Foundation of Scientific Computing

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Module No. # 01

Lecture No. # 08

Today, we are going to discuss about, begin our discussion by talking about, wave packets, which are essentially regular and non dissipative; that means, the wave packets will keep appearing in space and time, at regular intervals.

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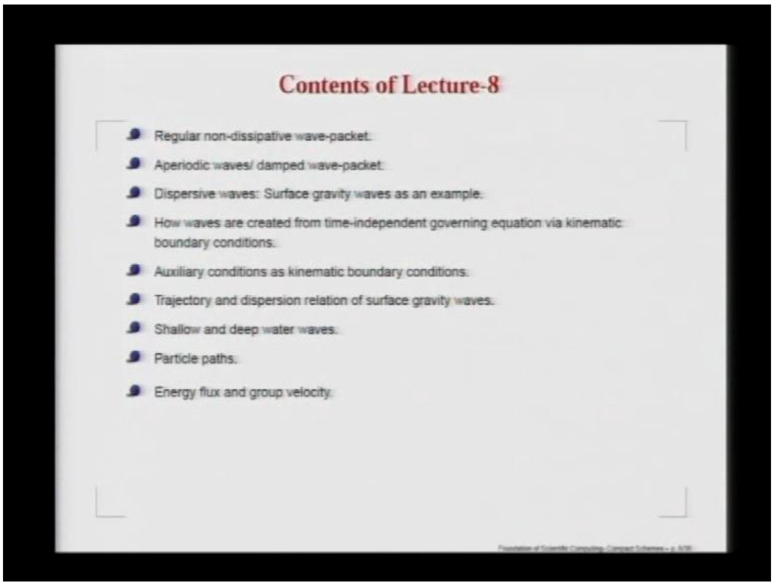

This will be followed by wave packets which are essentially singular in appearance, in the sense that they will be aperiodic. This can also come about, because of the damping present in the system and as we said that when it comes to waves we do not distinguish between hyperbolic or dispersive wave.

We draw an example of a dispersive wave from surface gravity waves, forming over liquids or fluids or water and what do you find? That the creation of the waves does not require the governing equation to be time dependent; that is why we have specifically chosen this particular problem of surface gravity waves because the governing equation is Laplace equation which is time independent.

The space time dependence of the wave propagation comes about through the boundary condition and how this boundary condition affects that is what we are going to talk about with the help of these kinematic boundary conditions. This auxiliary condition sets up the waves and then we will talk about specifically various forms of surface gravity waves, how the trajectories are of individual particles.

We also would like to relate its space and time dependence through what is known as a dispersion relation. That is a relation between the wave number k of the circular frequency, omega. This is what we would establish for those surface gravity waves. We can further classify the surface gravity waves into two depending on the depth of the waves. We are talking about either shallow or deep water waves and we will see how the particle paths are different for shallow and deep water waves.

We will also notice one particular aspect of this dispersive wave is the existence of group velocity. Group velocity is distinct from the phase speed because the group velocity is important; because this is the speed at which energy flux is created in the flow.

Basically, I thought that you are familiar with waves but, it is mostly done in the context of may be optics or acoustics. So here, I thought I given an example of a mechanical system waves and fluids happen to be very well researched topic; people have been looking at with much longer than any other fields. Probably you could be able to relate to other fundamental concepts of energy and its propagation etcetera. So, I thought in geo physical fluid dynamic example of the quite in order.

Please note down that I am mostly following this number 1 and it is a very decent book. There is South Asian edition available also in the market. So, if you want to have a book of this type dealing with fluid mechanics engineer; it is a beautiful book you can really take a look at that.

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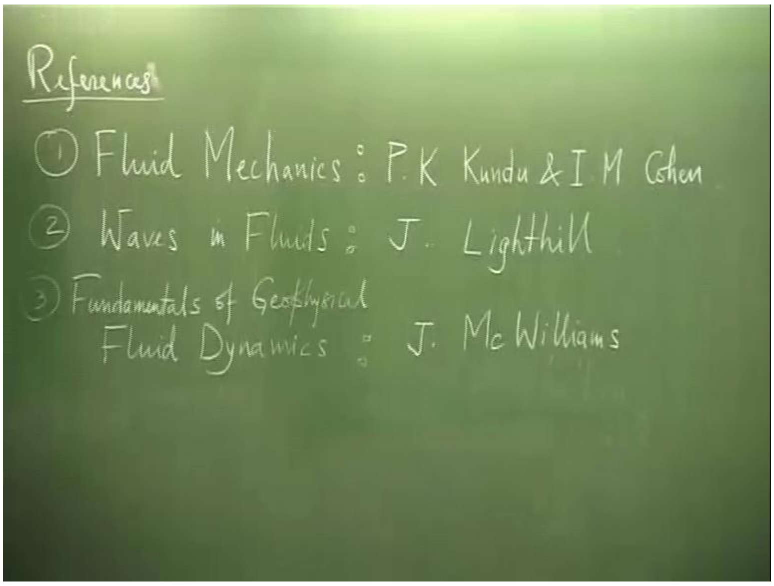

So second one is a classic, this written by J. Lighthill. We owe most of our current state of knowledge to people like Lighthill (( )) those you have contributed in the last century. Of course, prior to that you have giants like Stokes, Bucyrus, Raleigh, they have contributed but, this one should be more than adequate for you to understand what we are doing and some of the things I am doing it from the numerical prospective. So, you will have to pick it from some book on numeric.

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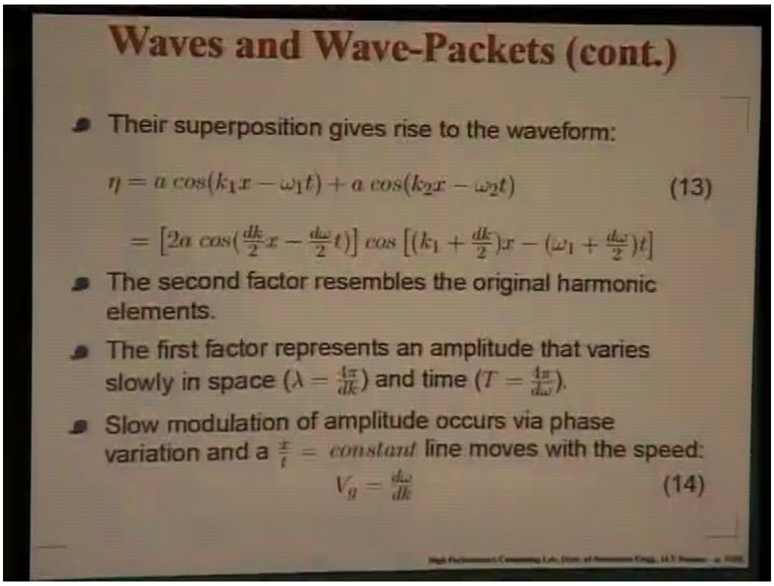

Yesterday, we are talking about system were we have not necessarily a single harmonic content but, you would have the presence of a continue of wave number and if you have that then, we were trying to figure out what happens when 2 neighbors interact. So, the neighbors here are characterized by the wave number k 1 and k 2. The corresponding circular frequencies are given by omega1 and omega2; their differences are given by d k and d omega.

So when we look at their super position, we find that it is composed of two factors. So, first factor gives you amplitude that very slowly with a wave length of 4pi by d k and the time period of 4pi by d omega whereas, the second factor is nothing but a small variation of the original harmonic element which are given here.

So what we notice that two neighbors interact via super position to give you a slow modulation of amplitude and that gives you a kind of a phase variation given by this argument and if you want to track constant x by t line that you can see, you would be doing it by following d omega by d k.

So, what we find that the amplitude moves with this speed right, this Vg and since the amplitude is proportional to the energy, the other way energy is proportional to amplitude squared. So, this velocity is the characteristic velocity of the propagation of energies (Refer Slide Time: 07:00). I will show it with the help of some examples why that is true in a little while.

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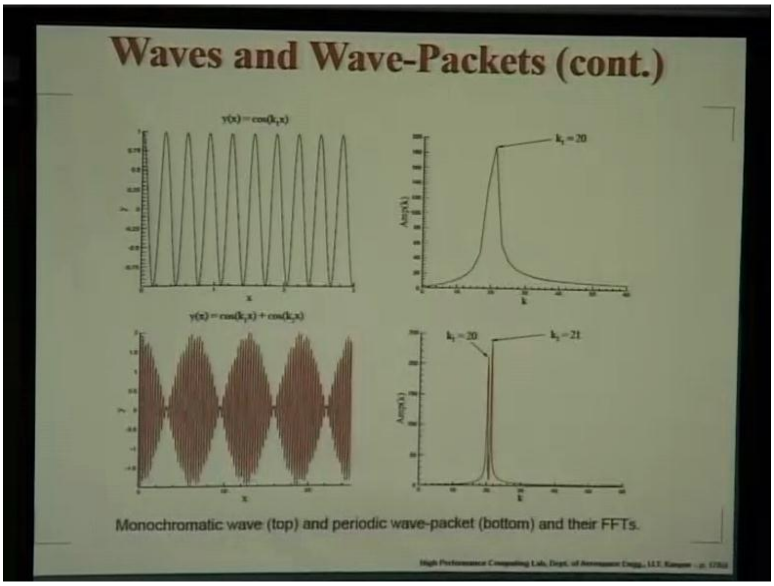

First of all, let us try to distinguish what we get by looking at waves and wave packets. So on top what you are seeing is a simple harmonic wave and which is characterized by let us say this wave number that we have plotted here for k 1equal to 20. This is the way the signal varies with phase that cause k 0, k 1 x is plotted here.

Now, what we did? As shown in the previous slide, if we look at interaction of two neighbors that is what we try to do here. So, what we do is we take two neighbors of wave numbers 20 and 21 and see what happens and this is what you see; ok (Refer Slide Time: 07:52). So this is what you are seeing; that is embedded inside each of this packet you have a harmonic variation given by this because that is characterized by some wave number, that is 20.5; those are those rapid variations inside and the amplitude envelope is given by a wave number 1, the difference between k1 and k2. So, this is what you would see.

So, if I go back to the communicating signal then, which one would be better? If I want to send a signal of let us say wave number around 20 or 20.5, which one would be better? Should I send it as a wave or should I send it as a wave packet? Why?

You see the presence of this nodes and antinodes. At antinodes of course, amplitude doubles whereas, at the nodes they come down. So, you will have to basically see the energy carried is one in this rules, you can compare. I can compare that if I try to send two signals separately at 20 and 21, what will be the corresponding energy and I can find out what will be the energy here, if I send them.

So, what we make? What is a conclusion? That you may not be able to get a tremendous benefit because energy is proportional to amplitude squared. So, here it is a fourfold increase at the antinodes whereas, it can come down to almost 0 in the intermeeting period near the node. So, you have to do that accounting and come back and tell me which one is better; I will look forward to that answer. So, that is the one way of communication where we can send of the monochromatic waves or we can send it as a kind of a packet.

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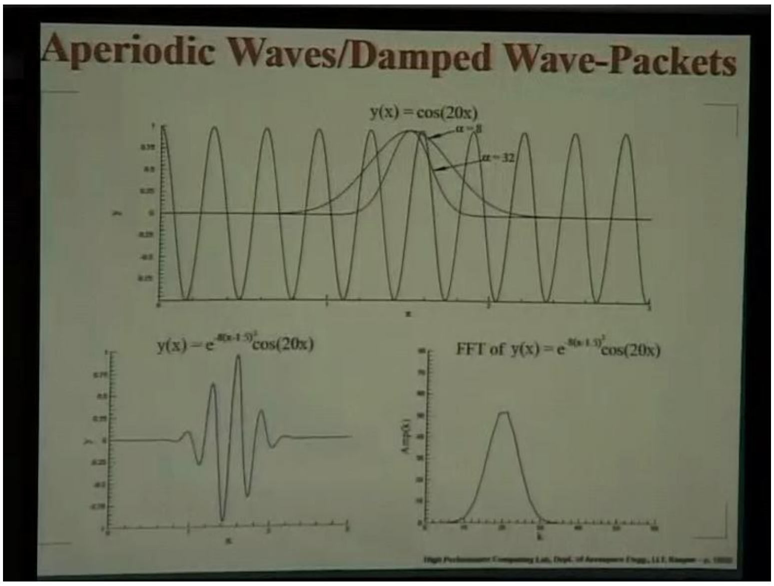

So, there is another way signals are quite often transmitted. Those are when this signal is embedded in amplitude envelope, which renders its aperiodic nature. So what happens here, we have the basic signal for 20 x let us say; then, we modulate its amplitude by this Gaussian Function, then what happens is the signal information that you are carrying still corresponds to 20 x, but it is amplitude decay.

So what happens as a consequence? You have almost zero amplitude outside this single packet; so this is now from a multiple packets of the previous slide, you have come to a state where you have single packet and by designing your system by choosing this exponent here I have shown you as 8. If I would have increase the damping and brought it to 32, then I would find that it will decay much more rapidly.

The envelop size comes down in proportional to this exponent. So, what we find that such a signal if I try to pass through continuum I would be spending less energy because if I try to send this original signal it is going to continuously start for attenuation and decay, various other sources of decay; whereas, such an aperiodic wave or a damped wave packet is going to be rather very good to do that.

In fact, later on will come back to another topic where you will see how signals are even transmitted as a single wave, solitary wave which may actually look like this envelop and that is what is called as soliton. We will talk about soliton and you would note that soliton is quite often used to transmit optical communication signals by pulses.

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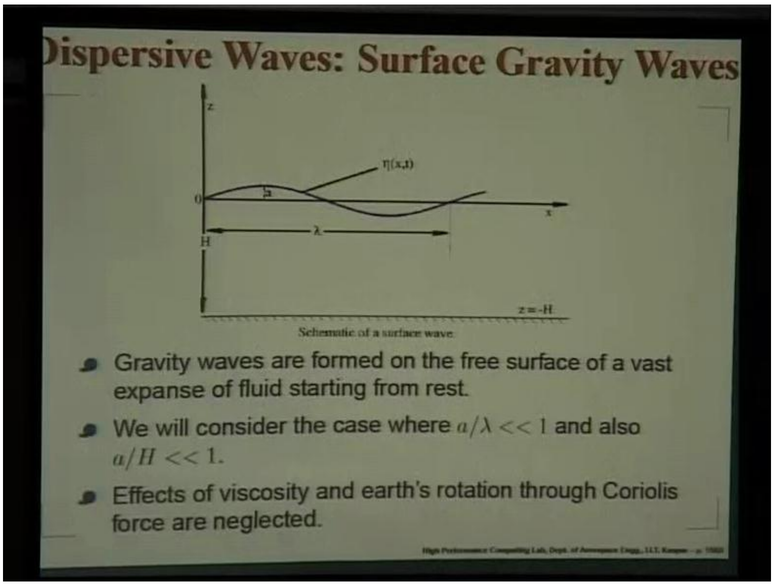

Now, let us get into some detailed look into one such mechanical waves, that is created over a vast expanse of fluid and you have a fluid at rest, then you just give a kind of a disturbance on the surface. So the surface will be deflected. Let us say, we have tried to create a single harmonic wave here, whose wave length is given by lambda and this surface wave that we are creating on the surface is firming over a layer of liquid of height h and the deflected control is given by eta x and t.

In this exert plane we would like to figure out, what happen? How these waves are generated? What are their properties? That is the topic of our discussion today. To make the arithmetic, the mathematics simple, what we would be looking at is really a small amplitude wave. So, small amplitude means the slope of the wave is small that is given by a over lambda, a is the amplitude of the wave. So, that is small and we would also be talking about this amplitude are so small that they are insignificant as compare to a depth of the liquid.

To make things further simpler, we will ignore the effect of viscosity. So, no viscous losses and we are talking about surface gravity wave and if we` look at the once that you generally see in lakes, rivers and ocean if we are not looking at very long wavelength cases, you can really neglect the earth rotation effect that shows up or to the coriolista.

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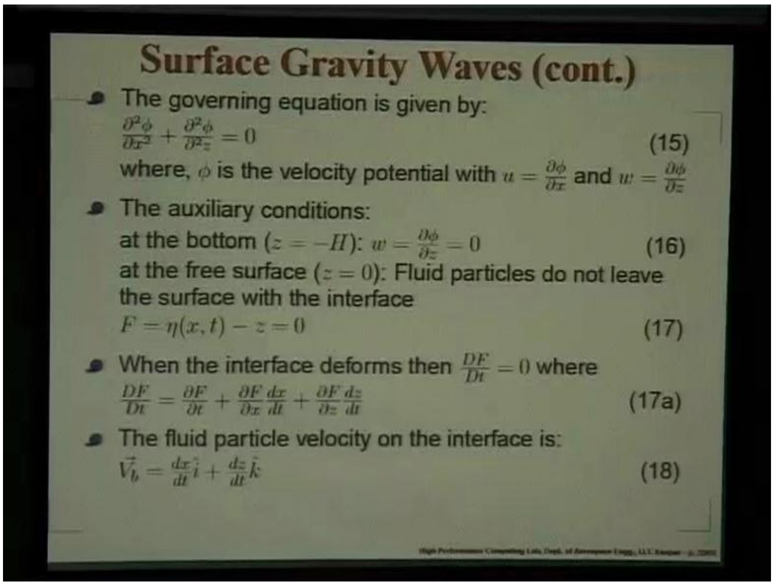

Now, what do we get? The governing equation here comes from the mass conservation equation that is delta at v equal to 0 and where v is essentially for rotational flow. We can write it as a gradient of a potential pi, so it is here and you get equation 15. The Laplacian of 5 in that exert plane is going to be 0 and the velocity as I noted here, the components are given as the x component del phi del x and the z component is del phi del x (Refer Slide Time: 14:42).

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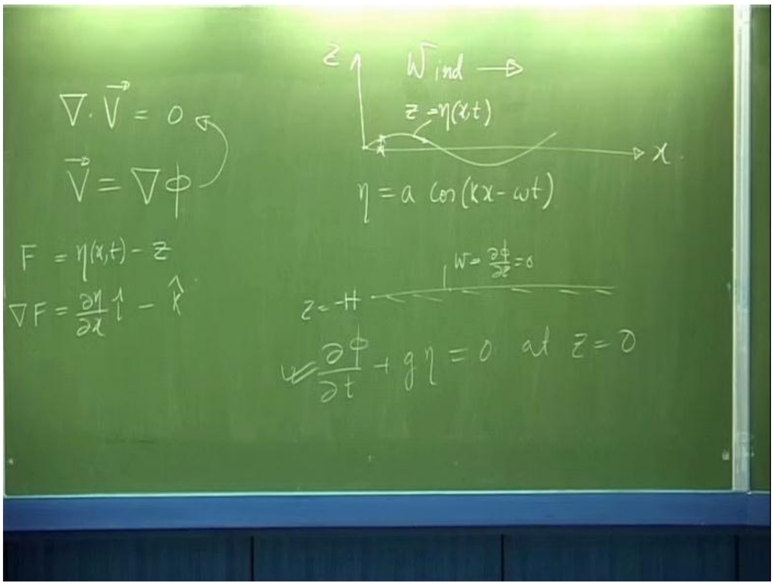

Now, you are considering formation of wave over a flat surface. This is my mean unreformed datum and in this direction we are plotting z. This is wave, let us say the wave is forming (Refer Slide Time: 15:15). Basically, we have this surface at z equal to minus h and that is at the bottom, what do we expect there? There should not be any normal component of velocity. Since, we are doing in this analysis it will allow a tangential component but, no normal component.

The first condition 16 corresponds to 0 normal velocity at the bottom of the bed, while at the free surface, at the mean position we will just simply say the fluid would not leave the surface, it should not go out and form a bubble here floating in the air; that of course is possible and feasible. People do it in multi phase pros but, we will not talk about it because here of course, ostensibly we will have wind air blowing. We do not worry about its contribution to the dynamics of this wave in the analysis that we are doing.

So at the free surface we will say that the fluid particle could not leave the surface and this interface equation as I told you is given by eta xt that is your z. So, what I can say is I can define a function F which is nothing but eta xt minus z equal to 0 that defines the interface of geometry.

When the interface deforms then, what we would expect? That it is total derivative would be equal to 0, where the total derivative is given by the local path, local time variation path. The other one that could be cause because of the motion of the fluid itself that is the convective path; where of course, you know that the fluid particle velocity on the interface, that could be written in terms of its components dx dt and dy dt like this. So, what you notice that this equation that 17a, we have written actually can be recognize as a vector equation of this form. So, gradient F. vb plus del F del t is equal to 0.

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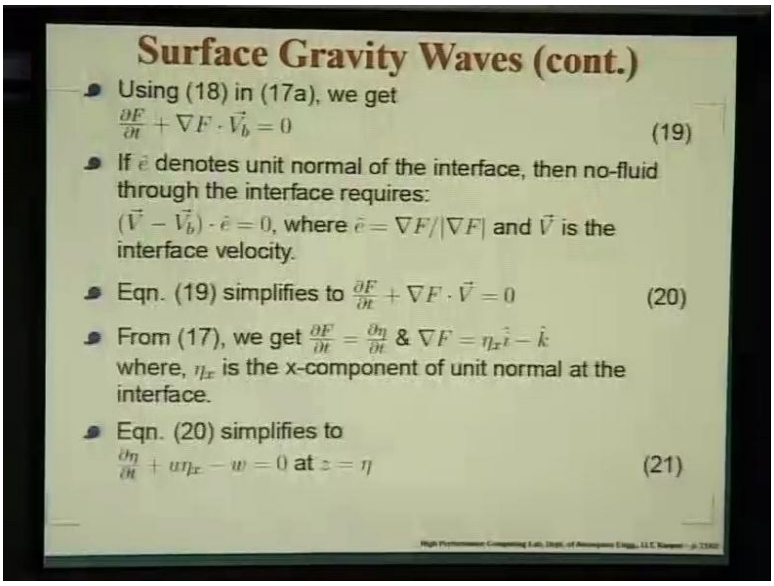

Now, if I define e cap as the unit normal on the interface then, the condition that we have said that there is no fluid through. So, what will happen is the velocity of this fluid particle that is vb and v is the velocity of the interface they would essentially be same. So, there is no discontinuity. So, that means the dot product of that with respect to e cap equal to 0, where of course, if F defines the interface e cap is nothing but, gradient of f divided by the modulus of that gradient.

So, we can see that at the interface v equal to vb. So, we can replace that vb by v; basically, trying to mix up Allerian Legrangian description that we have decoupled here, it was coupled because vb was Legrangian velocity whereas, this v is the corresponding Allerian discretion; so that is what we simplify here. What we also note that F interface was given by eta x t minus z. If, I try to find out what is del F del t, it will be simply nothing but del eta del t and the gradient F you can see is going to be eta x. So, that will be del eta del x and this path will give me just the k component.

So, del F del z will give me the k component and that is what we have written here, gradient of F is eta F is that of particle F and plug it in there, then you get this expression and this is at the deform height (Refer Slide Time: 20:00). So, it is to be applied at z equal to eta.

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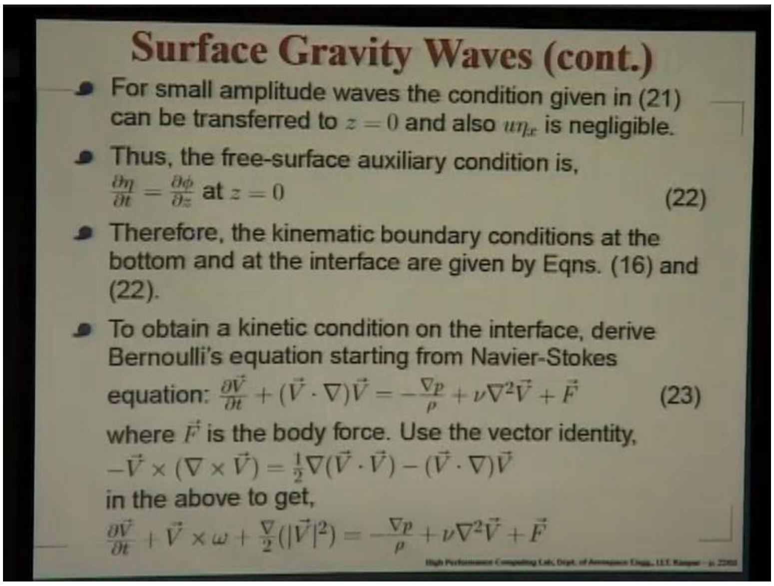

As I told you that we began this analysis, saying we will be doing some kind of a linearization looking at small amplitude waves. So what we would be doing is, we will be making some additional simplification at this condition that I am applying at the deform interface would be reflected back on the mean interface; that is what we are saying that condition will be transfer to z equal to 0.

If you look at the last equation, this middle term u into eta x is a kind of a product term every quantity here is small. So, del eta del t is small, w is small, but you look at this, this is a product up to small term. So that also we can neglect in a linearized analysis and that is being said that u eta is negligible.

What happens is the condition that we had written del F del t equal to that gradient of F times v equal to 0, simplifies to this condition. This is a consequence of linearization and transposing the boundary condition at the mean interface. So with those two conditions as additional approximation, we arrive at this auxiliary condition at the mean interface.

Now what we have? We have two conditions: one condition is as we wrote it down that w that is equal to del phi del z equal to 0; that is your equation 16 that we are referring to here. In addition, we have this condition del eta del t at the mean interface. These are two kinematic conditions.

Now, in addition we would also like to derive some conditions which originate from the governing equation. Those are called the kinetic conditions and that comes out from the governing equation simplified for the present purpose. So governing equation, generic equation is the Navier-Stokes equation and with all this assumptions that we are making about irrotational flow, small amplitude waves etcetera will allow us to simplify to Bernoulli's equation. So if any of you are interested, these are the steps that you follow.

So, equation 23 is your Navier-Stokes equation and to simplify, what we have done here basically this convective actuation term that appears here is replaced by this vector identity. You can notice that this is nothing but gradient of v square by 2 and that is what is written here and this term del cross v is the vorticity omega itself, that's what we get. This is another way of writing out the Navier-Stokes equation that we keep solving all the types in fluid mechanics, aero dynamics etcetera.

Now, what we are going to do is we are going to simplify this equation. Simplification will come about by omitting the viscous term that is explicitly there and then we will also say that the flow is irrotational, so omega is 0 and this term will also drop out and b has written as gradient of a file.

So, this equation that we have (Refer Slide Time: 24:09) we will write it as del del t of gradient of phi and this term we can put it on the right hand side. Suppose the body force is coming through the gravity. So that f is nothing but gradient of gz and this is what happens to your Navier-Stokes equation.

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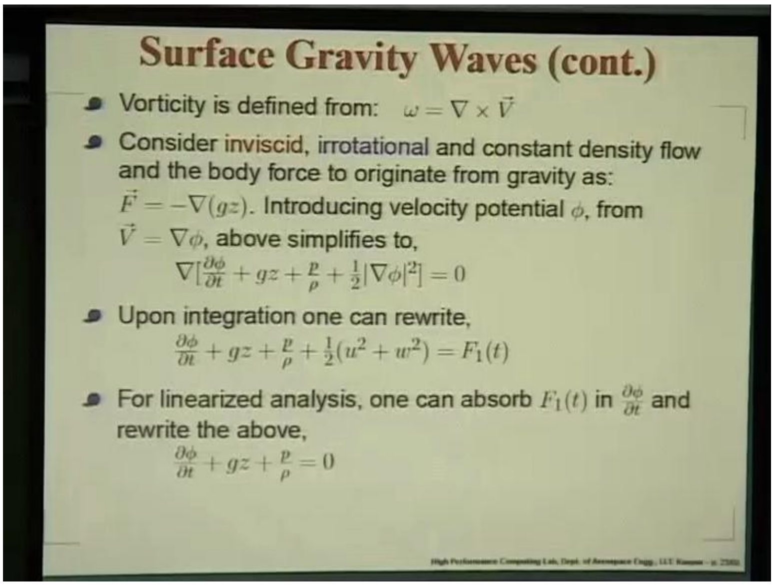

This term comes from the del v del t term, this comes from the body force term, this is the pressure gradient term and this is one path of the connective actualization which survive after irrotational approximate (Refer Slide Time: 24:20).

This is what you have is the special gradient of that quantity is 0. So, you can integrate and this is what you get. This is the general formation of Bernoulli’s equation that you may have done, if you have taken any course on fluid mechanics before, this is little more general because we are talking about the unsteadiness of the flow as well.

Now, what you can see that if you are doing a linear analysis then this could be actually put in this term, we can redefine this phi; we include that F 1 term and of course, this two term are basically non-linear because u square and w square, so that you omit and you get this as that equation. So, this is your Bernoulli’s equation.

Now, I was expecting that someone - some of you- would quiz me on the fact that here the governing equation or Laplace equation. What happened to the discussion that we had made about boundary conditions? How many boundary conditions we are talking about? Why we are going through all the steps? Various conditions we are seeing here.

What is the domain? See we are looking at this few problems in the exact place. If, I want to compute, so in the fluid path z is restricted between the deform interface and the bottom, what about x? Anyone, what about x? What is the range of x that we are talking about here? Web length, basically, we are talking about waves being getting created. So, it is a basically unlimited domain problem. It is the absolutely known limit.

What we are looking at in a sense, unbounded domain in the x direction and we all looking at system which is not visited upon by losses, because this terms have been omitted. We can afford to take a very large domain and then what happens is we can look at the dynamic of the flow in the z direction and that is why we are struggling with getting some condition here, some condition there (Refer Slide Time: 27:04).

So in a sense, in the back of our mind, we are thinking in terms of some kind of separation of variable and that is why we are looking for some kind of two sets of condition, one at z equal to 0 and another is z equal to minus h; that is what we are going through and you are seeing that needs little bit of care here.

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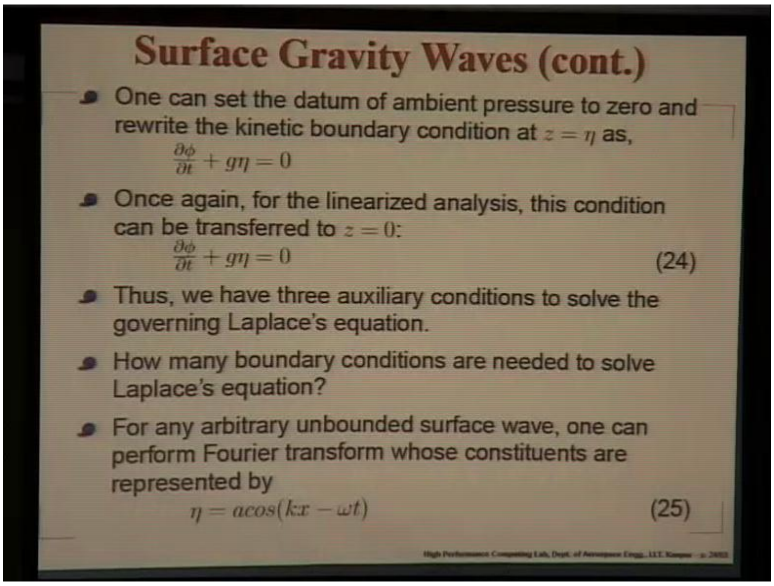

What happens is we also look at the pressure variation in terms of some datum. Datum is given by rules the z, what we could do is (( )) above in the air, part of the problem. We could put that pressure equal to 0. So, that is your datum.

Then what happens is the exertion of pressure would come through this term; one is due to the variation of the velocity field because of this unsteady deformation here and the unsteady deformation also brings about the change in potential energy. So, that comes through that term. What happens is, because we are doing linear analysis and we said that we are going to transfer the condition from z equal to eta to z equal to 0. These are the fix.

What happens is now we have three auxiliary conditions to solve the governing Laplace’s. Now, I already answered this question, so you know the answer that we are thinking of solving a problem which is unlimited in the x direction. That is why, we are using a trial solution of this time a cos of kx minus omega t. So, x can go from minus infinity to plus infinity. We are not restricting there and a is some kind of amplitude of that surface wave and eta is that surface interface displacement.

Now this is rather interesting, you know many times in computing just because some terms do not appear explicitly, we cannot forget about it and that is why this example is also very good one because we saw the governing equation is a Laplace equation.

There is no mention of the time variation, but we know the problem that we are trying to solve has this unsteady variation of the interface; so surface waves are getting created. So, that is what you will have keep in mind that just by looking at a governing differential equation, do not make up your mind in thinking what you are solving.

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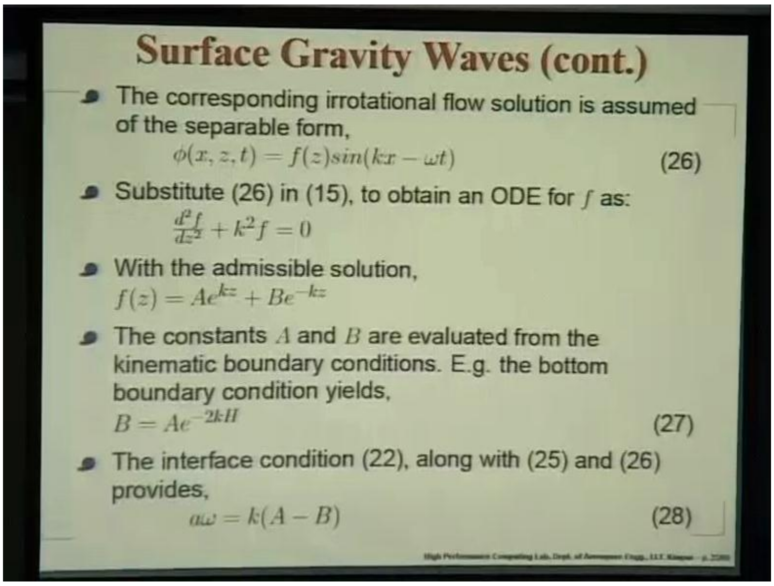

Here is a case, time is implicit and that is why what we have done is we have used that Trans solution where time appears explicitly like a wave part that is what we saw in the d'alembert solution. If I have a wave, it will have the argument going like this, a combination of x and omega t. They would not appear separately, they have to come together to have a space time variation as a wave.

Now, if I look at this space time dependent problem, what I could do? I told you that I would like to write it in terms of a wavy part; that is the wave propagating in x and t and its amplitude would be a function of z. It will depend on at what height you are looking at. You substitute this in your Laplace's equation, you get this ODE that would give you step forward the solution F of z combination of a two exponentials. These two constants can be figured out this condition that del phi del equal to 0.

If I plug that in what I would do is that should imply del phi del z equal to 0, would imply df dz equal to 0 at z equal to minus H. You plug that in will give this solution, this condition relating d and a, because also that we have worked out the interface the interface condition was given by what? We wrote it down that the condition that we would like to apply would be del phi del t plus g of eta equal to 0 at z equal to 0.

What I do? I have the Trans solution phi, so I try to find out what that del phi del t is? That will give me minus omega F and g of eta will give me g and a cosine this, so that would give me this solution (Refer Slide Time: 32:00). All of you can see that so, if basically application of this condition using the expression for phi here and eta as we have written down. We substitute and you get the equation there.

So, you have obtained the value of a and b as written here and put it back there, because you know what that solution is A e to the power plus kz plus b2 minus kz. We substitute it there and you would find that they will give you this hyperbolic function, cosine hyperbolic function and sin hyperbolic function will come (Refer Slide Time: 33:05).

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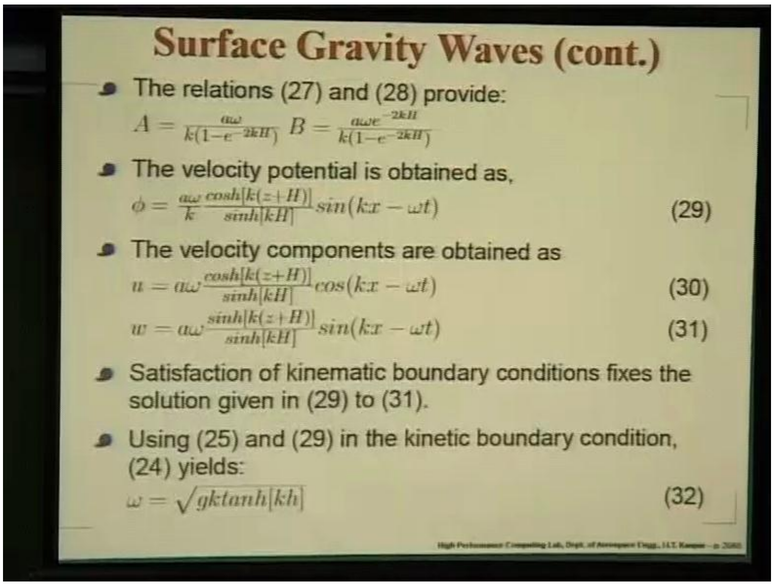

Having obtained the velocity potential you can calculate the velocity by differentiating it with respect to x will give you u, differentiate this with respect to z, you will get w and what is left? That is of course kinetic condition; the kinetic condition would come about I think, I have misguided you that is what happens then you are in a rush sorry this is not this is the kinetic condition and I mix it up for you sorry for that mistake let me just work out how do I get that…

See, that the kinematic condition that we had at z equal to 0 that was written down here 22. So that was your del eta del t should be equal to del phi del z and eta of course, is a cosine kx minus omega t and phi, we are writing as A e to the power kz plus B e to the power minus kz and sin kx minus omega t.

Now, del eta del t would give me minus plus A omega and we will have to apply that z equal to 0, that will give us A of k minus B k this will approximate as it is. Suppose that gives that condition 28, we just nock this off, so you get A omega equal to k times A minus B sorry for that mistake (Refer Slide Time: 35:50).

Now, we have taken care of both the kinematic condition to derive A and B. What is left now for us to really explore the kinetic condition? The kinetic condition is a Bernoulli's equation that we have written g of eta equal to 0 and we have the expressions for phi and we have the expression for eta. We can figure out del phi del eta would be minus A omega square by k and cosine hyperbolic k times z plus H by sin hyperbolic kH and cosine k x minus wt.

So, what you are getting from here that will be minus a omega square by k and there is a part, I have not written that; take down, I will write rest of that and plus g of a equal to 0 (Refer Slide Time: 37:50). That is basically done tells you what you are going to get is this, we are going to apply at z equal to 0. This will simplify to cosine hyperbolic kH. What we are getting is a here also goes off, so we are going to get omega square equal to gk and sin hyperbolic by cos hyperbolic that is given tan hyperbolic of kH.

This basically tells you what this relationship between omega and kr, so what you do? You write down this expression as we have done here in 32 that relate your omega with k. So this is what we called as the dispersion relation. Why did you call dispersion relation? We will shortly discuss what we notice is, we are relating the time variation with the space variation in the k omega place, your governing equation did not have that information because that was a Laplace equation.

See, the reason that we went to k omega space is because our physical space does not allow us to have that information. There is no explicit dependence on time from the governing equation. How did we get this? All of this came through those kinematic conditions and the kinetic conditions which was time dependent.

The boundary conditions - time independent boundary conditions - give rise to this. So, this is what we have been talking about for quite some time now but, space time relationship is often obtained not from the governing equation but also through the boundary condition. Here is a crystal clear example of how it happens.

Here this is a mistake, this should be capital H as I have done it there, this is a correct expression gives me that is why I have not loaded the load case after keeping all this throughout all this small mistakes and then, it will be loaded may be tomorrow (Refer Slide Time: 40:20).

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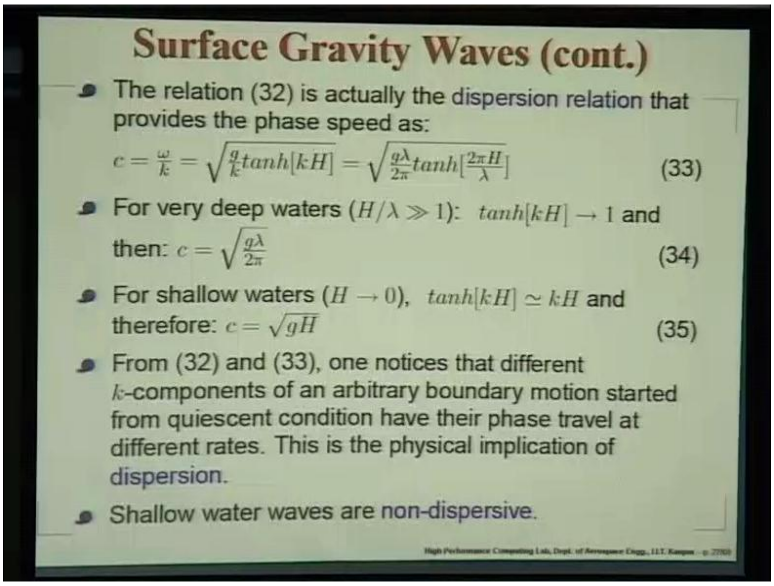

So, this is what we call of the dispersion relation and now, we should be talking about what dispersion is, but for this purpose gravity wave you can talk about the phase speed. Phase speed is by definition omega by k, so we get this. Now, you can also distinguish two cases of dispersion gravity wave. Consider the case, where it is forming over a very deep layer of liquid, so that H by lambda is a very large and then what happens? sin hyperbolic kH takes the asymptotic value of 1 and then this part is 1. So, you get t equal to g lambda by 2pi.

So that is for your deep water wave and for shallow water on the other extreme would be where H is negligibly small. So that tan hyperbolic kH can be approximately kH and if you do that you will notice that this kk will cancel out and you get this. Now, I waited this one to explain you what is the meaning of dispersion that you would see if you compare this expression given in 34 or 35? In 34, what you are seeing that c is a function of k.

So, different k component will have their phase varying at different rate whereas, if you look at the shallow water case, all k components will go at the phase, so what happen? Initially, if I create a small disturbance somewhere, all its harmonic component will travel with the same speed a shallow water case. If it is a shallow water case, then whatever may be I have created arbitrary disturbance like this. Let us say this is my say x and this is let us say f of x.

Now, given this function you can immediately convert it into a wave number component. You would probably find that this is a sort of a function like this or whatever it is let us say, it is like this band limited, but it is essentially near the i. So what is the situation of the shallow water wave? All of the k components will be travelling with the same speed in terms of its space.

Now, what will happen? You would notice that this initial disturbance each harmonic component going with the same speed that in later time, you will see there will be a same phase relationship as there were x t is equal to 0, that means what? Initially If I have a compact disturbance, it remains compact; it does not spread apart that is your dictionary meaning of dispersion, it has not dispersed.

So, that is the reason that we have not mentioned what exactly we mean by dispersion. Only with an example here, we can very clearly see that if c or the group velocity becomes a function of k, then different component will travel at a different rate and initially whatever may be the their locations are, they will disperse with respect to each other that is what we mean by saying that shallow water waves are non-dispersive.

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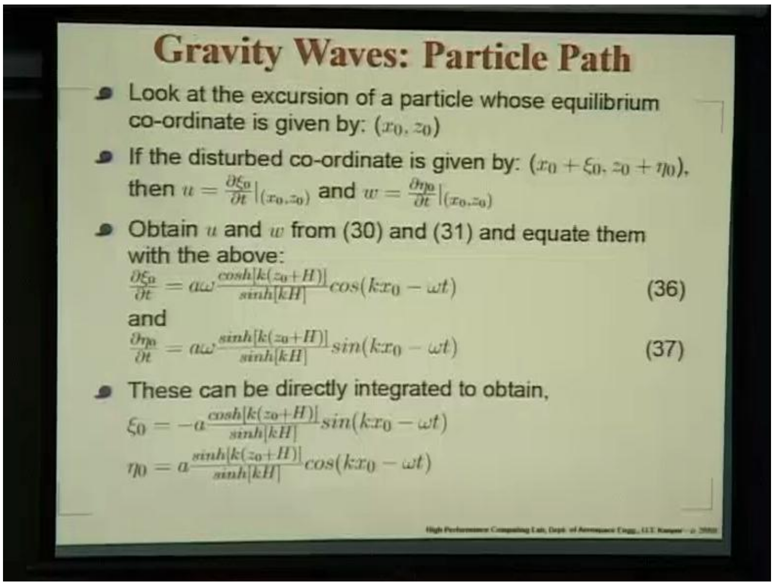

So what happens? In almost all physical system which displays the presence of waves, you have this first responsibility to really find out whether the system is dispersive or non-dispersive. Then, you can come out with your competition strategy to follow that physical trend. This is the one of the way that we go about the computing. Now, let us look at some of the other interesting aspect of gravity waves. We have the expression for phi well it is some are there, I have written del phi del t but phi is there.

So, what we could do? We could try to find out, what these particles are doing on the interface or anywhere in the fluid for that matter that we can find out by fixing our gauge or an equilibrium point which I may call as x0 and z0. Let us say it has been port out and the port out coordinate is given by x0 plus xi 0 and z0 plus eta0. This (()) of course, comes about associated with a velocity u that is the partial of xi0 with respect to delta t and w is a partial of eta0.

Why did I write partial here? Why should not I write the ordinary derivative? See, this is something like your coordinate and then, finding out its time derivative. Basically, what we are doing while we are noticing the coordinate of the particle, but we are still keeping x0 and z0 as fixed so, we are looking at that particle whose equilibrium location was given by this. That is why I wrote this although it is a kind of Lagrangian description but we still talk about a partial derivate.

Since, we have 5 expression, we can get u that is this; that is, your u velocity that we have noted and similarly, the w velocity is this and this are related to this del xi not del t del eta0 del t. So, what you can do? You can integrate and you get this two expression say for integration would give you those two coordinate and the locus of xi0 and eta0 will be obtained, if you eliminate 3 from this solution (Refer Slide Time: 47:15).

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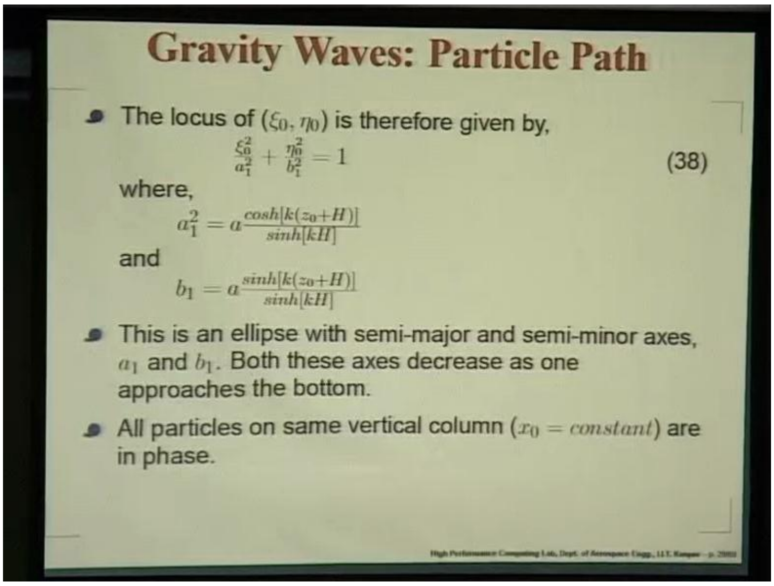

If I just simply find sin square kx0 on this, so that is why we have eliminated equilibrium coordinate and t and that will be all given in terms of z0 and A as amplitude of the wave. You can very clearly see this defines an ellipse; the ellipse is given by this semi-major and semi-minor axis. You notice that as you go towards the bottom of the liquid, z0 is going to progressively go towards minus H.

So what you would point that both these acts are going to decrease, because is the argument comes down both cosine hyperbolic as well as sin hyperbolic reduces. You can also see at the bottom of course, it will all become 0; whereas, if you look at all particle from same vertical column, there all travelling would be same phase that is what is also implied in this equation of ellipse because they are all in same phase.

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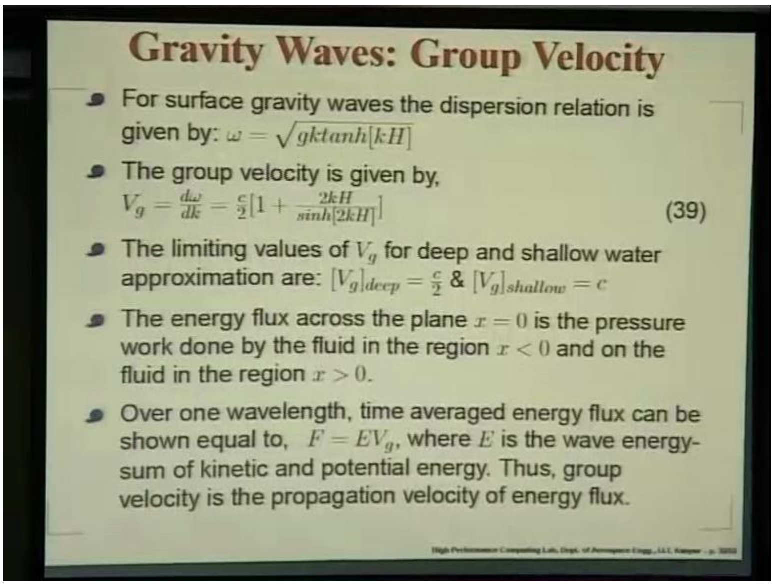

So, coming back to our introduction to group velocity, we have obtained dispersion relation, which is omega equal to square root of gk tan hyperbolic kH; differentiate it with respect to k to get the group velocity. If you want me I can derive this, but it is straight forward, you can work it out. Now, you can again look at this group velocity value for deep and shallow water wave.

For the deep water wave of course, kH goes to infinity, sin hyperbolic kH goes to infinity and the second part will go to 0. You will get the group velocity are simply c by 2; whereas, on the shallow water waves approximation limit sin hyperbolic 2kH would approach 2kH, this will give you 1, 2 we will cancel it 2 and you get this.

So what happens? You notice that the energy in a deep water waves travel slower as compared to the phase speed whereas, for the shallow water wave they are just the same. You also notice that problem that we had looked at that 1d wave equation, if we would have looked at solutions from amplitude cos plus cos kx minus omega t. This also will give you omega equal to kc. If you calculate d omega dk, we will find that is equal to c so, that is here.

So this is an example, where you can see that Vg does not depend on k. Here is a model equation which you can imply to figure out, how your numeric is, because your physical group velocity tells you that if you create some kind of initial condition, it should remain the same; it will simply translate to the right.

If you try to solve it to the bad numerical method, you can see that even if you give a sort of a wave packet it will break upon because of the problem of numeric. That is something we should look at, so group velocity is a very good indicator for us to judge if our numerical method adopted is a good one or not, because that is the speed at which the energy of the system ((travels)) that is much more important for us to understand.

Now, this path let me see, if I can quickly zip through and I explain to you what exactly we are looking at, next I made a statement here, what the energy flux is happening when we have this kind of a wavy disturbance at the interface?

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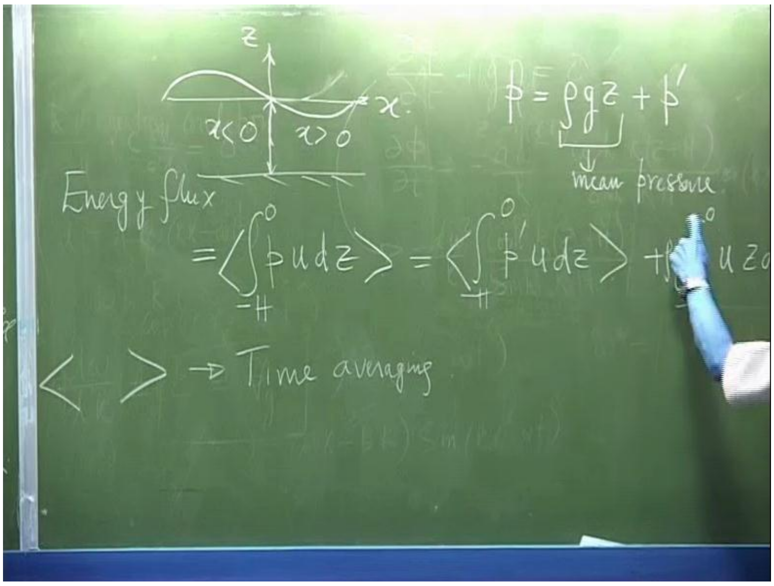

If I look at a wavy interface, the energy flux would be created because there is a pressure excursion, so pressure is varying as you go along in the x direction. So, what we could do? If we keep our attention fix at a particular say perfectly harmonic component and this is your x equal to 0.

So what happens? We have seen the energy flux across this plane will be the pressure work done by the fluid in this path and pressure, the work done on the fluid on this path. This is your x equal to greater than 0 and this energy flux; we can work it out in terms of pressure times, velocity integrated over the whole height.

You can imagine this is your bed; you are basically integrating over this whole range. That is what this limit tells you, this is the special time velocity gives you the energy flux term. If we are talking about a time averaging, so this angular bracket indicates a time averaging operation.

Now, pressure as we have noted comes about from rho g sorry rho gz plus set of fluctuation term. This is your rho gz term plus some fluctuating pressure. This is your mean component (Refer Slide Time: 55:26). If I substitute it there so, I will get two components minus H to 0 so this will be fluctuating part and get as shown.

If I do a time average of this, it will reflect on this and this time average itself. If I am looking at perfect harmonic seen over one wave length, this will go to 0. So, all we need to do is look at this and if you recall the Bernoulli's equation, we wrote they are the fluctuating pressure was given like this and whatever we add here g eta (Refer Slide Time: 56:20). This is the mean part that is, what we have written there that part is got.

So if I leave it like this, p prime works out as rho del phi del t and phi expression that we have written down and what we will find that no this to this result this is here will be minus (Refer Slide Time: 56:40). Since we have the expression for phi, we will get p prime would be equal to minus rho time del phi del t, we have that expression; we will get rho a omega square by k.

So basically, if I look at this flux term will be p prime times u, so u also we have written down the expression before, we have the expression of u here. So, we can plug that expression for u and we have the expression for p prime and we look at this (Refer Slide Time: 58:15).

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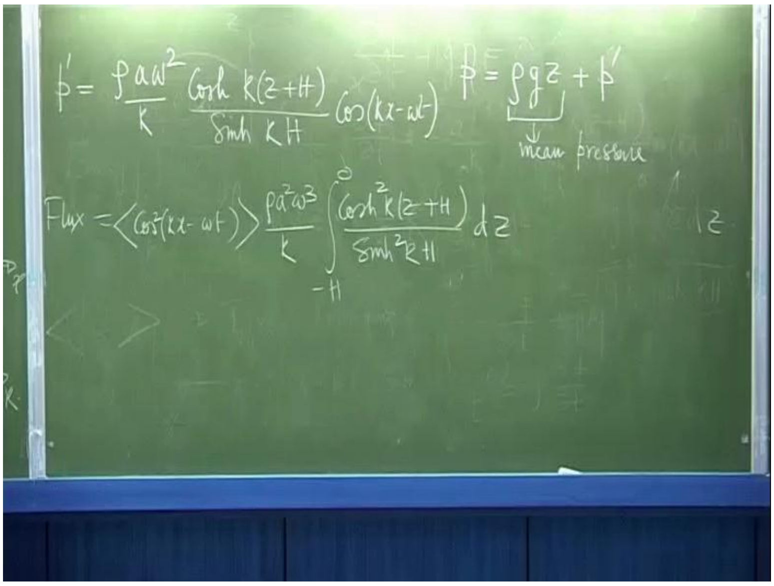

So that will find both of them will contribute a cos of kx minus omega t that is the time varying path, time averaging will relate to that; so that will be cos square one coming from p prime, one coming from u that will be that and we have rho a square a will comes from u, also this and this expression would come here. In addition all just please allow me to write this term, sorry, this will here contribute to that (Refer Slide Time: 59:05).

Ok, I think, I will stop here; I will wrap it up in the next class. So, we will work out couple of more steps on that we will continue.