Foundation of Scientific Computing

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Lecture No. # 15

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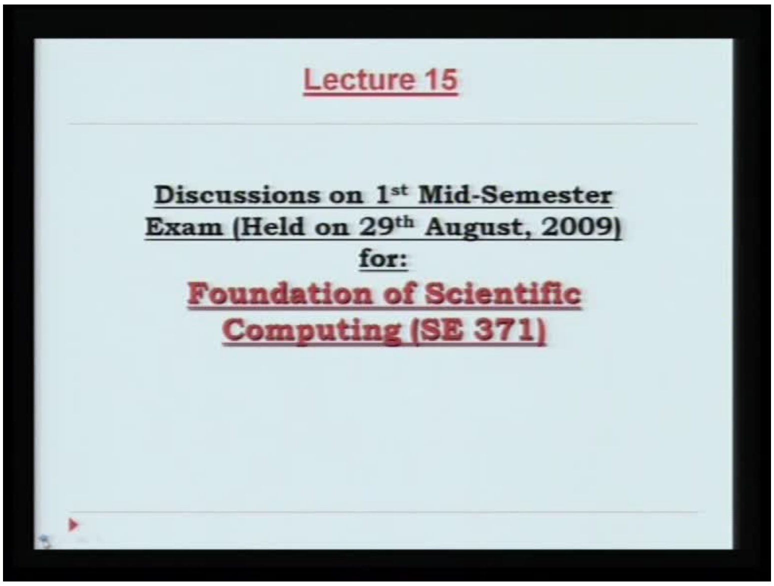

We have just finished our first mid-semester exam, held for this course, and today is our fifteenth meeting, and I want to discuss about the questions that were asked for this mid-sem.

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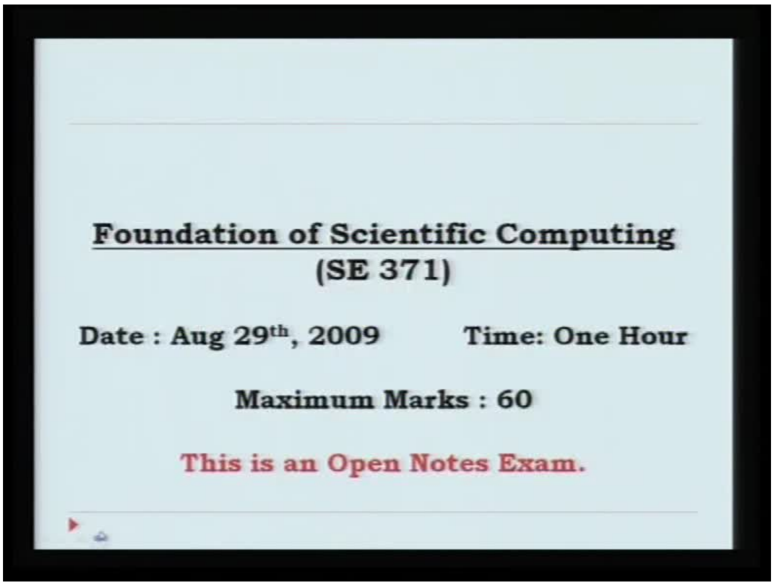

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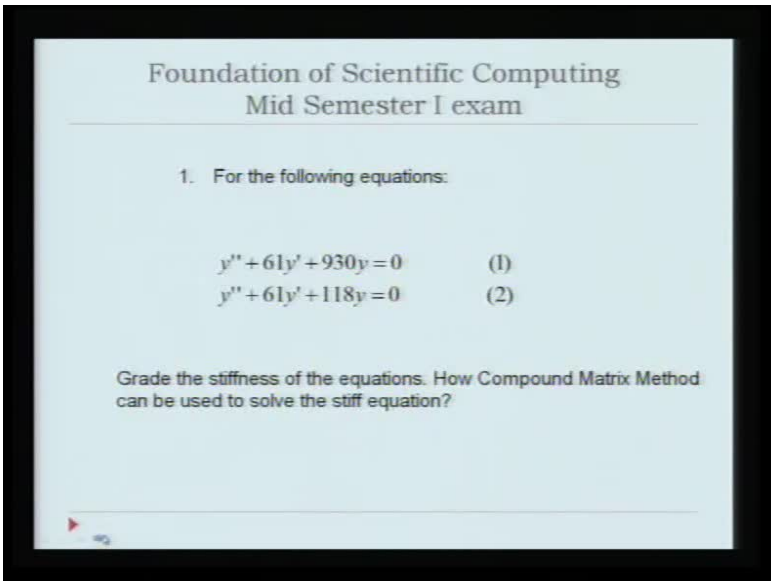

So, let us make the viewer familiar with what type of effort is needed in learning in this subject, in terms of this module question paper. It is for one hour, and as usual, all the course, all the exams that are given for this course are open notes, so that nobody has to struggle in memorizing formula, and we have asked four questions, the first question shows two similar looking equations, that one is a regular equation, another is a stiff differential equation.

So, basically I have ask the student to really grade the stiffness of the equations, and we have talked in great detail about compound matrix method, so how would one, so one of the stiff equation out of one of two, that should be, so it is actually, it is very easy to for me to say, that equation two is stiff, but that is ((no audio from 01:47 to 01:54)).

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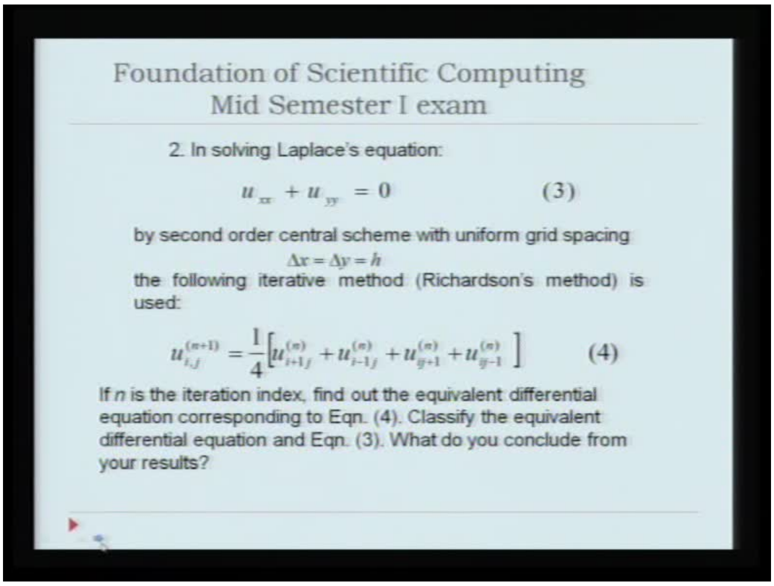

How elliptic equations are solved, so we have talking about simplest of the norm, that solving the Laplace's equation by the iterative method due to Richardson. The governing equation is given in equation 3, the iterative method is given by equation number 4, and you notice that superscript n indicate the iteration index, and the question that is asked is, basically find out what is the equivalent differential equation are corresponding to equation 4, that one solves. And the question also ask you to really classify this equivalent differential equation, and the original differential equation give in equation 3, and based on this classification, what can student conclude, what happens in computing.

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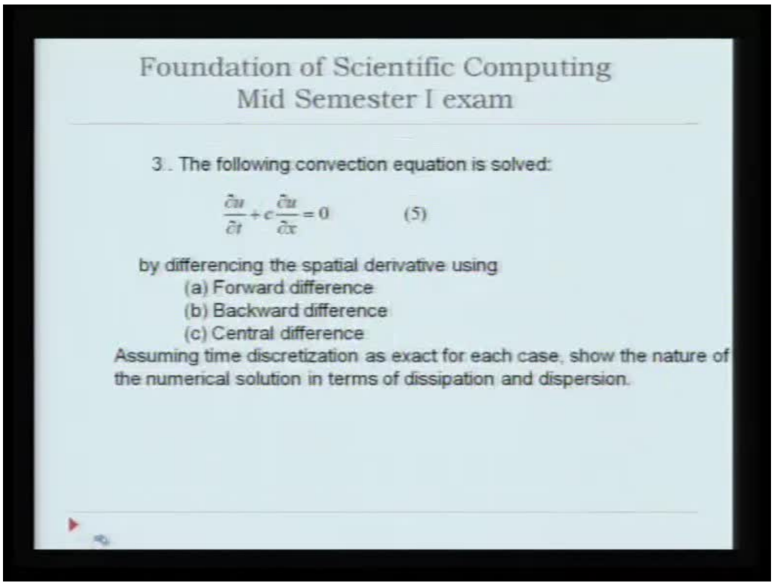

The third question actually is the simplest possible, one-dimensional convection equation given by equation 5, and this is a prototype model for hyperbolic p ds, this equation is used mainly, because it admits exact solution, so one can calibrate any method that is, chosen for solving this equation.

So, the question that is asked is, if you solve equation 5 by discretizing the special derivative by forward difference, backward difference and central difference, what is the kind of numerical solution, that you would be getting, and your input is required to help, what kind of numerical dissipation or numerical dispersion comes through in this three particular variants of the solution.

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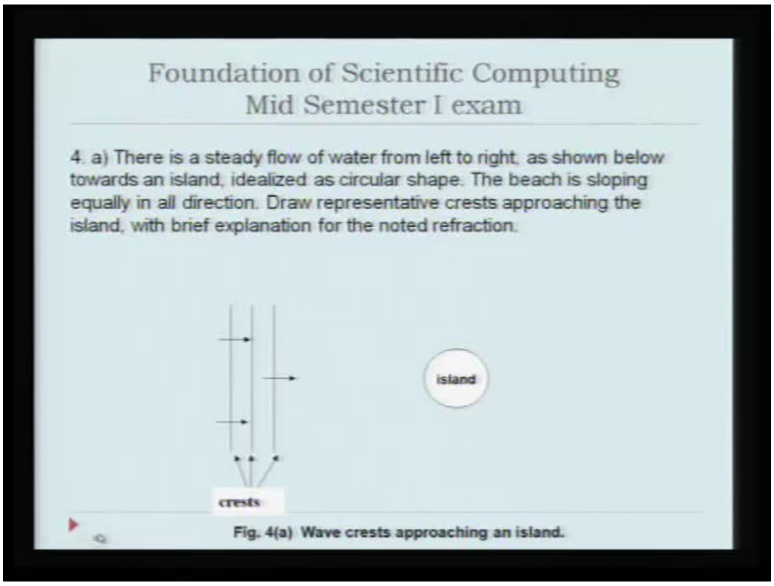

This is the question number 4, a basically ask you to explain, what you are going to see, when a wave front approaches on island, island has been idealized as a circular in shape, and we are also been told the beach is slopping equally, that means, you go further away from the island, it becomes deeper and deeper, and some represented crests, wave crests are shown which are kind of parallel here. As they approach the island, we need to find out what happens to this wave front, that is, what you will have to answer. This will involve knowledge of refraction of mechanical waves, as we have discussed in the class; so, this is what is being tested here.

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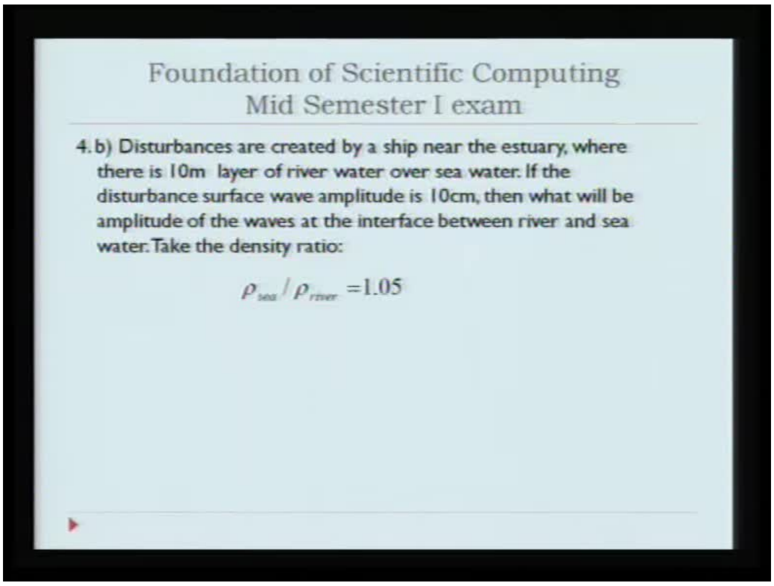

In question part b of question number 4, we are talking about a ship which is approaching with a estuary; so, of course, the ship is creating some disturbances, and because we are close to the estuary, so here we have two layers of water, the river water, over the sea water.

Now, if the disturbance surface wave amplitude is roughly of the order of 10 centimeter, then what will be the amplitude of the waves, at the interface between river and sea water; you can take the density ratio of sea water over river water as 1.05.

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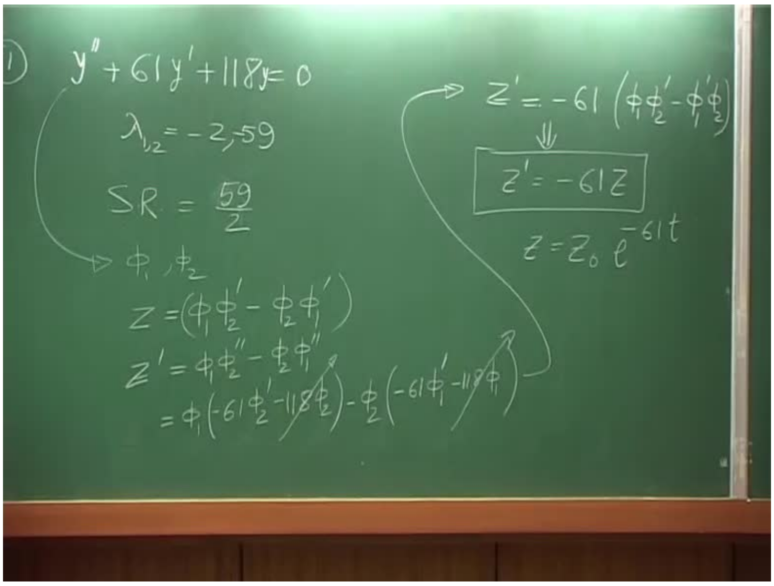

I think this part all of you have understood, that the second equation as the stiff equation; in fact, this is a, such a synthetic problem, you can make out that minus 2 and minus 59.

So, how do you define stiffness ratio, naught e to the power, but just the ratio of the two, so, either you write it the maximum by minimum; so, that is why you say this is stiff equation; so, how do we go about following it.

You have two modes right, so two modes, means, this will, let us say, call those modes as phi 1 and phi 2 right. So, if I have these, what is my solution, see this is one thing in all the problems, people have fail to interpret what is a solution.

Solution is anything, that is one order lower than that is given; so, the solution should be like y and y prime, but some of you, actually I think try to relate it to a fourth order equation, why should you, why should we, so basically I can define a compound matrix variable which will be nothing but, who is adithya sood is, is anyone here.

I think he has done it rightly, so, if I have define this, this is nothing but anyone of the first variable, that is what you would have; so, what you do is, we just simply z prime, and if you do that, you find this will be nothing but (Refer Slide Time: 08:22).

Both phi 1 and phi 2 are solutions of this, so what I could do here, I could write this as phi 1 into if I put in on the other side, that will be 61 phi 2 prime, I think there is y here, I missed it minus one, 18 phi 2, and from here phi 2 same thing. You can see that this will cancel out; so, from here, you are going to get z prime is equal to just a moment, yes.

(( )).

Pardon.

(( )).

Speak loudly please.

(( )).

I just simply differentiated it, because there would be a phi, one point phi, two point, that will cancel out is not it, so that is your equation known. This is what I said, that this start of with a second order equation, you end up with a single equation, that is it, and that is very simple to solve, that would be some z is equal to some initial solution times e to the power minus 61 t, so that was the first part; everybody clear about it, it understandable now.

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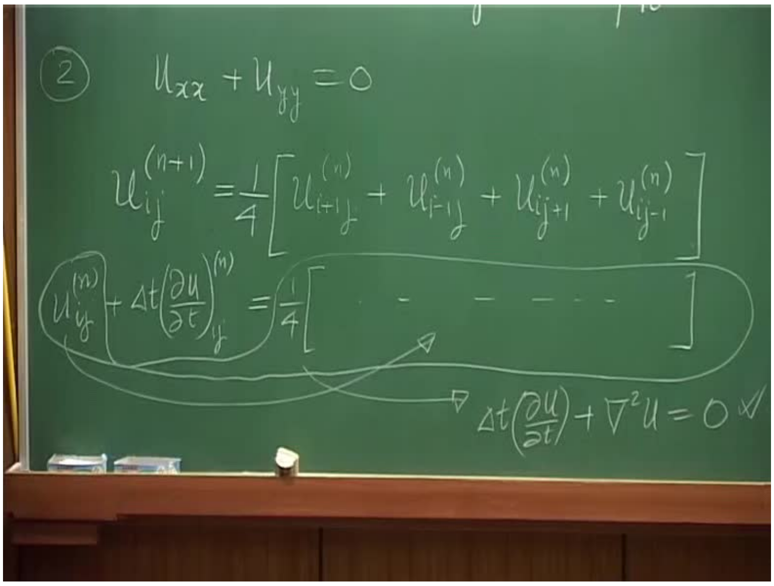

And the second question I think this is somewhat tragic, because I told you in the class, that I will ask you this question, it was not that a well kept secret or anything, I told you that I am going to ask you this question, some of you did come and discuss with me and I did provide answers, not given the complete worked out solution, but I gave how to do it. So, we said that solving this Laplace's equation by this method, which is called a Jacobi Richardson method, we solve it iteratively.

So, what you do, it is rather simple as I told you treat n as something like a pseudo time, if you do that, you can see the left hand side, I could write it as u i j n plus some pseudo time step delta t, and this will be del u del t evaluated at the same point, at this terminal run and of course, there would be other term, we will eliminate, we will just neglect them and then I will write this.

Now, you can very clearly see that, when I write this thing together, like if I would not have written n plus 1, if I would have written n, what was that, that was the discrete form of the Laplacian.

So, from here itself, you can very clearly see, that this equation is nothing but this coupled with this is the Laplacian; so, that is on the right hand side or left, so what I am going to get that would be delta t, say something like this.

The whole idea is that, when you actually try to solve an equation of this kind, you converted into a pseudo time dependent equation like this, and you are interested in the study state, when this term drops out when del u del t is zero, then you get back to your original equation that you set out to do.

So, now this part is easy, so you could get the equivalent equation, so we try to solve this equation, that in effect, we are solving an equation of this kind; now, I asked you to classify these two equations, this is very straight forward, you know that d y d x will be plus minus i that you get.

How about this part, this part is I did not doing on the black board, but I give you enough hint to really go about solving it; so, if I want to solve this equation, once again please do note, what we mean by solution, the highest derivative that appears in the solution; so, when it comes to time, solution means the unknowns would be u t.

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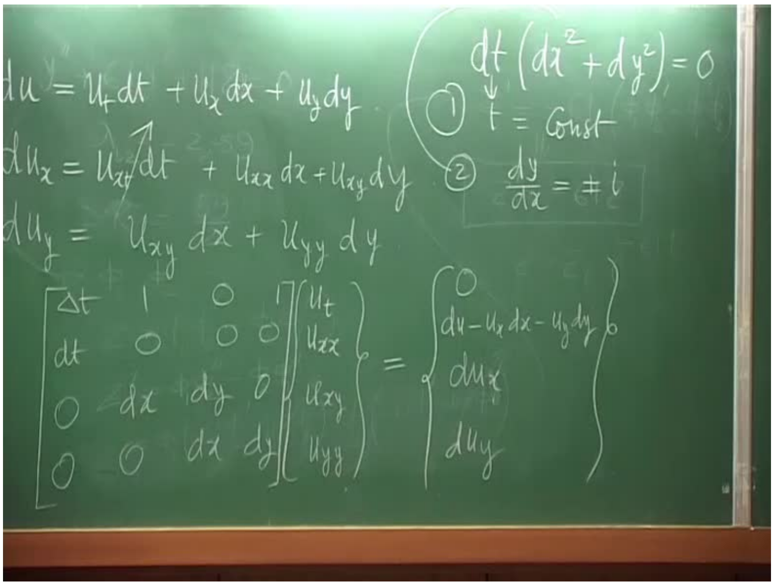

So, if I want to write down the corresponding auxiliary system, then what should I do, I will write this as u t d t plus u x d x plus u y d y, what else is our solution, u x u y also are our solution, so if I write this, what should I write this is a bit of a catch is there. As I said, I will say that I will not write this, most of you would be tempted to write like this, but it would not be correct why.

You can very clearly see that, this is your governing equation, so there is a barrier beyond which we should not go any further.

So, if I look at this, so del u del t is the, is the Laplacian, so if I take this particle derivative, what am I getting, I am taking a partial derivative of this quantity, that is going to increase the order right; so, this is not to be taken, so this, this is not permissible, so this is something which we do not have as a solution.

Solution, this does not qualify as a solution, because this increases the order of the system, so same way you get this, I think rest of it is very simple, you have four equations, 1, 2, 3, 4 is try to solve for the unknowns.

So, unknowns are u t u x x u x y u y y, so if you look at the first equation u t, of course, has a delta t, u x x as 1, u x y as 0, and this is 1, and if I look at the please be careful I am not very sure about the sign, and make it proper, I think that might be I think that is.

So, if I know now look at, so on the right hand side, of course, there is nothing for this equation, from this second equation, my unknowns are u t, so I get a dt here, and what about the other, there is nothing else.

So, here I will get d u minus u x d x, that is the second equation and the third equation there is no term of u t, u x x has a coefficient dx, u x y has a coefficient dy, these are the 0. So, now rest is very easy, all you will have to do is take the determinant of this, put equal to 0, and what you are going to get is this, this so of course, one of the characteristics as d t equal to 0, means t equal to constant; so, this comes from here and this one gives you, so what does it mean. It means that its parabolic in time elliptic in space; so, that is the, that is the answer, so about the three question, I do not think most of you had any problem, except some you still have to understand, what you are trying to do, you look at what you are doing numerically, and then you can revert back to the corresponding differential form.

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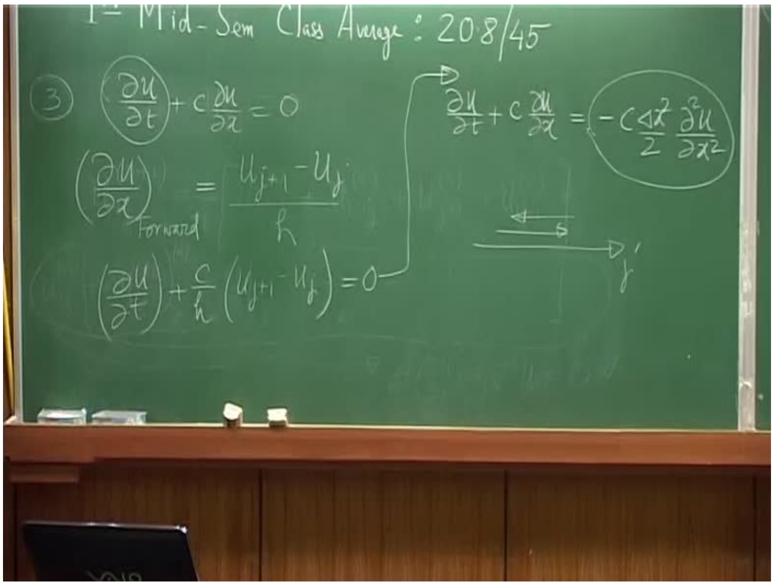

If you do not do that, then you are doing something totally hypothetical, and which, which actually leads to the totally the wrong conclusion, say for example, if I will just show you, one of it, firstly you like told you, that the time derivative is exact; so, there was no need to temper with it; so, there is no need for you to say, I have taken forwarding time, etcetera, etcetera; so, we are making that assumption that this is ok. Suppose, I do this, what I should write, say it is a j th node, so I will write like this. So, basically, numerically you are treating the equation like this, this has no error and this has c by h.

Now, from here of course, you can write down the Taylor series for this, and you can see u j will cancel out, what you are going to get del u del t plus c del u del x, and the next term will be delta x square by 2, if I put it on the other side, that will be c delta x square by 2, this is we need term that is all you need to worry about. And the sign of this will alert you that this is not proper, because instead of dissipating, it is actually going to amplify the solution, and I have also reasoned it out heuristically, that if my information is going from, so if my this is my j direction, so information is going in this direction, but what am I doing here, and sending the information in the wrong direction, that is why it is failing and for good reason.

So, if you could just, any of you could write this, and say look this is on physical, that is why this solution will grow up, I would give you full credit for your understanding.

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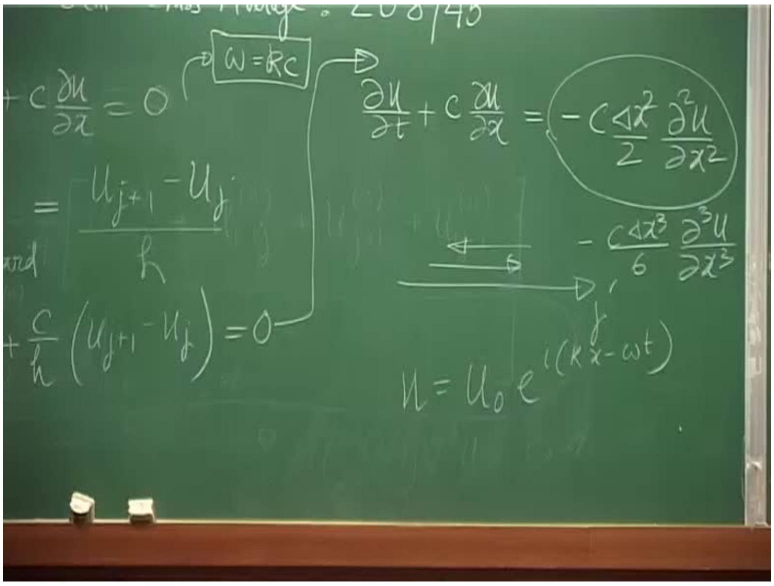

However, I wanted also ask you to write in terms of dissipation and dispersion; so, you can see this is not a dissipation, these are what we called as a anti diffusion or anti dissipation, then of course, there would other term which we did not write, but there will be there, but they are not the leading term, the leading term is this; so, that way if you would have done it the backward, then it will be u j minus u j minus 1, and then, this sign would be plus, and then you would have a dissipative solution.

The third part is when you do the central differencing, of course, you will find, you will write, of course, here minus j minus 1 by 2 h, and you would find the leading term is this; so, that is your dispersive solution. I expect is you to take the trial solution, some u naught e to the power i k x minus omega t, and find out the relationship between k and omega, that is your dispersion relation to tell you what happens. And you would see for the central differencing, you will find that usually theoretically speaking this equation have a dispersion relation omega equal to k c, this is your theoretical dispersion relation.

In the forward and backward difference, you will find that, this part is there plus an imaginary part, those imaginary part implies, there is numerical dissipation either plus or minus, that will determine what it is, while this one you will find, that you will get a pure real relation, this one plus something that you can see; it will come from here, that will depend on the spacing, that will depend on delta x square ,so that is that, so it was quite simple.

question 4, I thought that was really something you could have looked up in the slides, they are all there, while the island part I did on the black board, so there is something that you should not feel surprise, because it was done on the black board.

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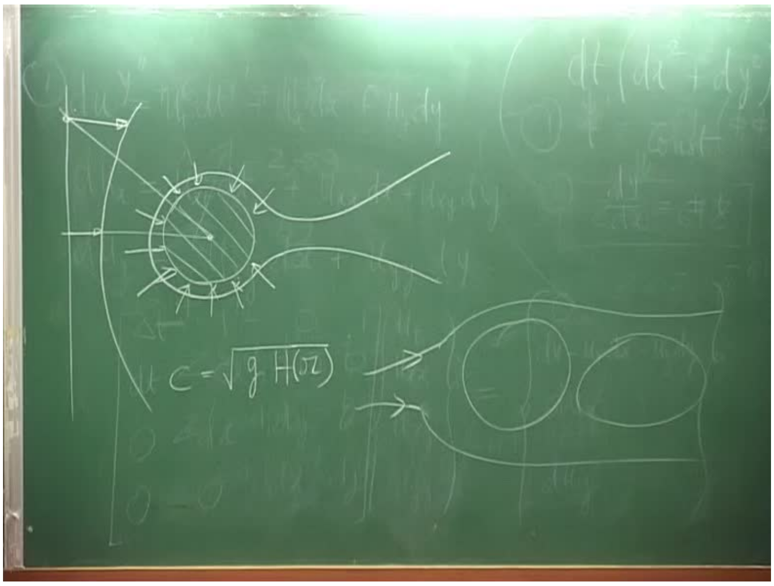

What I said if you recall, that if I have an island of this kind, there is this bottom of the shelf is receiving as you go away; so, if I have a straight crest, what I should be looking at, I should be concerned with the phase speed not the group velocity, because you are focusing on the crest.

Crest defines what, the condition of the phase, crest and trough, so if you also look at their, if you are very close to the island, so it is now can be treated as a shallow wave and then c is (Refer Slide Time: 25:29). So, if I have the in homogeneity, it should be like this, so what happens is, if I measure the distance with this and this, so of course, this is a larger h, so c will be larger, so the crest here will be moving larger distance compare to here.

So, what you are going to get, you are going to get something like this. What is the interesting, though that as you come close by you will find, that this will come like this; so, the crests are all coming like this, so quite a few of you have actually drawn the correct picture, there is no problem there, but we understand, that is what is happening.

This is very interesting you know why, suppose you take a cylindrical structure, put on in a river, then if around that, if there is a in homogeneity, because of this you will find, actually the water will also in the backside, will push towards the cylinder, but if you put the same cylinder in the air, what you would find, there would be a sort of a dead water zone, there would be nothing here, but here you have seeing that, it is actually going back towards this cylinder; so, this is a very interesting observation, that you can notice; so this is what is somewhat different from this water wave v is a, v is this what you would see in fluid mechanics related to, let us say homogenous case, if the, if the height would not have change, then you would have seen a dead water region in the back.

And 4 d path, it is straight from your notes, so I am not going to talk about, you have seen we have developed the dispersion relation, there are two modes what we called as a barotropic or surface mode, and we have also seen the baroclinic mode; so, in the barotropic mode what did we find.

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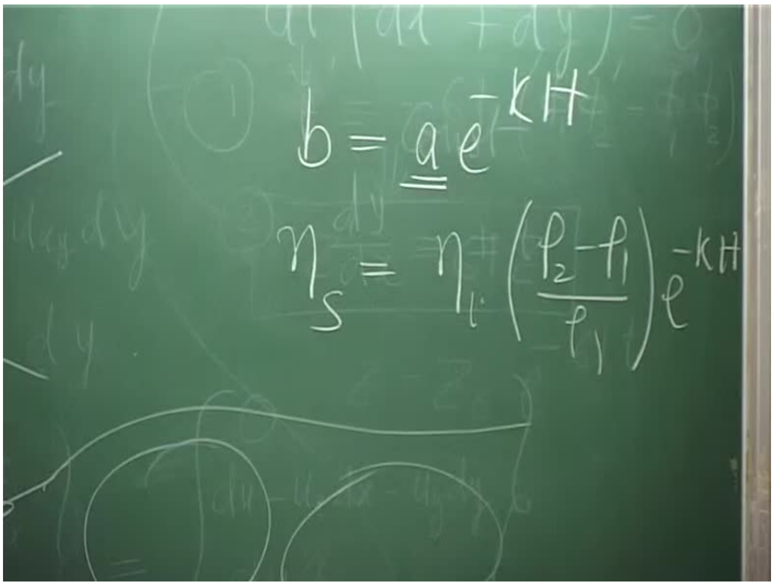

We found the amplitude actually scales like this; so, if my surface wave as an amplitude a, if I go down at a height h, that will be that right; so, that is one thing you have seen. The other one is, of course, was also given to you, there would be, I do not remember the formula, but it should be something like this. If you look at this, then what you find a very, very interesting thing, that the internal wave amplitude is going to be very large compare to the surface wave.

In fact, you know this was a kind of a mystery to all these people entering the hard work, they always use to find that, as they come to the port through the estuary all of a sudden, they experience a very large drag, why, because, although on the surface you may create, let us say small web linked disturbances, like I will say 10 centimeter may be 20 centimeter or something.

In your case, you must have seen that the interface was what something like 2 meter, times some e to the power some factor, so different k would go at different growth rate, but this part itself will show that, this eta i is something like twenty times, so what happens is the ship is gently going on the surface, but down below of the interface, you are creating massive waves, where would that energy come from, it is from the engine room of the ship.

So, this was quite confusing for sea farer for centuries, till there can scheme on explain this, it comes from here. I think, I have consume quite a bit of time, as I explained class average is there, I want you to come and collect your answer book from the lab on Wednesday anytime, I do not wish to waste our time here, instead we go straight to what we want to do.

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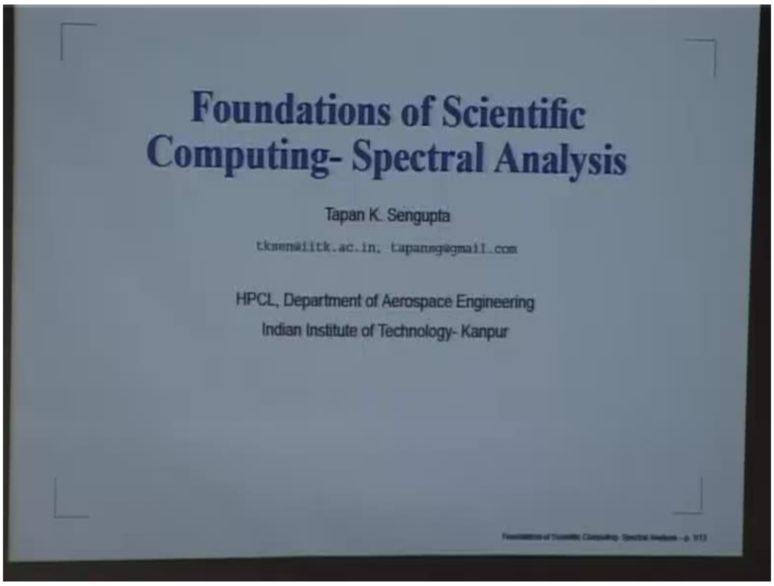

So, this is what we have been doing in the last few lectures, last lecture actually, trying to figure out, if we can understand the scientific computing in a better way, how do you do better way, you know, there is a bit of a, sort of a story telling here, if you are talking to another of your colleague, and you say look I use finite difference method, and that your friend will say, oh but I use finite volume method, and you know finite volume methods are better than finite displacement.

So, this kind of childish rivalry has prevailed for ages, so we decided to put a stop to this; we say look if you are making some such claim, we should be able to figure it out, what it is. So, we need to find out a common yardstick, so that common yardstick, we decided to adopt is in the spectral plane.

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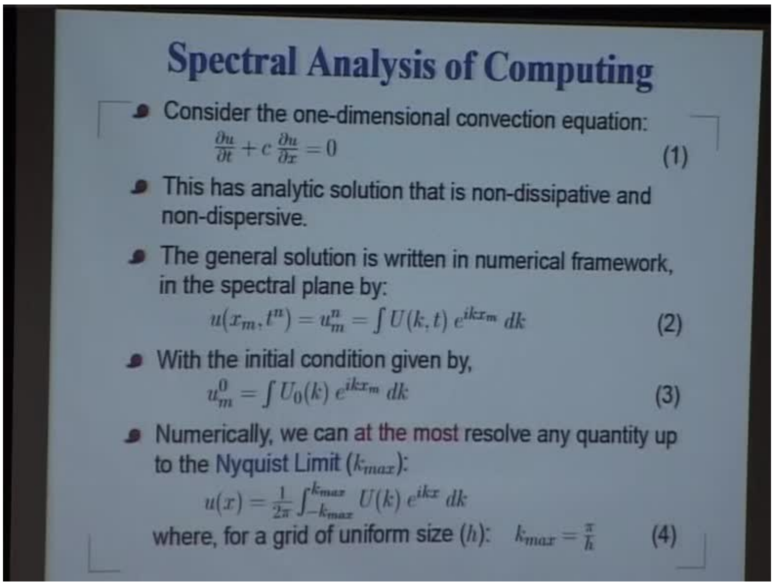

So, if you give me an any method, I immediately go to the case phased omega case phase, and compare your method with any other method, and I will tell you whether your claim is justified or not; so, that is why we want to do this part. So, this is a very, very interesting bit, and let me also tell you that this is what we have been doing over the last ten years.

So, people had some misgivings in the beginning, but, now I suppose everybody is falling in line, they realize that, there is no escape from the scrutiny; you have to subject yourself to the test. So, let us take up a very simple equation, this point it simplicity, as I have mentioned, this equation is a very fantastic equation, because this as an analytic solution, so you cannot escape by saying oh my problem is so complex, so you cannot, even know, you have no business judging my system, I said, no, here is a system which has a exact solution, you compute this, if you cannot, then keep quite; so, it is as simple as that.

So, what is the property of this analytic solution, we have already seen, it is non dissipative, that is what we did in the mid sem. We showed that, if I take one sided difference, then I violate this, either it is it becomes dissipative or it becomes amplified, and I mentioned also in the last class that amplification is better, why, because it will just glow up on your face.

So, you would know something is gone totally wrong, but if it is attenuated, you still can try to do little bit of hand waving and saying, this is, this is the physical reason, that is why it is happening but fortunately enough, this equation does not give you that middle room; so, you will be able to say whether your solution is really dissipating or not. And of course, this equation has given us the dispersion relation, here omega equal to k c, and that tells you that d omega d k is equal to c, so group velocity is equal to the phase speed, so you have a non dispersive system.

Now, if I look at such a system, and look at the solution in the discrete space, as we do numerically, so I am looking at the m th node in x, and let us say at a time t n, so we can conveniently also write it with this kind of subscript superscript notation. What we do here, though we write down this quantity, in case space, while keeping the t as a dependent, independent variable, as it is written here. So, all we are saying that the solution would be summed over all possible k for that time t; I will be able to tell you the state of the solution.

So, this we have also noticed, that what this equation does, it takes the initial solution, and it slowly convicts it to the right with the velocity c if c is positive, if c is negative it would go to the left does not matter, but what is important is, the structure of the initial solution; so, the initial solution tells you what is the range of k, that you have in the initial condition.

So, once for all you define the spectrum, what is the k wise distribution of the solution? Once you have that you also know, because of this non dissipative non dispersive property, that same spectrum will be there for all time to see, it is only that the disturbance will simply bodily move, but if it was compact, it will remain compact, it will not spread apart, because there is no dispersion; so, we look at this initial condition.

Now, this is something we discussed in the last class, that if we choose to solve this problem, when a, in space with a equal spacing, let us say h, then I require three points to define a way; so, three points means two delta x, within two delta x, I will get a complete wave, and why do I need 3, because I need to fix the amplitude, I need to fix the wave length, I need to fix the phase difference.

So, that is why I need 3 points, so if I need that, so my lowest wavelength is what this two delta x, so lambda minimum is two delta x, so what will be the corresponding counterpart in terms of k, k will be 2 pi by, so 2 pi by lambda minimum will give me k maximum. And that is what you have written it down here, this is called pi by h, and it is also call the nyquist limit. As I have told you, that if you are looking at time series, and if your data are sample at your interval of delta t, you immediately say with this kind of a sampling, I can resolve only this much of frequency.

So, the same story we are narrating here in space; so, nyquist limit appears either in space or in time, but the concept is same, the moment you tell me about your sampling rate or you tell me about the discretization are going to come back and tell you.

Look theoretically speaking this limit for this integral should have been from minus infinity to plus infinity but numerically I am restricted by this nyquist limit. So, you understand this is one of the story of finite resource computing, that you do not have unlimited resolution; resolution is determine by the nyquist limit as we have written here.

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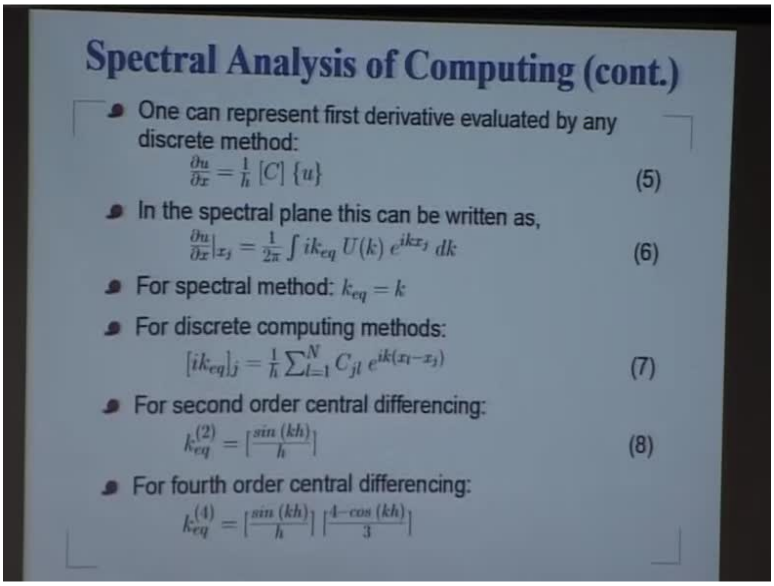

Now, we have also seen that, if we are trying to calculate the derivative by explicit method, what do we get, we write it in terms of the function at different nodes, and if I want to do that, what I would do, I would write that as some constant matrix c operating on the whole vectors.

See the unknowns are u 1, u 2, u n, so that is what I am writing here, that is your vector, and c would be depending on your method, for example, if I would have done this, c would have at on the diagonal element, it would be having minus 1, as on the super diagonal element, it will be plus 1, everywhere hence it will be 0. So, if I let us say do central differencing, then I will get j minus 1, so diagonal would be 0, super diagonal will be plus 1, sub diagonal will be minus 1. So, you choose your method, so we will not worry about what method you are doing; we will just simply say you have fixed a c matrix and is a constant matrix, the coefficients are like constant, 1 by h minus 1by etcetera, etcetera.

Now, so this derivative, suppose say I am evaluating at the j th node is given in terms of theoretically speaking, all u is, so u 1 to u n, all the points theoretically can take part is not it, that is the whole idea. Suppose, I choose a method, where c has non-zero entry across the whole row, so you understand what this is, this is the vector, so if I look it the j th entry, that would be like j th row of c multiplying by the whole u column.

So, I can choose a method, where the j th row is non-zero for all elements, so that means, what I am evaluating the derivative at the j th point in terms of all the points in the domain, so that is up to you.

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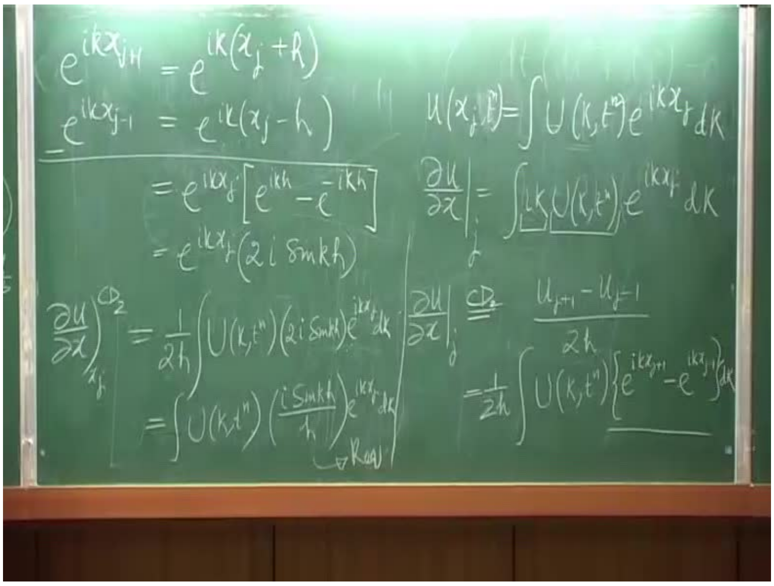

Now, if I look at Fourier transform, and try to write down the derivative, what do I do, what do we do, I am writing, let us say in terms of u k t e to the power i k, let us say I am interested in finding out, this u at the node at j and time t n, so I will write this time is t n, and here I am writing i k x j d.

Now, if I differentiate this equation what do I get, I will get here from here i k, of course, u remains as it is and e to the power i k x j d k. So, you see, if you want to find out the derivative of the j th point, where do you get your information in the right hand side, only from the j th point itself is not it.

However, look at this expression, I am actually taking information from all the array, that is what it means, that is what it means, so it seems that every time, I take a discrete method and violating one of the fundamental tenant of Fourier transform, that this is local in nature is not it, if I want it at the j th point, I get the information from the j th point itself.

Whereas, when I do discrete computing, I am violating that local principle, I am getting information from the whole range, and then what happens is, we are violating the property of the locality of the solution; there is another thing that is happening.

In spectral method, so this is like your spectral method, we are defining the problem in case space, I am taking the derivative; so, taking the derivative is equal to looking at the Fourier transform, multiplying it by i k.

However, if I let us say, do if, I do a discrete computation, let us take an example, let us say, we do it by second order central differencing schema, I will call that as c d 2, so what I would write u j plus 1 u j minus 1 divided by 2 h.

So, I would write 1 over 2 h, what do I write u j plus 1, means, I am putting in here x j plus 1, so I can use that expression; so, I can write it like this, so the first part u k t n, they remain as it is, I have, now for this I will write e to the power i k x j plus 1, and from here, I am going to get e to the power i k x j minus 1, and this we are summing over all possible k.

Now, if you look at this e to the power i k at j plus one, if I take uniform spacing, then I can write e to the power i k x j plus 1 is x j plus h; so, I, of course, same way you can write e to the power i k x j minus 1 will e to the power i k x j minus h. So, this quantity in brasses here; so, when I subtract this, then what do I get, I can take the common part out, and I would write e to the power i k h minus e to the power minus i k h.

And what is this, this is agree with me 2 i sin k h, so what actually as happened here, from here, then I can write in a discrete form, what I have done, I have done a c d 2 formulation, then I get 1 over two h, then u what I get, I keep it and from here, I get 2 i sin k h e to the power i k x j d k.

So, now, you compare this expression with this what is the essential difference; in this case, every Fourier component was multiplied by i k. Here, what happens, every Fourier component is multiplied by i sin k h by h, so what I could do, I could write it like, I will put this thing inside, I will write this, this I will write i sin k h by h and then e to the power i k x j d k.

So, what actually has happened by discreet computing, I did not multiply the Fourier component by i k, instead I multiplied by i sin k h by h; so, this quantity which is different from k, I will call that as k equivalent, that is what you are seeing here, this is what we have done.

So, what you notice, that even though we could refer back the point there, in terms of x j but my k has changed to k equivalent; if I would have done a spectral method, then that k equivalent is k and that is what it is.

Now, what you can do is, you can work out this expression for i k equivalent from 5, what would you do, see what we have done here, we have written it down in terms of j plus 1 and j minus 1, because of particular stencil that you have chosen.

Suppose, I take a general stencil like this, then what I could do is, I could refer each and every point here in the array, you want to, even I can refer it back to u j; do you all follow, let me spend a little more time here, this is important for us to understand.

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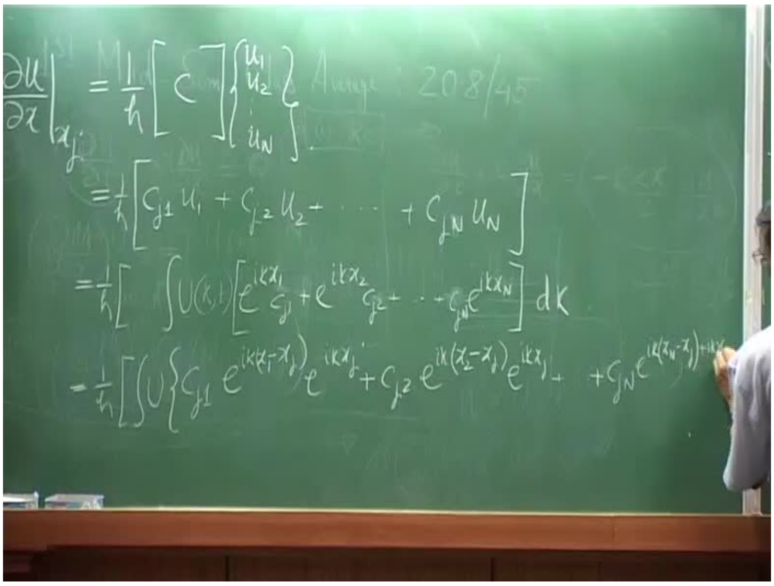

See what we have written there, let us say I am doing it at x j, and what I have written here one over h c matrix multiplied by u, so u as like this, so if I expand it what do I get one over h, I would look at the j th row, so I will get c j 1 u 1 c j 2 u 2, although we have to c j n u n.

So, each one of them, I could write by the Fourier series; so, what I would do, I will write this as c j 1, what is e 1 is u of k and t, whatever it is e to the power i k x 1, that is what it means, u 1 is the value of u at x 1, so that is that, and what about this, this will give me.

So, what I want to do is, yes, pardon, the constant coefficient would all be going here, you are absolutely right; so, this will be c j 1, this will be here c j 2, and here you will get c j n, you will get all of it there.

Now, each one of this point, what I can do is, I could write it as 1 over h, u I will write c j 1, this one what I am going to do is, I am going to write it as e to the power i k x 1 minus x j times e to the power i k x j. Same way, I will write the second term c j two, I will write e to the power i k x 2 minus x j into e to the power i k x j, all the way up to c j n e to the power i k x n minus x j and plus i k x j, the whole thing is this.

You know why I did this, it is pretty much operant to you, because you are trying to derive the derivative at the j th point. So, we should be able to write it in a Fourier series with the face part given by this, you see everywhere, I managed to bring in e to the power i k x j.

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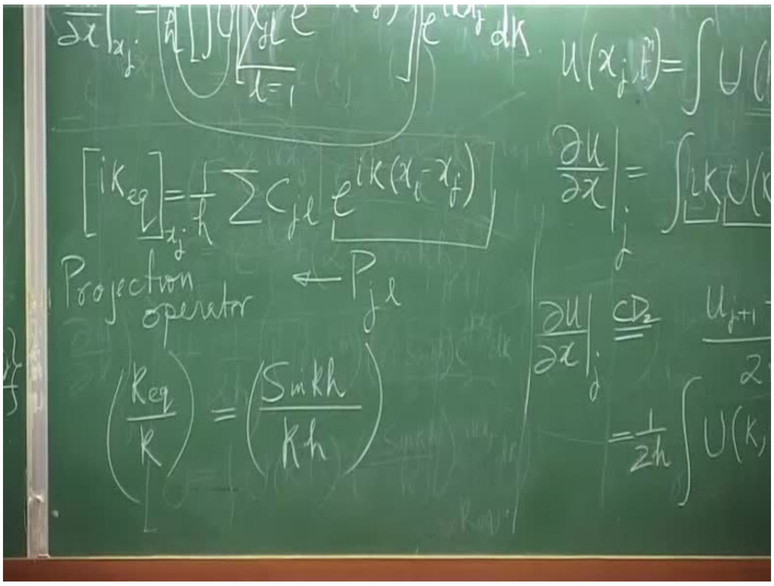

So, what I would do, then I will write del u del x j at at j th node that would be 1 over h, then u, and what I am going to get here, I will write c j, I will write, I, let us that create confusion c j l into e to the power i k x l minus x j, so what I am doing here, I am writing this whole thing as a kind of a series l going from 1 to n, so that I could do.

Then, of course, this whole thing is multiplied by e to the power i k x j d k. Now, can you see equation seven in front of you, there it is, c j l e to the power i k x l minus x j, that is this quantity; so, this is your anything that is multiplied with the Fourier transform is your i k equivalent.

So, for a general methods, see that is why I do what like to talk about any specific method, let us keep it as general as possible, and then you choose your method, and then you will be home, and write you will have no problem to, so basically what I have done, each and every point I have projected it to the x j point.

So, sometimes I would call this as p j l, and I will probably use a fancy word called a projection operator, you know, when things are very simple, the mathematician follows invent a jargon, so I also decided to call it a projection operator, but you understand what is the basic idea, I have projected every point to a j th point, because that is where I am looking for the derivative.

So, one thing you have found that this method, which we call as a metric spectral analysis method. This is going to be a function of the node is not it, that is what it is write, so if I change x j, then I will have a different point. So, what have been able to do is, now take a rather general problem, where we do not wish to say, what we have done at each and every point, but as we have already discussed that what we do in the middle, may not be amenable towards the end, because of boundary interviewing there.

So, what happens is, those c matrix entries will have all those information built in there, and once you have it, you can actually find out for each and every point, what is the kind of a resolution you have gotten. Do you, do you not see this, why I am calling this as resolution, because the spectral method multiplies the amplitude by k; so, instead by all this discrete method, we are not getting k, but some k equivalent, so what I could do is, I could define a performance parameter, which I will call it as k equivalent by k.

And for the c d 2, what did you find, if you recall that was sin k h by h, I have derived by this, this function looks quite familiar to us right, this function looks quite familiar to us. So, this is your c d 2, well that is what it is, you are seeing here, so I wrote it c d 2 with this supper script inside the bracket two, but is going to give you that and how does it look like.

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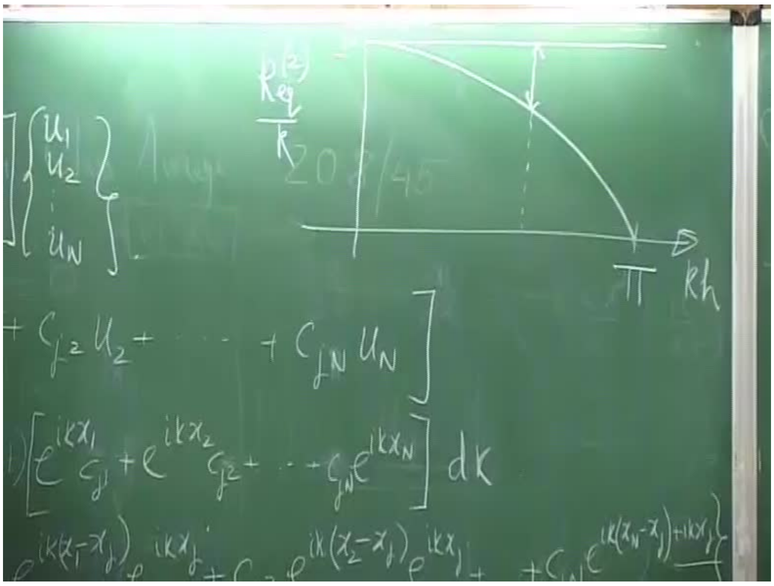

So, what I could do is, I could plot k equivalent by k, for this second order method, and on this side I should plot k h, why I am doing k h, because this a universal number, because k maximum could be pi by h, so k h you will go from zero to pi, so you understand the reason why we do this.

How does this function derive sin k h by k h, all of you know its starts up from one, and then, it goes like this. What does it tells, it tells us the following, that if I do a second order central difference in, different k is weighed differently right. For various small values of k, this ratio is one, so this is almost exact; so, for very small values of k, we are going to have a absolutely correct resolution.

However, as the wave number increases, we are going to see that, this value is coming down, and it is zero at pi, so what is happening is ideally you would like it to be there. It should be one all through, but for any k, you are actually having a loss of resolution given by that bracketed quantity.

So, you see this is the story of discreet computing, that whatever you do we will have to accept loss and the loss is more at high rate number, means, smaller scales, so I think, we will stop here, we will continue and finish this tomorrow.

You may decide to work out this expression, and come back and tell me, if you have gotten it, and I will send you an email about collection of your answer book, any time you can come on Wednesday.