Foundation of Scientific Computing

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Module No. # 01

Lecture No. # 09

We will continue our discussion on surface gravity waves. Today, on the 09 lecture we will talk about the pressure fluctuation in the surface gravity waves and show that the energy flux and propagation is related to the group velocity.

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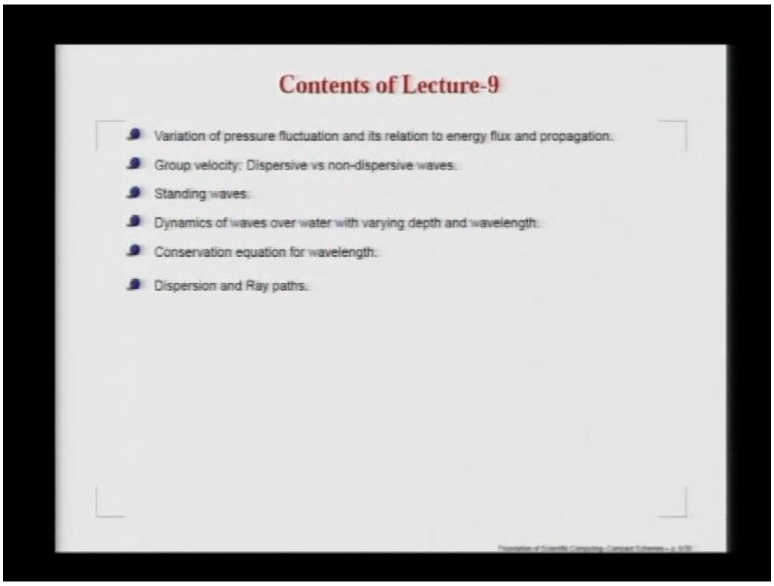

We will use the group velocity to characterize waves as either dispersive or non-dispersive waves. If the group velocity of all the constituents is same then, we will notice that they are called non-dispersive system and in contrast, if the group velocity keeps changing with wave number we have a dispersive wave system. This is a very interesting topic in various aspects of applied physics and mathematics. We will talk in great detail about the dispersive waves.

When it comes to surface gravity waves, if it is formed in a confined region, then we would also notice that we can obtain standing waves. We will discuss in detail and show how the dispersion relation becomes degenerate in this case.

This will be followed by our study over dynamics of waves, over water with varying depth. We will notice that the waves those are formed will vary greatly in terms of wavelength and frequencies. We will be able to show you that whatever may be the wavelength, we can write out a conservation equation for wavelength.

Talking about ray - optics was developed for studying. Optics - we will talk about dispersion and ray paths for dispersive ways. One of you at the end of the class ask me little more about deep water waves and shallow water waves, what are the particle paths, there were some questions regarding that. So, I thought, I will just simply go through it once again.

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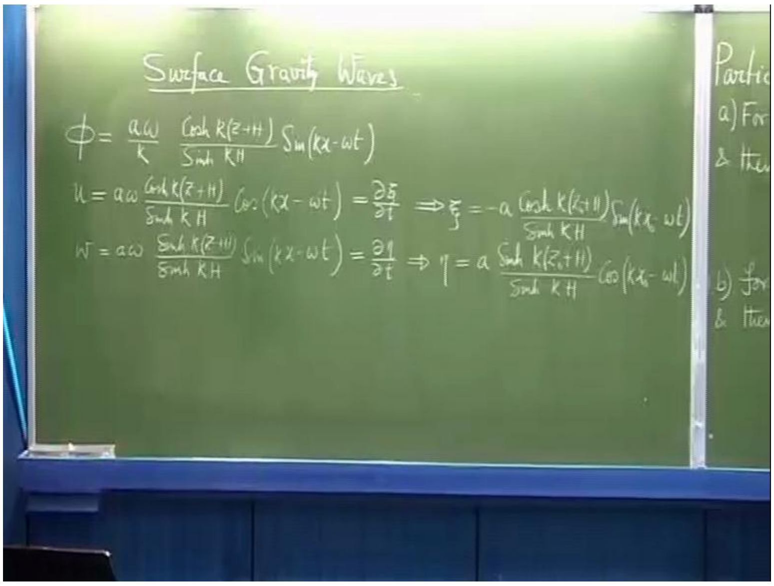

We look at these surface gravity waves as a consequence of the linearised irrotational approximation and then the flow field is defined uniquely by only one unknown that is the velocity potential phi, which is given like this. So, this is your a travelling wave solution, this have been obtained by phi. You can successively take derivative with respect to x and z to get the velocity components. From the velocity components you can equate it to its Lagrangian description, which upon integration gives you the departure of the particle paths from its equilibrium condition, it is given by x naught and z naught.

So take a look at this, this is the way it looks like. Then once you eliminate this x naught and z naught and t you end up by getting an equation of an ellipse here.

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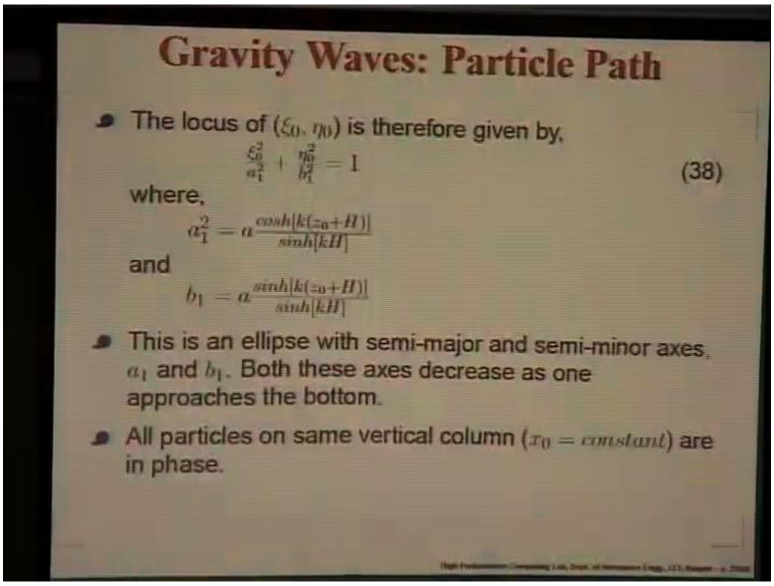

This is the general form of the semi major axis (( )), once again there is no square here; that is a mistake; I will correct it once again. So basically what happens is that is a generic expression for the semi-major and semi-minor axes.

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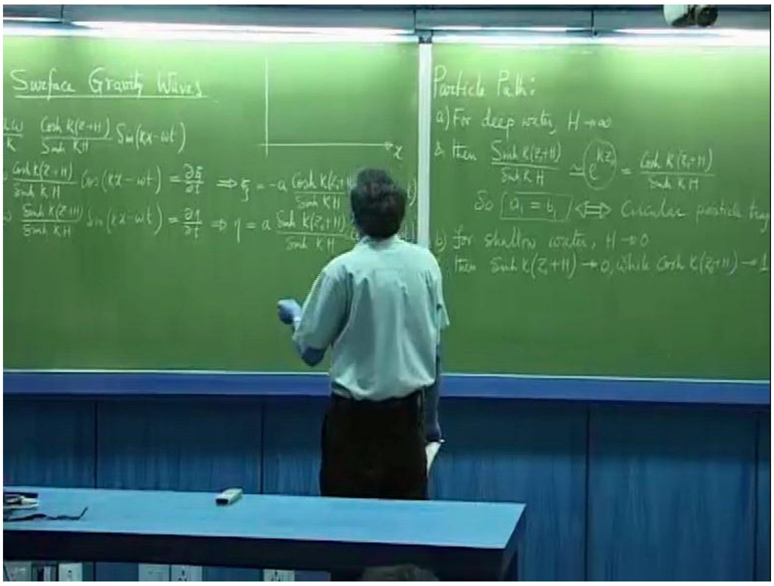

However, if you look at specifically for deep water waves then H would approximately be 10 into infinity of very large. Then if you write these in terms of the exponential form you can see that for H very large this quantity sine hyperbolic k into z plus z naught plus H by sin kH. This part would go to this value and so the other one would be that cosine hyperbolic k into z naught plus H by sine kH. So, what happens as a consequence, A1 is equal to B1.

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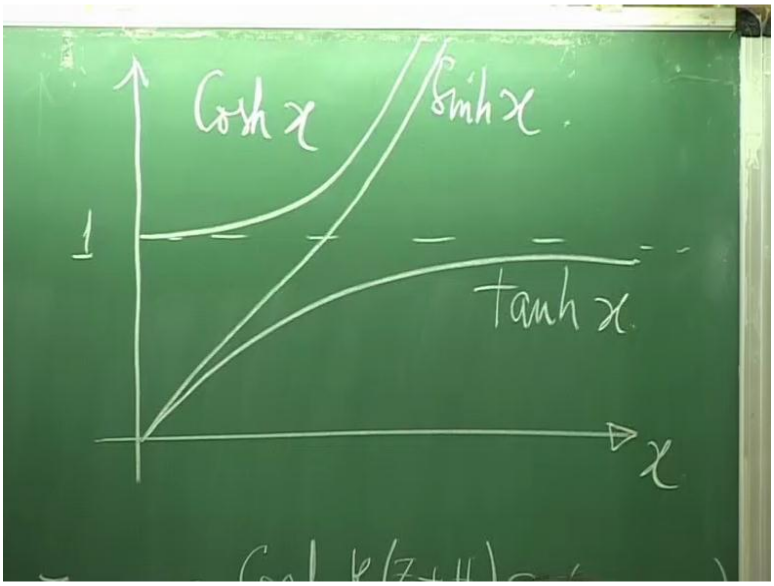

Well, in case if you have forgotten, probably none of you have. If I plot let us say this hyperbolic functions, then what I find is that cosine hyperbolic begins with 1 and then it goes to infinity; that is your cosine hyperbolic x. If you look at the sine hyperbolic x, it goes like this, so that is your sine hyperbolic x; that also asymptotes to infinite value. Whereas, the tan hyperbolic k actually this goes like this.

Now, you can very clearly see that when x goes to infinity both of their asymptotes becomes together and you get this A1 equal to B1. So, what you get in a sense is a circular particle trajectory.

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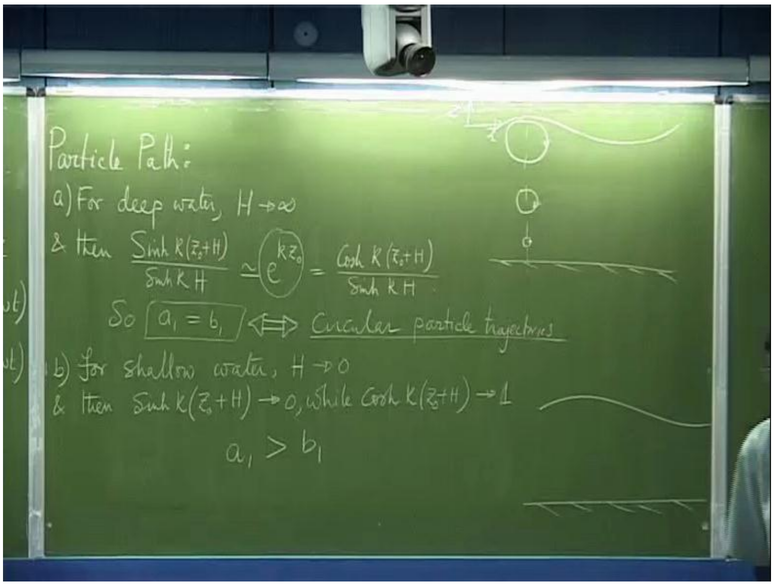

If I have to plot these particle trajectories this is the bottom of the bed, then say this is the surface gravity wave, then what you are going to see is the clockwise trajectory. As you keep coming down, this is your datum for the z axis, so as you keep on coming down you are going to see circles of smaller and smaller radius that is because of this factor coming down e to the power kz naught will keep on coming down, because z naught is negative there, as you go down it you will get this.

This is how you get and that is what we also wrote here. All particles from the same vertical column will be in phase, the trajectory defined by these particles will describe smaller and smaller circles for deep water wave. Same way, you can look at the shallow water limit for which the H itself will go to 0 and then you can very clearly see that sine hyperbolic component will go to 0. Well, the cosine hyperbolic component goes to 1. That is the consequence of that figure that we have drawn.

What happens, you can see this is your A1 and this is your B1. What you are seeing is A1 cosine hyperbolic part would be much greater than B1. What we are going to see in this case is, of course we are going to see elliptic trajectories with the semi major axis elongated as compared to semi minor axis. You also note that the semi major axis this part would not change very much with z naught, because they will be in the close vicinity of one itself.

So what happens as a consequence is here as you have seen the amplitude actually keeps coming down and it goes to 0 like this, this is what you get. In this case, if I similarly try to look at envelop of the semi major and semi minor axis I will find that A1 would virtually remain same. As I keep on going down this will become thinner and thinner, and so and so forth. This is what you are going to see in the shallow water wave.

Now, what is a deep water wave? That is what we need to find out. We can draw information from these trigonometric hyperbolic functions. Whenever, you have kH greater than say 1.75 you will notice that tan hyperbolic will rapidly approach one. You look at your dispersion relation that is where you would see the tan hyperbolic part comes and it is a kind of saturates to that. There you would see that this will be kind of value, so your kH greater than 1.75 is a kind of a threshold below which you can talk about intermediate range.

If it is far below then you will say it is shallow water, but anything above 1.75 you could classify it as a deep water wave, this is something that you would know. Now, a bit of trivia or information, if you look at ocean waves, if you are very close to the continent what is called as a continental shelf then there the depth of water is roughly around 100 meters or so. But, if you go to the open ocean little further inside the deep sea, there it would be something of the order of few kilometers, something like 4 kilometers or there about right.

This surface gravity wave that we are talking about usually corresponds roughly about lambda is of the order of 150 meter. If you talk about h of the order of 100 meters even in the continental shelf, we are not talking about very close to the beach but little further, may be few 100 meters away from the beach, then you would be seeing this kind of depth and there the wave of this kind is qualified as a deep water wave.

For all practical purposes the waves that you would see in the sea excluding the one very close to the vicinity of the beach front can be classified as a deep water wave; this is what I told you. Probably your exposure in optics and acoustics you have talked about longitudinal wave and transverse wave, but this is a mechanical wave and it has got this interesting property that is the trajectories are not necessarily defining a vertical or a horizontal motion, but by the combination of the two you get either circles or ellipses.

One thing now what we could do is we could also work out how the pressure varies. So for the pressure variation you can use what we have already derived in the Bernoullis equation.

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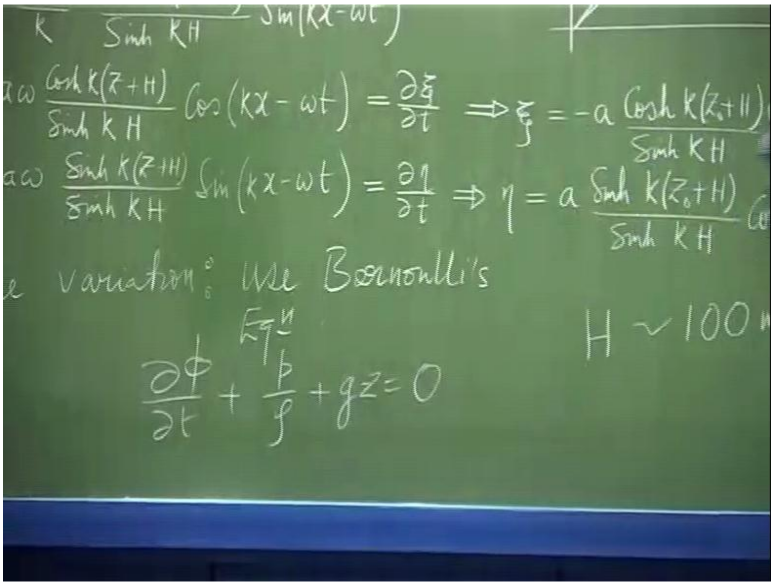

What was Bernoullis equation? Of course, take a look at it, it is an unsteady flow, so you will have this term and then we will have p by rho plus this plus a constant which we can conveniently set the datum to 0. Now, what you can do is you can define the pressure fluctuation, what do I mean by pressure fluctuation? It is the departure of the pressure from the undisturbed value of the surface.

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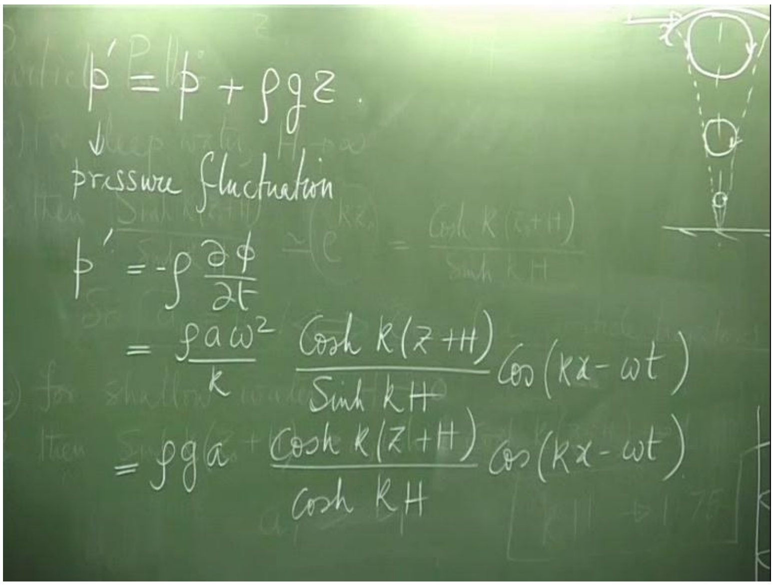

Basically, we will be talking about p prime is the pressure fluctuation that should be the actual pressures that we have written in the Bernoullis equation. Then we will be doing this, because minus rho z was the actual pressure undisturbed case from hydrostatics; that is what you get. So this is your pressure fluctuation, it is easy for you to see that from here that this two together is going to give us p prime.

So that is what we are going to get p prime by rho is equal to minus del phi del t, so I will write this as this. I have obtained the expression of phi, you can calculate this as rho a omega square by k and cosine hyperbolic, this into the phase part. If I use the expression that dispersion relation, it tells us omega square equal to gk tan hyperbolic kH. I could simplify and then I could write this as rho ga, and cosine hyperbolic k z plus H, all divided by cosine hyperbolic kH, of course this remains the same.

Having obtained the pressure fluctuation we would like to talk about something which is of immediate interest to us, namely how actually the energy propagates? Why we are trying to talk about this is simply for the reason that we have defined a new quantity - group velocity. I do not know how many of you are very intimately aware of this quantity, I find that main stream literature on the book they do talk about group velocity, but do not emphasize its physical importance that is why I would like to emphasize it a little more.

Now, having obtained this expression what I could do is I could try to calculate the energy and the rate at which the energy flux is propagating in the fluid that is the very important consideration.

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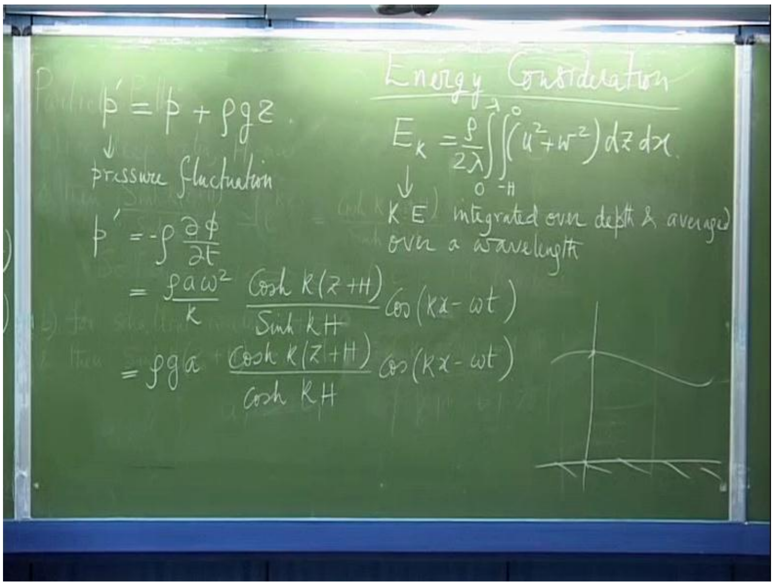

So, let us talk little bit about energy consideration. If we try to talk about energy, I think we have briefly talked upon it. We have gone through this expression for dispersion relation, calculated the group velocity. We have obtained the limiting values for the deep water waves, shallow water waves. What we noticed that the deep water waves goes at half the phase speed, while the shallow water waves go at the speed of the phase.

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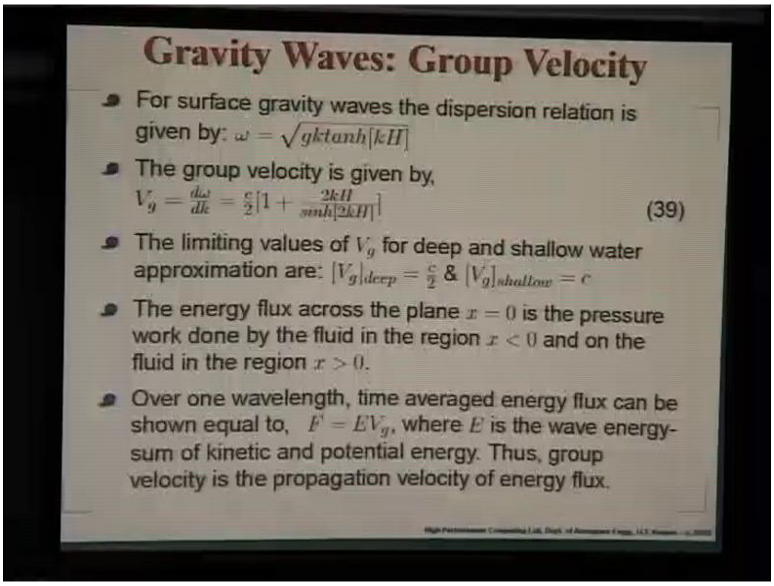

Now, we did talk about this energy flux and I did make this observation. Now I think that a deeper appreciation of that last bullet would be in order for us to appreciate what really is the true role of group velocity. Unfortunately, this has been lost not only in the precision of the physics of the problem but also in competition. This plays a major role that is what we are going to see as we go long.

So, having obtained this solutions given in terms of phi and the velocities, one thing of course we can talk about is the kinetic energy, which I will call let us say E of k. How will I define this kinetic energy? Well, at any location I will calculate the kinetic energy say per unit volume. Then what I am going to do is I am going to average it over a wavelength and integrate it across the whole depth.

Basically, we are talking about a surface gravity wave like this, so I will look at it and integrate the energy from the top all the way to the bed. So, basically we will also be integrating it from minus h to 0, this will be taken about this.

So, you can very clearly see that this dx integral is actually taking the average over A1 wavelength. We can say that this is the kinetic energy integrated over the depth and averaged over a wavelength, so there you have it. We have this expression for u and w, we can plug it in there. I will take the liberty of omitting the steps and will tell you that you would get the x and z integrals, they are decoupled and one of which the x integral will give in terms of a times cosine cos kx minus omega t or sine that has been squared, so that would give a kind of a expression rho g into eta square bar.

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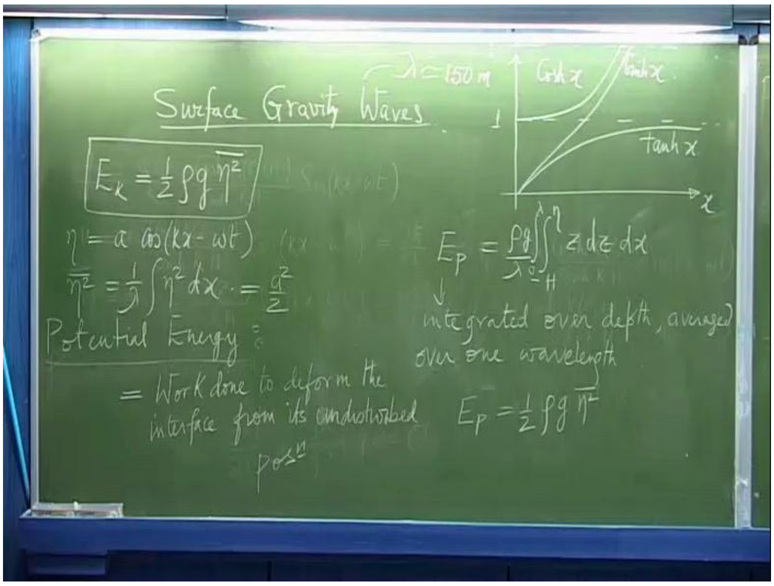

So this is like mean square wave elevation that is your eta square bar x. So, I have eta as the wave elevation, I will square it and I average it over 1 wavelength that is what we are writing. Since eta is a cos kx minus omega t you can work it out, it is rather easy, what is this going to be? 1 over lambda eta square dx that is the definition of eta bar square eta square bar. If I do that I will plug it in here and then I will work it out, I am going to get a square by 2, we are very straight forward.

Now that is story for kinetic energy, we are looking at very genuine case of lossless wave propagation and that would allow us to calculate the potential energy. These are the major players in defining the energy of the system because there are no losses, no heat addition and etcetera. We can talk about the dynamics in terms of kinetic and potential energy what will be it? Well, very simply stated, this will be actually the work done to deform a phase surface interface from its undisturbed position n.

So that is easily obtained again, I will call that as Ep, what would that be? Well, once again I will write the pressure, work done that I will find out for the deformed case going from the bottom minus H to eta - the deformed phase, rho gz will be the pressure that will act over small element dz, I am integrating over the whole thing, as we have done it before we will integrate over x and make an average.

It is the same context like kinetic energy was integrated across the depth and averaged over 1 wavelength, so potential energy is also are written in the same bases. It is the same thing integrated over depth and averaged, so there you have it.

Once again, these are simple algebra you can do it by yourself, what you would find? You will find that it should be also equal to half rho g, you can anticipate that. You are going to get a steady state where you are going to get a constant amplitude waves, what does it mean?

Like what we talked about earlier, it would mean a perfect balance transfer of energy from the potential to kinetic like your simple pendulum. It is a non dissipative system that is what you are seeing here also.

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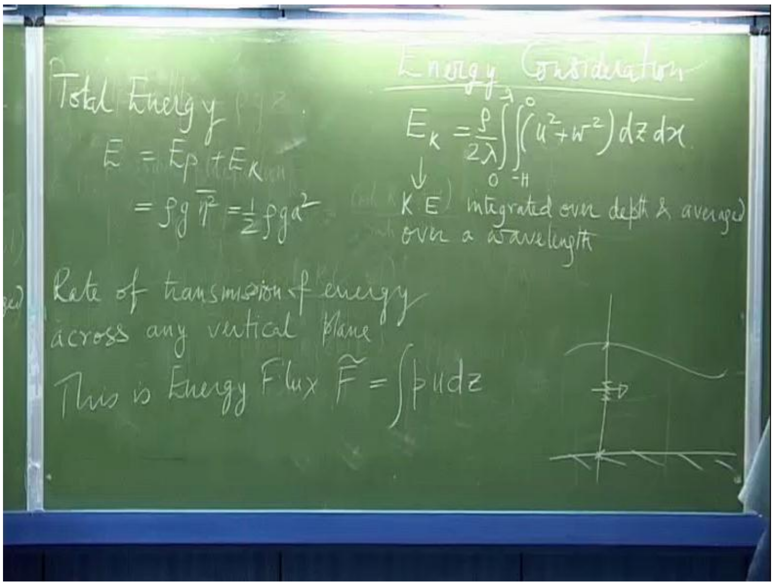

No price for guessing that the kinetic energy and potential energy should look like the same. Having obtained this what we could do is we can talk about the total energy of the system, well that would be sum of these two, you find that should be equal to rho g square bar, that should be equal to half rho g a square.

Next what we are going to look at is, this is the total energy of the system, if I integrate it over the whole depth that is what we are going to get. So, if that is the energy we want to find out when the wave is propagating – travelling what is the rate at which that corresponding energy is going as a flux?

If I want to calculate the rate of transmission of energy across any vertical plane, rate of transmission of energy is basically the flux and this is done; what we mean by energy flux? This energy flux which I will indicate by F tilde that would be nothing but what would it be the pressure velocity work.

Some of you would be familiar, some of you may find it little tricky, but that is essentially the thing. If I look at dz element, p into dz is the force acting and if I am looking at the rate at which it is going, the work done would be that force and displacement is dx. If I am trying to calculate its weight that will be dx dt. So that dx dt - that rate is given by this and this is your force p times dz, so that is you flux; that is the way the work is done per unit time for the displacement dx.

So, work it out. Now, I have just erased, but you can look at your class notes that p prime, this p is the pressure so I can write it in terms of the fluctuation. So, what I would get is F tilde would be equal to p prime minus rho gz time u dz, so it has two components. One would of course be coming from the fluctuating pressure part and the other part (Refer Slide Time: 29:42). So this you understand, so this is going from minus h to whatever eta or 0, I mean probably take the corresponding linearised version and we will do that.

So, you can set the limit here minus h to 0 and this is the other part, it could be minus h to 0, we have rho guz dz. Now as we have done before we have now reported quantities which are averaged over 1 wavelength, so we retain that spirit and define F. So, that is basically flux averaged over 1 wavelength. If I look at that expression that would be nothing but, F would be nothing but, I will integrate F tilde dx and this is what we mean that bring us like quantities.

What we have talked about the energy? We are correspondingly finding out the flux which is averaged over 1 wavelength. What do you find when I am trying to average this quantity over 1 wavelength? What will happen x dependence comes from here in u and u expression that you saw was in terms of cosine kx minus omega t? If I took an average of cosine function over 1 wavelength then that will give me 0, so this part is not going to contribute because the average in process in x direction on u over 1 wavelength, it will yield a null value.

So, what you are getting here is you are getting this expression 0 to lambda and the z integral minus h to 0 and we are going to get p prime u dz dx. So, you have the expression in front of you and we can plug those values of the quantities we already have.

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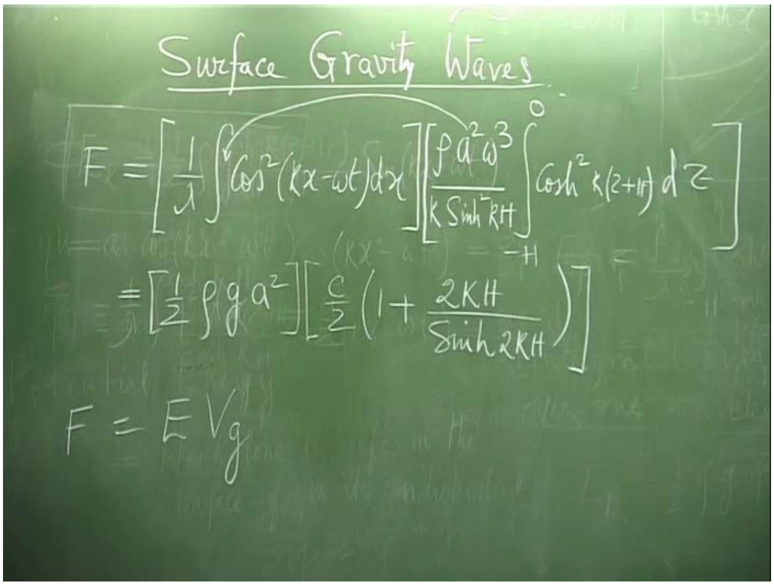

We have the expression for p prime that you look at it today, you would note that for F we have written the flux, flux would then have the x dependent part that would have cosine square kx minus omega t; this is your x integral. So that part is multiplied by all those associated quantities rho a square omega cube by k sine hyperbolic square kH; we will be performing the integrals of minus H to 0; we will have cosine hyperbolic square k times z plus H.

So, this times dz there are this two quantities, I have decoupled the x and z integral. Once again you can figure it out, this will be half, well of course this is a square, bring it here that will give you a square by 2. In place of omega cube I can use the dispersion relation; I can take omega square and put gk.

If I do that I am omitting steps, you can do it by yourself or if you are lazy you can look at the book, there you will find that this expression is given by as shown (Refer Slide Time: 35:00). So essentially, after all that what you are seeing that F is equal to this; your E you have just now derived, the energy total energy and this expression is Vg.

So, what you are seeing? What we expected to see was that the energy is being transported across any vertical plane is given by the energy that is being transported times a velocity and that typical velocity happens to be the group velocity, not c. So this is the physical implication of group velocity, it basically tells you the speed at which the energy propagates.

This we have shown it for specific example of surface gravity waves, but if you also recall when we looked at the superposition of two neighbors, there also we see that the amplitude propagates are Vg and V related with that amplitude square to energy. So you have seen the same thing, this is rather very important we have given a physical system and we are asked to compute how the energy is propagating by that system, then we need to know the correct velocity scale.

See what happens, many times we are so much preoccupied which our knowledge of simple linearised waves where Vg happens to be equal to c and then you do not have to distinguish between Vg and c. You always draw a general conclusion out of a mistaken notion that Vg is always going to be c and that is not the case.

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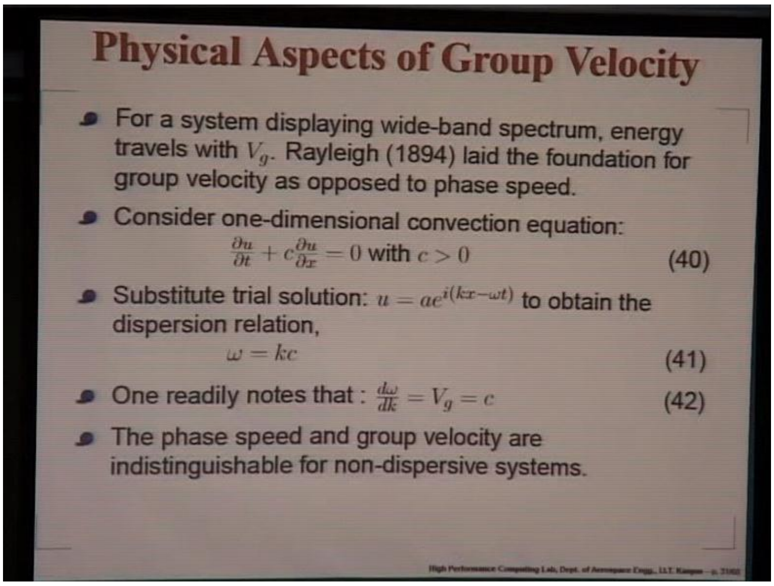

So, what happens is in many of the physical systems we will be looking at, we would be finding out the speed at which the energy propagates. If we are able to calculate that carefully; I mean that is a very important concept; tell you what; this was there in an original paper by Hamilton. You have heard of Hamilton in mechanics let us say Irish mathematician.

Hamilton actually in one of his paper talked about energy propagation speed, but it is probably Rayleigh who did a very good job and laid the foundation of group velocity. He was the first to tell that group velocity is different from phase speed. Now as I was telling, you if you are looking at 1d convection equation with c positive, you have substituted trail solution. A trail solution would have amplitude, a phase part and then plugged that in over there, you get this dispersion relation.

If you take the derivative of omega with respect to k it gives you the Vg. In this case, the phase or the group velocities are the same; this is one of the properties of non-dispersive system. Whereas, in large number of situation where you come across that is including the surface gravity wave what we see? We see that Vg is not equal to c because the phase speed itself is a function of k.

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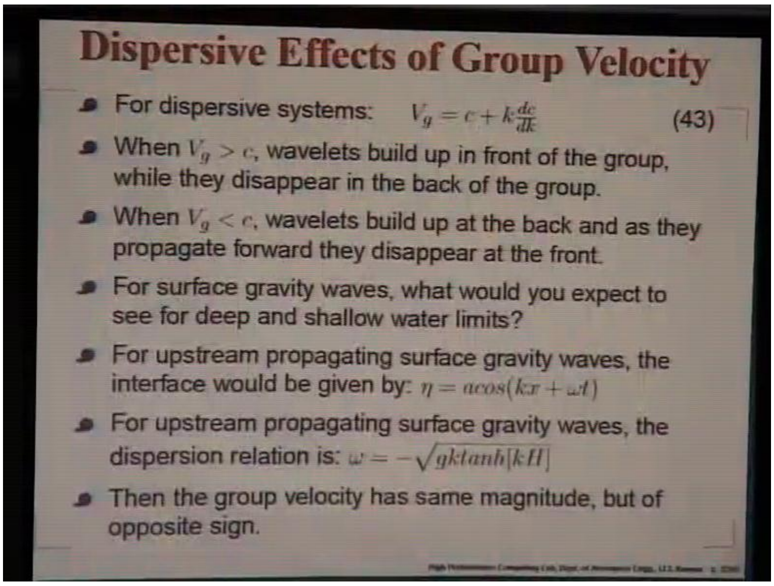

This part is non zero that makes Vg different for different k. What happens in consequence of that? An initial impulse is given to the system, fragments itself into various components, this will be travelling at their own velocity and you are going to see the different k component going at a different speed.

Now, what you notice that the phase is defined by phase speed c, but if the energy is propagating at a speed which is greater than the phase speed then what will happen? You would see that small wavelets riding on the overall wave packet. They will actually build up in the front because they are going at a faster speed compare to the phase, you will see them building up in the front of the group. Because, the energy is not there what will happens? They will disappear from the back.

This is something we will have to realize that if the group velocity is more than c then you will see the wavelength building up in the front, disappearing at the back and vice versa. When the group velocity is less than phase speed then you will see that the energy is lagging behind, while the system has the propensity to show that the phase can go ahead; that is what will happen. You would see that the wavelets will fall back, because there is no any of the energy to support them. So, they will keep disappearing in the front.

Next time, when you are near a pool of water, you decide to drop a stone and you can see which case depends on the depth of the pool that is, you are looking at on that type of wave that you can create. So, that is why we raise this question, take a look at it. Now, one thing I wanted to tell you is that so far we have been talking about waves which are going in the positive x direction that is why the phase was defined as kx minus omega t.

Now, suppose we are try to discern what is happening with gravity waves which are propagating in the upstream direction, then its general description would be given by the similar expression but you see the xt dependence on the phase is interface by a plus sign, this we will indicate that the wave is propagating upstream. What happens, you go through the same analysis that you have done? You would find the dispersion relation a p h with a negative sign, it is just that difference.

Now, what do you find? Calculate the group velocity by taking it is derivative with respect to k. You would find the magnitude will remains the same but it appears with opposite sign and there is nothing wrong in that simple observation.

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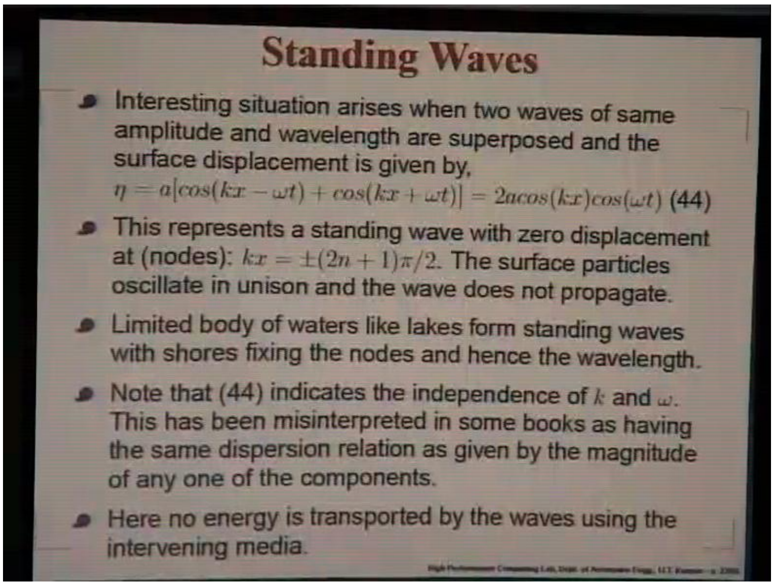

However, that is where something happens interesting, if we have two waves one is going in the positive direction, the other one is going in the negative direction, if they decide to superpose, then what happens? Well once again, simple trigonometry will tell you this. Here is an interesting scenario, the system depends on x and t, how? They are disjoint, they appear differently. We also notice some very interesting property that around kx equal to plus minus is odd multiplier of pi by 2, you will have eta equal to 0.

If that is the case then these locations are the nodes with 0 displacements. What happens here? I would get a kind of a space dependence that is given by cos kx, I have that but the time dependence is a simple harmonic, so what happens? All the point would go up and down in the same phase that is what we meant is here by the surface particles will oscillate in unison and the wave will not propagate.

I am particularly bringing it to your attention, because if you look at that book by kundu and cohen, I think that is where they have made a mistake by claiming that there is a group velocity that is given by either the upstream propagating or the downstream propagating path. But that is not true; the energy does not propagate, because you do not have a space time dependence appearing together. You need to have them in a composite function, which will be appearing together like what we have seen k x plus minus omega t.

If I have a limited body of water like lakes or ponds then the standing waves get created, why because, if you recall the d'alembert solution we talked about the Cauchy problem, we took the interval going from minus infinity to plus infinity and if I created a disturbance somewhere then it can go in both the directions.

Now, if I intervene by putting in some kind of a boundary what will happen? Those waves will go there and reflect back. Then the incident and the reflected wave can create standing waves and what happens? The dynamics would be given by this because the nodes will be fixed at the shores and that will determine the fundamental wavelength that is the longest lambda by 2 that you can get between the two shores, you will get half the wave; that is what you can get.

From this, we also summarize that k and omega are not related. So, we do not have a dispersion relation, they are independently appearing their and as I said that books tell you that dispersion relation remains the same, but it isn't true. Here, no energy is transported by the wave which is in the intervening media, standing waves has this property. Now, there are some other interesting properties of surface gravity waves. If I create an impulse on the surface of the liquid then what will happen? Simultaneously many harmonics will be created.

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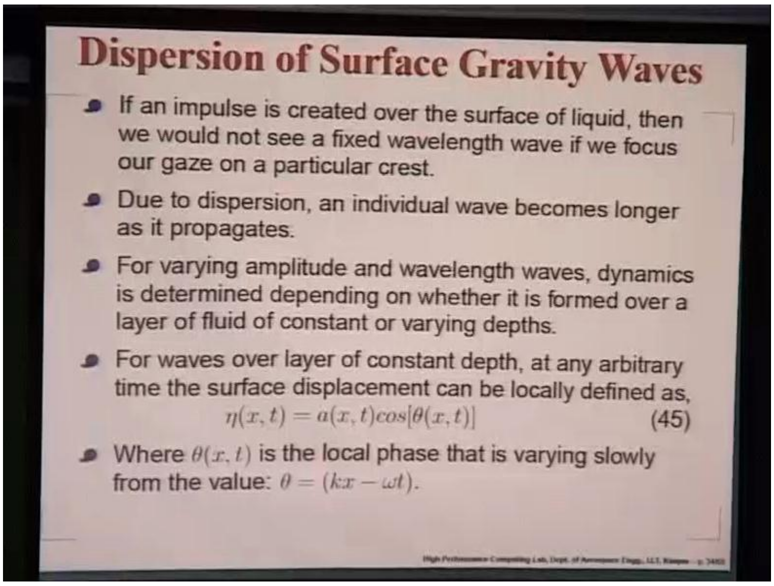

This is the case of a dispersive system, what will happen is that different k component will travel at different speed, what you would see as a consequence, if I give you a compact disturbance source with time? It will enlarge the region over which the disturbance is left.

Now, you have the expression for Vg. You would take a look and you would notice that depending on the value of k you are going to see different parts going at different rate. What would you see at the leading edge? If you look at the expression for Vg you would notice that the smallest k travels at a higher speed. So, near the leading edge of the disturbance packet you would see the longer wavelength disturbances.

Initially, I may create disturbance which have a compact basis but then with time it will not only stretch. You will also see the longer wavelength to the right in the system that is a generic statement, but further refinement occurs when we start looking at the dynamics of the disturbances as seen. We are looking at those disturbances forming over constant depth or a varying depth. In fact, this kind of variation of depth can add to very interesting situation.

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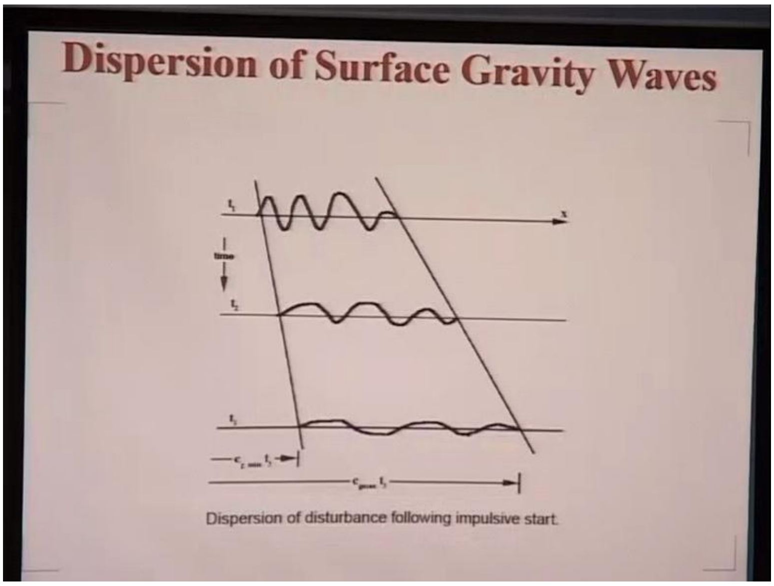

I can show you this picture and this is what? I was telling you probably this is not drawn very carefully and that shows up what should have actually happened in this figure. The longer wavelength part should be on this side.

This is the disturbance, this is the x axis and the time is increasing like this (Refer Slide Time: 48:40). So, with the mean passage of time the disturbance is stretched over larger and larger x. This part would be dominated by longer wavelength smaller k. So, this is not drawn very carefully as I should have done it, now what happens? We have seen that it is no more a monochromatic wave.

So, what is happening is, we are going to see some kind of a displacement, which you would like to say that is a slowly varying function of x and t, and slowly its changing. The same way the phase would be written in terms of theta, which is also a function of x and t, how different is this theta of x and t from its slowly varying function? This will still be written here like kx minus omega t, but the important point to recall here is that k and omega they are changing with x.

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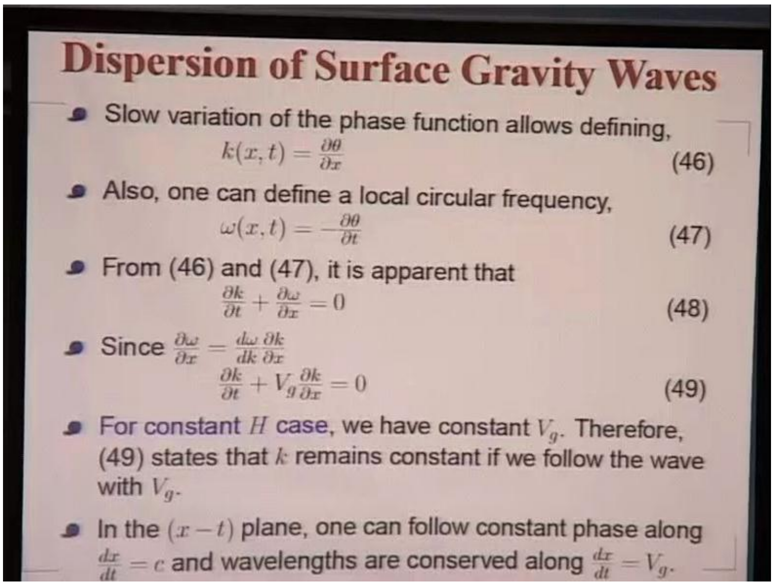

They are no more in a fixed entity like what we have talked about when we looked at monochromatic wave, so what happen? If I have a theta which we wrote like kx minus omega t then well I could take a derivative of this partial with sorry d theta dx, a slowly varying function I would just write it as k. That is what we have here that k of x comma t would be obtained by looking at the variation of phase function with x. The same way, I could differentiate this with respect to time and that will approximately give me minus omega.

This is all happening because we are talking about slow variation. If that is the way you look at the displacement, find its variation with x and t, talk about a local k and local omega, then you can further differentiate this one with respect to t, this one with respect to x and you see the right hand side will add up to 0 (Refer Slide Time: 51:00).

So, you see a very interesting property that for such slowly varying waves the del k del t plus del omega del x goes to 0. Since, this del omega del x we could write with the use of chain rule in terms of d omega d k into del k del x. This d omega dk is a group velocity, so what we are noticing that for slowly varying waves is that k is governed by this equation. This is like your convection equation that we had obtained for displacement of other physical variable.

So, similar such thing is being noticed here for the wave number k provided, we track it at the value of Vg and see if we are going at the speed Vg then we would be always looking at the same k, because this is nothing but the total derivative d dt of k. This tells us that if we decide to track the same value of k then we should be fixing our gauge and the gauge should move at Vg.

This kind of confusion occurs if you have noticed some of these waves in pond or confined space. Then, most of the time what we try to do? We try to follow a crest or a trough something of that kind, then what happens? You see the following things; one we have already said that as it goes outwards what will happen? You are going to go over to longer over longer length, how? Then all of a sudden when you have energy cannot keep phase with the speed Vg that it has, and then it will disappear that is what you see.

However, if you want to follow the same wavelength component then you really must move your gauge with this velocity Vg. Now, if I go back to my xt plane then I should be looking at two sets of lines; one is given by dx dt equal to c, those rays will tell me along which c is constant. There is the other ray path that will draw that it is given by dx by dt, given by Vg is the line along with k remains constant.

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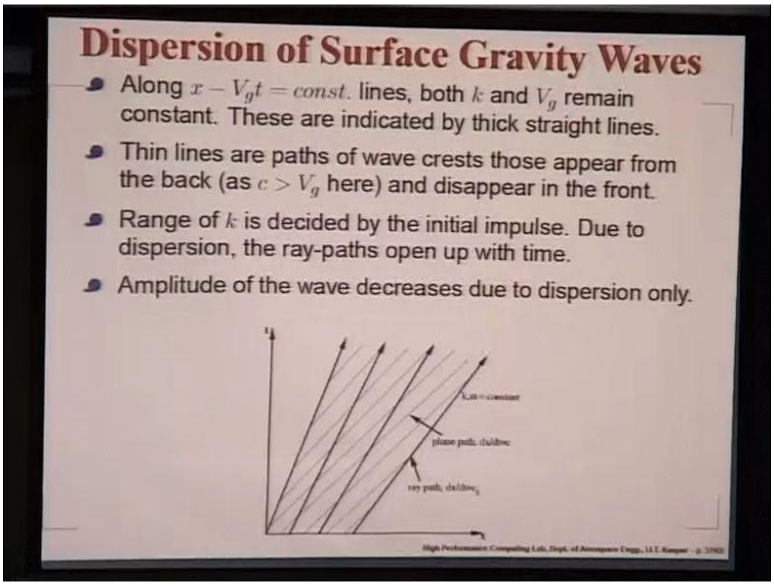

What we find that at time t equal to 0 I have created some band of disturbances in this range of x with the passage of time that band widens because of dispersion effect and then what we can see? We can see that the ray path given by dx dt equal to Vg that defines where I would see the actual disturbances, because that is the speed at which energy is going.

So, the extreme of that Vg range will determine how this envelope is going to enlarge and this thin lines tells you along which your phase is constant. If I am looking at surface gravity waves what happens is, along this line k and Vg remains constant. These are the ones those are indicated by the thick straight lines. The thin lines and paths of wave crests those appeared from the back because of why? We are looking at let us say either of the case; suppose we are looking at deep water wave then c is of course greater than Vg and for deep shallow water c will be equal to Vg.

So if we are looking at deep water wave c will be greater than Vg, then what will happen? You will see that crest will appear from the back and then they will disappear in the front that is determined by the range of Vg - admissible Vg. This range of k is of course decided by the initial impulse, how I have distributed the initial energy across different k that I can get it from the initial condition.

What happens later is a story that is dictated up on by the dispersion, as we note here the ray paths open up with time and this is a very important observation. If I create a disturbance at one point with time what happens? You are going to see constant itself although we are talking about one d wave propagation, but here x is more like your arm in a cylindrical quadrant system. So, the same energy is going to be distributed over larger and larger area.

So what happens? The amplitude must come down, when you see such waves amplitude is coming down it is not a viscous effect, it is not due to loss that the amplitude is coming down, it is because of distributing energy over larger area. That is another nice property of dispersion and what happens, because of dispersion amplitude keeps coming down as time progresses. We will have a better and better support for your theory, because amplitude comes down and that was our starting assumption for linearization.

So, I will stop here and we will continue may be couple of more lectures and talk about certain very interesting properties of this.