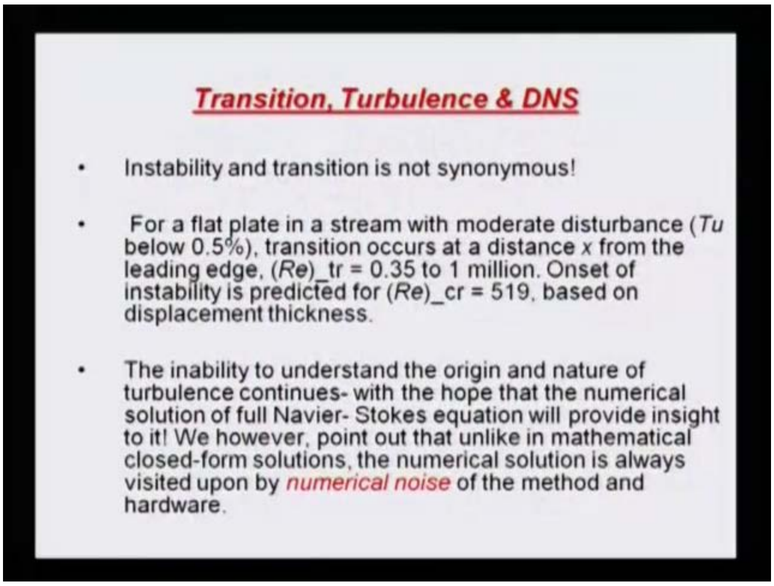

So, if you look at that, that value is definitive, whereas transition can occur. It could be a variable thing; it could be between 0.35 and 1 million.

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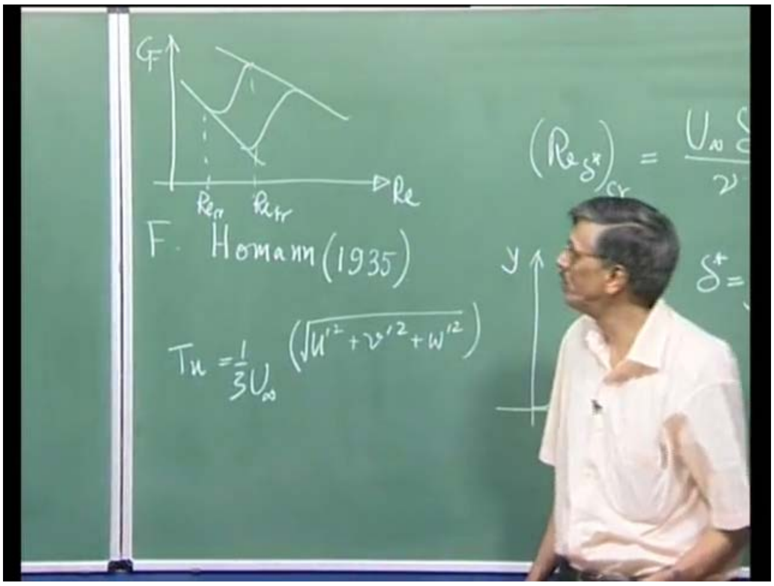

That we also showed in yesterday's class where we plotted Re versus C F. If you recall, we had this. This is the laminar branch of the solution; this is the fully turbulent branch of the solution. Depending on disturbances, you could go like this or you could go like this and so on so forth. So, that is why this is what I will call us Re transition and this I will call let us say Re critical, but this disturbance level itself can be a function of the particular experimental device.

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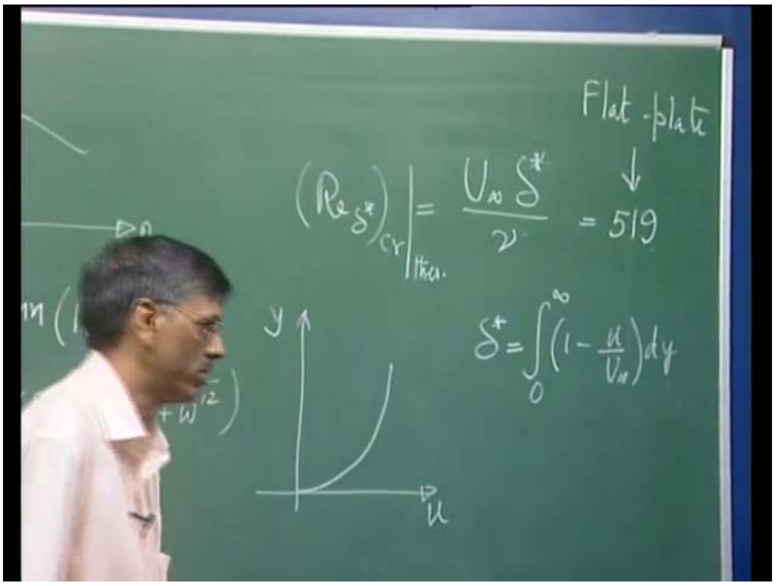

So, this Re critical that here, we are talking that this is experimental, and this is what we are defining here; this is a theoretical. So, this is like your transfer function telling you that the flow is ready. If Re delta star is 520, the flow is ready to become critical. If you provide the disturbance, it would show the signature of those disturbances.

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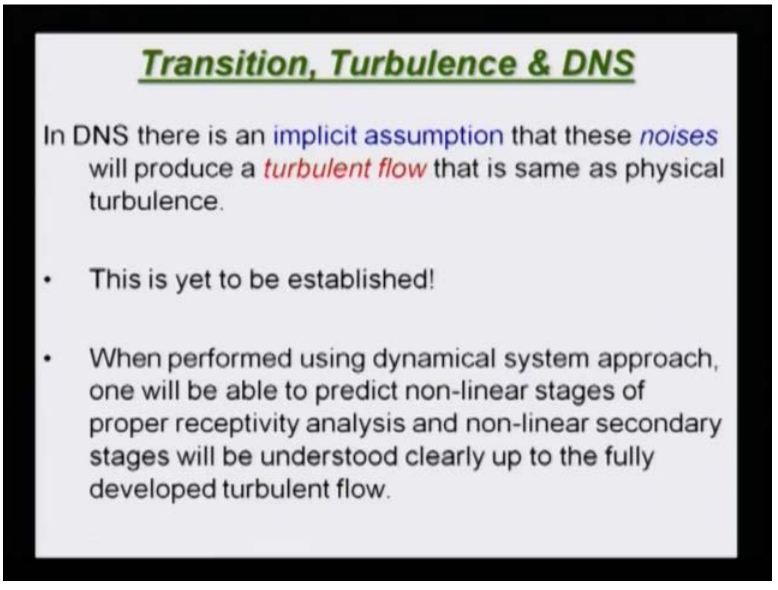

So, what we do is we actually also should understand that we are still unable to bridge the gap in our understanding between the difference of origin and the nature of turbulence. Everybody hoped and continues to hope that if you can somehow solve the full Navier-Stokes equation, you will get the insight to the story. This is a matter of hope you know.

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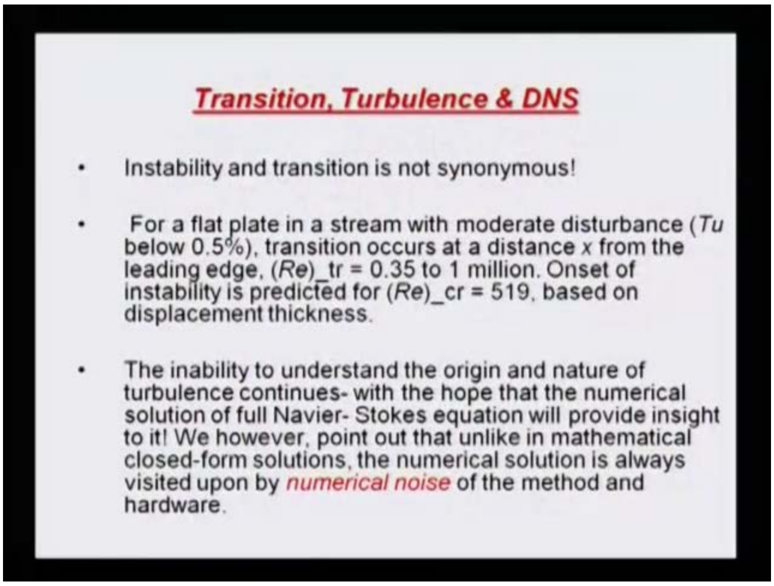

However, I would like to or we should point out that unlike in mathematical closed-form solution, numerical solution is always contaminated by noise. So, whatever we are saying here, the variability of the transaction point and the criticality depending on disturbance, that also may affect the Navier-Stokes solution.

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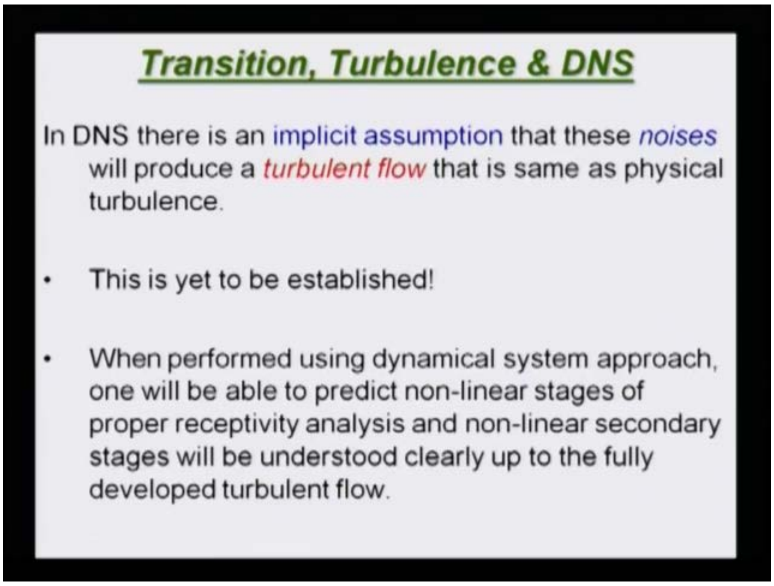

So, this is something we should keep in mind and that is what is being said in the next transparency that, when we are doing the simulation of Navier-Stokes equation without any modeling. So, we are solving the full Navier-Stokes equation without any models. That is what is called as direct numerical simulation or DNS.

So, in DNS, people hope there is an implicit assumption that all kinds of noises that you have numerical noise, the errors we talked about round-off error; we can talk about truncation error; we can talk about various other sources of error. They all will trigger the flow to a turbulent state that is same as physical turbulence, and let me tell you very categorically this assumption has not been shown, and if we hope that, it will be correct.

So, this is more like a hope or belief which is not been proven yet. When actually we saw Navier-Stokes equation using dynamical system approach, we can still study this let us say, for the given background numerical disturbance level, we can study the linear stage, the non-linear stage. We can perform all that. In fact, we will see some such results during the course of 6.2.5.

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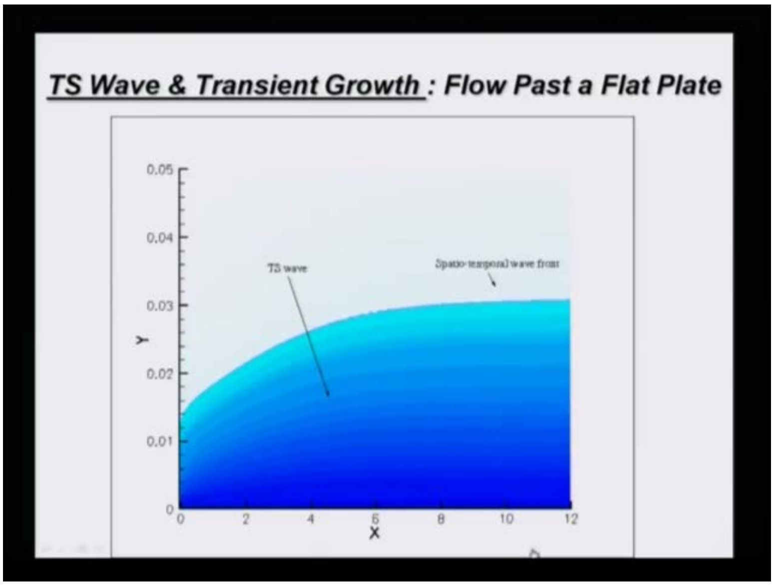

So, this is what we could do. I will show you now something that I want you to see. So, this is the simulation, this is the simulation of the flow. Now, what you are seeing here is this is a flow over a flat plate and there is some kind of an exciter somewhere here and that exciter creates a so called Tollmien–Schlichting wave.

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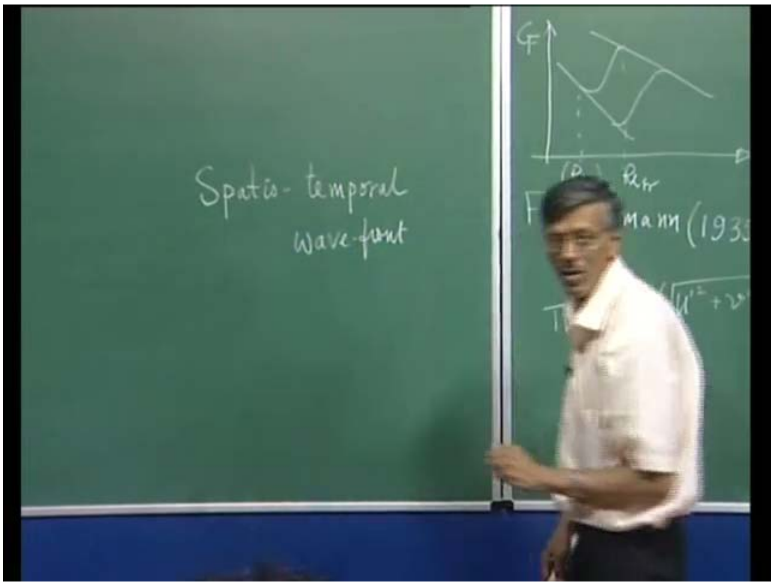

So, those are the somewhere here. So, the TS waves are here, but what you notice? Ahead of the TS wave forming a wave packet, there is something much bigger than that. Let me use this word little loosely those disturbances which go at the front or they are like tsunami kind of a thing; they are only few in numbers - three four peaks and valise, and this is what we called as Spatio-temporal wave front, because these are quantities go like a wave front and their growth rate is spectacular. These growth rates are significantly larger than what you see for the Tollmien–Schlichting wave.

Actually we perform these calculations in all. This is based on Navier-Stokes solution but we also did corresponding study using those linear parallel flow assumption theories, receptivity analysis and we did show how this spatio-temporal wave fronts come about. Even when the flow is shown to be linearly stable, multiple modes can interact, and this what people have also talked about as the transient energy growth mechanism and we have seen that this wave front can really go very high.

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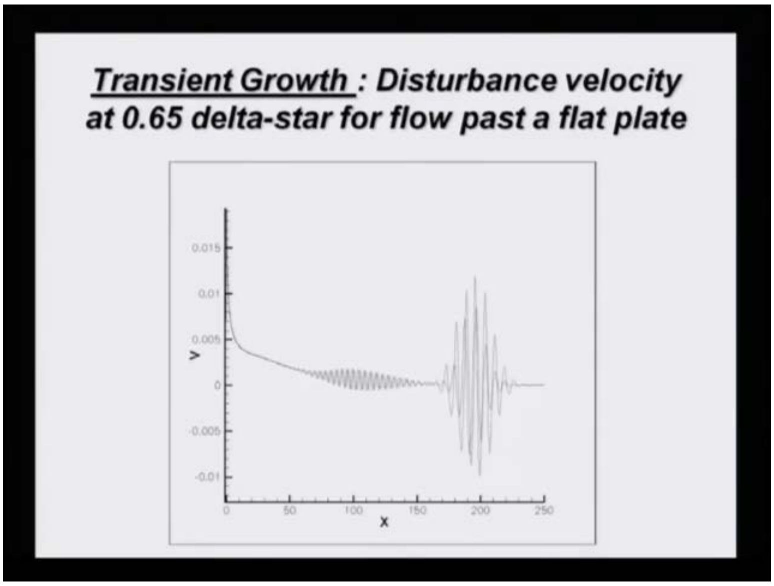

Now, what we could do is - we could see similar such things. So, this is what we did not show in the last picture. Now, you are seeing. So, this is your TS wave packet and this is your spatio-temporal front, and you can see what I just now said that these wave fronts could be huge and this is kind of a picture that you can get from a very high quality Naver-Stokes solution. So, it is something we must keep in mind that what we get. So, here, we are seeing simultaneously few things that we have been talking about for the last few days. There is the TS waves and the spatio-temporal growing wave front. So, if you want to call this, this is like your bypass event.

If this amplitude grows very large, that can itself trigger the flow to go from laminar to turbulent state. So, this is, here, the non-linear effect would be significant because amplitudes are far higher compared to the trailing TS waves. So, this could be also an example of a bypass mechanism. So, that is what we talked about earlier also. So, we had seen some of those things.

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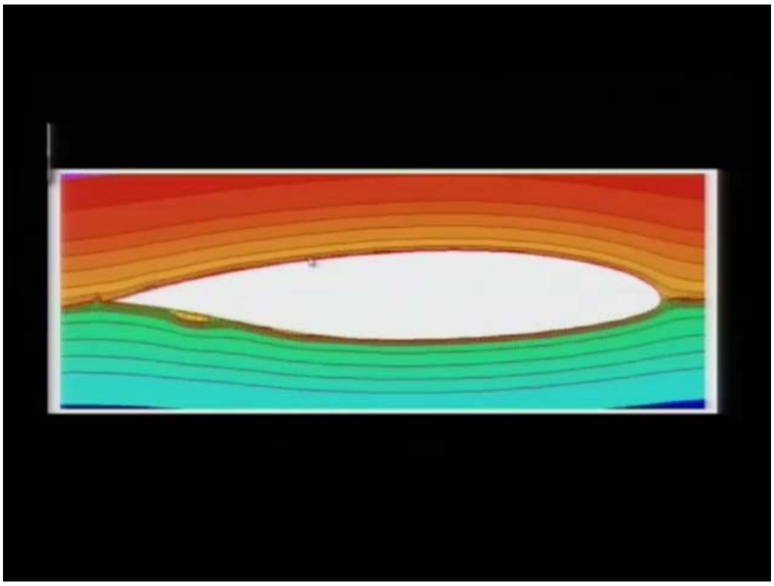

Let me now show you yesterday you recall, we saw that Honda airfoil, and what we see is that if we perform the simulation for this flow going from right to left and this is the simulation for a Reynolds number of 10.3 million.

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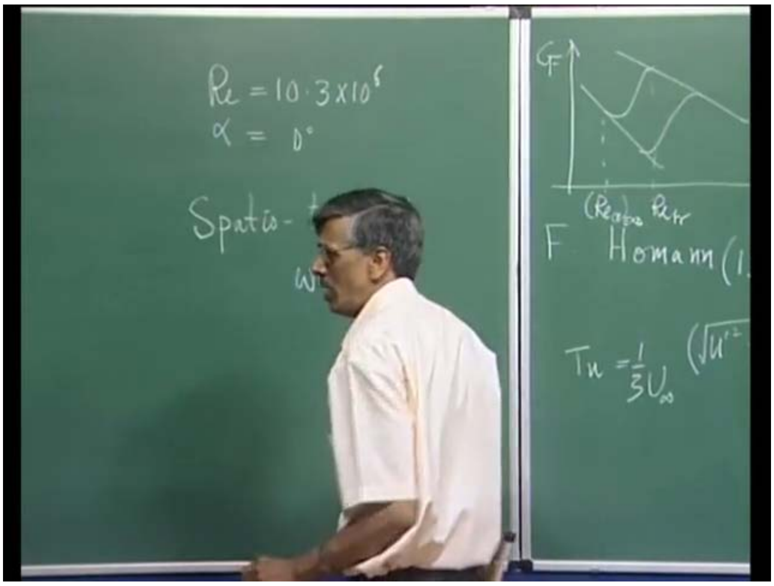

So, this is the cruise Reynolds number for that Honda jet and this we can compute without any models, we can do it, and this is at a zero angle of attack. So, the flow comes without any inclination zero angle of attack, and what you notice here that the flow kind of the boundary layer remains attached up to the middle of this airfoil. Up to this path, the flow is perfectly steady, but what you notice is that in the second half of the airfoil, you are seeing some small separation bubbles and they are unsteady, they keep moving, they keep moving and you see the same thing on the bottom side also.

So, what you are noticing that you are not seeing any Tollmien–Schlichting wave, and if you calculate this drag of this airfoil, this is significantly higher than what you get if the flow have to be completely laminar. So, this is another mechanism. This happens because up to this path, flow experiences favorable pressure area. Beyond this path, flow experiences adverse pressure gradient. While in the favorable pressure gradient region, the numerical noise does not do any harm, but the same numerical noise shows this kind of attribute of bypass mechanism. I will call this as a bypass transition.

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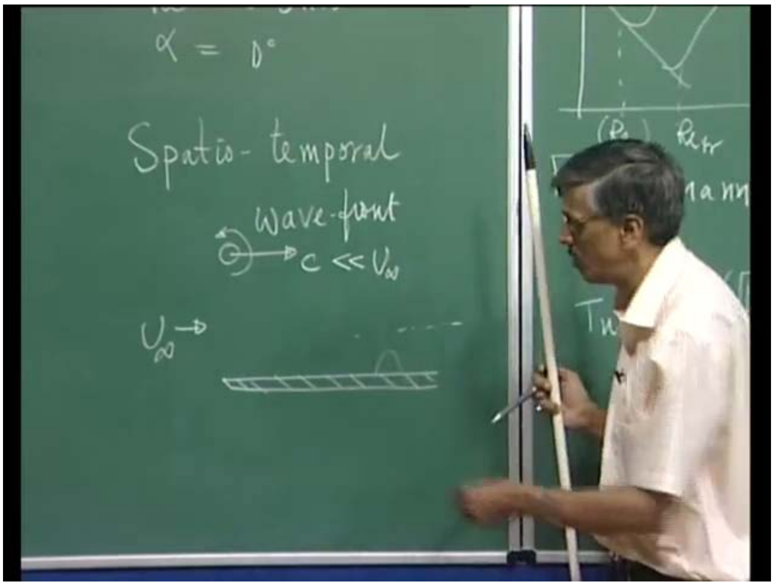

We will also see in later part of the course that we can have other kinds of vortex individual stability. Where if I have a flat plate flow, and then, if we are talking about the scenario by this that we have a flat plate, I would not show anything right now, but let us say this is the age of the shear layer and let us say we have a vortex and that is conducting at a steady speed c which is different from say u infinity; u infinity is speed at which the flow goes from left to right over the plate and that is how you get this boundary layer. If c is a much smaller compared to u infinity, you would notice some kind of bubbles forming and they also propagate like what you have seen here on top.

So, this is what we called as a vortex-inducing instability because we have a definitive vortex there and that is linking with this boundary layer. So, there is this. These vortexes actually destabilize this shear layer. So, this instability is called as the vortex inducing instability. So, we have various mechanisms by which we can see this.

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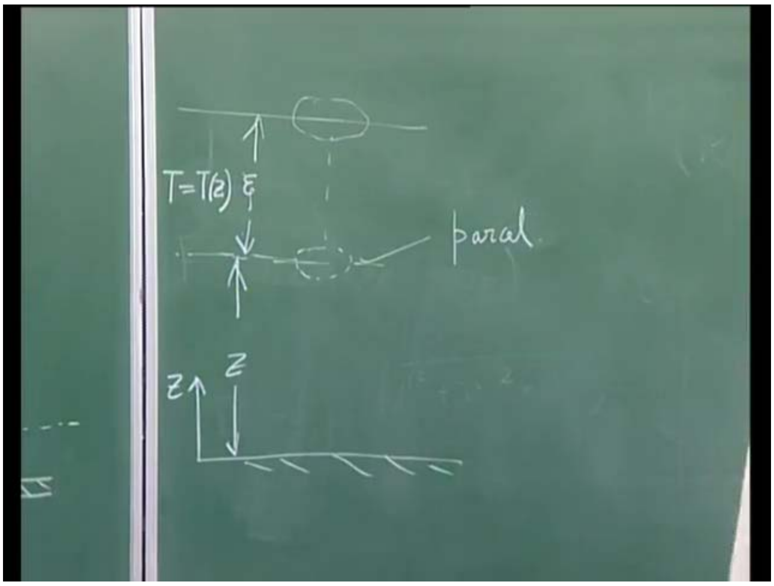

So, having given you this overview, let us now talk about a very specific case where we try to work out the details of what we mean by stability and instability, and I have drawn this example from something which we can relate to is the stability of the atmosphere itself, and how do we study this? Well, we study this in the following framework that, we have a still atmosphere, there is absolutely no motion. Then, all the quantities the properties of the atmosphere is going to be a function of height, we know that, and then, what happens is if I identify, let us say we fix a datum here and we fix a axis z axis perpendicular to this data, and let say this is at a height z. What is this? This is a parcel of air. Say somehow we take a small sample of air and color it. So, the properties remain the same with the ambience; so, it is an equilibrium state; this parcel is there at an equilibrium state. Now, what we do is we make this parcel to go up to another height. Let us say this displacement is xi. Since the temperature here is changing and we know what happens in atmosphere as we go up temperature falls.

So, what happens is - let us say this T is a function of height z. When I have displaced this parcel to this height, what happens? The air here that is carrying all the properties from here will be different from what is the condition out there because of this temperature differential. So, what would happen? The parcel will experience a force of buoyancy, because of the temperature difference, there would be intensity difference and the density difference will cause the air to move.

So, that is precisely what we are saying. Now, the ensuing motion that we will see of the parcel, we want to see it as a dynamical event. What is the difference between dynamical and static event?

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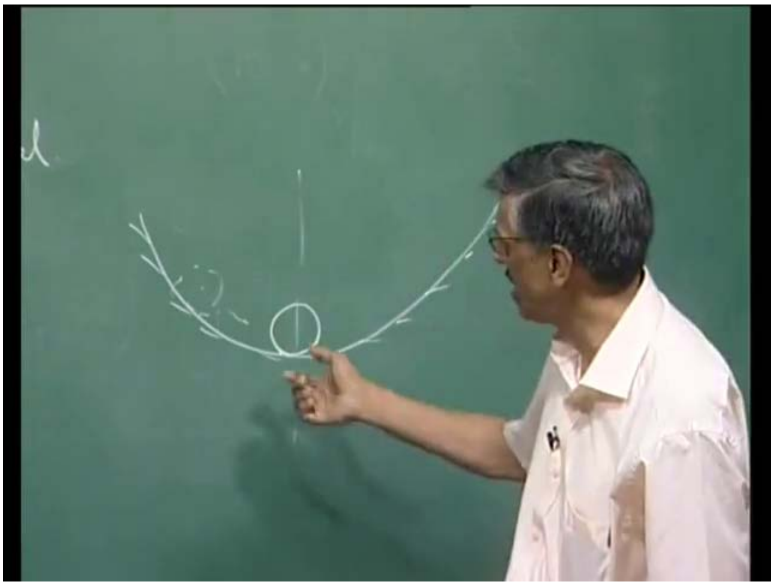

You all know in your mechanics course, you have always talked about this famous example. What is this example? You keep a ball here and it is in equilibrium state. Now, if you move it up there, what happens? It comes back and then we say it is stable, why, because in the disturbed position, it has the tendency to come back but we do not study its time-varying motion. Even if we do, we know that it will because of the friction, amplitude of oscillation will come down and it will settle to the lowest point. What is the property of this lowest point? That is a lowest potential energy configuration. So, system wants to reside in lowest energy configuration that we all know in high school physics.

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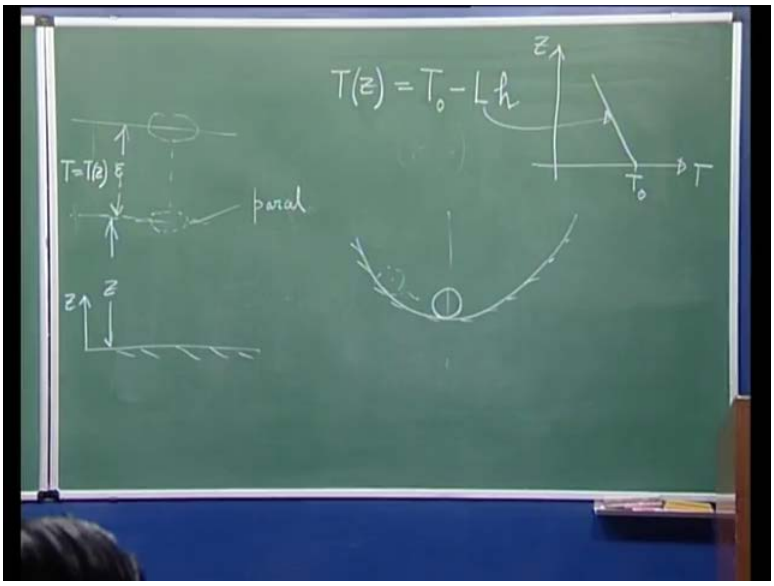

So, we can do a similar study actually. If I disturb it from here to here and let us say if I call this variation of temperature as given by T 0 minus say some height h, I am calling it, that is, its linearly falling. So, basically what we are talking about this T if I plot the height versus T, so its start off with some value T 0, and then, it is falling off like this. This rate at which it is falling is given by L.

So, with unit height, the temperature falls by L. So, it starts off from a datum level T 0 at falls off; we can study the instability. So, what we could do is - basically we can study its static stability. I will invite you to do that. What you could do is you can try to find out how the property varies with altitude. What we will do is we will see what happens. Suppose if I have displaced it from here to here, we will work it out. I will leave it as a reading assignment; you will find it in many books.

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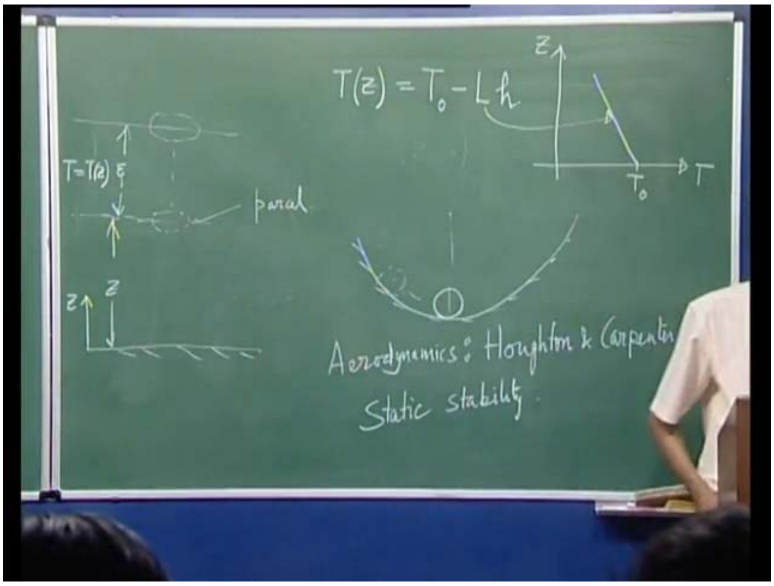

Please do read the following reference. You will find out this book on Aerodynamics by Houghton and Carpenter and find out static stability. So, what we are going to find out? We are going to find out that in this displaced position of this packet, whether it has a tendency to come back here or it has a tendency to go further up, if it has a tendency to come back to the basic equilibrium state, then we will call that as statically stable. If it stays there and does not do anything, then we will call it as neutrally stable, but if it continues to float up further up, then we will say it is statically unstable.

We are not very interested in that so that you can study by yourself and find out. What we are studying here is the dynamic stability; that means what? That, if I identify this packet by, let us says its density. so let us now talk about the properties.

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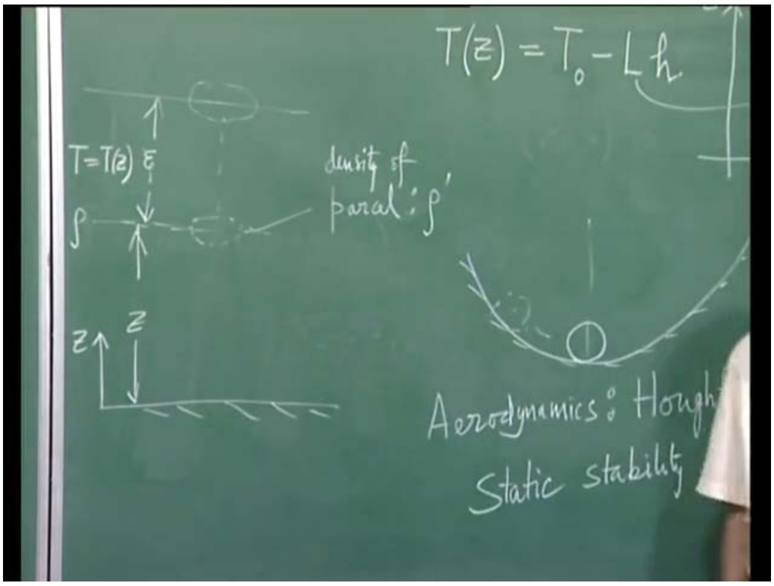

Let us say the density is given by rho as a function of I, whereas density of the parcel, this I will call it let us say rho prime.

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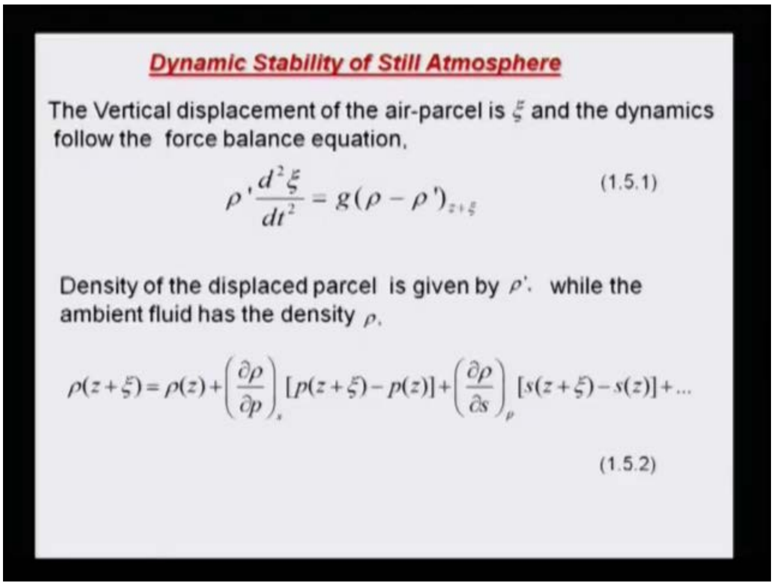

So, if I now talk about a unit volume, then its mass is rho prime and we have moved it vertically and it is going to experience a vertical force due to buoyancy. So, if I want to study its dynamics, I will write down this. So, this is mass times acceleration on the left hand side, and what is a buoyancy force? g times rho minus rho prime and this is where we are trying to find out at the disturbed location. The disturbed location is z plus xi.

Now, we want to solve this equation and see the detailed motion xi as a function of time. That is your dynamic stability, whereas what we have talked about the static stability is in the disturbed position, we just simply see its tendency whether it can recover back or it goes further away or not. So, that is the essential difference between static stability study and dynamic stability study. We are pursuing here a dynamic stability study. Now, density of the displaced here as I told you is given by rho prime; ambient fluid as this. So, what happens? The density of the ambient fluid at the elevated position can be written in terms of a series like this.

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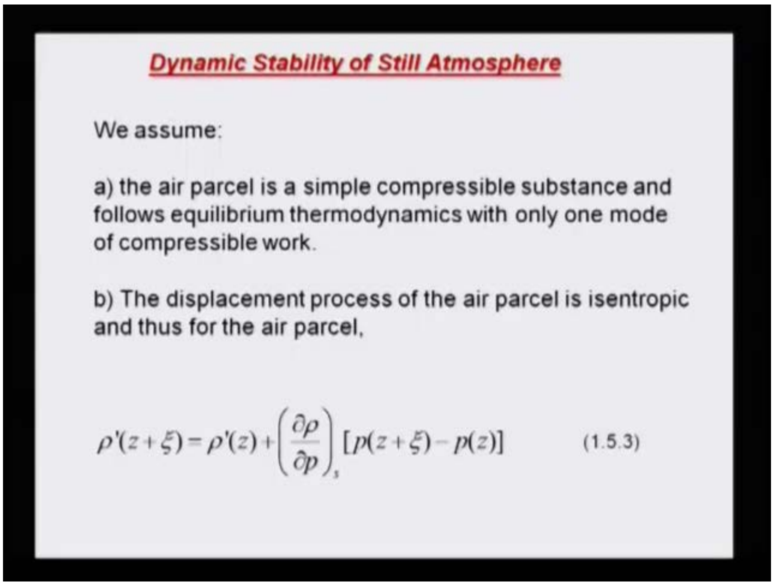

Now, why did I write like that? That would require some understanding of thermodynamics. That part follows from equilibrium thermodynamics. If you all recall, we have all done thermodynamics. We have forgotten but it is time to flip your old books back and find out. We are talking about the air parcel to be a simple compressible substance means, it is compressible. So, you can do work by either compressing or dilating it. So, that is what it is and that is the only mode of work that we are talking about.

So, how many degrees of freedom we have? From the thermodynamics, we would have 1 plus 1 means the possible modes of compressible work plus 1. So, in this case, the possible mode of work is 1, so, what I call as a degree of freedom should be equal to 2. What does it mean degree of freedom? It means at any thermodynamic property can be written in terms of any two other thermodynamic properties and that is what we have done in the previous slide.

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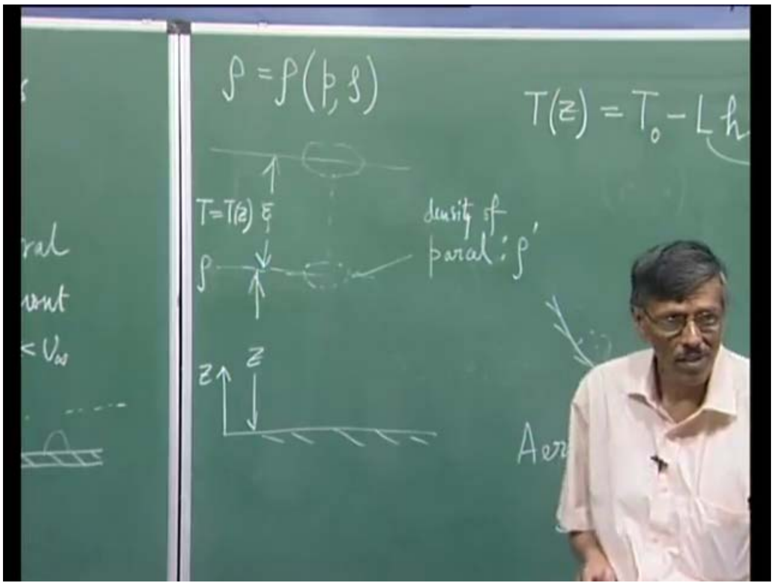

In the previous slide, what we did? We said that we will write this density here as a function of two variables and we are talking about these two variables as pressure and entropy. I have written the specific property, that is, I have written it in lower case. So, what we find that we will use these two as the independent variables, pressure and entropy.

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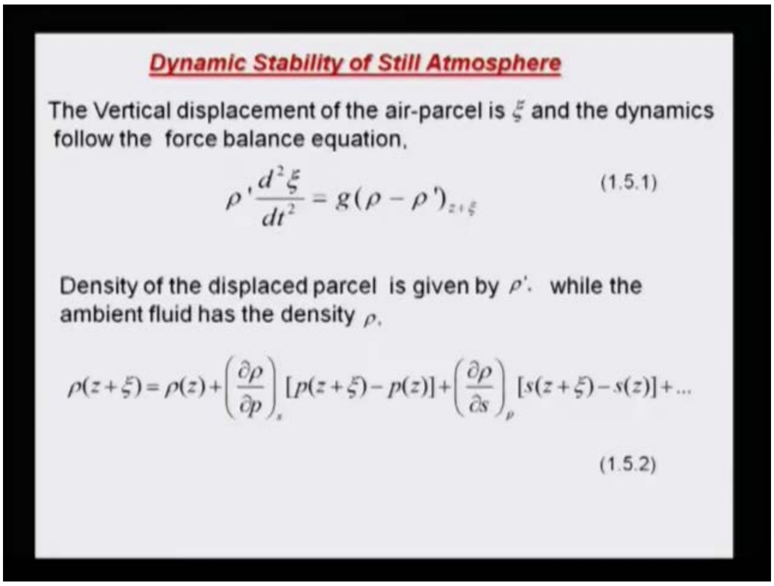

Then, what we do is very easy. I could see its variation of the density with height, I will say the density at the undisturbed position rho at z plus how the density has changed due to the pressure variation? So, that is this part - del rho del p times delta p, and delta p is what? p at z plus I minus p at z. Same way I could also write the density gradient with respect to entropy and change in entropy. So, what has happened is I am looking at the ambient air and this is how the density varies with height because of this thermodynamic description.

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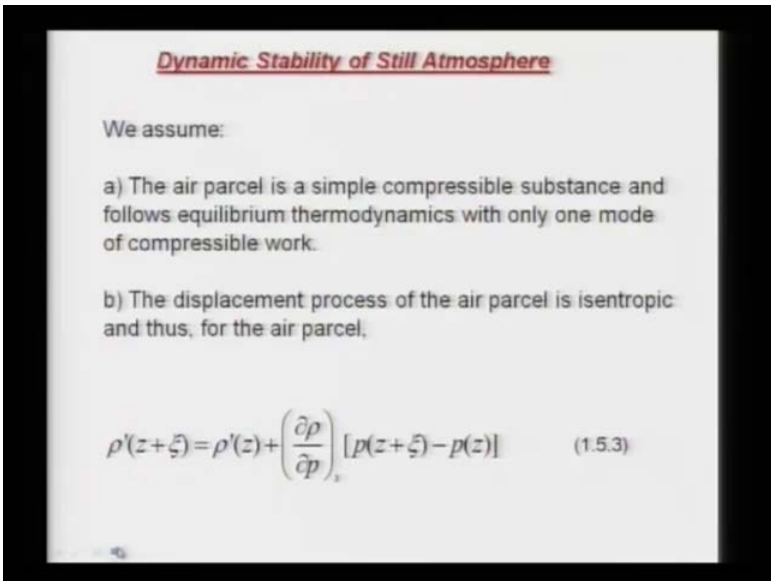

Now, we also make a second assumption that when I move this parcel from this height to that height, I did it rather quickly; I did not allow the parcel to do some kind of a heat exchange with air. So, that is a kind of a adiabatic process, and if I do it reversibly, then I can call that adiabatic process as isentropic.

There might remain some kind of a confusion. If I say if I do it very fast, that militates against our concept of reversibility. In reversible thing, what we say that we do it very slowly. At an every step, we have equilibrium, but let us not get into that debate, let us just simply say that this displacement process of the air parcel is isentropic. Then what happens? I can also similarly write the variation of rho prime with height, but now, what has happened, because this isentropic process, I have the second part missing.

Del rho del s times delta s. So, delta s zero; so, I will have that. So, that is what we have written here. Please make this correction this is del rho prime, but if I look at rho and rho prime to begin with, are they different? They are same. So, that is why I did not write that rho prime there. So, the gradient of the parcel and the ambience are the same, because we have taken the same thing, we have just simply colored the parcel to identify its motion. It is almost like a flow visualization in a column. You basically identify the same fluid but just simply color it. So, here also we are doing that experiment. So, you can see that this is the variation that we see here for the parcel in the previous transparency in 1.5.2; we have seen the variation of density of the ambient.

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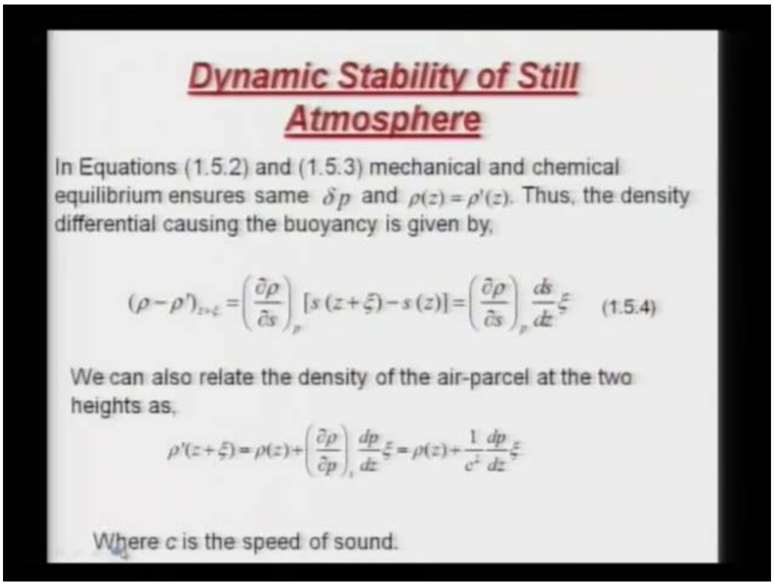

So, now what we could do is we could calculate the driving force that was proportional to rho minus rho prime into g. So, that is what we are doing. At the displaced position, what is rho minus rho prime? That is given by this. That the way the entropy changes with height for the ambient air, because that is what we are trying to study. We are not interested in studying a process; we are trying to study the substance. See, the process is what? Process is I have the parcel I move it quickly isentropically. That is understood, but we are saying the ambient air itself may have a entropy gradient and that is what we have written it down here.

Now, you are seeing that the buoyancy force actually depends on this del rho del s into delta s and that delta s itself I could write it as ds dz times xi. That is what it is; it is a chain rule. That is what we have done. Now, what happens here? Earlier I wrote down the density gradient. This is going to be rho prime. Please understand that this is rho prime, but then, at z, both are same. That is why I wrote it as z itself. Then what we have written here? del rho del p times delta p. Delta p again I can write it as dp dz into change in height xi.

So, what happens? I can write it as rho of z plus what about this del rho del p at constant entropy? It is 1 over c square; the speed of sound it is defined like that; speed of sound is defined like that. So, what I am seeing that at the elevated height, the density is related to the density at the lower height plus 1 over c square dp dz into xi.

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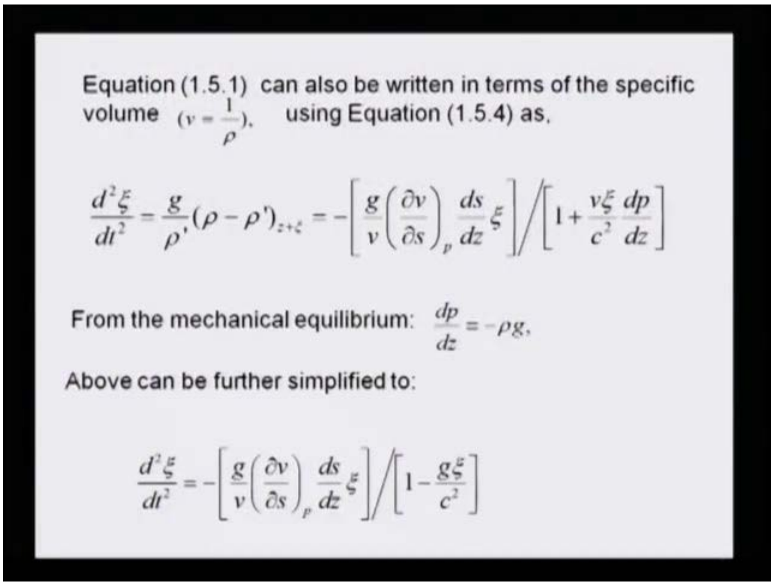

Now, having done this part, what we could do is we could do some additional manipulation. Say like for example, what I have done? I have introduced a specific volume which is nothing but reciprocal of density. What is specific volume? Volume per unit mass. So, that is your specific volume.

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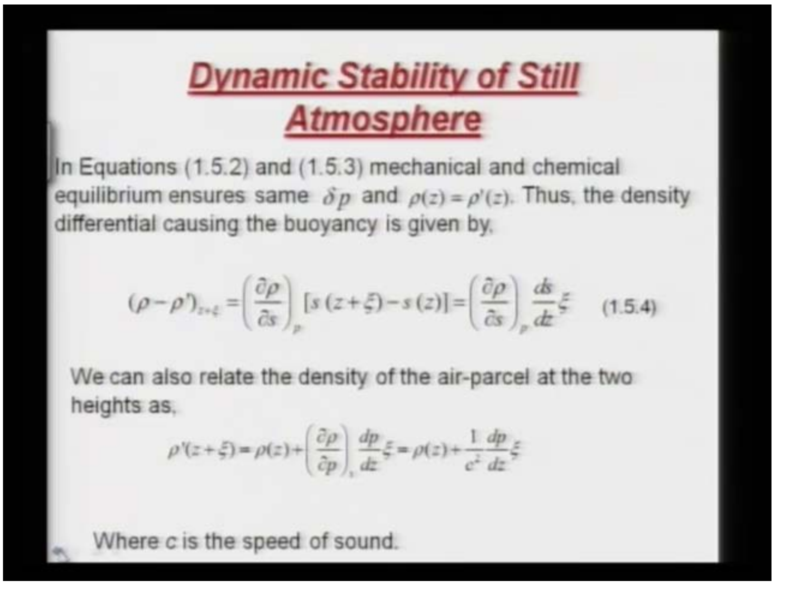

So, what I get is from my governing equation was rho prime d 2 xi dt square. So, let me be very slow here for you to understand. So, this was rho prime d 2 xi dt square into the buoyancy force. So, I have brought that rho prime down here. So, this is what I get. We have just now work this quantity out. So then, what do I get? Remember, this is rho prime, and that is nothing but if I see the previous transparency, there I have worked it out - rho prime at z plus the lower expression. So, if I write this as a rho prime by rho, I will get 1 plus 1 upon rho c square dp dz xi.

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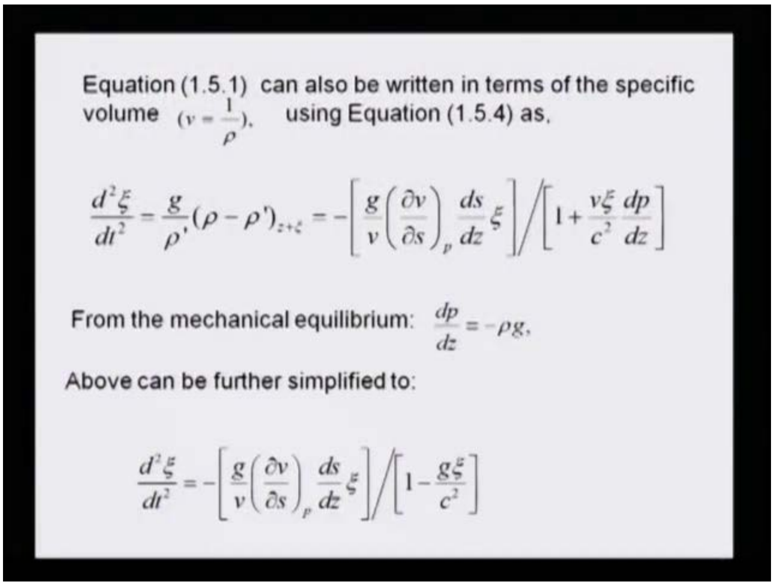

So, this is going to be in the denominator of the governing equation. That is what we have done, and what we found is a numerator was rho minus rho prime that was given in terms of g into del rho del s into delta s and that was given by this. Now, what we have done? We have done some bit of simplification because rho itself was 1 upon on v. So, del v del s by one upon v square into minus sign comes and because we have divided both sides by rho, that is why we again get g value.

So, this is what we get. Now, if I look at the parcel itself, what about its mechanical equilibrium? I can consider a cylinder on the lower side. I have a pressure p upper side; I have a p plus dp dz into delta z. So, if I look at the equilibrium in the vertical direction, if the forces are balanced, then the weight of this column should be equal to the pressure gradient and that is what we are getting. If this height is dz, so delta p must be equal to rho g into dz; so, that means dp dz is equal to minus rho g. So, what we have done then? We have, we can go ahead and further simplify this. Replace this dp dz by rho g here and that is what you can see here. So, that is what we have seen there that we could simplify that way.

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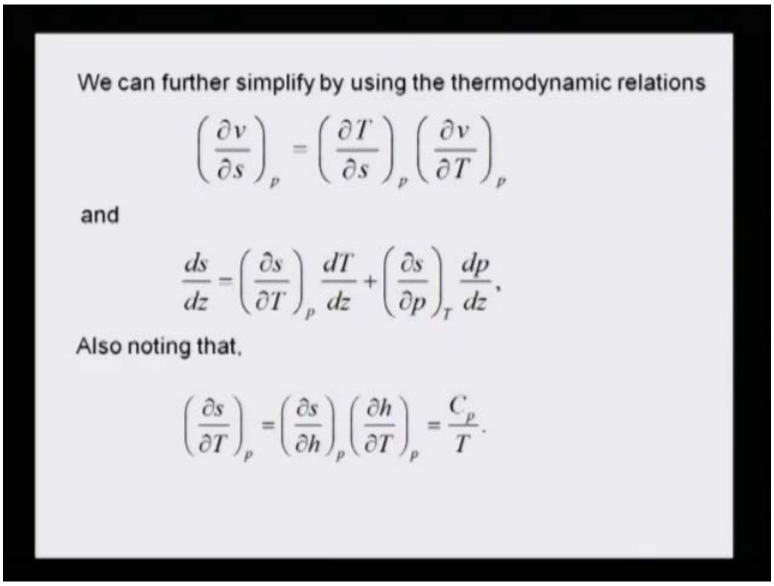

Now, there are lots of further simplifications possible. We want to write down the quantity in terms of measurable things. Measurable things are certainly not entropy and specific volume could be measured but certainly not entropy. So, what we do is - we use various kinds of thermodynamic relation. We have this quantity in the numerator - del v del s at constant pressure. That we can write it as the chain rule del T del s times del v del T at same pressure rate constant, and we also have ds dz; ds itself s also we could write it in terms of 2 thermodynamic properties.

So, let us write s as a function of T and p. We can do that. If I do that, then ds should be equal to del s del T at constant p times dT plus del s del p at constant T times dp. So, now, if I try to find out ds dz, I will get this.

Now, in this quantity, I have this del s del T. I could write del s del T in this particular fashion del s del h and del h del T at constant pressure; h is what? Specific enthalpy, specific enthalpy. So, that part itself h is what? c p into T. So, this quantity is c p and this quantity is 1 upon T. So, we are doing all that tricks that we have learnt in our equilibrium thermodynamic course by just simply manipulating and we can also use Maxwell's relation.

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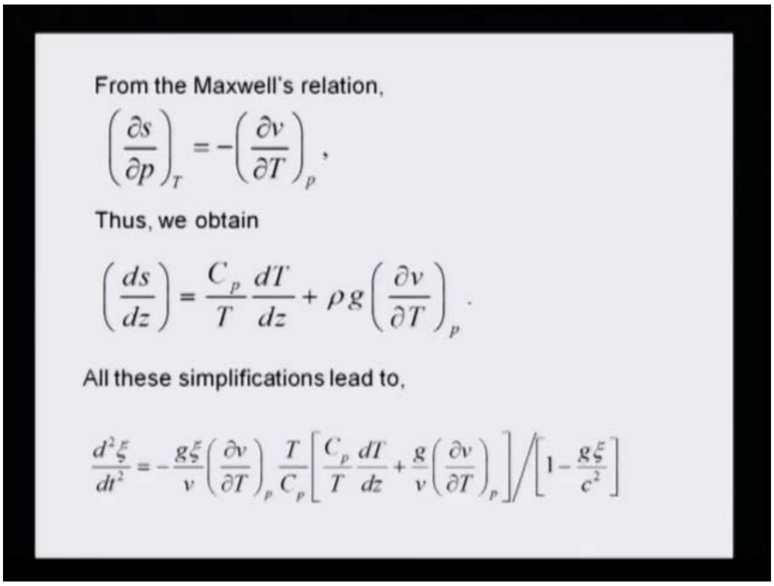

One of the maxwell's relations tells you that del s del p at constant temperature should be equal to minus del v del T at constant pressure. This is something which we can do in the lab. I can go to the lab; I can measure the pressure; I can measure the temperature; I can measure the density. So, this quantity is knowable, while this is something which we cannot measure in the lab experiment. In fact, Maxwell's relation gives you a tool to find this quantity in terms of quantities that can be measured. So, if I use that, I get ds dz is equal to in this particular fashion. So, this is the simplification. We can substitute all of that in that relation and this is what we get. So, we are making some progress; we are simplifying the quantities.

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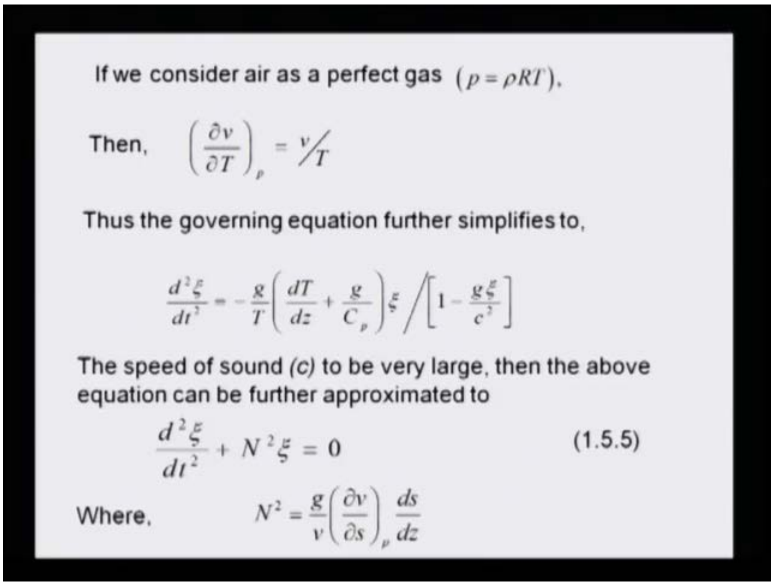

Now, if you consider air as a perfect gas, then p is equal to rho RT, and then, what you can do is - you can find out del v del T; del v del T will be nothing but v by T, and if you substitute that equation, this equation simplifies this. So, even if you are looking at the motion of a simple parcel of air, the dynamics is coming out to be quite interesting here. Now, if you, for the sake of simplification, you consider that c is very large. Incompressible flow if you consider c is infinity, but if you do that, then you can perhaps omit this term so that denominator becomes 1, then you have an equation of this kind. Now, we have made some progress. So, it looks like a simple harmonic motion. Except the fact that, this is not necessarily a constant N square is given by this expression.

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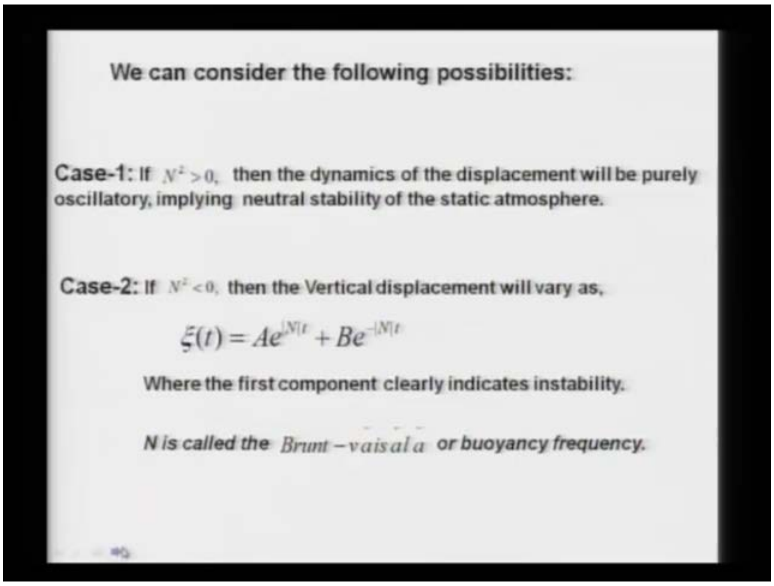

So, this expression, this was done by two Scandinavian scientists and this N is called the Brunt-Vaisala or buoyancy frequency, because it comes out in a simple harmonic motion a square. So, it has the dimension of a frequency. That is what you get for your simple harmonic motion of a pendulum. So, that is why N is called a frequency or the Brunt-Vaisala frequency.

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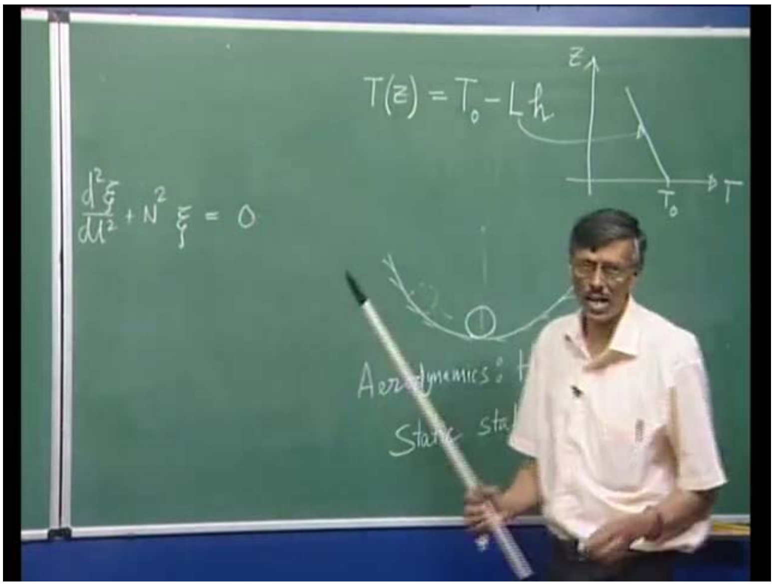

Now, if you look at the previous expression, then what did you have? We have obtained the governing equation as simply this d 2 xi this, and as we discussed that N is not a strictly a constant. So, what happens is you can have the following possibilities. Suppose I am looking at different altitude in the atmosphere, I can locally measure N. Suppose it is so happens that N square is positive, that is very comfortable scenario. Then we will have a pure simple harmonic motion if N square is positive. This will be a simple harmonic motion, and what kind of a motion will that be? It will have the same amplitude simple harmonic motion.

So, amplitude does not change with time. It will be either sine or cosine combination of the two. So, what kind of stability we will be talking about? If the amplitude does not grow the xi is the displacement it is a disturbance. So, the displacement does not change with time, remains same. That would be a neutral stability, all of you see that. If the amplitude grows with time then, will be having instability. If the amplitude became with time, we will have stable condition.

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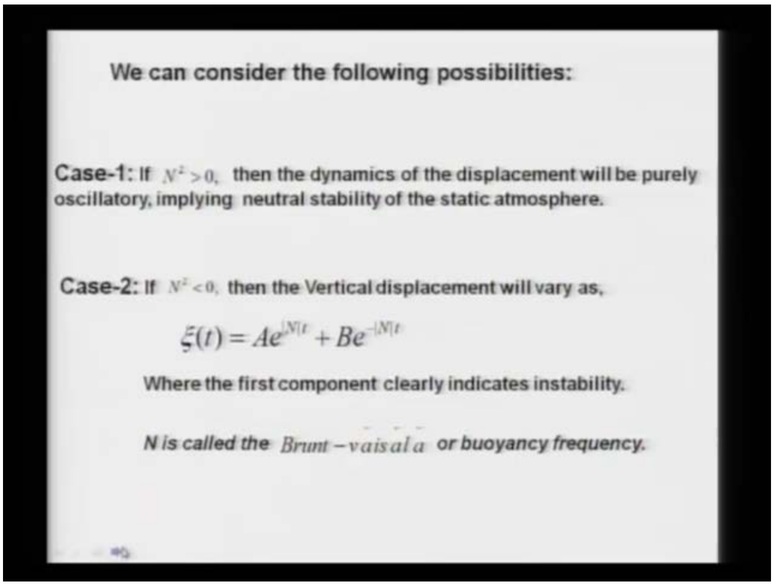

So, what we are seeing that if I have a per chance n square greater than 0, then we will have a neutral stability of this static atmosphere. I think we will stop it.