Foundation of Scientific Computing

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Module No. # 01

Lecture No. # 02

This is our second meeting. Today we will talk about the following topics as listed here.

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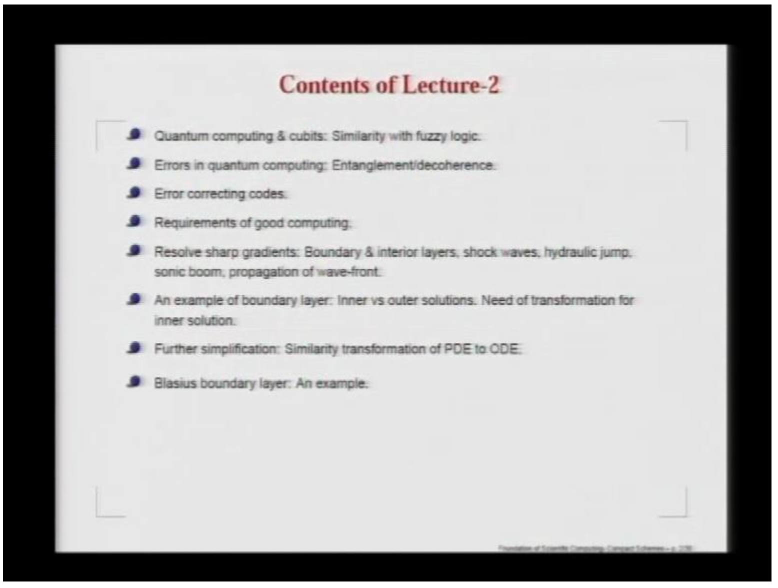

We will spend a little time talking about quantum computing as one of the student have shown some interest to know about it. Then we will talk about the various pluses and minuses of quantum computing, the errors that ((bedevils)) this subject area of quantum computing, we will talk about the various developments that are taken place on the theoretical side of this, which talks about the error correcting codes.

Leaving aside this futuristic topic of quantum computing, we are going to talk about the scientific computing in the classical sense and that will be beginning with a discussion on requirements of good computing.

One of the requirements is listed here as a resolving sharp gradient; this comes about in resolving various boundary and interior layers shock waves, hydraulic jumps, sonic boom, propagation of wave fronts and this will be followed by an example on boundary layer.

We will talk about the structure of boundary layer; why we need transformation to resolve this inner layer which is close to the boundary. The essential goal in this whole approach was to bring about simplification which will allow us to go from a partial differential equation to ordinary differential equation, so we will spend on that. We will be finally finishing the discussion with a description of Blasius boundary layer.

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In quantum computing you actually use what is called as qubits. Qubits are basically quantum states of microscopic system like electronic state of atom or ion. So, this basically is a microscopic state of atom or ion, whatever we do. It immediately suggests to you that quantum machines would be best suited for problems of that kind; for example, you want to simulate some quantum system then you should use a quantum machine. Or maybe the niche areas where it would find niche activity would be in chemistry or it could be in material science.

For example, people have tried to investigate high temperature superconductivity and this is a potential area where we could really work. Of course, people have a lot of hope that you would be able to get some. Two things I just want to request to all of you that since we are recording it, we request you to please be in time. So that, once the class started that kind of distracts everyone.

And, do not hesitate to ask any question at any point in time. Otherwise, I will go very fast. If I have some feedback then I would know where to stop, where to pause where to emphasize.

One of our students asked me about quantum computing. I collected some information and I am sharing it with you. What happens is, in quantum computing you do not depend upon the binary state of the qubits, it does not depend on either 0 or 1; it could be a super position, it could be anywhere in between 0 and 1. Some of you may have known this is what is done in fuzzy logic; you can see them all in your camera, washing machines, they are used already. But, that is not microscopic property. I mean they are totally classical operations fuzzy sets and logics, as defined by Zadeh, it is been there for last 40, 50 years.

Basically in q machines - quantum machines, you depend upon what we may call as a super position of states, but by definition quantum mechanics is a very hyper sensitive system; you measure something and every other thing is affected. So, what happens as a consequence? As a consequence you get what is called as entanglement. Say, you are trying to compute something using one qubit, but one qubit cannot work in isolation, it is going to bring in, because of the quantum property other qubits also into picture and that is what contributes to what we call as entanglement.

Please do understand that these are all theoretical construct. Quantum physicists are thinking of potential pitfalls when they are figuring out where the potential difficulties would be and they are trying to unravel some of those difficulties. So, entanglement is one such difficultly which people are expecting and that it would happen when you have a practical computer using quantum principles. Then this dependence of super position of state would automatically lead to entanglement. Once you have entanglement means, it is interdependence of qubits; the qubits are dependent on each other.

Basically, this is also a positive property because, if you identify a particular qubit to do some operation, the other qubits are also taking part in that computing through the quantum property. So, it is almost like similar to your parallel processing. Same problem is being addressed by many qubits simultaneously; that is what is happening. You can say that this has some relation to parallel computing, so this is sort of a simile, it is not exactly used in parallel computing but it has the potential of doing that.

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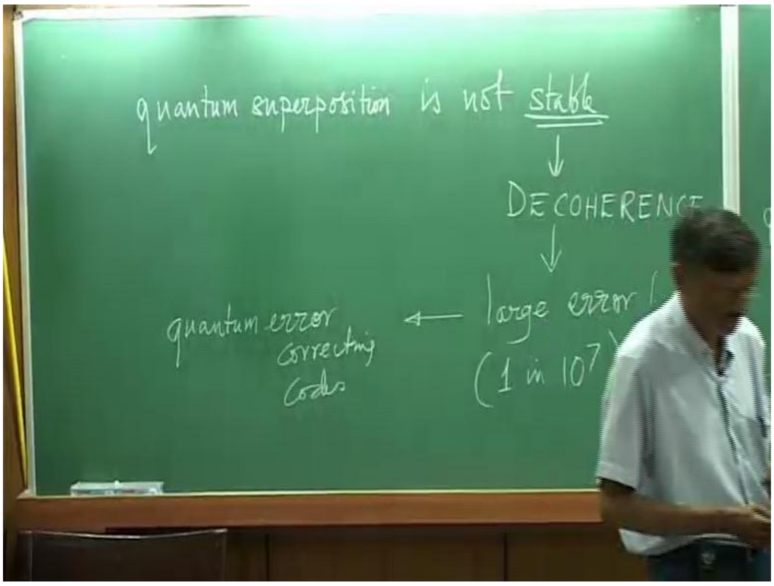

There is one aspect that really kills the joy; it is basically the quantum superposition that we just now talked about and it is not stable, this is a major issue - stability of computing (Refer Slide Time:08:58). When you try to compute something, you would like the system to behave in a stable fashion for quite some time, but in quantum computer the problem of stability is a major issue, because the moment a qubit interacts with its surrounding. As I told you, a quantum principle expects that everything would change, system is not in equilibrium, I mean it has changed by itself, this is what is called as decoherence.

This is a major issue of quantum computing, it is the essential property of quantum state. The way they behave leads to decoherence, computing would be quite unstable; as a consequence, you will have large error. How large? That is the question. Say for example, if you look at the gating operation in a transistor you get an error of kind 1 in 10 to the power 14 operations that is why you are often advised to reboot your system.

If you continuously keep working, it is quite likely that you would make error in that with that kind of statistics 1 in 10 to the power 14; whereas, quantum computers would give you an error which is 10 million times more. This is also something that is agitating the mind of people working in this area, so people have started working on what is called as quantum error correcting codes. For this lot of work has been done in 1990s, lots of papers have been written; but the bottom line remains the same that we have to wait probably another decade or more before we start seeing quantum computers.

So, I do not know if you have followed recently the news, couple of thing were announced last week. One was by this group from PPFL in Switzerland; they made an announcement in a conference in Oxford, that they have been able to simulate human brain. Now, what does simulation of human brain means? That was a very interesting news paper quote; that it is something like 10000 laptops working together; well I did not get much out of it.

The other thing that we probably have seen in the news paper is, some US group has come out with a paper in general biotechnology or biological sciences or something. In that they have talked about the bacterial computing. They have used E.Coli and tried to solve the problem which is called the travelling salesmen problem; you have heard of it, right?

You have cities and a sales man wants to visit all the city, we try to minimize the path taken, minimize the time taken, all kinds of possibilities. Although they wrongly attributed it to a Hamiltonian path problem, it is not. It was actually travelling salesmen's problem. But they also said they could do with about only three cities; so for three cities actually you do it on the back of an envelope, right? You do not need to go to a computer, but it is a beginning anyway.

So there are lots of things that are happening in computing; but as far as this course is concerned, we stick to classic computing with very deterministic methodologies, very definitive problems, which have been explored for long time. So with that I suppose to give you a brief introduction to what I talked about yesterday, you should wrap it all up. Let us look at some real serious business, so we will start the second module of our course after the introduction.

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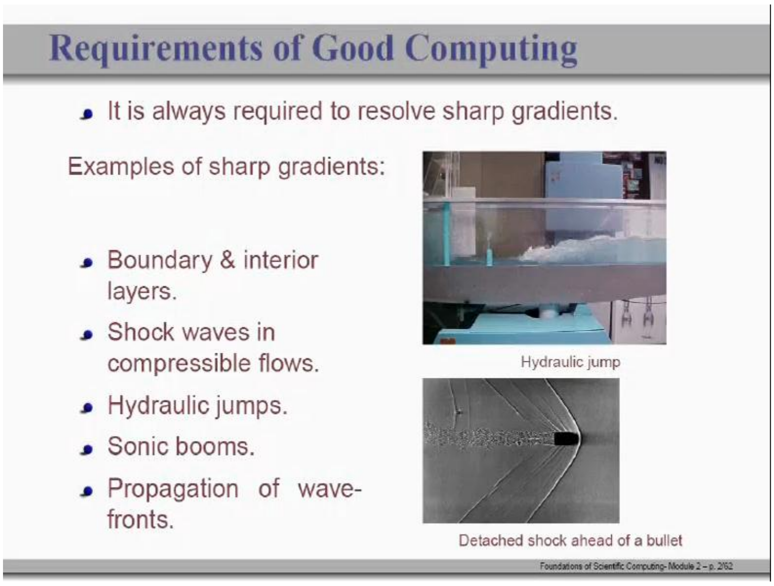

What are the requirements of good classical computing? Now, the first and foremost is that when you look at the system behavior that is, the property as given by functions usually have sharp gradients. So, we need to resolve those sharp gradients, there are some examples taken from different branches of engineering or science, whatever we call.

The first one is familiar to you; probably you have taken courses in fluid mechanics. You see that if you look at flow over a body, very close to the body, near the boundary, you have a sharp gradient. The flow increases rather sharply near the body because at the body the velocity is 0, but then it keeps growing very rapidly, so you call that as a boundary layer. If such phenomena is happening not near the boundary but in the interior of the domain then we call them as interior layers.

These are classical problems that people have been looking at it for almost 100 years - now, a little more than that; this thing is an aerospace engineer or people working in Bel Sticks have been worrying about shock waves. Here is a picture of a bullet, the bullet is going from left to right and you can see a sharp front of discontinuity what is called as a detached shock wave.

Across the shock wave the flow property jumps discontinuously and that is an example of a sharp gradient. Then if you look at hydraulic jump which is shown here in a lab scale, so what is happening here? It is a channel where water is flowing from left to right and there is this hydraulic jump. It is almost like a shock wave, so across this jump the flow properties again change discontinuously, this is something probably some of you have noticed what is called as sonic booms.

If you have seen this high speed aircraft flying overhead and especially you would notice the rattling of the windows, you would see that there is a distinct pattern of the signal - the motion that comes about and it looks like N wave. The path following some variable would have some kind of a signature like this, so it will be quiescent, then it will peak up, then again you will have this kind of things, this is what is called as variation N waves. Those are called as sonic waves – booms.

In all disciplines of physics and engineering, wherever you see waves, you have the wave fronts propagating. These wave fronts also represent some kind of a discontinuity that is the borderline between the signal and no signal.

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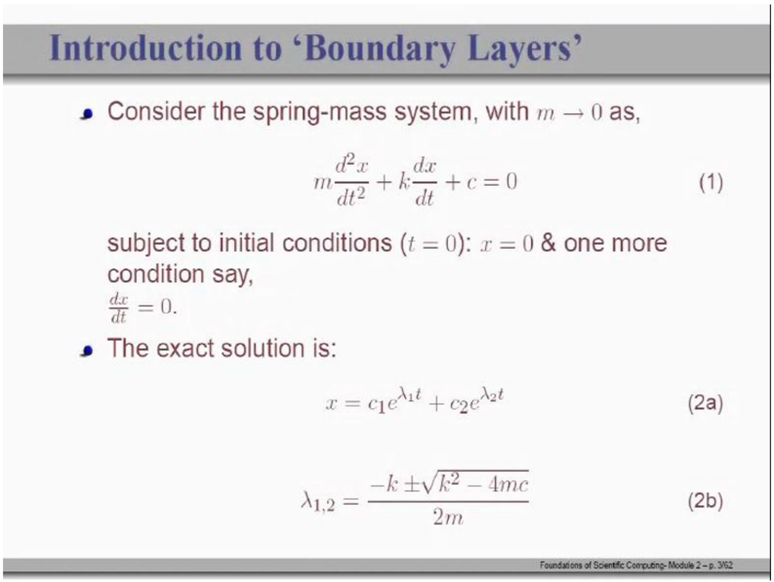

Let us try to understand what this boundary layer is and where this sharp gradient comes from. Let us fall back upon the old spring mass system which all of you are familiar with. Unfortunately, if you notice here k and c has been interchanged, here k is the dash pot here and c is the spring. This the other way, I have taken it for both and I am just following. This is for Schlichting's book on boundary layer, so I was just following that (Refer Slide Time: 18:02).

Basically, what you are looking at an equation of this kind is a very simple a second order ordinary differential equation, but there is a twist in the tail here that here we are considering a system where m goes to 0. Of course, if n is a regular quantity, if I just simply take a basic solution as e to the power lambda t then of course this will yield a quadratic constant coefficient o d and those two quadratic will yield two characteristic exponent lambda 1 and lambda 2 given by here.

As you can see if m goes to 0 you really cannot directly apply this, so what you do? What you do is that you can see that a naive approximation applied on the equation - governing equation by switching this term off, but we will not do, why because, it changes the order of the system.

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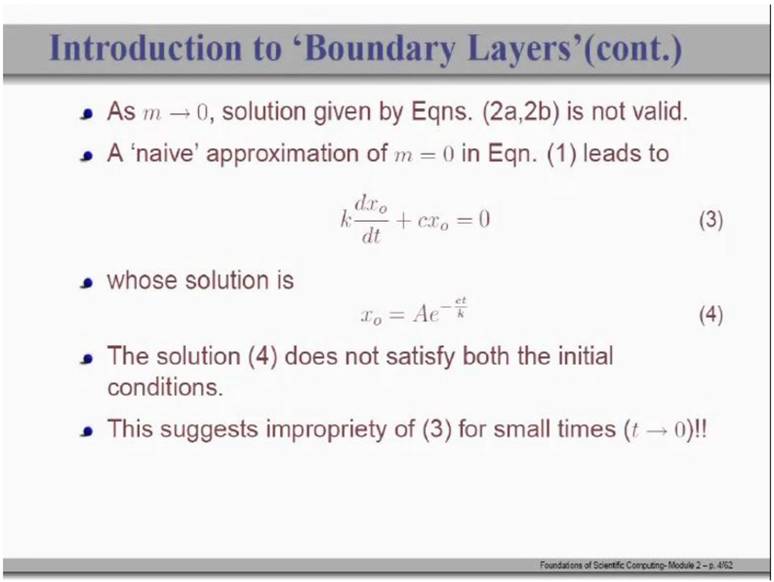

But you notice that it would immediately yield a solution which is purposely written with a subscript o to imply; that is, I will explain to you what I mean by that and that would be (Refer Slide Time: 19:59). This is a solution, it will not satisfy the initial condition; initial condition as we may have shown in the previous slide was that t equal to 0, you have x equal to 0 and let the velocity is also 0. If I look at this solution this satisfies neither.

Apart from the fact that the order of the system has come down so you cannot satisfy two initial conditions, you are hoping to satisfy at least once that and hope is also dashed, because this does not satisfy even that.

This shows that this kind of a naive approach will not do, so we may have to do something different, because the impropriety of that solution actually is close to t equal to 0 from the beginning of the system. So what we could do is we could do some kind of a transformation, we telescope this time. We take the small time t, we divide it by m which is itself is a small quantity. So, I am basically stretching the time, this t star is basically the stretched time in terms of the basic time given as t (Refer Slide Time: 21:13)

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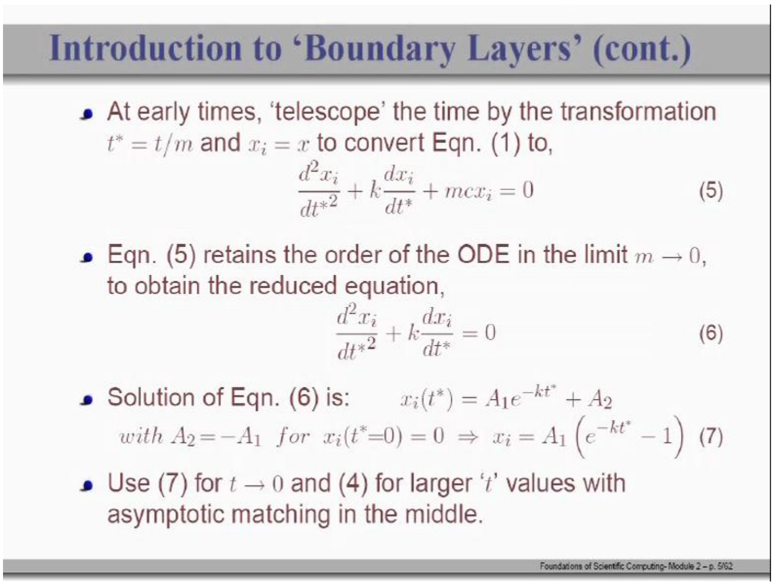

Now, if I call this solution as xi or x then what happens is I could transform this equation. What you are noticing here; xi is denoted as the solution and we are using t star as t by m. Then what happens is any derivative would be nothing but d by dt star and that would be give you as 1 over m. So what you can see here is that you are going to get m times 1 over m square, and x I am calling it as xi, this will be dt star square, this will be your k by m dx i dt star plus cxi equal to 0. You can see this one over m goes here, so m comes there, these two terms are divided by m which may be going to 0 (Refer Slide Time: 22:35).

What you have achieved here? You can probably look at the limit where m going to 0 then that last term drops out. There is a consequence, if you retain the order of the system and the reduced equation is still a second order equation and then you can solve it. It is quite easy if one of the exponents is 0, the other one is of course minus k, so you get the inner solution given by A1 e to the power minus k t star; please note this t star plus A2. If you apply the initial condition that would give me a 2 equal to minus A1 for this 0 displacement condition and then this is the solution that we get. What we have seen is that we have contrived here a method by which we can obtain a solution which should be valued for small time. So we have two sets of solution, one is valid for small time, which we just now wrote as x inner, which is shown by this red line. The other solution we call as the x outer line 4.

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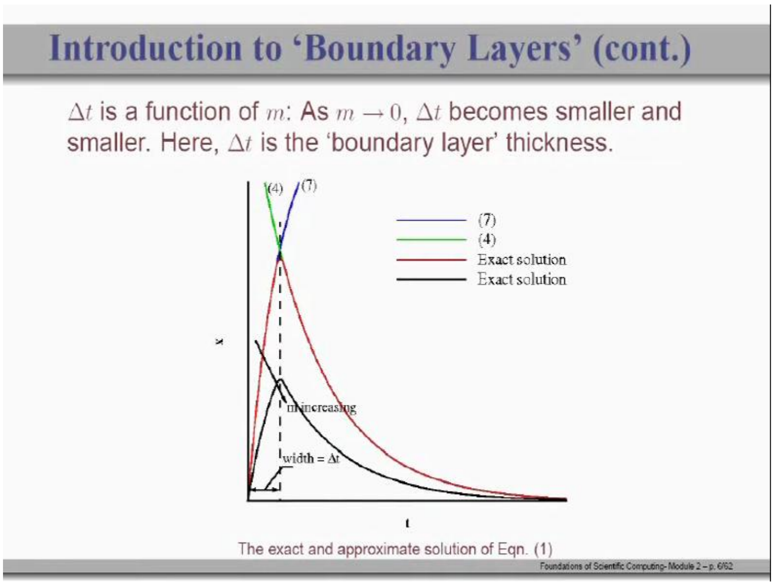

Right now, you can see that there is a region over which this xi is valid and there is other region outside where the xo is valid. For a moment, instead of starting to think that replacing this t by some special coordinate like space; then we are having two regions; one very close to t equal to 0 or x equal to 0 and 1, which is little away. So that is why, which is close to the boundary or which is close to t equal to 0, we are calling it as a inner solution, so we have a inner solution.

The same way the code gives you what is valid in the outer part, this is the basics of asymptotic theory. What you do is you split the original problem into two parts, you get the solution for the inner kernel and then you get it for the outer part and you blend it. Of course, we can get a exact solution for which we have shown here for couple of m's. As we decrease m what we find? The solution becomes steeper and steeper and this width over which the inner solution is valid becomes narrower and narrower and that is the reason that why you call this as a boundary layer. It is a very thin layer adjacent to the boundary. We have seen that there are many such situations where this boundary layers do occur, not necessarily only in fluids, but let us show you how we can exploit this property of the solution in simplifying some problem.

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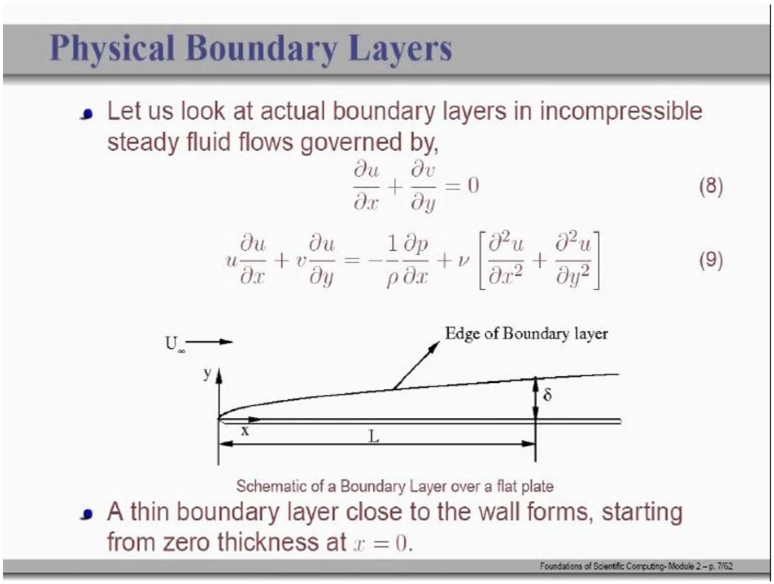

This is again a fluid dynamical problem, what is happening here is you place a flat surface or a flat plate as we call and it is exposed to uniform flow u infinity. Then what happens is you are going to see the velocity in the stream wise direction to have a very specific structure. What we are looking at is we have a plate here and if I try to plot, if I fix a coordinate system x and y then x component of velocity will grow like this (Refer Slide Time: 26:25).

The place where it actually reaches almost its outside value is what we are calling as the edge of the boundary layer. What happens is, if I join those points for different location I get this age of the shell, so this boundary layer is thin and it has got a 0 thickness at the leading edge, but as you go down you will see that it slowly increases. However, this thickness of the boundary layer delta is significantly lower than l like the property that we discussed from the previous solution.

Basically, one of the features of the solution is that l is much larger than delta and these are the governing equations that you are familiar with. This is the mass conservation equation or equation of continuity that we call and this is one of the momentum equations.

When we look at this solution that we call as the Navier-Stokes equation, Navier-Stokes is the holy grail of fluid mechanics. People have been trying to solve this problem for ages. Almost every (( )) dipped their toe and burnt and left the field, all of you probably are not quite familiar (Refer time: 28:00).

But Heisenberg's PhD thesis was on fluid mechanics, he got so disappointed so he left and found at quantum mechanics. Einstein also had looked at it in the context of Brownian motion and he actually had some post doctoral student like (( )) by looking at fluid dynamics problem. Everybody's only goal is to get some solutions of Navier-Stokes equation.

The problem is very simple, the nonlinearity that is the momentum equation. This nonlinearity does not allow us having a close form solution. So, that is why these days many of us try to solve the problem using computers. When we solve Navier-Stokes equation without using any additional assumptions or approximations we call them direct numerical simulations, so that is a huge area of activity.

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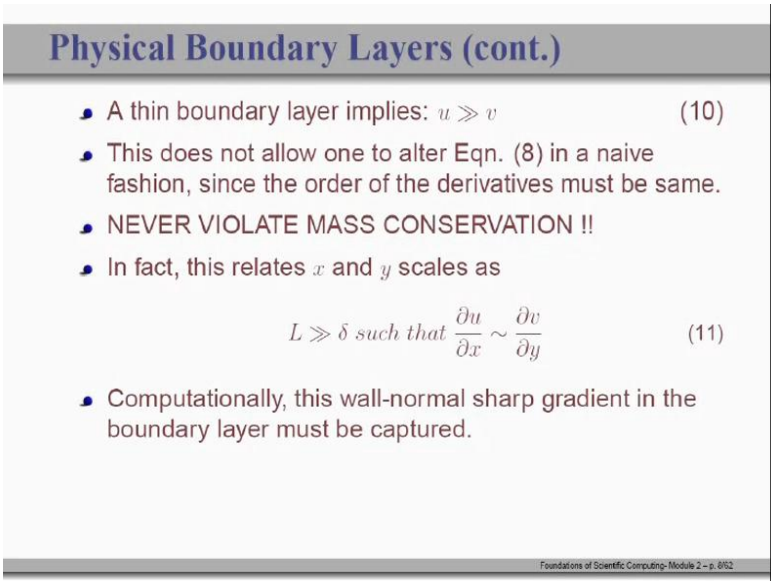

But, let us not digress; let us look at what happens to a simple flow like this, so we notice that a thin boundary layer, which is growing near the wall, which has a 0 thickness at the leading edge of the plate. Well a thinness of the boundary layer implies that if I plotted u here; the x component of the velocity and this is y that's how it is looks like. What do you notice? It is thin, why because the x component of velocity is much larger than the y component right.

Any particle when it moves in it is converted faster in the x dimension than in y that is what it makes those layers thin. So, one of the consequence of that observation is u is much larger than v, the same way we have already noticed that l is much larger than delta. Now, if I have such a flow what we noticed is that mass conservation is given by this measure, this is for incompressible flow (Refer Slide Time: 30:30).

What we have just now said, if u is very large compared to v then you may be tempted to eliminate this term in comparison to this then that would be a mistake. Why because, what we are looking at are not the velocities but their gradient? One of thing that you would never do in any physical activity - physics activity is to violate conservation principle, so you better be careful whenever you make some approximation do not violate any conservation property. Here I am warning you that this equation represents mass conservation, be very careful.

What happens is that this and this - two condition are also synonymous to the following this del del x, any stream wise gradient is much smaller than the one normal gradient. If you do that then you would see one property that both of these terms are of the same order. Basically, what we are talking about x is of order one, all of you are familiar with this order business and order of magnitude analysis that we talked about here. y inside the shell a is of the order delta that is what we talked about.

So what happens, if I look at this, if two are of the same order what I find is v is of the order of whatever may be the scale for u velocity times delta by l, so that is why it is of order delta, you see where delta appears explicitly there. U is order one, l is order one, v becomes order delta, so this gives you v as order delta, so at least we have made use of the fundamental logic, we have seen what is the consequence of what we notice in the experiment that we cannot eliminate one term at the cost of the other. We have to retain it, otherwise we will be violating mass conservation.

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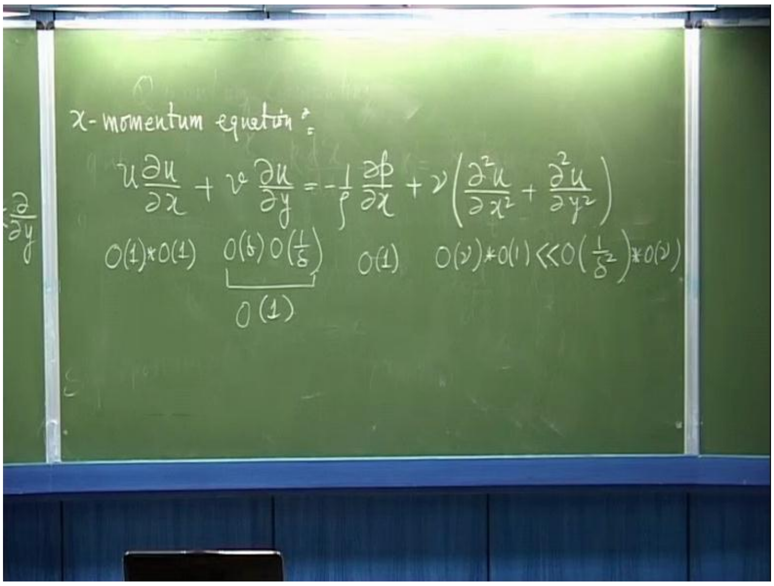

Let us look at the momentum conservation principle or the momentum equation. If we are looking at steady flow this is your non-linear term which we talked about. This is the pressure gradient term; this is what we have in the Navier strokes equation; the diffusion term. I have written down here for you one of the momentum equation. Now, let us go through this order of magnitude analysis what we get.

U is order 1, as we have notified earlier this would be again order 1, our derivative also would be order one operation, so we will be having a order one term. What about this, v is order delta that is what we noted here. What about this term, del u del y. del u del y could be a term which is order 1 over delta, because things are happening much more rapidly in y scale and that is in the denominator, so that is why it should be back. What it means? It means that if delta is small this derivative is very large, to compensate for the smallness of v the product taken together it becomes of order 1 term.

So both terms are of same weightage, so we cannot dump one at the cost of the other. Now, pressure gradient is the gradient, the pressure or force that we are applying pressure differential to get the flow on. So this is some kind of a forcing term, this is the driver of the system, so this better be order one, so we do not tamper with it, because this is what is driving the system, so this is the giver of the energy, so it drives.

Now, when I look at this, what do I get here? Well, I get this quantity, what is this? This is a kinematic viscosity and when we are looking at this property of fluid we know that this is a very small quantity of sample.

This nu is by itself is quantity which is much smaller than order delta quantity. So, what we have here is this is order 1 quantity, what about this? Right, you agree with me, all of you have no problems. Here, you notice that this term is significantly larger than that term right.

So, what you can say as a consequence of this observation is that this can be sacrificed. Now, if you have noticed analogy between this problem and the spring mass system that I discussed little while ago, what is the connection? Both the equations had a small quantity multiplying the highest derivative. That is why in that spring mass system we could not make that naive operation of m going to 0. By dropping the highest derivative term we were unable to satisfy the initial conditions, the same thing here, if we look at it that nu is multiplied with the highest derivative term - the diffusion operator.

We cannot just simply wish away, in fact this takes us to the history of the subject. If you look at the fluid mechanics as a subject people have had a long thought that viscosity is such a small thing. You know that fluid dissipation as exemplified by this constitutive property nu is much larger than other dissipative losses like in a structure. You construct a civil structure there also you have dissipative losses if there is some motion of the structure, but those dissipative losses are 3, 4 orders of magnitude higher.

In contrast, in fluid mechanics what you find that the dissipative losses are much smaller, so is it a good thing or a bad thing? As far as the problem solving is concerned it is a very bad thing, because you have to take care of all possible modes those would be excited in a system, because they have not damped out. But, in most of other situations you would notice that this dissipative losses even say electrical system omit losses is massive.

This quantity in fluid dynamics is such a small quantity that it allow you to retain all the modes that means trouble, because you have to be talking about very large degree of freedom system. So, as we have said that this smallness of this quantity contributes to the formation of the boundary layer, the property of the solution comes when the heights derivative is multiplied by a small quantity that is exemplified here. Although this analysis - order of magnitude analysis tell us that we can drop this term in comparison to the second diffused term.

So, what we get? As a consequence, we get that equation what I have just now written down and I could do the same exercise with the y momentum equation, I do not wish to bother you; you can look at that equation that would be this (Refer Slide Time: 40:59). We would notice that all this individual terms will be one order lower compared to the x momentum term. See on the left hand side these are all order one term, you will find individually this is order delta, this is order delta, so on and so forth.

As a consequence you would find that all these terms or what we call as subdominant that is the mathematical term; all these terms are subdominant and you end up by getting this that dp dy del p del y is equal to 0, what does it mean? That across the (()) the pressure does not vary; that is a very good, because without that I suppose the subject would not have developed because people where earlier measuring the pressure outside pretending that was the pressure on the surface and that pretension turned out to be correct via this analysis which was only reported in 1904.

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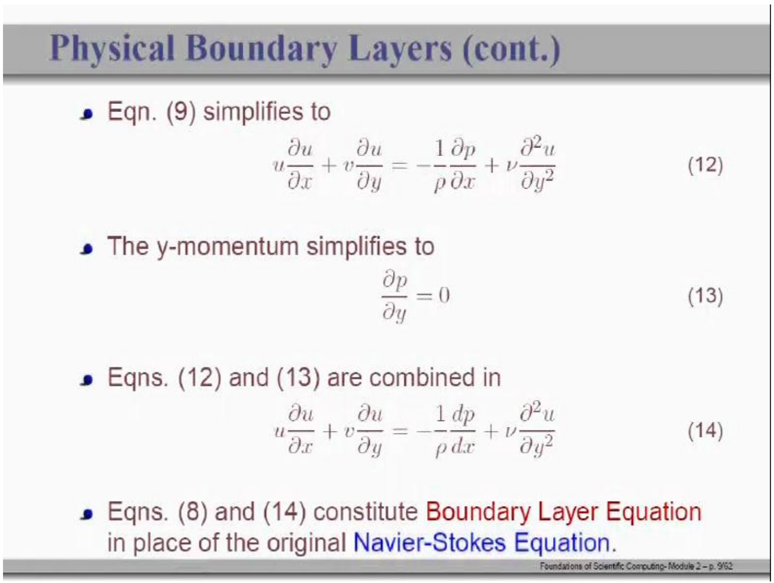

It is not a very old trick but what happens is that this two momentum equations simplify like this, taken together if this derivative is 0. I can make this as ordinary derivative and this I have already commented upon by saying that this drags the flow. The pressure gradient drags the flow and we get such structure of the flow inside, how do we get it? That is the thing that we are going to discuss shortly.

So what happens, we started off with the Navier-Stokes equation and we get this equation plus the conservation equation that I showed you del u del x plus del v del y equal to 0. That constitutes what we will call as a boundary layer equation, so what was the big deal? Well, this big deal was a single achievement by Ludwig Prandtl in Gottingen in 1904. As I told you, he published his paper and nobody noticed it. For nearly 7, 8 years people thought it was just a simple piece of work, some 7 or 8 pages paper.

It was not very important and then people realized that this simple trick actually converted the nature of the problem mathematically, if I look at full Navier strokes equation then what we require to solve it? It is a time independent problem so we do not have to talk about initial condition, but if I look at the flow inside the domain it is driven by the boundary condition, that is what we call as a boundary value problem in red. Boundary value problems are those that where we are dictated up on by the boundary condition.

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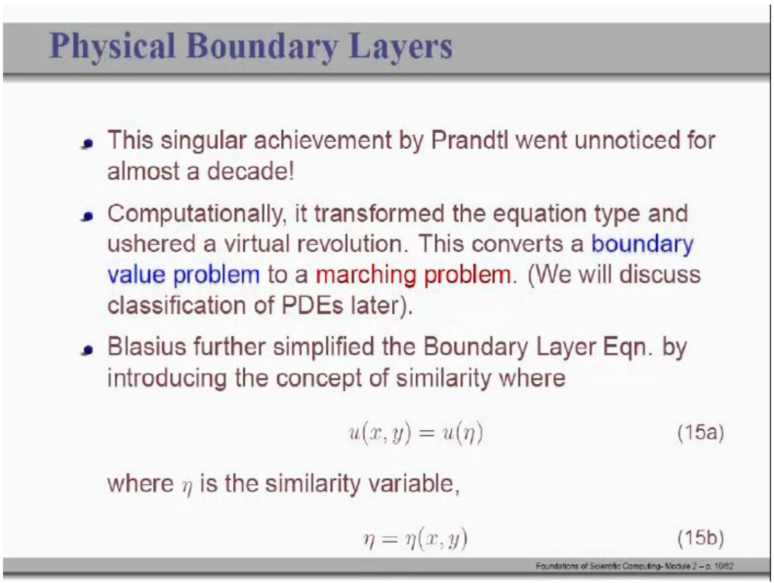

By dropping this term it actually change the nature of the problem from a boundary value problem to an initial value problem. Initial value problem or a marching problem is something like this that if I look at this boundary layer growing over this plate then in a sense saying that that equation allows us to get the solution in station 1 then I can get the solution in station 2, see that is what we talking about.

If I prescribe this thing to you, if I know whatever has happened previously in some station here that would allow me to evaluate all these quantities in the left hand side, the solution of this equation will tell me how this u structure is versus y; that is what we plotted and that would look like this, so that is the solution coming out from this. We will talk about this, but the essence of the activity is that we have really converted the boundary value problem to a marching problem, so I will solve this, next I will solve this (( )) if I have this solution, I (( )) and there goes the story.

So what happens is we are going to talk about this classification of partial differential equations probably in the next module. But, you understand that this is a revolution because think that you have a domain and you want to solve a problem. You have problem with let us say 100 points in the x direction and let us say 100 points in the y direction, so if I am trying to solve the Navier strokes equation then I would have unknowns which are 10 to the power 4.

So if I convert all these differential equations into linear algebraic equations that what we do in computing, we will see that what we would find that we will have to solve in setup linear algebraic equation with the metric size 10000 by 10000; or it is not a very trivial issue if you look back in 1960s that was considered very formidable. Yesterday I told you about benchmarking all the computers. Now we do with two million equations right that is considered as the low end of the job, but instead of solving this problem - boundary value problem involving 10 to the power 4 unknowns if you try to solve the marching problem then you would be solving problem with variables of dimension 100. So, this actually you could do it even with a desk calculator that is what the way it happened.

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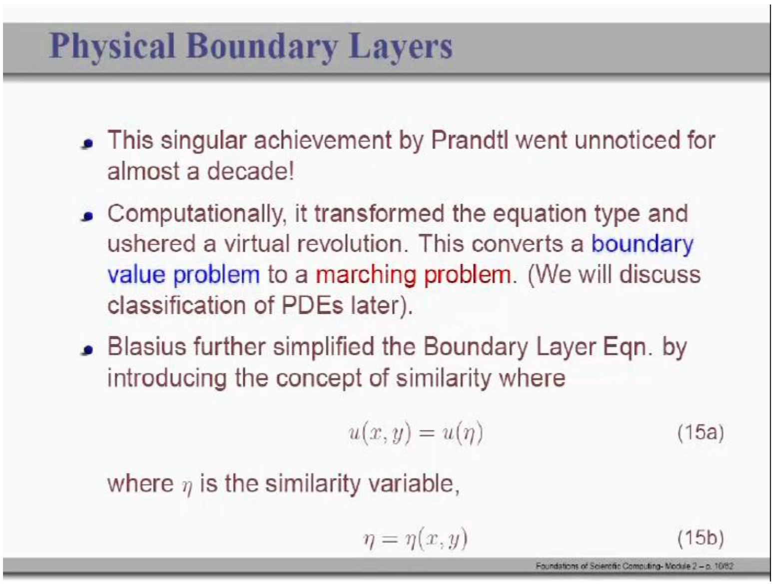

Some of the students of Prandtl namely Blasius, they were the pioneers in extracting the benefit of conversion of this boundary value problem to a marching problem. People were able to get the solution of the viscous action for the first time in the history of fluid mechanics, this assured in a totally a new era. Paper in 1904 written by Prandtl changed not only fluid mechanics and also mathematics irreversibly. I think you will find millions of mathematicians make a living out of asymptotic theorem, they did.

This insight into the nature of the solution lead us from what we call converting an elliptic partial differential equation into a parabolic partial differential equation, we will see that. The genius of Blasius did not stop there, he was a clever fellow, so what he said that I will try to even do something better. He said that if I look at the boundary layer developing then I could find out some combinations of x and y in such a way that the solutions for that fixed value of that combination will determine the solution uniquely there.

So what he did, he said that look I will find out a combination of x and y and I will call that an eta, so that the solution will not be dependent on x and y separately and that could be written down as a function of a single variable eta, whenever we can do this we call this as a similarity transformation. What we are saying is that this part is similar to some other point here which may be another point here. So, we could probably draw a line across which the solution has the same value that is given by this u of eta.

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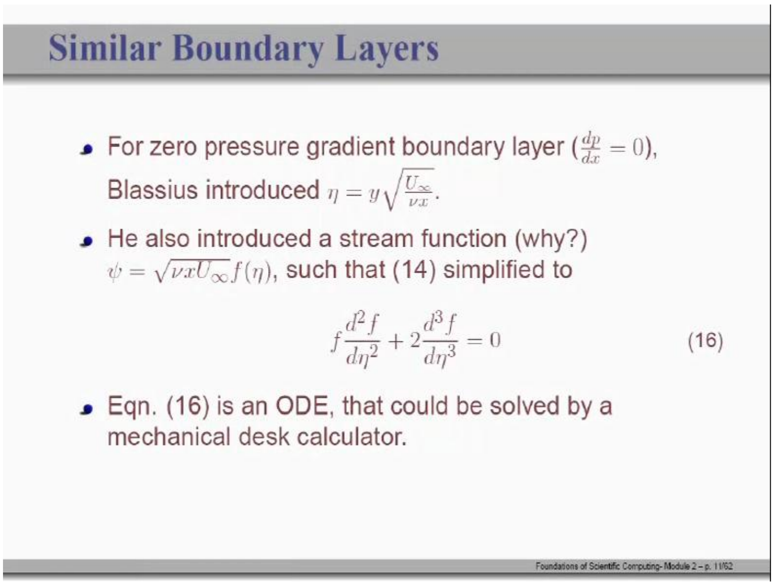

Let us see how it goes. Blasius actually introduced this eta in terms of this combination, so what is the idea? The idea is same that what we did with the spring mass system, wherever we have problem we stretch the quadrant. Same thing here, if boundary layer is forming very close to the wall to very small thickness of y we stretch it open and that stretching is achieved with this. I do not wish to get into this, there is a formal subject called a lee group transformation that is where you basically come out with all this similarity transformation.

This is a very involved subject by itself, so you can see what each paper can do. It has given many branches of physics and mathematics over the last one century, but once you do that you can actually get the solution which was earlier a function of x and y, now it becomes a function of single variable eta. We can now reformulate, rewrite that equation not in terms of x and y but in terms of eta then we would have done a wonderful thing. What we would have achieved? We would have converted the PDE into ODE.

So, you see that was the really remarkable insight of Blasius, when he talked about it. Then he also introduced what we now call as a stream function psi, give it all along, so what happens here, we write it like this. f is some kind of a non dimensional function and the scaling is done with respect to this, why do we need psi? Very simple, I told you never violate mass conservation. For this problem if I introduce the stream function, I am guarantying mass conservation. This is ironclad guarantee that you would not fall through the trap of violating mass conservation by the introduction of psi.

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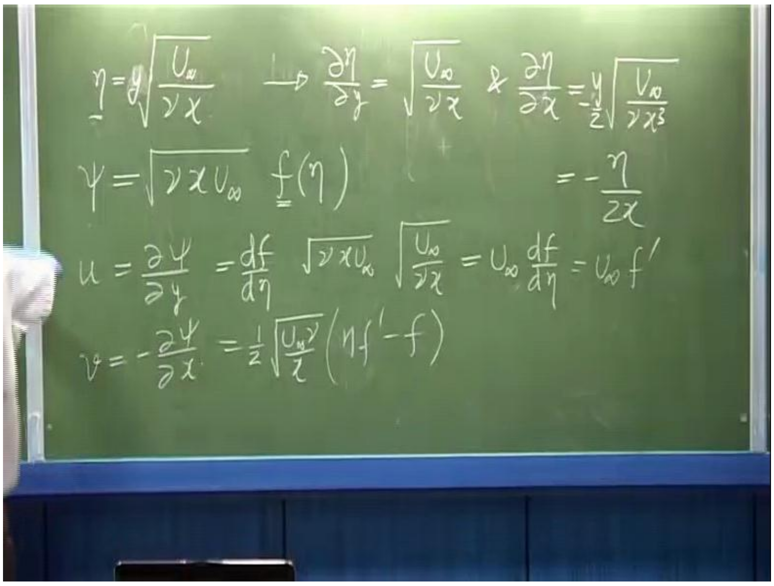

Once you have psi you could get the u component of velocity that is given by this and v component of velocity that is given by this. If I do this then what I am trying to do is getting eta on f in this equation. So, what I would get from here, you can see del eta del y is given by u infinity by nu x and del eta del x, we will have (Refer Slide Time: 53:34); so basically this works out to eta by 2x.

I can use this, I will differentiate this with respect to eta that will give me df by d eta times d eta dy that is this expression. I have already this quantity sitting upfront and then I have this u infinity by nu x, so this basically gives me u infinity df by d eta or I could get simply write it with a prime conserves space. By the same way we can get this expression for v, I think we can work it out and we will see that this is going to be half (Refer Slide Time: 54:44). Now you can keep doing those manipulations working out the various terms in the boundary layer equation.

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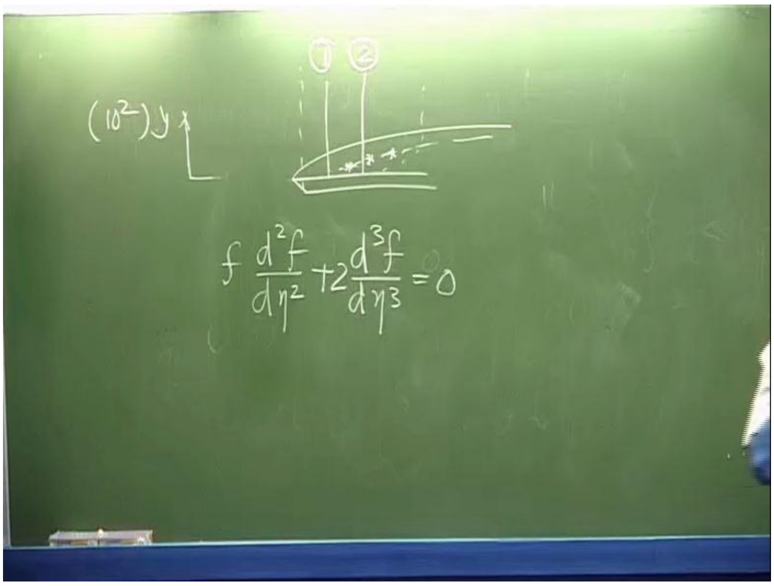

You can plot those quantities, eventually you will get the equation that I have written and you can work it out by yourself. Let us say sequence of observation that led us from a non-linear PDE to a non-linear ODE. The good news is as I told you that I could perhaps use a mechanical desk calculator to solve it and people started solving it.

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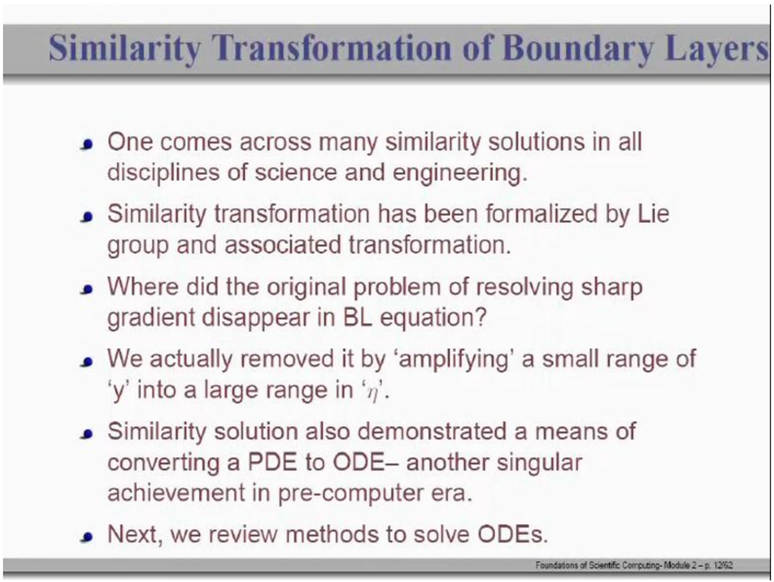

These are some observations that we noticed that similarity solutions are not restricted only to fluid mechanics, we would see all disciplines and they have been formalized by lee group. As I told you that we really stretched that y into eta and thereby that sharp gradient problem has been resolved by having this simple ODE. This similarity solution demonstrates a means to really converting PDE into ODE, this is another single achievement in pre-computer era; I mean this was a big deal that we can talk about, so basically done. One of the properties we just now talked about resolving sharp gradient; let us do all this, so we have seen that there are ways and means by which we can simplify the problem and up with ODE.

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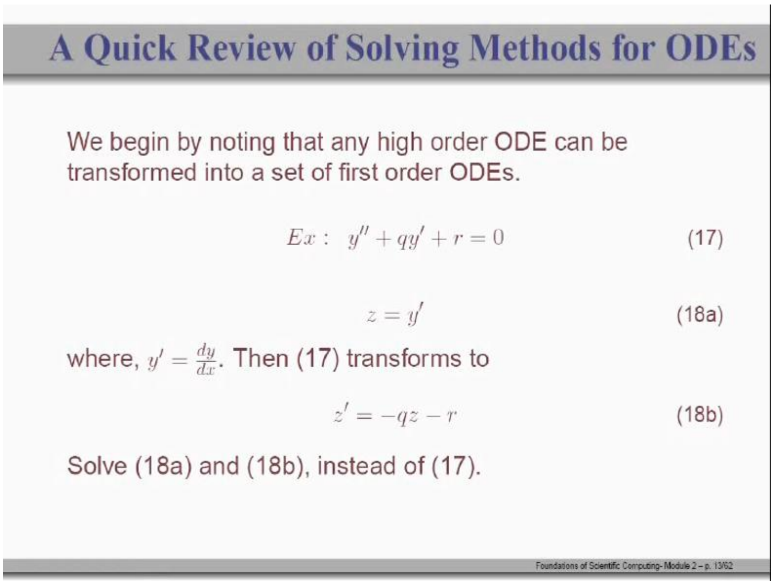

So what I would do is very quickly give you a review of solving ODEs. I am sure all of you are familiar, is there anyone who does not know how to solve ODEs? Do not bother we will follow it as we go around.

One thing about ODE is the study, it is rather simple because it can take any order ordinary differential equations; you can cost them in a set of first order ordinary differential equations. So, if I have a method for solving a set of coupled first order ODE I am mostly done, I can use that same technique for solving all ODEs that makes the job rather simpler; for example, look at here at y double prime plus q y prime plus r equal to 0, so set z equal to y prime, then you'll get z prime equal to minus qz minus r. Basically it is a 18 a or b, you solved it instead of 17 that is all is the idea.

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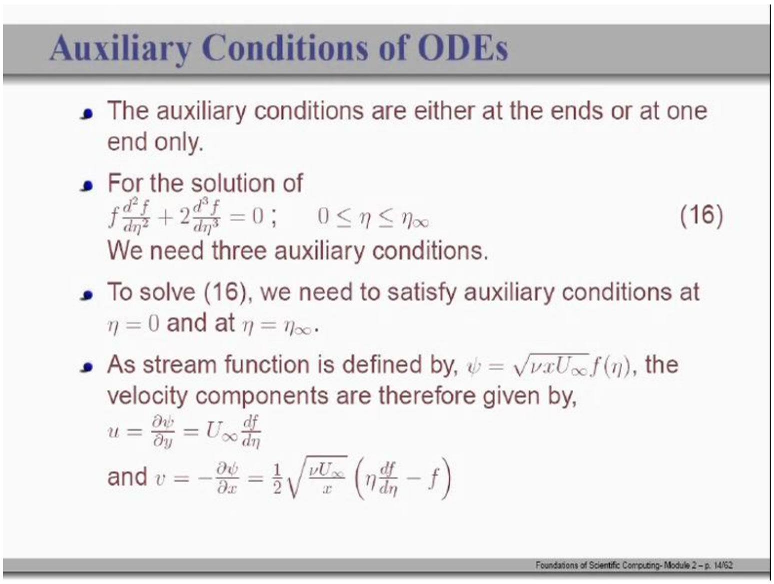

Even when you are solving ordinary differential equation you need to have what you call as auxiliary conditions. Why do I use a generic term, auxiliary condition? You are familiar with boundary condition and initial condition, right? Auxiliary conditions are basically a compilation of all. In some problems you will require both this types - initial and boundary value problems, in some cases you just require the initial conditions; in some cases you just require the boundary conditions.

For example, this Blasius's equation that we have is a third order ODE, so we need three auxiliary conditions. I think will just stop here; we need to really find out what are these conditions; how do we solve it? We will wrap it up in the beginning of next meeting, thank you.