Foundation of Scientific Computing

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Lecture No. # 35

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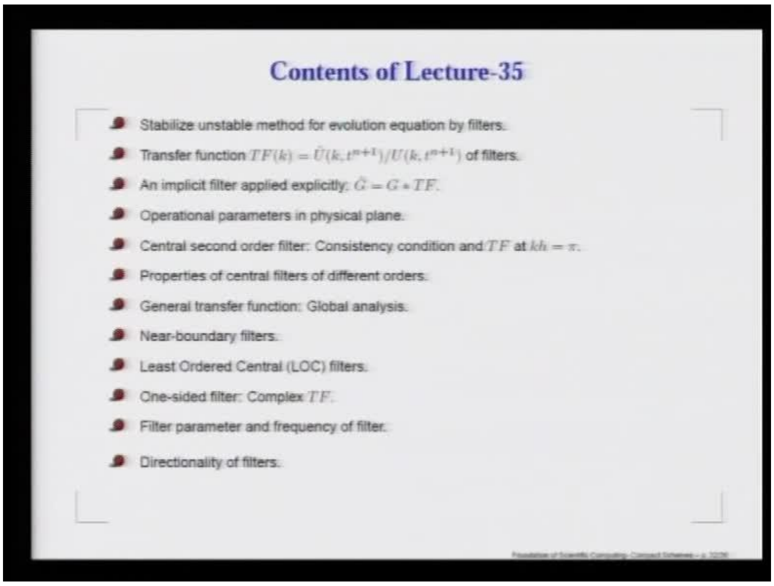

Lecture 35 and we continue our discussion on stabilizing effects of filters. We can pick up any unstable method, and we can actually design a filter in such a way, that we have a stable method. The property of the filters is defined in terms of what we call as a transfer functions, which is the quotient of the Fourier Laplace amplitude, after and before filtering; and this kind of implicit filter that we are applying here in explicit manner, alters the numerical amplification factor G, by a convolution with a transfer function, to give us altered numerical amplification factor upon filtering. And we try to figure out what are the operational parameters in the physical plane itself. And, as an example, we show how second order filter, central filters are designed. This requires satisfaction of a consistency condition and prescribing a transfer function at a particular value of k h, which happens to be at the Nyquist limit of k h equal to pi.

Subsequently, we discuss the properties of those various order central filters, and we do it by adopting once again, a global analysis; and we notice that, if we are to be solving a non-periodic problem, we need to adopt, once again, near boundary filters revert back to a higher order filter in the interior; and, as we go near the boundary, we can keep on reducing the order of the scheme, and this is what is called as a Least Ordered Central filters or LOC scheme. This is also, again, given by Gaitonde and his colleagues; we will show how this can be effectively used.

We also talk about the various filters that one uses near the boundary; these one-sided filters actually leads to a complex transfer function, and this can lead to alteration of dissipation and dispersion properties. This is what we discuss in detail that filter parameters are important, but we also note that we can play around the frequency of filter; that means, how often we perform the filtering can determine the quality of the solution.

But what is more important that, at times, we notice that in a multi-dimensional problem, the filters applied in different direction works differently, and this directionality of filters in association with the physical nature of the problem can be a major source of error, and this is what we discuss on this finally.

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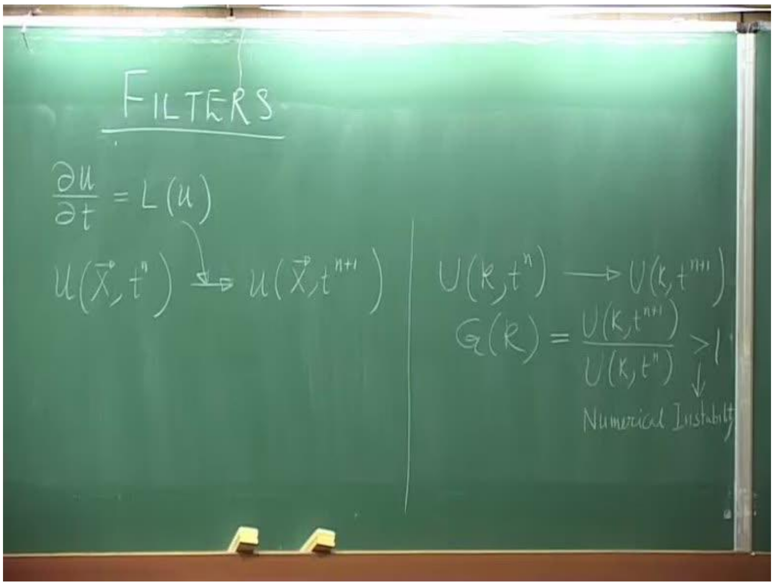

So, let us start on our discussion again, on filters. I suppose you have also seen the assignment now. Now you know what filters can do; your assignment tells you that, if you have an unstable method, if it uses a filter carefully and make it stable, right? So, that is precisely what we are trying to do. So, suppose the basic principle remains the same that, if you have an evolution equation of this kind, so, what you are trying to do is, you have some solution, let us say, depends on space and on some time, let that be the t n; and through this equation, you actually arrive at like this, okay?

So, we actually march in time via, some algorithm, and using this equation, we arrive at the new time-step. It may so happen that this process of direct application of the method on the differential equation may lead to numerical instability; like your assignment. I have purposely suggested that you take a method which is inherently unstable. We know that we, we usually would see, that if we go to the corresponding k plane, so, this is your physical plane in the k plane, what you would be looking at is the u function for a wave number k; and this would take you to the next time-step. And what we define this method was in terms of the G, the amplification factor which we called it as a function of k, as the quotient of u of k evaluated at the advanced time level divided by the predecessor.

So, this is the the definition, and your assignment tells you that we have purposely chosen a method which is greater than 1, right? So, this is your condition for numerical instability; so, for that k component, you are noticing that the method is unstable, right? If the method is unstable, for any k, then of course, the overall method would not be workable; but then, what we are saying is that, we would like to use a filter; and for the filter, we will define what we defined the day before, was the a transfer function, right? We did talk about a transfer function here, via use of some filters which is given there.

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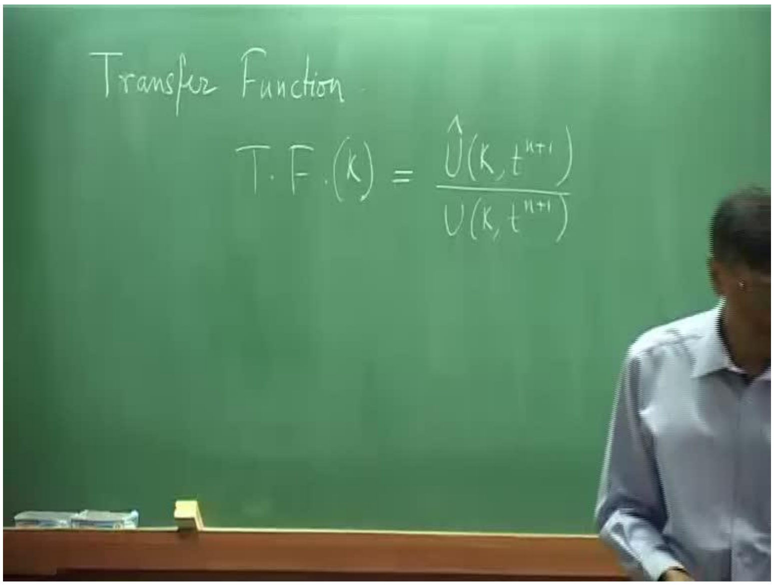

So, this transfer function what is this transfer function? This transfer function we would define, we take whatever we have obtained via the time integration, so this is your time integration step. By the time integration step, you have to write that u of k k at t n plus 1, and the filter takes that solution and operates on that, given by this. So, I will call this as… So, basically we are using an auxiliary function which we call it the transfer function and we have noted how that comes about this implicit equation.

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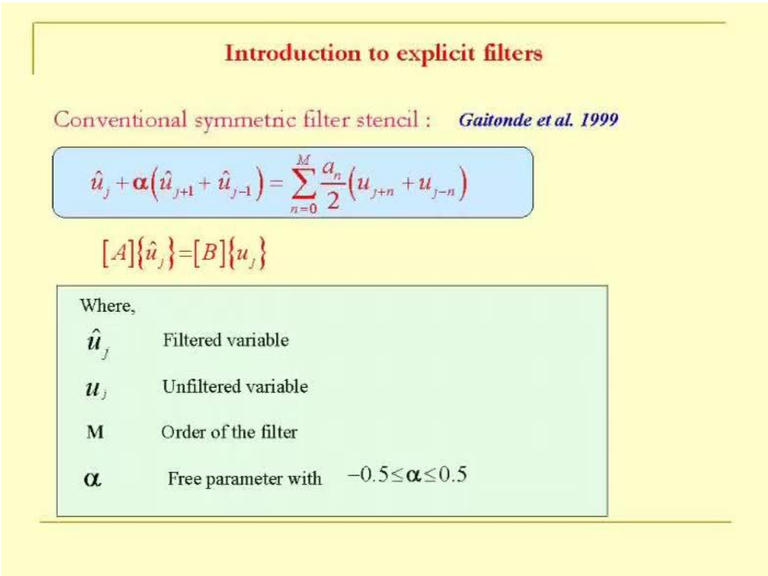

Please note that this equation is implicit, and this is exactly like your tri-diagonal system that you would like to use because of ease of operation; but we will call this filter as explicit, why? Because we are getting the solution here, and then we are explicitly applying a filter characterized by this transfer function; so, we are basically going to multiply the obtained numerical solution at the advanced time level with this transfer function to get a solution filtered solution which is little more well-behaved; that is the whole idea; more well-behaved in the sense, right? Now, to begin with, it is greater than 1; we would like to bring it to exactly equal to 1, that is essentially is your assignment.

So, you have to take an unstable method, you have to choose this transfer function very carefully so that you get a perfectly neutrally stable algorithm; so, that is probably the best way of understanding what filters can do for you. So, basically what happens here is that, this is the accepted solution, right? This is the raw solution upon the application of the numerical method. You take the raw solution, convolve it with the transfer function to get a solution which will be acceptable; that is your numerator, right?

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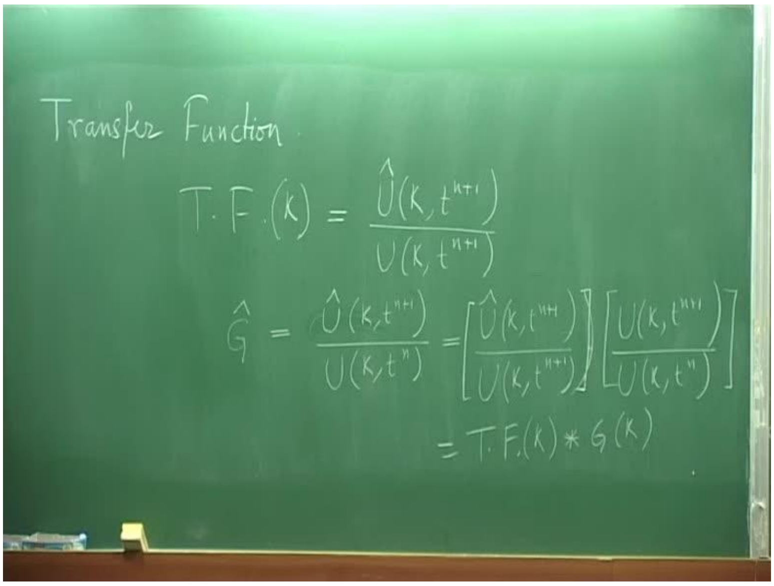

So, basically, then we can define G hat. G hat is essentially the composite gain function, amplification factor, so this is a combination of... Then, we will we will write it like this; that G hat, we will write it like this, that this will be the accepted solution at the advanced time level divided by the solution, that you had started off before the integration, right?

So, what you can see is that, this we can simply write it as u hat k of t n plus 1 divided by u at k t n plus 1 times. Now, of course you can see this quantity here that we have written here is here, the transfer function, right? So, that is what you are doing; times this. What is this? This is our original amplification factor, right? Now you know what is to be done; very easy that you have a G of k, which is greater than 1, and you will multiply with t f so that this remains well-behaved, right?

For your problem, what has been given has been given a packet. So, the packet is fixed at a single wave number centered around a single wave number, right? That is what you have seen; that k h basically defines the central wave number. So, all you need to do is find out for that value of k h. What is this quantity? You design this, so that you get this G of hat g th or k equal to exactly 1, right?

I thought I will explain to you, what is in essence you are expected to do in your assignment; and now, the main question that remains is, how do you design that transformer, right? How do we do that? And that is what we have discussed in the last class; that we obtain the numerical solution those are there on the left right hand side, and then you apply a filter of this kind; essential idea remains, that you end up solving a linear algebraic equation of this form with a being, again a simple tri-diagonal scalar matrix and that is that that is.

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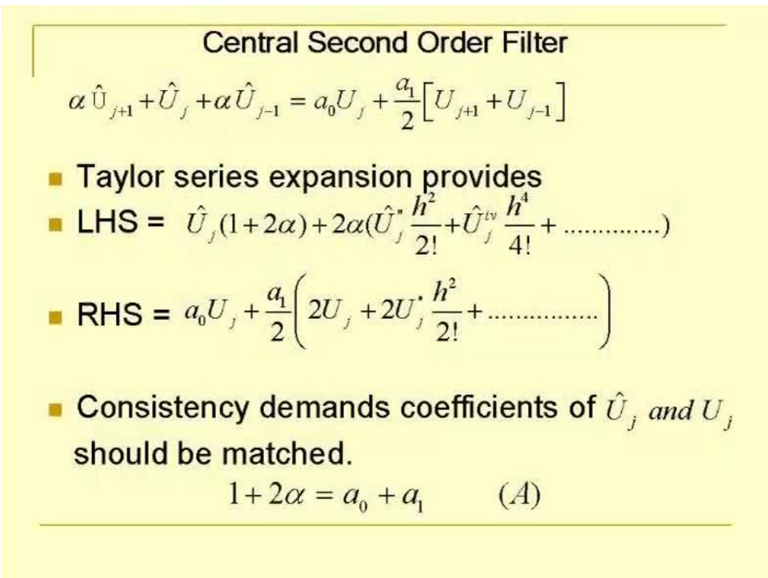

So, essentially, then we have noted down that these operations are occurring in the physical plane, and this view that we are taking, is in the spectral plane, right? So, we need to work out what and how these things are related. You notice that you take the integrated solution and go on to the right hand side, and make these operations in sequence, right?

You start with a 0, then you will have a 1, a 2, etcetera, all the way up to a of N, and that N was defined as the order of the filter; and on the left hand side, you have this coefficient alpha, which we call as the filter coefficient; and we actually identified the range of alpha to be between minus half and plus half, why? Because, we wanted to make this e matrix diagonally dominate so that we do not get into any numerical problem, while via this equation.

However, I will not give you an answer of, but I would ask you to show what happens when I choose the alpha equal to half plus half. We will see that it will not filter anything; that means, the transfer function for alpha f equal to half is 1, right? I think it will be good exercise for you to prove it.

For any value of m, for m equal to 1, that is what we have shown here, is the central second order filter; and in your assignment, you can actually make use of second order filter, so, do not have to do anything fancy. We can do much more higher order; we can take higher order filters, but we would restrict our attention to the second order filter; and you you can see, what does this central mean. The central means, that the coefficients on either side, they are symmetric, right? About the diagonal, on the left hand side, you have both equal to alpha; on the right hand side, you have both a by a 1 by 2.

And, what you have noticed here, is that a Taylor series expansion gives you this kind of an expression on the left hand side and right hand side; and we demanded that consistency should require that at the basic level, u j must be equal to u j hat and that would mean the coefficient 1 plus 2 alpha must be equal to a0 plus a1; that was what was required at the basic minimum level or called we called it as the consistency condition.

Now, what we could do is, we have three unknowns alpha, a 0, a 1, and we wanted to keep alpha as a kind of a free parameter. We do not wish to query, fix the value of alpha, so that we are left with no control. We want to control the performance of the filter to the choice of at least one parameter, and let that be that alpha; that means, that we need to figure out a 0 and a 1, and this is one equation, that we can use for evaluating those couple of unknowns. The other one, we decided that the transfer function should be equal to 0 at the Nyquist level; and, how do you do it? Well, we have written down that equation, that we see it here.

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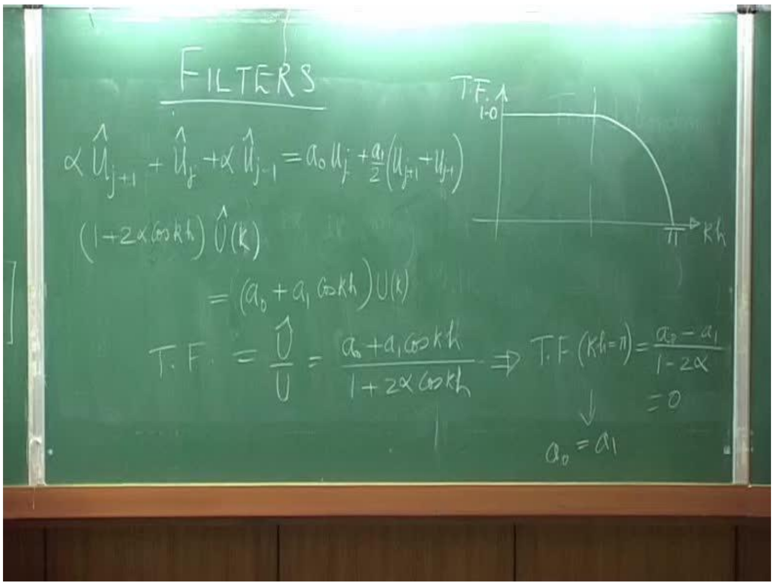

So, this is basically the... At least do understand, that this is written in the physical plane; this should be all lower case. Although the slide shows there to be upper case, but this is what we are doing; so, this is your filter.

So, you use the actual presentation of this and then you can immediately see that this becomes 1 plus 2 alpha. So, this will give us cosine k h, because this will give e to the power plus i k h; this will give e to the power minus i k h, add it up, you will get 2 cos k h that we need to do; and we would have u of k e to the power ikhj; that is what we are doing. This should be equal to, on this side, we will have a 0 plus… If I (( )) again, I am going to get a 1 cos k h.

And that will be again u of k, so, this is d k; and this also would be evaluated at the same node, and this is what we get. And basically, this is our U hat, and this is U, so you can see if it works for all k. So, we can get rid of this, equate the integrant and this; since they are operating on the same node, so, we can get rid of common path; so, this is what we expect. And you can very clearly see, the transfer function for this second order filter would be equal to U hat by U, and that is equal to 1 plus 2 alpha cos a h divided by what we have written, a 0 plus a 1 cos k h.

One of the functions of these filters, is to basically stabilize the computation. And, by now, I think we all agree that most of the time in numerical computation, the problems arise at the highest wave number, right? So, that is what we decide some qualitative feature of the transfer function, like that. Also, we need to keep in mind, so, we like to do that, and what we expect transfer function should be of that nature, which will not alter the solution at low k; so, it should remain flat, equal to 1. And, what we would like to do, it should attenuate all the higher k h component, and this is what we expect to happen; that the transfer function should be equal to 0 at k h equal to pi, and that is what we are talking about.

So, at k h equal to pi, so this we will basically tell you that k h equal to pi should be equal to 1 minus 2 alpha. I have dome [fl] Okay, so that is what I was myself getting surprised, so, we would get done. The numerator, we get a 0 minus a 1, and the denominator is 1 minus 2 alpha, and that also tells you why you do not want alpha to be exactly equal to half.

What happens if alpha is exactly half? In addition, if I take alpha is equal to half and demand the transfer function at k h equal to pi, it becomes the indeterminate form. But you can work it out and show that, that becomes 1, but that is just what I am telling you for a second order filter. But the thing is that, I ask you people to look at, is that, for any order filter, you can show alpha f equal to half; we will give you transfer function equal to 1, you should be able do that.

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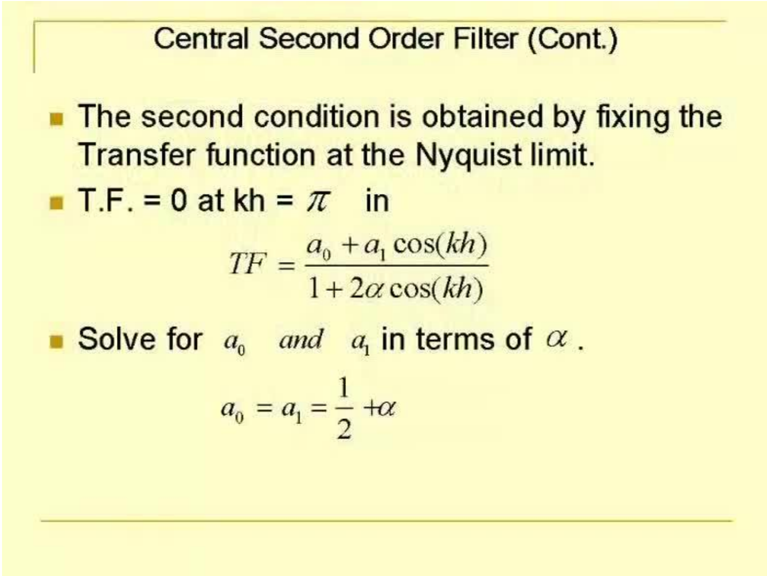

But then, if I want to do this equal to this equal to 0, then of course, I require the numerator to be equal to 0; that gives you a 0, should be equal to a 1, right? And, if a 0 equal to a 1, then look at the previous, I mean, the equation here. If I put, that equal to that, and this is what we get; a 0 equal to a 1 should be equal to half plus alpha.

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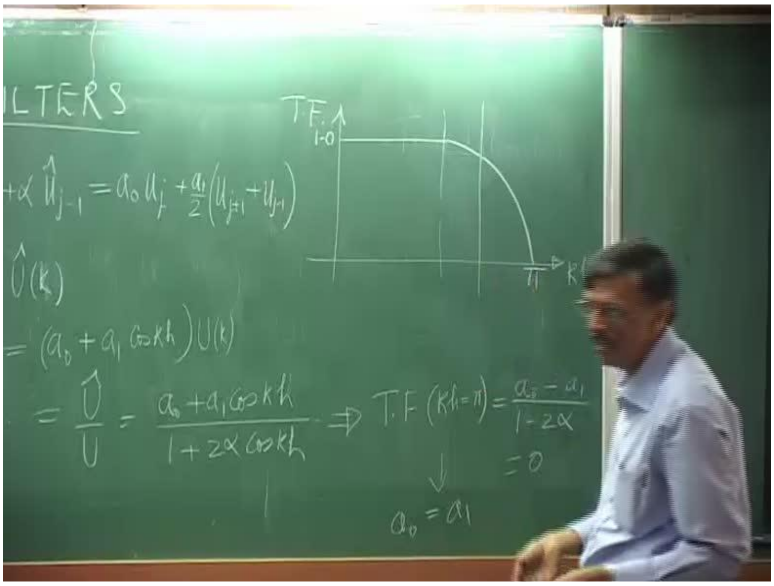

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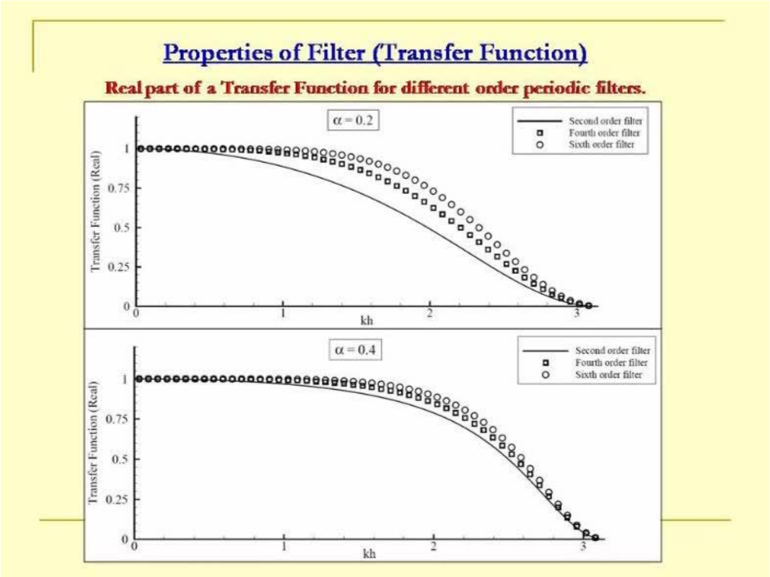

So, now you can actually realize that in your assignment; what you need to do is, basically find out, you know, the value of k, where your wave packet is. And next thing that you would like to do is, figure out the value of alpha, where, for that value of alpha, you should get, say, suppose this is my k h, I choose some discretization number of times; I have not talked about how many points you should take, right? Maybe, you know you are choosing some point here; you have chosen h in such a way for the wave packet, that you are here; so, this value you know. So, if I know what this is, let us say, this is 5 9 5 5, and then, if I know this is 1.05, and if I multiply by 0.955, do I get 1? No. Then what I can do is, I can keep plotting these causal functions for different values of alpha, right? And that is distinctly possible for any order filter that you do; and this is kind of a result that you are seeing here; that look at the solid line for a moment. We are focusing upon second order filter, those are given by the solid lines; and these are the two values of the filter transfer function for alpha equal to 0.2 and alpha equal to 0.4

What you notice is that, for smaller values of alpha, transfer function starts deviating for 1, earlier than what you get to see here, and what did I say? That, when you approach alpha equal to 0.5, what did you get? You will get this solid line to go straight up to 1, and at the Nyquist limit, it will just fall off to, equal to 0; that is your box filter, that is what is 1, associates with spectral method.

We have talked about it, right? A discrete method, we keep on attenuating the solution smoothly; but when we adopt spectral method, then, there what we notice is that, it works like a box filter. So, you do not do any alteration for all possible k h; and once you have up to the grid resolution, that is, where it falls out, so alpha equal to 0.5 would take you along a line which will go straight all the way up to 5, and then it falls to 0.

But anyway, you are noticing that your control in solving the problem revolves around two things; one is choosing the value of appropriate alpha, the other thing is h. How many points you can take? You know, but, given in your assignment, I have removed this second degree of freedom. If I prescribe the value of k h, so you are not given a value of k; you are actually given a value of k, why did I do that? Because, we have noticed that all our numerical properties depend on this non-dimensional k, right? That is the k h, so, that is what we have done. So, all you need to do in your exercise would be, to choose appropriate value of alpha so that the crossbar function multiplied by the unstable g of k; this right hand side should give you an exact value of what? At wave packet term.

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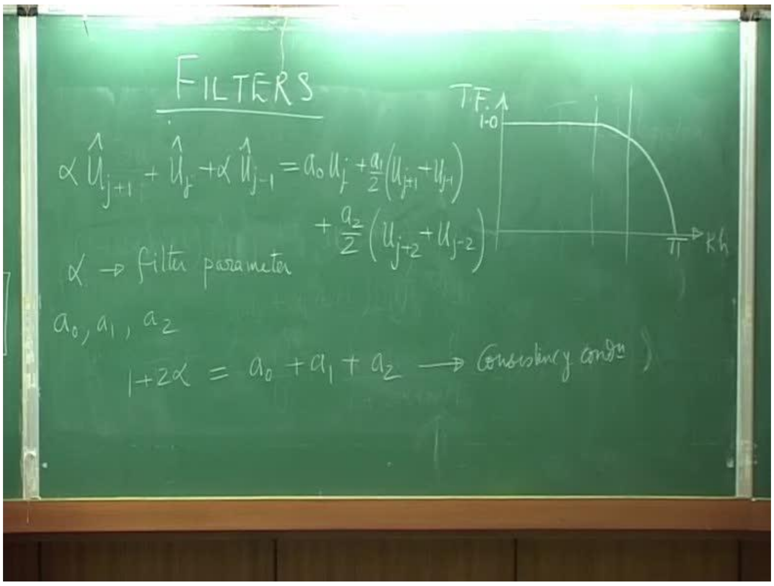

Now, this is what we, just now, talked about, is for a second order filter. So, suppose we want to look at higher order filter, what do we do? Well, we keep taking more points, more points on the right hand side right; for example, if I want to add on an additional set of terms, I could do it like this. I would write it a 2 by 2, and then I will write u j plus 2 plus u of j minus 2, right? So, what will that do for us? It will do a couple of things for us. Now, keeping alpha still, as the filter parameter; so, we are going to choose it freely by on our own account that leave us with the task of evaluating three unknown coefficients a 0, a 1 and a 2, right?

What will be the consistency condition here? Left hand side, we will have, still have the same thing, 1 plus 2 alpha, what do we get here? a 0 plus, right? Is that so? This is the condition that you must satisfy. So, for this, we are talking about a higher order, higher than second order filter. Now, talking about this order business, I suppose, you would all realize that you all realize that, that order comes from this Taylor series expansion, right? In this exercise, we have just simply have done this; so, the next level of terms which are unbalanced for the second order; that is why we called it as second order filter.

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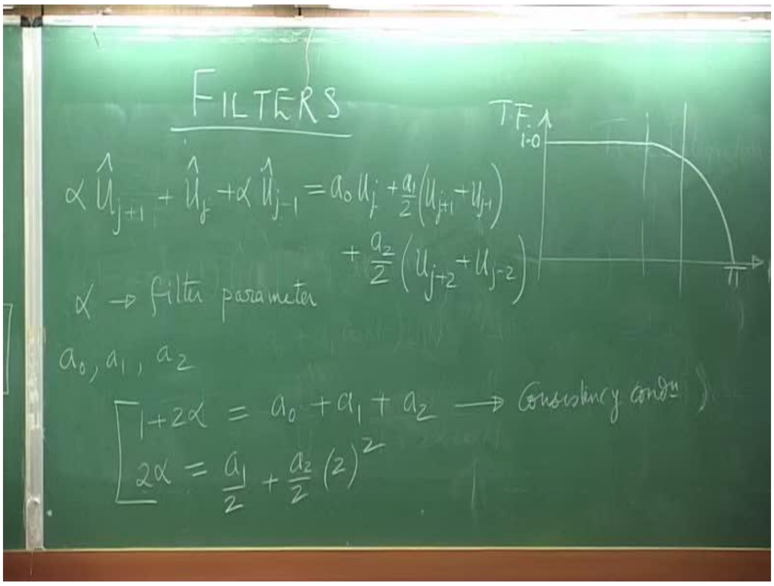

In the next level, what we should be doing actually, we would be equating the coefficient of the next set of terms. So, what should I get for the next set of term, that should be the coefficient of u double prime, right? If I look at that on the left hand side, I will get 2 alpha, right? And, on the right hand side, what should I get? a 1, right? Isn't it a 1 by 2, right? And from a 2, I would get what? a 2 by 2 into 2 square, right? So, that is what you have to do; so, for the next higher order filter, you would be actually, be satisfying these two equations; and then, what remains to be done forcing the transfer function at the

Nyquist limit equal to zero; that will give you the third equation. And then, you will be solving for those three equations to pick a 0, a 1, a 2, in terms of alpha.

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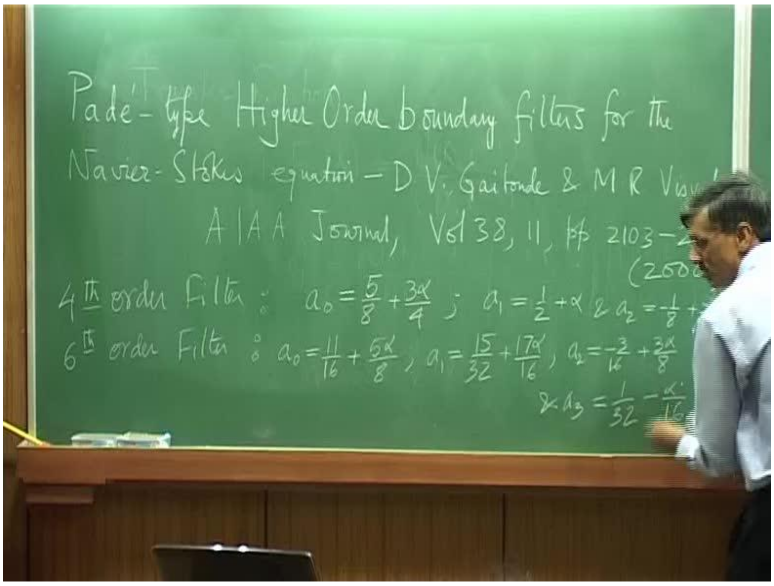

In fact, let me give you this reference, where all these things are given; all, this coefficient as a function of alpha is given; this paper is by again, the same group. They have done most of the initial development; this is in AI, a journal. You can take a look at this, where these people have really worked out order filters, and just to review at least one more set. So, we will, the next order filter would be a fourth order filter; that is what we have being looking at here.

These two equations plus the transfer function at pi equal to 0 would give you a 0 as pi by h plus 3 alpha by 4 a 1 would be equal to half plus alpha and a 2 is minus one-eighth plus alpha by 4, right? So, we have this fourth order filter defined by these three coefficients in terms of alpha, and we have actually also seen the fourth order filter behavior.

And you notice that the coupling of the terms on the left hand side and on the right hand side ensures that the order of the filter always increases by 2, by adding one set of extra term on the right hand side right; that is what we have done here. The moment we have added this a 2 term, we have gone from second to fourth order, right? And those are the coefficients that you get; and well, let me just fill you with this information. And I will not give you the rest; you can take a look at the original source material and then we should have the additional coefficient a 3 and that what (( ))

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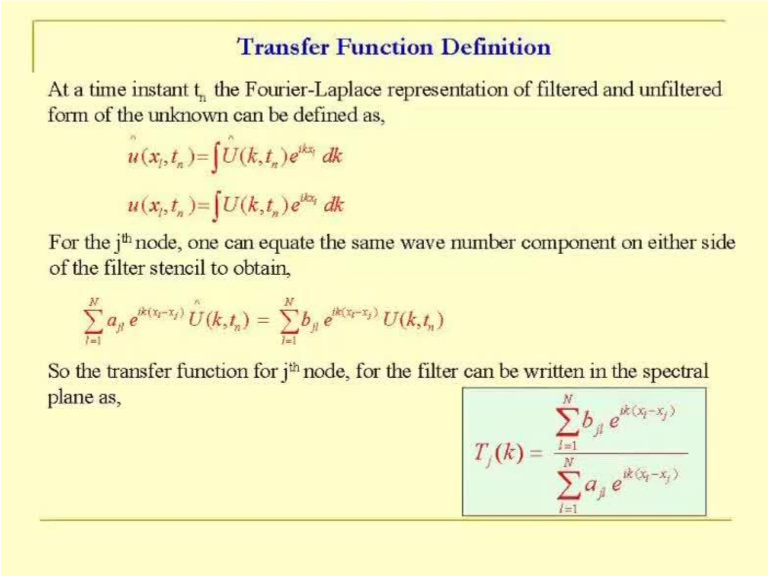

Yes. Is there any question, any observation, any anything that you would like to share? Tell us. No? Not related to anything of this kind? Fine. So, if I basically look at this, this is the generic equation that we are talking about in the tri-diagonal framework; but we can actually make it even more general by looking at expressions of this kind. As I told you, we write it like, a times u hat is equal to b times u; that is the nature. So, that actually works out, and expression of this kind; and we have written it out in the k plane, right? We have written it out in the k plane so that, we you can get this. What what is special about this compared to this? What we have been looking at? What is the difference?

The difference is the previously what we are doing. We are doing a kind of a local analysis, looking at the j th node only; and, whereas here, what we have done? We have been able to integrate the whole domain together and obtain the transfer function on a node by node basis, right? We can get the all the transfer functions for different j th node by an expression of this kind.

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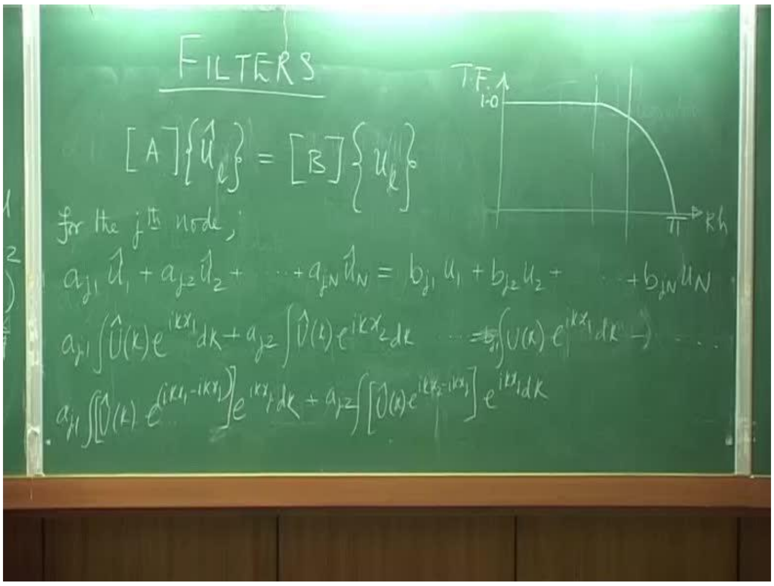

So, whatever we have done, if we can write it down for the full domain, right? We will write it down for the full domain in the following manner, so we will not write it like this. So, what we are saying is, we are writing this a times u j matrix should be equal to b times this vector u j.

Now, as you can see that what we could do is, we this is an implicit equation; suppose, I am looking at say, the j th node; let us call it a different running variable u l, so, if I am looking at the j th node, I would be looking at the j th line entry of the a matrix multiplied by all the u hat l, and that should be equated on the j th line entry on this side.

So, that basically would give you j 1 u 1 plus, let us say, a j 2 u 2 etcetera etcetera; and let us say we have total n number of points in the domain, so, we will be writing a jn u n; so, that is your left hand side; and on the right hand side, we will have similarly, b gj1. Now, this is u 1 and so on and so forth.

Now, what did we do? What we originally did for the analysis of one compact scheme, the same thing we do, right? We can refer everything back to our representation in the k plane; so, what we could do is, these are sort of constant coefficients, so, they they could remain as it is; and this, I could write it as, say, u of k e to the power i k x 1 b k, so, that is this term. And then, I could write e a j 2 and this, we are writing u hat, and you will write u hat of k e to the power i k x 2 d k and so on and so forth, equal to the same thing;

we can do it on this side. Well, you can keep the b j 1 outside and you will get u of k and e to the power i k x 1 d k and so on and so forth.

So, what I try to do is, I can we try to use here, that same idea of projecting this phase into the j th node phase; so, what would we be doing then? Just, simply write the same thing; we will write u hat of k ,and this I will write it as x 1 minus i k x j times e to the power i k x j d k. So, that is what we are going to talk about. So, we have a term of this kind here, well, let, and from here also, we can write down similar things; we will write u hat of k e to the power i k x 2 minus i k x j and then, e to the power i k x j d k.

So, we can write all the quantities in terms of the j th node phase, and then of course, you notice, that this is what you are going to get, right? On the left hand side, we will get a j l e to the power i k x l minus x j, and then, this is u hat, let us say evaluated at t n; that is what you are going to get; sum it over all possible l's, right? That is what we have done here; the same way, you are doing it on the right hand side.

So now, since you have obtained this expression for the j th node in mind, so the corresponding ratio of u hat by u should be the transfer function t, if I was call it t of k. But now, it is specifically done for the j th node, right?

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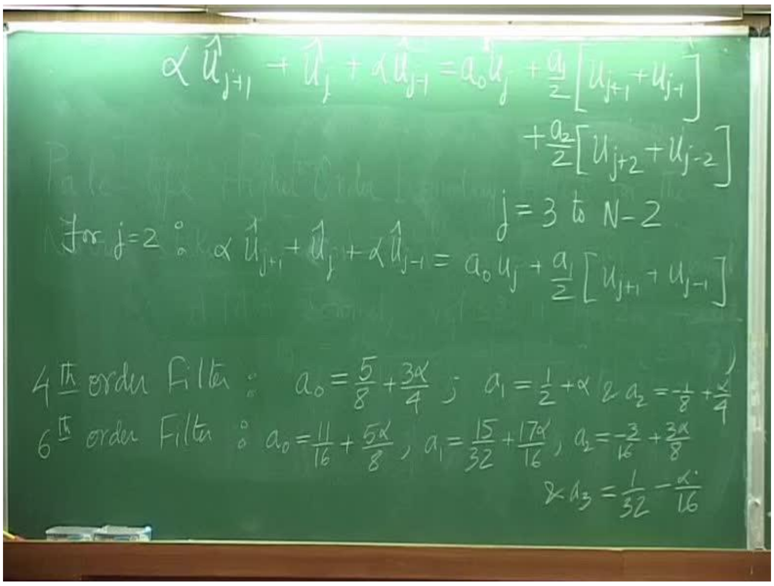

So, what happens is, you notice that whatever the filter formula that you choose, eventually you will have to write out a complete set of equations; and if you have noticed, for your second order filter, we did write it like this. So, you can see that we can start using this expression from j equal to 2, all the way up to n minus 1. One thing, we must also realize that in many of the physical problems, for in all physical problems, you would require boundary conditions, that means what? At j equal to 1 and j equal to n conditions would be given to you either in terms of the functional form or in terms of the derivatives, right?

So, you can do that. So, if you look that, then there is a need for applying this formula from j equal to 2 to n minus 1 only. So, when I am trying to use a second order filter like what we have written out there, we need to apply it from j equal to 2 n minus 1 only; so, we can clearly write this equation, right? Without any problem; because, we can use the same stencil for all the interior points; there isn't any ambiguity there.

However, however, what happens is, if you want to be little more ambitious in terms of higher order and would like to go to, let us say, fourth order filter then, what would you do? Well, as we have written, we would then be adding this next pair of term, right? That would be this, a 2 by 2; and then, we will write u j plus 2 and this (( )) j minus 2.

Now, you can see that this equation, whether you are doing a second order filter or fourth or sixth or any order filter, what do you do? In the left hand side, all is remains is the tri-diagonal things; because, we do not want to increase our computational overhead, the left hand remains the same; it is only the right hand side that points; those are used in filtering, keeps increasing with the increasing order of the filter. So, in the fourth order filter, we need to take j plus 2 and j minus 2; but, then we have to now, think of the following, that we can apply this from j equal to 3 to n minus 2 only; what happens to j equal to 2 and n minus 1? We cannot use this expression; we have to do something more. And doing something more, means the left hand side would be just the same.

So, all we are looking at, is basically asking for for j equal to 2. What should we, will, be doing the left hand side? We will keep it as it is; because, that does not cause any problem on the right hand side. Of course, you have to be concerned about what you could do. One possibility is, you revert it back to second order filter, so, then you have no problem.

So, basically, then what we are saying is that, we are applying a fourth order filter from 3 to n minus 2 at j equal to 2, and j equal to n minus 1, we are going back to second order filter. And then, once again, we have no ambiguity; and what is it called? Yogesh? Lowest order compact is, its people have all kinds of mouth filling names for it.

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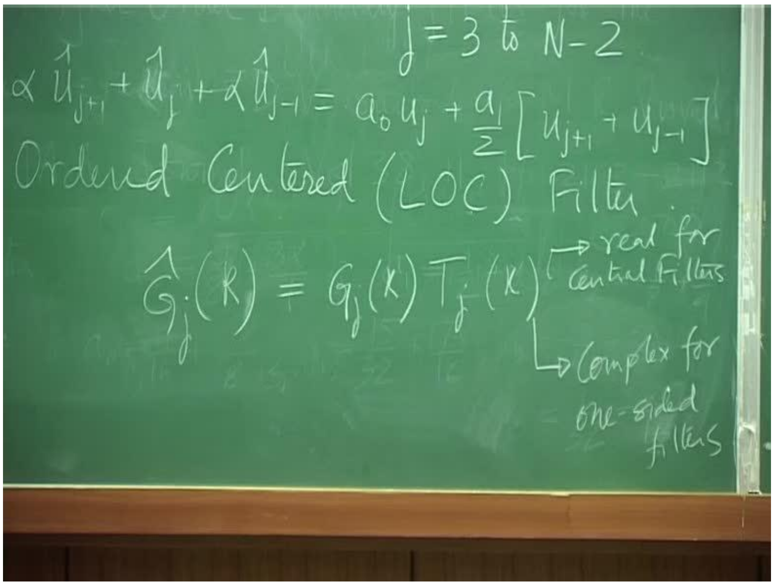

So, this is what is called as least ordered LOC filter; this is what they call. So, you keep on having all is everywhere, the central stencil. Why are we so particular about central stencil? I think any one of you would be able to tell. If I do not have a centered stencil, what other options can I have? I can have a one-sided one; if I use a one-sided filter, what will happen? You have seen the Taylor series expansion for the second order filter; we had always second derivative, fourth derivative and so and so forth.

The moment we do a sort of a one-sided filter, we are going to get also these odd terms; and what those odd terms would do? They would do, apart from adding dissipation, it can add to first derivative; if it is a first derivative, what we call this as that is convection, right? That is error convection equation. We have always seen del u del x is a convection term, but if I have a third derivative, fifth derivative, we call that as dispersion term.

So, basically a convection term is a special case of a dispersion term, but in actual balance, we always will call the first derivative as the convection term; and that would do what? Well, you can understand what what happens. What we have seen is G hat was, let us put it as j now, we also know how to do it for the full domain analysis; and we could write like this, that would be g of the original numerical method times this. So, if I take central filter, t is real for central filter, right? So, it is real for central filter whereas, this becomes complex for; if I call them as one-sided filter, that is what it is; for one-sided filter t of j becomes this.

Now, if you recall for all those error analysis that we had done, we had seen g itself can be complex, and its real and imaginary part fixes the phase shift, and that helps us in finding out that c of n, remember the numerical phase speed. So, suppose you have struggled to get some good combination of k h and n c to get correct value of g j of k, but somehow, it becomes slightly unstable, then you are trying to use this filter to bring it down to its neutral case.

But then, if this is becoming complex, it is actually puts you in a dilemma; because, your original g of j is correct, but, now that you are trying to add a complex transfer function, that will shift the phase relationship of g imaginary and g area. So, you know there might there would be a some sort of a conflict, and it is for that reason that we always try to avoid using one-sided filter, and these LOC filters are one way of getting around, getting around.

However, we will talk about time, permitting that sometimes we may actually, intentionally design upwind filters for better properties, so that, if we have time, we will get get there.

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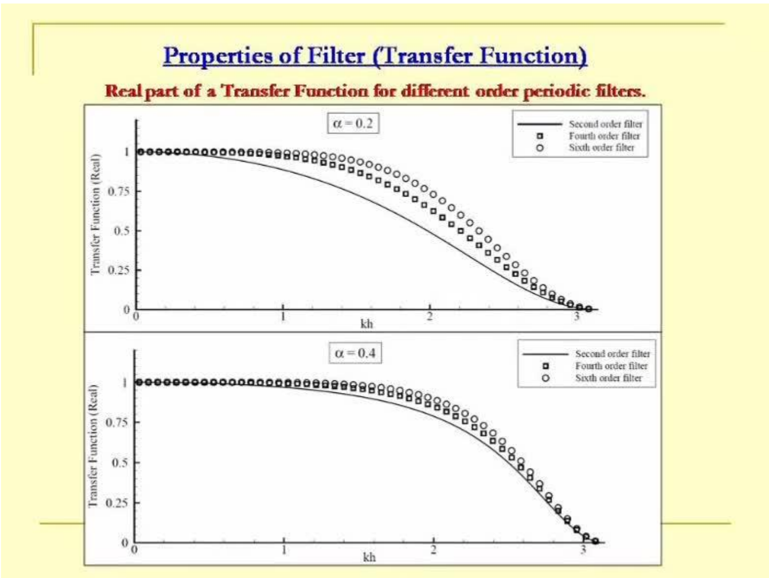

So, what happens is, the now, you can take a look at this figures, and you have in front of you, the transfer function for second, fourth and sixth order filters; and they are shown, say for fixed value of alpha, as the order of filter increases, you have lesser filtering at low k m, whereas, the decent is rather rapid at i k h, right? So, the sixth is the this hollow circles. So, that is what you are seeing; that it does not alter the original amplification factor; may be although you have to one. But then, its starts dropping of, whereas the fourth order filter, this may start happening at 0.6, 0.7; and the second order filter, it may start happening from 0.3 itself, right?

So, you can see that is where the order of the filter comes into picture. However, when there is another way of altering the filter property; it is by increasing the value of alpha, I mean, changing the value of alpha, and what you are noticing here? The increase in value to 0.4 actually improves; well, depends on what you mean by improvement. Here, we have talking about improvement in terms of not interfering with the original numerical method, it only should interfere at i k h, so in that sense, alpha equal to 0.4 would be considered an improvement over alpha equal to 0.2

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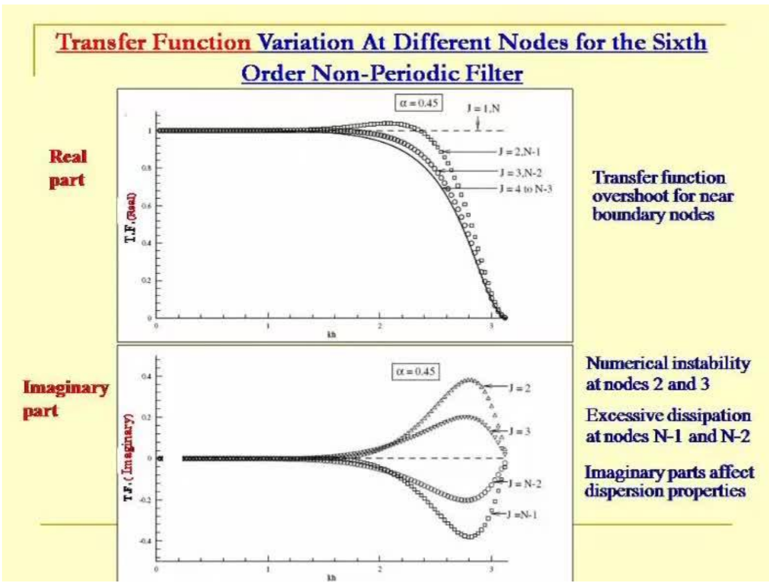

So, this is what we could do; and now, there are ways of, as I told you to not adopt central filter, but have upwind filter. And, people have suggested this paper itself, suggest a host of them, unfortunately though, they did not do the full domain analysis, right? They did not do the full domain analysis and that is why they have had no clue of what is happening for the filter in the full domain, and because we have the, where (( )) you really analyze, and that is what we have done.

We have shown you here, the transfer function, the real and imaginary part with one-sided filter; and you notice that j equal to 1 and n, we do not alter the function, right? Because of the boundary conditions, we do not want to alter; so, that is why we have just shown here an imaginary line at 1. So, that is what happens; that means, at 1 and n, we do not interfere, but at 2 and n plus 1, what happens? We see some kind of an overshoot.

So, what happens is, your desire is to reduce the amplitude at high k h, but, for j equal to 2 and n, you can see that there is an intermediate range over which, instead of attenuating the function, this transfer function actually amplifies it, right?

Now another thing that I did not talk about is that, this is a push processing operation, right? We are doing numerical calculation after every time-step; we can use a filter, so option remains with us; we can also not use a filter after every time-step; so, the frequency of filtering is also an additional degree of freedom in your armory, to actually control the quality of solution.

So, what we are noticing is that, if we do filtering at every time-step, then you would notice that at j equal to 2 and j equal to n minus 1, this filter can actually amplify. So, if your original intension is to make an unstable method, stable; you are noticing that at j equal to 2 and j equal to n minus 1, you are not getting that. So, you have got to remember that, and the imaginary part, of course, will tell you what is happening. This will actually bring in the first derivative and this kind of operation attenuation is due to the even derivative; and these kind of values that you are seeing in this lower frame, causes instability, right? We do not want them to be there.

So, what happens is, basically then if we try to use some of this filter, boundary filter as suggested in this paper, then we notice, that for j equal to 2 and j equal to 3, we actually end up having numerical instability as opposed to what was our initial intention; to stabilize a computation. It actually destabilizes near the boundary whereas, of course, on the other side, you have the complementary phenomena; it actually attenuates much more severely.

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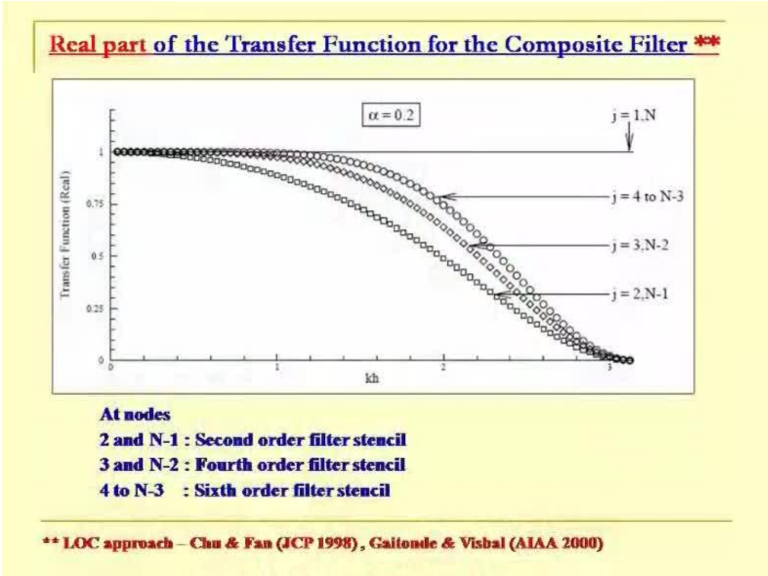

So, basically I am not going to go much more deeper into it, just to tell you that this is that LOC approach that what we wrote there; least-ordered, centered filter operation that one can do. So, basically, this is the way that we have plotted this figure, that at j equal to 2 and n plus 1, we have used a second order filter; at j equal to 2 and n minus 2, we use a fourth order filter; and rest of the points, we have uniformly used the sixth order filter; and the corresponding centered filter behaves like this.

Now, this is much better than using one-sided filter, right? We do not have to worry about numerical instability; you may lose some bit of accuracy at lower k h near the near boundary points, but by large, it looks pretty decent; and of course, you have additional control over alpha equal to 0.2