Foundation of Scientific Computing
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Module No. # 01
Lecture No. # 11

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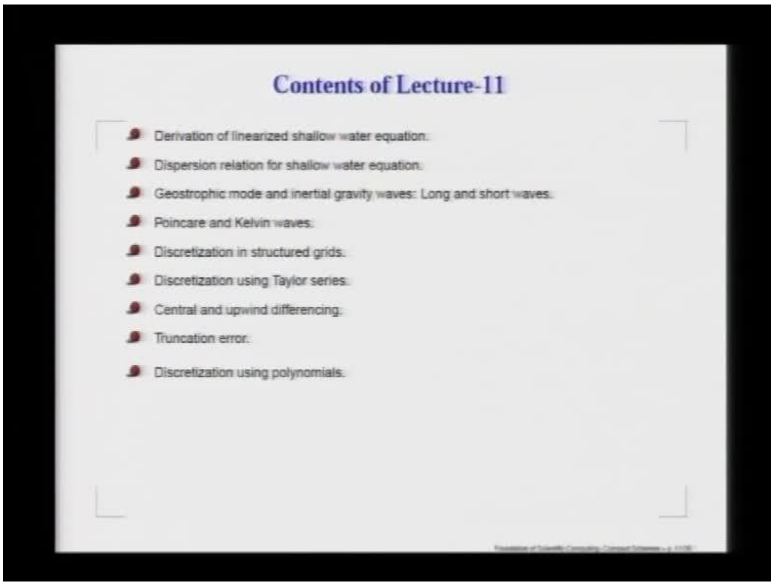

Today, on this eleventh lecture we will start deriving the linearized shallow water wave equation. We will then follow it up by obtaining its dispersion relation, and talk about long and short waves, and look at its various limits in terms of Poincare and Kelvin waves. This will essentially finish our exposure to the topic of waves and then we are now going to talk about solution methods in a numerical sense.

So, we will be focusing our attention in this course, mostly with numerical methods, which work with a very well structured discretized points; so, that is what we are going to talk about here - discretization in structured grid. This will be done using either the Taylor series or by polynomial methods, but first we will begin our discussion using Taylor series. We will introduce various types of differencing or discretization, there would be some discretization which are central in nature; so, the discretized equations have some symmetry or if they are asymmetric we will call those as upwind differencing.

Now, we will next touch upon truncation error of these discretization methods and we will show, the different types of discretization through a truncation error have different numerical effects, and finally we will be showing the equivalence of discretization of via Taylor series with the polynomial expansion of the function in a discretized grid.

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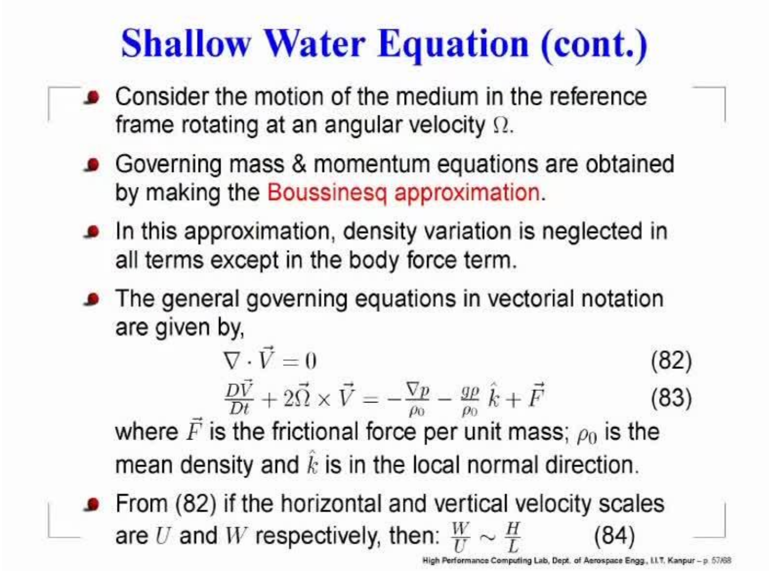

We started looking at shallow water equation; we mentioned that this will be equally applicable to the motion in air or in ocean if you are looking at geophysical fluid dynamics, that is where this equation takes a very central place. How do we look at this equation? As I mentioned on top, that in studying this you have no other option, but to position yourself on a moving train of reference and for earth this movement is very simple, you are moving at a constant rpm.

That angular velocity is given by this (( )) and to make matter simple, because the temperature excursion that you see in atmosphere, I mean, in the weather, the system is rather not very extreme, not very large.

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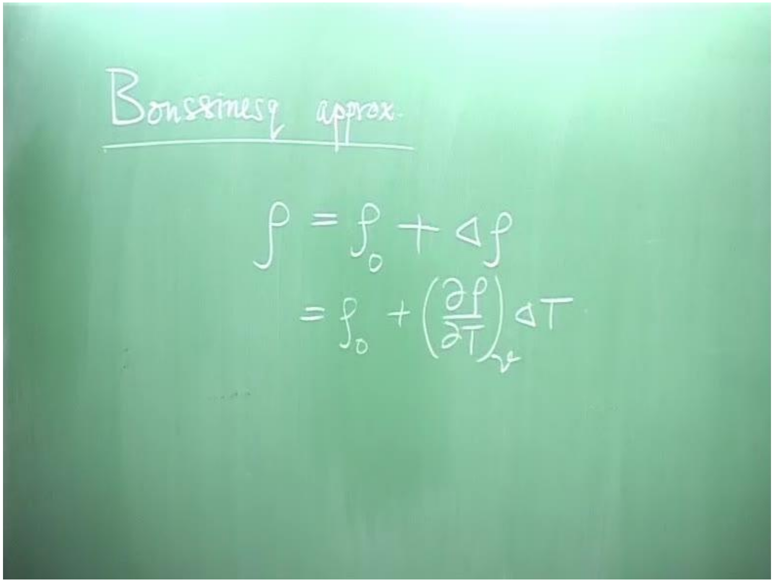

And in such a case, on a local time scale variation the temperature differential is small, so we make what is called as a Boussinesq approximation. So, Boussinesq approximation implies, basically, if I take the density rho, then I will add to it, is in the sense, nominal value rho naught; we will write it has delta rho and this we can write it has del rho del T times delta T.

So, any temperature variation will bring about change in density and this density variation is kept only in the body force term; so, the body force is due to gravity, so that is rho g acting along the k unit vector, so, that is where we use the actual density; in rest of the terms, we keep the nominal value that rho naught, that we are mentioning here.

I also say it just now again, that we are looking at the equations of motion in the rotating frame and these are the two equations in vectorial notation for the mass and the momentum conservation. Since none of you pointed out to me or asked me why do I have the Coriolis term here and not the centrifugal term, do you understand what I am trying to ask you?

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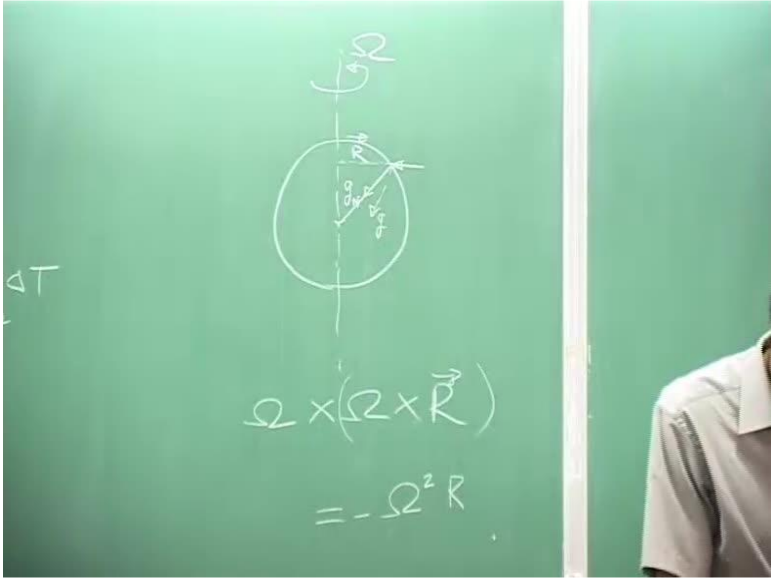

I am asking you the following, that let us say, this is your earth rotating with this angular velocity here, then if I look at a location here which is at a distance capital R from this axis of rotation, then there would be a centrifugal term which should be like this. So, if this is the R vector, so that is the term that you have and if you do not value some vector identity, and you can show that this is going to be nothing but…

So, what we actually do that centrifugal term or centripetal term, whatever the way you would like to call that, is included in g, how? If you notice that Newtonian gravitation would be always directed towards the center of the earth, so if I call that as g n, so that is acting towards the center, and this force that we are talking about here - this will act in this direction, right? So what will happen? The resultant would be g, and that is the g that we have written.

So, this g incorporates, both the gravitational attraction term plus the centrifugal term; and we noted that we are looking at the dynamics of a layer, which is rather thin in the depth while the horizontal scales are much larger. So, that is shown here on the right hand side - the depth scale by the horizontal scale is related to the velocities in the vertical direction in the horizontal plane - and it shows that the velocity normal or in the radial direction could be considered insignificant because H of L is insignificant.

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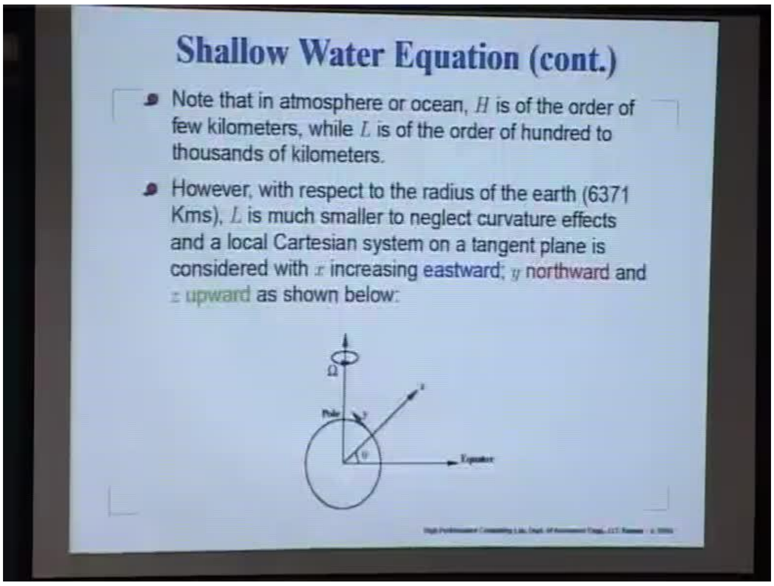

So, we justified by noting, that if you are looking either at that atmosphere or the dynamics in the ocean, H is of the order of few kilometers while L is of the order of 100s to 1000s of kilometers, and with respect to the radius of earth, even this horizontal scale is much smaller. We are talking about, you know, predicting the dynamics over 100s of kilometers to even a 1000, compared to that 6400 is quite significant.

So, if we are allowed to make that assumption where you can neglect curvature effects, then we can associate a Cartesian coordinate at any point. So, suppose this is where you are located and you are trying to solve the flow equations, then you can draw out a local Cartesian frame, where z will be of course the radial direction away from the surface, y would be towards the north pole northwards while x should be in the eastward direction; so in this case, it happens to be perpendicular to the plane of the figure and theta is the latitude - defines where you are located, right, where…

Yes

Last time said that (( ))

Yes

(( )) resultant of (( ))

(( )) k cap is (( ))

No, this k cap would be in that direction, in the direction of g.

So, you are right, so when we are actually getting this z axis - this is opposite to the g, not g n, not the Newtonian gravity term.

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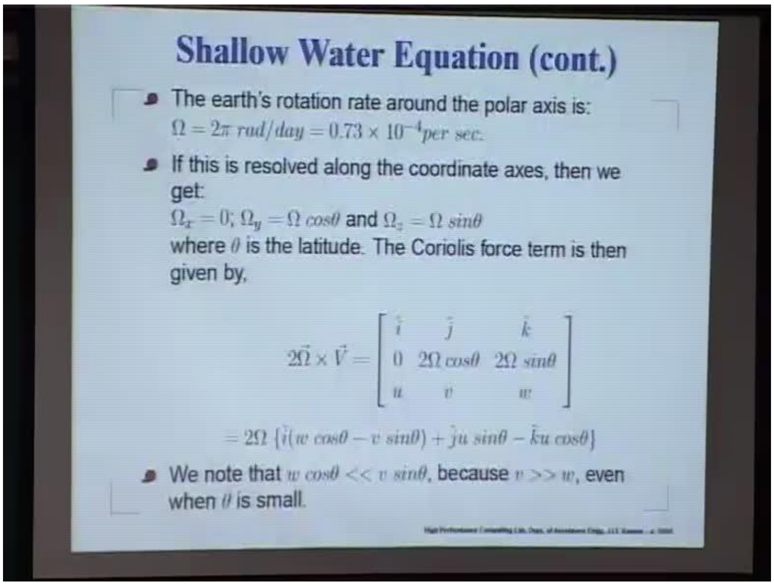

So, having decided upon this coordinate system to work with, we have also noted the earth’s rotation rate, is a very fixed amount, he was right - two pi radian per day and what happens is, that is, in this direction, in the polar direction. So, we can decompose it in the x y and z direction, and as a consequence, we notice that in the x direction you get nothing, because it is a perpendicular to the plane whereas in the y direction, we have omega cos theta and in the z direction we have omega sine theta.

And with theta as the latitude, you can evaluate the Coriolis force terms by looking at this vector product and this is what you get all the three components present there. At the same time, we have already noted that the vertical velocity scale w, is significantly lower than the horizontal scale. So, therefore, we could neglect w cos theta term as compared to v sine theta in this i component. So, we can make some kind of a significant and simplification there.

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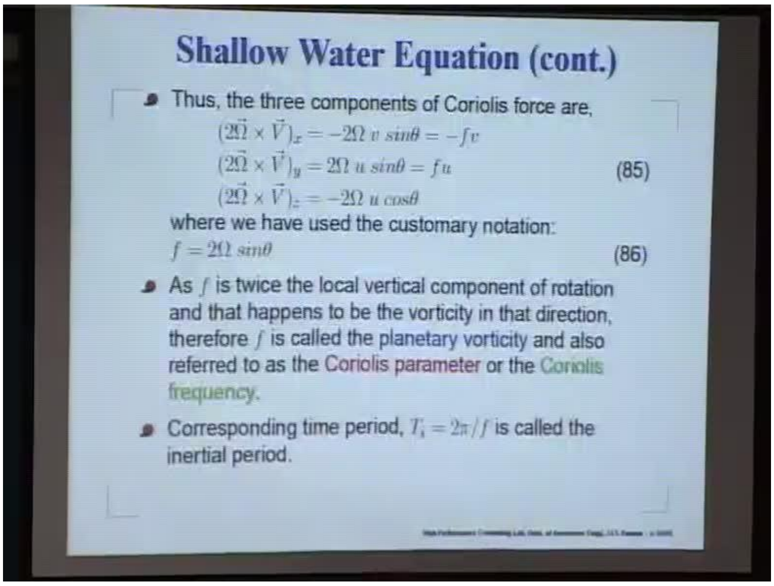

Now, so, we also adopt the notation where we write 2 omega sine theta is f and this happens to be twice the local vertical component of the rotation rate, and that is exactly equal to the vorticity; so, we call this f as the planetary vorticity or the Coriolis parameter or alternatively, also it is called the Coriolis frequency. And since it has a time scale of 1 over T, so we can work out a time period corresponding to this f parameter and that is what we call as the inertial period.

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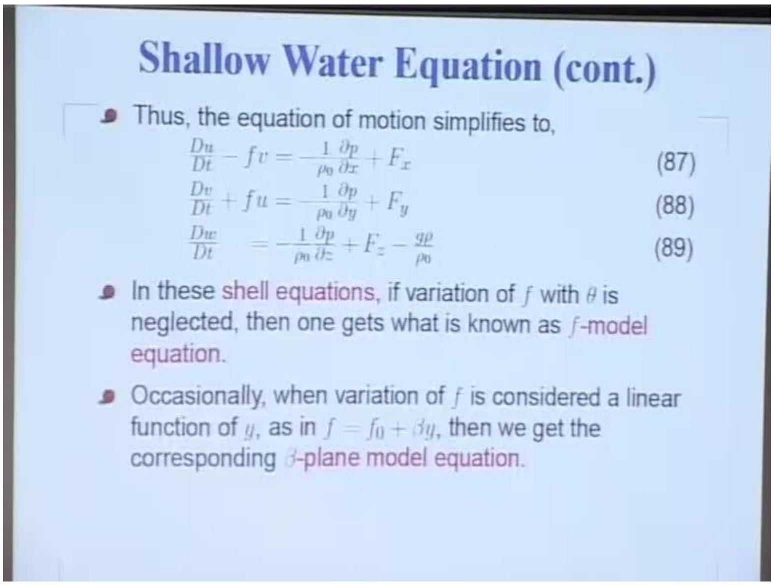

Now, then what we could do is that we could write down the equations of motion in this particular fashion. So, this is the substantive derivative in the x y and z direction, to that we have added those Coriolis terms in the x and y, in the z it is negligible, whereas these are the pressure gradient term which is driving the flow, and F x and F y are bottom friction term that we have model. In this description, again you can think of two variations, these are written for shells.

And if you decide to investigate in a narrow region, then you can afford the luxury of assuming f as constant and if you do, the corresponding equations are called the f model equations. When you are little more adventurous, you want to look at a larger area where latitude varies the significantly, then you can look at your central latitudinal position that determines the Coriolis term f naught, to that you add the variation of this f with the y direction. And when you do that…

So, this is some kind of del f del y term; so, this is why this can be taken as a locally, as a constant and when you adopt that model, we call that as beta plane model equation.

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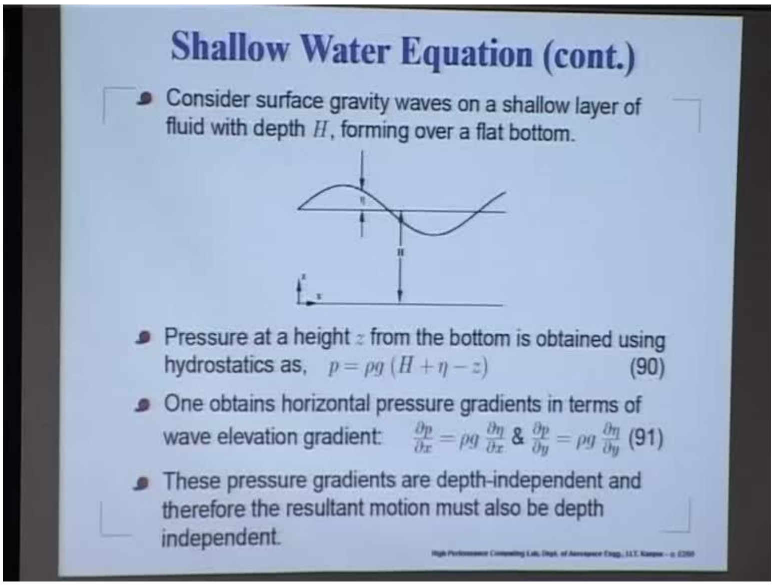

Now, coming to the study of the shallow water equation. What we can look at once again? A single harmonic description, may be, a wave amplitude given like this, with the amplitude given by eta local variation and H is the mean altitude of this interface, and if I fix the coordinate system like this - so, z is a pitch perpendicular to this, x is along this direction.

Then pressure at any arbitrary z location is given by this, that is straight forward and that also, this equation 90 helps you in defining the horizontal pressure gradient in the x and y direction in terms of the wave elevation, right. del eta del x and del eta del y will define the pressure gradient and this pressure gradient is considered as independent of the depth at which we are noticing it.

So, the corresponding del p del x would be independent of depth. As a consequence, any motion that is created by this pressure gradient, also would be considered most height independent, right, that is the point we are making right at the bottom there.

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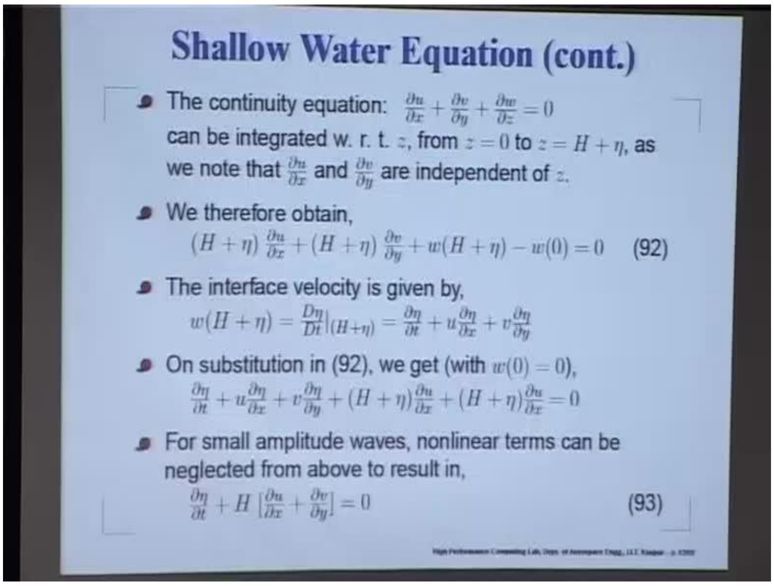

Now, if we now look at the continuity equation in this Cartesian frame and we can now integrate this equation with respect to z; considering the del u del x and del v del y are independent of z, and then we should integrate it from the bottom z equal to 0 to the deflected interface, that is going to be the mean plus the elevation. Having integrated that, we get this and if we consider a smooth plane bottom, then this itself w at z equal to 0 can be taken as 0; that is what we do subsequently.

Now, then what we need to do is, get some estimate of this w velocity at the wave interface and that can be obtained from this kinematic condition, del eta del t. And this del eta del t at H plus eta would be that local acceleration term del eta del t plus the convective acceleration brought about by the u and v velocity there.

Substitute this in equation 92 and you get that and I still have not corrected it, but I hope the one in the website that was posted yesterday, you would see that is correct one; see the last term here, this should be del v del y. So, please be aware of that, it is already corrected in the notes posted in the course website.

So, if we are looking at small amplitude waves, we can neglect non-linear terms. Non-linear terms are these ones - u del eta del x v del eta del y eta into del u del x eta into del v del y; and if we ignore them, this is what we get. So, what you are seeing equation 93, is nothing but your altered mass conservation equations for this shell equation; so this is simplification that has been brought about.

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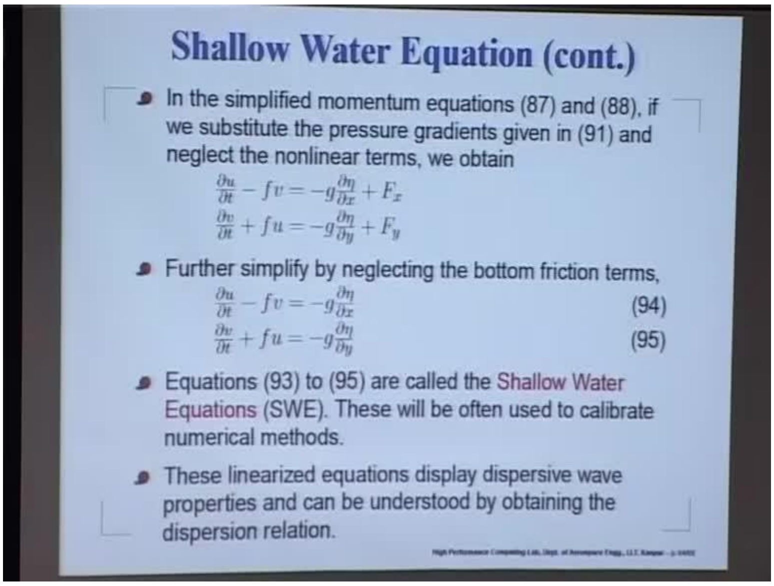

Now, looking at the momentum equation, we have replaced del p del x and del p del y by this wave elevation terms and these are, of course, our those bottom friction terms. If we neglect those bottom friction terms, we get this and what have we done here? We have in expanding the substantive derivative, we have also omitted those non-linear terms. The non-linear terms were there like, u del u del x v del u del y, that has been omitted here and same thing about this.

So, now, what we have is very simple looking three equations - one of which we saw right at the bottom here and the other two coming from the momentum conservation. So, these are linearized equations where curvature terms have been dropped out and we has been written in a locally acceptable Cartesian frame; these three equations are, what are called as, the shallow water equations.

We actually are going to use this equation to calibrate numerical methods. Now, why do we do that? Because we have seen that other equation that we have talked about, that was about that d'Alembert solution of wave equation - that equation was non dispersive, that equation did not show any attenuation, whereas this equation shows dispersive wave properties and to understand that, it is indeed dispersive, so we try to obtain the dispersion relation.

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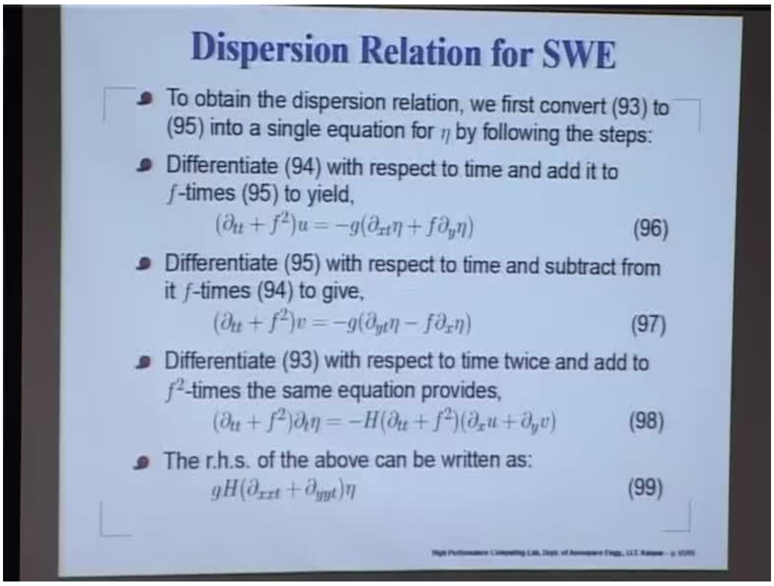

Now, let us go through this exercise step by step. What we do? We differentiate the x momentum equation with respect to time; so, it already has del u del t, so I will differentiate it with respect to time, so I will get del square u del t square.

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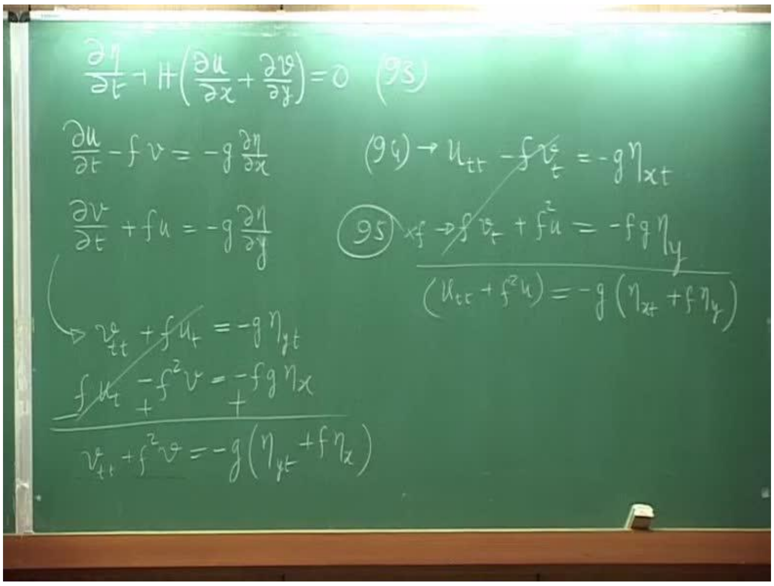

So, that is this term and to that what we are going to do is…; well if you want me to go slow and just… and let me also write down the mass conservation equation for this. So these are those three equations; O.K. so that is your 93, this is your 94 and this is 95.

So, if we go over slowly with this equations, what we are doing? We are differentiating this, so I will write it as u t t minus f v t should be equal to minus g into eta x t, right; to that we are going to add f times this, so I will get f v t plus f square u and minus f g whatever; it is y. So, if I add this, of course this cancels out and we are seeing u t t plus f square u, that is there and on this side we can see that eta x t plus f eta y; that is what we have written equation 96 as.

Now, the next step you differentiate this with respect to time that will give you v t t plus f u t equal to minus g eta y t. O.K. Now, what we are going to do is we are going to subtract f times this equation so that will be f u t minus f square v is equal to minus f g into eta x. So, we are going to subtract this; so what we are going to get once again, this cancels out, and we are going to get v t t plus f square v and take a g common, you get eta y t plus f eta x, right; so, that is your equation 97.

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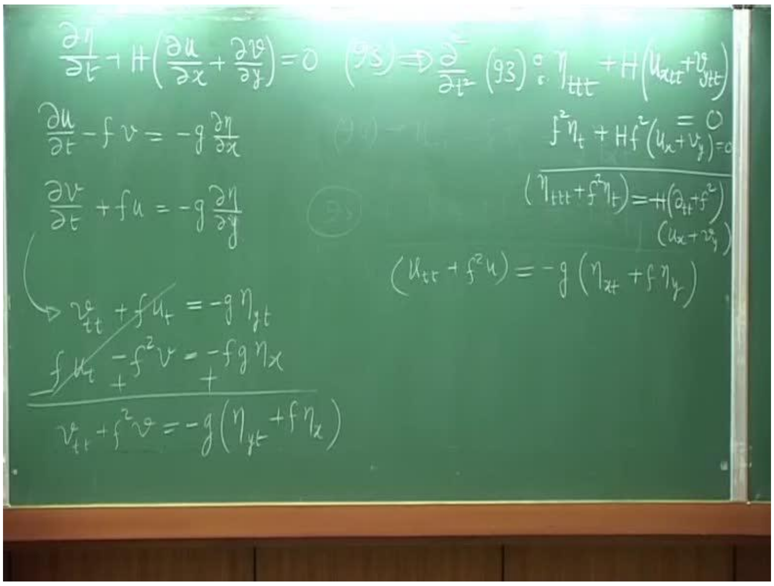

Now, the third step is to differentiate the mass conservation equation with respect to time twice, so if I do that, so if I do that what do I get? So, this is the one equation that we have, from here if I am doing this operation, then of course I am going to get here the third derivative and plus H of u x t t plus v y t t, that is our equation 98 and no, not yet; so, we have to add to it x square times the same term, right.

So, we are going to add to that; so, we will add f square eta t plus H square u x plus v y. So, if we add that we are going to get eta and post all the terms on the right hand side, and that will give us minus H and this four terms, we can club them together and we can see they are going to give us this operator operating on u x plus v y. So, those are the equations, right.

The last equation that we wrote, this one, we can make use of these two equations right; so, that is what we have here. We can interchange the time and x operation here, so that is del t t of plus f square operating on u; so, we can get that from here. The same way, del t t plus f square into v, that will take it from here and you will see there would be a cancellation and the only two surviving term would be nothing but g into capital H times this operator. So, as you can see if I take the time derivative operator outside, then inside I will have del x x plus del y y; that is nothing but your Laplacian.

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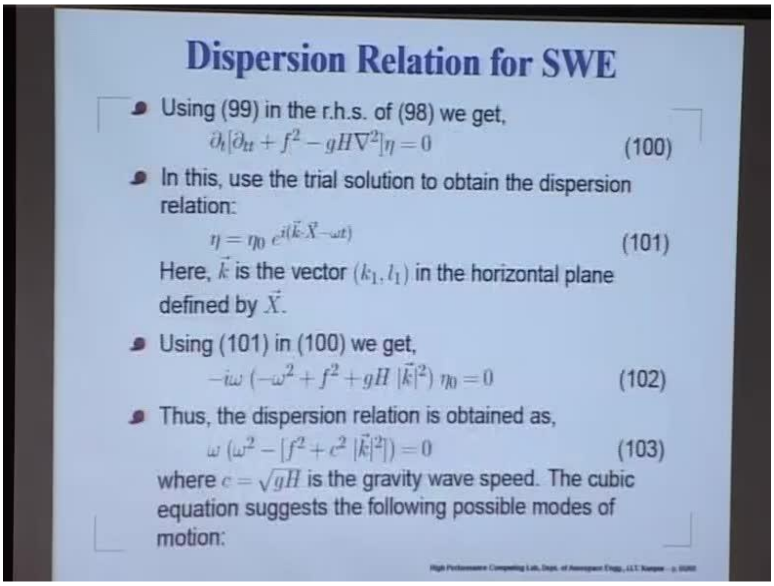

So this, this term is nothing but del del t of the Laplacian of eta, O.K. So, if I do that, this is what I get, so I already had this term that you can see coming from here; if I take del del t out, so that will be eta t t plus f square, that is what we have there - this two term, what did I do? So, we have obtained this equation.

So, basically what we have done so far? You can realize that we have written a single equation for the single variable eta instead of having three variables: eta, u and v. So, having obtained the PDE in terms of a single variable eta, that is given by equation hundred, what do we do to get the dispersion relation? We plug in a trial solution. The trial solution, since we are looking for a wave like feature, so that we will write it as k vector with a take in a dark product with the x vector; x is the horizontal plane in the x-y plane that we are talking about.

So, k itself will have a component k 1 and l 1 in the horizontal plane, then plugging this trial solution second derivative would…; so, this first time derivative would simply give us minus i omega; so that is there outside.

Now, the second derivative inside will give us, with respect to time, will give us minus omega square and f square remains as it is; and what do we get from the Laplacian? That would be nothing but that k vector’s modulus square, right; so, that is what we are going to get. So, we have the dispersion relation, right.

Now, since we know this g H is nothing but c square is the gravity wave, so we have just simply written it in terms of this; so, replace g H by c square in this equation 103. So, what you notice that this is a third order equation in time. So, that is why we get a cubic for omega and one of the root is very simply seen, that is nothing but omega equal to 0.

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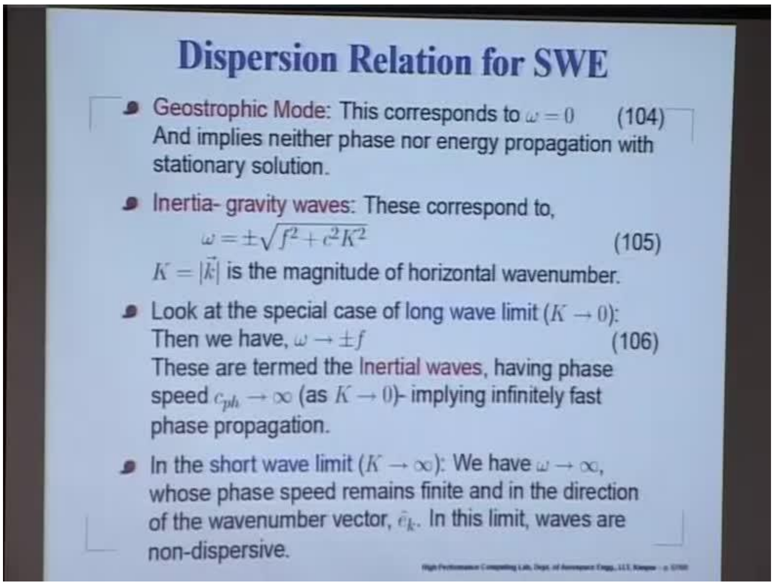

Let us see what we get one by one. So, the root that corresponds to omega equal to 0 is what is called as a geostrophic mode - this implies neither phase nor energy propagation because it is a stationary solution; omega equal to 0. So, there is no question of dispersion, whereas the other factor indeed gives us omega as a function of K in that horizontal plane. So, in that horizontal plane we can talk about the magnitude of this horizontal wave number that we are calling here by capital K; so that is what we get. These two roots give rise to what is called as the inertia gravity waves.

Let us try to understand what we are talking about here because if I look at long wave limit, that is, capital K going to 0, then you can directly see omega equal to will tend to plus minus x. And what will be the phase speed? I will be talking about the phase speed, will be omega by K and that phase speed goes to infinity because K is looked at in its long wave limit; K equal to 0.

So, such waves, long waves actually propagate very fast. So, if you are looking at sort of explaining the solution obtained from the shallow water wave equation, the small k limit would be, those crest will be going at an infinitely faster speed.

In contrast if you look at the short wave limit when capital K goes to infinity, then you can very clearly see from 105, that omega also will be infinitely large.

So, we are talking about very high frequency variation, right. If I am trying to find out what is happening in the order of meter wave length, so those corresponds to K almost going to infinity in the context of motion in the atmosphere or in the ocean. So, those will correspond to very high frequency fluctuations. What happens to the phase speed? Well, phase speed will still be finite because I will divide by K, so you can see it will still have a finite value. So, this is interesting.

So, if you are looking at events occurring in the atmosphere over the continental scale, those information propagates at very quick speed, right; whereas, if you are trying to locate what is happening in Kalyanpur, then you should be really looking at this kind of limit that we are talking about the local limit.

Those things will happen at a very large faster clip, but this events will go at a small speed, so you can see them convecting. So, if you see them that on one part of the campus it is raining, it may happen that the other part of the campus, it would not rain because those information propagate at a finite speed.

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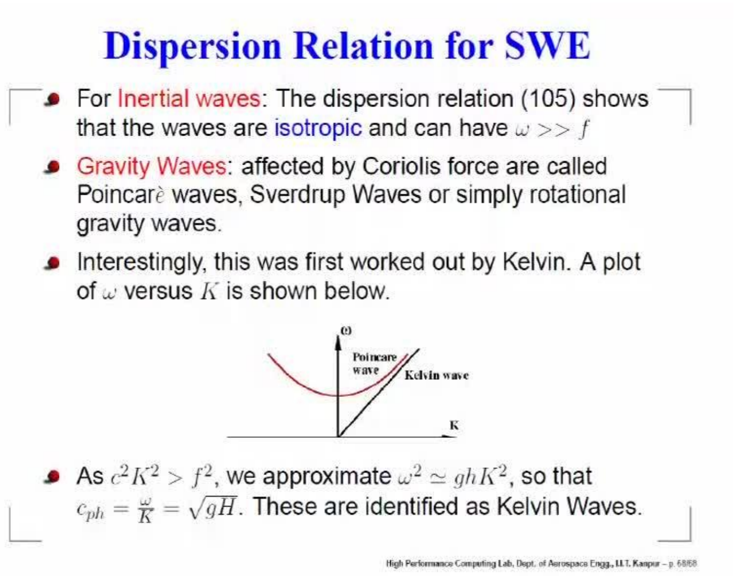

Now, so, we can look at some of the beautiful aspect - shallow water wave equation. If we are looking at inertial waves, we also notice that the waves are isotropic, you cannot distinguish between x and y direction.

And for those case we know omega is going to be large because we have omega is equal to square root of f square plus c into c square k square; so, omega is usually large. Whereas if I look at the gravity waves which are now affected by Coriolis force, they are called the Poincare waves. So, what we have done here is we have plotted omega verses K here by this red line, and you can notice that K goes large, your omega also goes large. These waves are called Poincare waves or Sverdrup waves or simply rotational gravity waves. Rotational means, because you have included now the Coriolis term which we did not do in the first part of the analysis.

This work was done first by Kelvin. So, at least as an afterthought, people have decided to call the large K limit solution as the Kelvin wave. You know this, these are very interesting topic; somebody does the work, somebody else gets the courage, for example, Bernoulli's equation was never written by Bernoulli, it was done by Euler. So, this is a subject called Eponymy - the assigning the credit to the people who have really worked on it and there is a law stated by a statistician called Stigler's law. The law states that very simply, the person who does the work, he never gets the credit.

So, here is an example of what Kelvin did and Poincare came much later, 50-60 years later, but people have started calling them as Poincare wave. Anyway, that is what happens. So, with this I think, I will conclude the discussion on waves and as an interlude will take up something which is rather very trivially simple and this is the topic of discretization.

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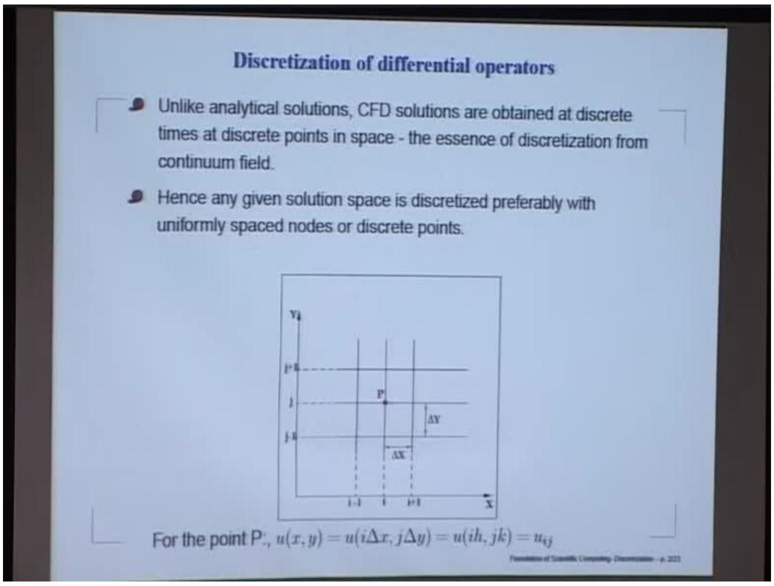

So, since we have already done a bit of it, I will zip through it rather quickly because I presume that you would find it rather, trivially simple.

So, any analytical solution, we define the points in the continuum and we get the solution; and each and every point, when it comes to computing, you do not have the luxury. So, what you do is instead, you try to find out the solution at discrete points and that is the essence of discretization, that you discretize the domain into lattice of points.

Well, I have shown here what is called as a structure grid where the lines are drawn with a very good structure in mind; people also do using unstructured grid we will not do that because structured grid analysis has gone light years ahead of unstructured grid methodologies and analysis. So, we will try to keep our attention focused on structured grid and try to find out what discretization brings to the table.

So, what we are doing? Let us say in the x-y plane, we divide the points in equispaced nodes, I have shown you some typical points i minus 1 line ith line and i plus 1 ith line; same way, we have shown you three such lines in the y direction j minus 1 j and j plus 1 at line and P is at the confluence of the ith and jth line. So, we write for the point P in a computational frame work. We will not be writing it in a that continuum description of continuous variation in x and y. Instead, we will say this is at the ith node, so i times delta x and y is j times delta y.

So, I may decide to sometime use delta x as h delta y as k; so we will may write it has i h comma j k or the alternative form that we have seen to use a subscript i and j indicating the nodal location of the point in question.

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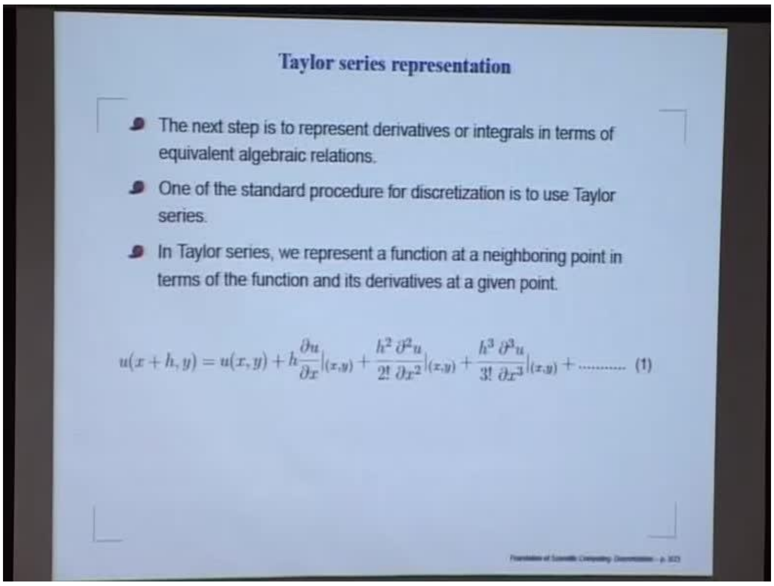

And this is what we talked about that the points at one node say, x plus h can be obtained from the information that has been already obtained at x y location in terms of not only the function, but it is all kinds of derivatives via this Taylor series expansion.

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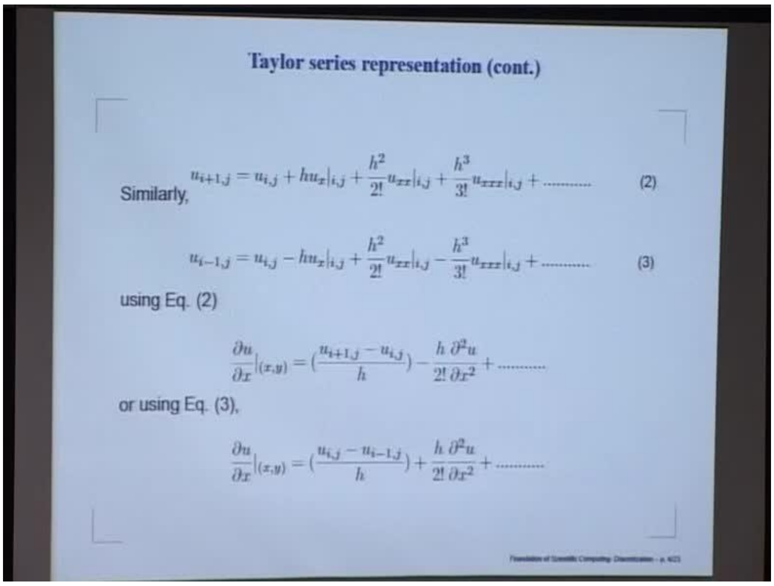

And what happens is, I can then write the function at x plus h in terms of the values at x and y by using this Taylor series. The same way I can find out the function values at its left neighbor point that would be i minus 1 j.

Notice that odd derivatives terms appears with a minus sign and that basically gives us the following options of evaluating. Let us say we are interested in evaluating the first derivative, so what you do? We will make an approximation by dropping out terms. For example, if I use the equation two to evaluate the first derivative, so del u del x, I could write it as u i plus 1 minus u i j divided by this h and the term then we are neglecting is given by the lead term here, that is this h by factorial 2 u x x.

Now, this is a troubling aspect of nomenclature; people try to describe the order of discretization by looking at the exponent of h. So, that is why this method is called first order method because the first term that is dropped out is proportional to h; the same way, if we would have used equation three to obtain u of x, then we would have written, transported this term to a left and this to the right.

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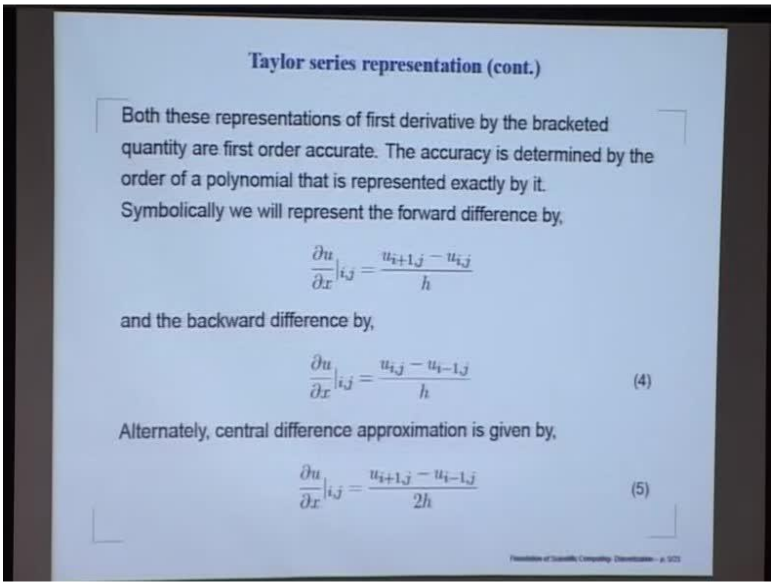

Then, I would have gotten del u del x as u i j minus u i minus 1 j divided by h and once again leading truncation error term appears with the exponent of 1 for h; and so these two representations that we have written are called the first order accurate method.

However, the accuracy should be determined by the polynomial which is behind this representation, that is satisfied exactly. For example, this way of writing the derivative at the node in terms of the node affront, will be called as the forward difference and if it is done with respect to point behind in the grid notation, then we will call that as backward difference.

Now, those two equations that we had written in the previous page here, if I add this two up, we can notice we need to subtract it. Then, what will happen? u i j term will drop out and this h u x term will add up; while the even derivative term are going to drop out, while the odd derivatives term remain and you are going to get this expression for the first derivative.

Now, what happens here? What is the lead term here? Lead term here is proportional to h square; so, according to the way the books write, it should be called second order accurate because it is proportional to h square. But if you look at in terms of polynomial, what will you get? The lead term is h cube, sorry, third derivative, the third derivative; so again that means, this expression is satisfied exactly by a second order polynomial.

So, as far as the first derivative is concerned, there is no conflict as such, the way we talk about the order and the accuracy, I mean, polynomial and the exponent of h happens to match together.

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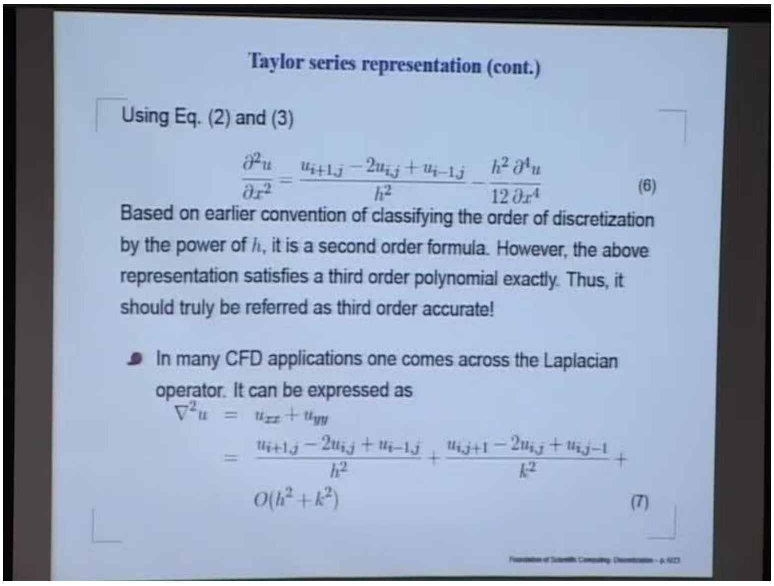

The problem comes when you start looking at the second derivative, you see the second derivative expression could be written like this. Now, this is where you notice that the lead term that has been dropped out is proportional to h square whereas the derivative term is related to a fourth derivative. So, if I want to call it in a classical way, this is a second order accurate formula, but if I want to refer it to in terms of polynomial, it is basically a third order accurate formula. So, that is the point we are trying to make.

Now, in many applications it is not necessarily that you will have to do fluid dynamics, anywhere you go, you would be coming across this Laplacian operator very often, and in the way that we have developed here, we would be writing this and we have already discussed it. Now, going back to this page, we can define the first derivative by any of these three representations, which one should we take that is the question; so, we must have some guideline.

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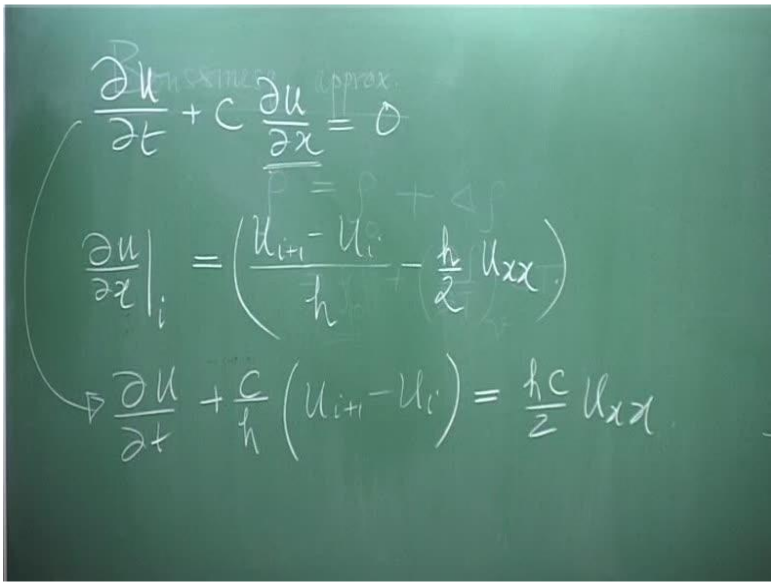

What should be the guideline that of course, comes from the job at hand, say for example, let us go back to our simple equation which we would adopt and keep using it quite often. So, if I am trying to obtain this term del u del x, so my question is, which one should I use out of this three? That is the question.

It is a very legitimate question to ask, if I were to use which one should I use? The one quick answer would be - choose the one which gives you more accuracy; so, you would like to choose the last one and that would happen to be a good answer, but it would still not reveal what we are looking for.

For example, if I adopt the first step, if I adopt the first representation, then what happens? See the term that we are…, suppose if I write it like this, so if I am writing it like this, at the ith node like this, what was the truncation error term? It is here right, so we have this as the truncation error term, right, so I will write this as minus h by 2 u x x, right. So, now, if I substitute it there, then I will be writing it like this; so, I will write c by h and I can post it on the right hand side and then I will get h c by 2 u x x.

So, adopting the first expression is equivalent to numerically representing the first derivative with respect to x by this stencil. We will write out a similar stencil for the time derivative, but then that would be equivalent to putting a term like this on the right hand side and what does it do? It dissipates the solution, right; the even derivative, the even derivative will always give you attenuation, right.

So, what happens? If I adopt that solution, the top one right, here that would be misleading because that would give us a solution which will be damping with time, right.

As I keep on…, but what we know from the understanding of the solution, what do we know? That this solution should not attenuate, whatever the initial condition I should give, that same solution would just simply translate by speed c. But if I adopt this, what I am going to see is well, it may actually go at the correct c, we will see that, that is also not correct, will come to that very shortly.

Above that we will see that amplitude of the disturbance keep attenuate with time; so that is the story of most of your commercial software.

Yes.

Sir, I did not get the point.

You did not get the point. The point is - this is your dissipative term, so what does it do? It dissipates; it reduces the amplitude of the solution.

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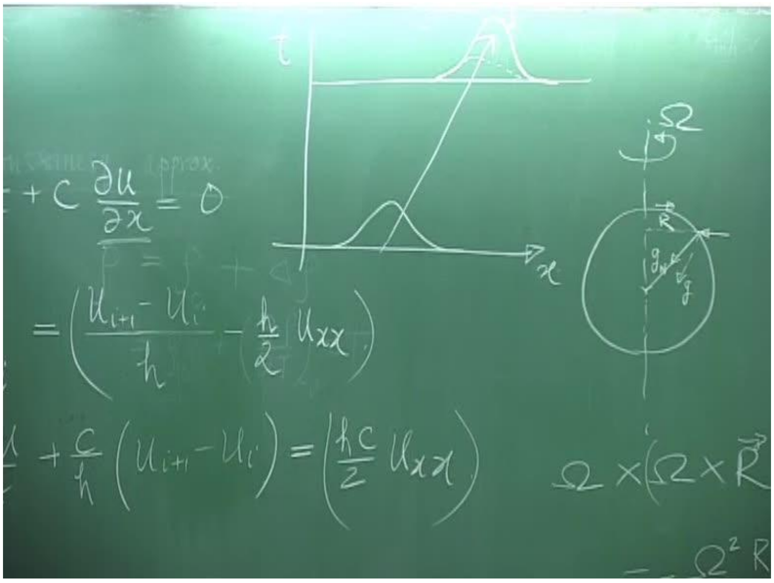

So, if I try to plot the solution in the x-t plane and let us say, I give you a solution like this at p equal to 0, at a later time, what do I expect to see? That this should go to the right at a speed c - that is what this equation tells you, right. So, I should actually get a solution which should be just that; so, there is no attenuation, there is no dispersion, whatever I have given, the same solution is bodily moving to the right. However, adopting this stencil would be equivalent to…, even though there is no dispersion because of this term, this will attenuate.

So, there are two things - one is, we are making the observation that there is no dispersion. We will see, almost every numerical method will have some dispersion; you cannot avoid it, O.K. We call that is uncertainty principle of computing, I will discuss it. However, if we remove dispersion effect, the presence of this term would attenuate that solution and you do it for some time, your signal is lost.

Remember that quotation I wrote from T.S. Eliot, I said you lose your information in noise; so, this is your numerical noise and you would be very sorry to hear that most of commercial codes actually do this kind of dirty trick. They are filled with numerical dissipation, so that the numerical instabilities are taken care of, but you lose useful information.

That is one of the reason I asked you not to use any package, you learn whatever you want to learn from the first principle and you would know what you are doing.

Now this is one aspect. Suppose, I would have taken this equation then what would have happened? If I would have taken the middle alternative here, what is called here as backward difference.

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Same

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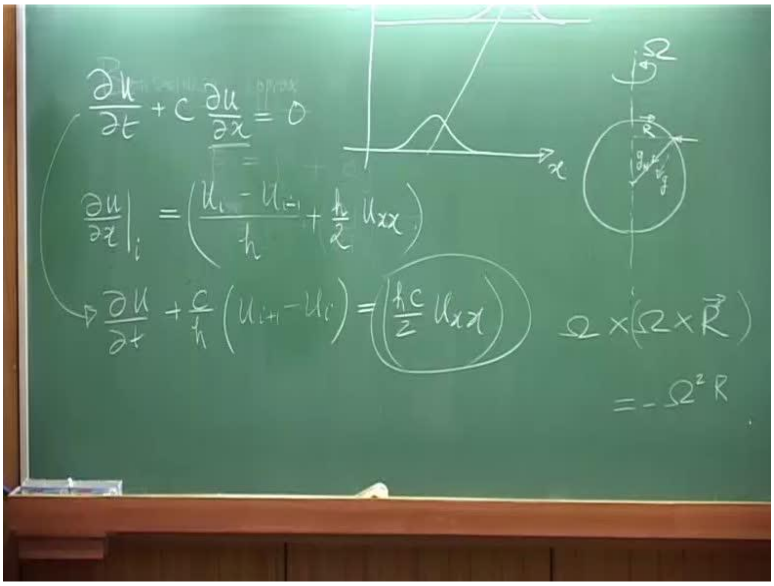

Now let us see, what I have here is like this. Now, it looks like almost same except the fact that it comes with a negative sign. What does it do physically? What will it do?

(( ))

Yes, instead of attenuating, it will amplify and you would have disaster in your hand, right.

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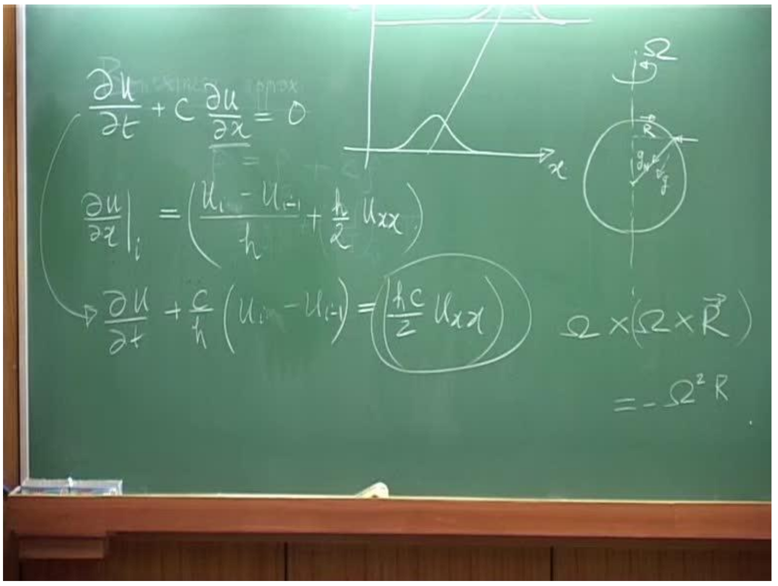

So you see, you, even for simple equation like this, you need to know how to choose your strategy. Now, you can see that this is what I will call as, because of this minus sign, I will call it as anti-diffusion. Many numerical methods are available and they actually give rise to this kind of problems, so you need to have the ability, unique ability, to work out what you are getting. Whatever method you adopt, you should be able to analyze it and one of the purposes of this present course is basically to give you that advantage of knowing how to do this.

Now, let us look at what happens with this solution? If I do that, I will write it like this and what term do I get on this side? What do we get?

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I am also forgot. Where did I, did I write somewhere? No, I did not write anywhere, but you can help me.

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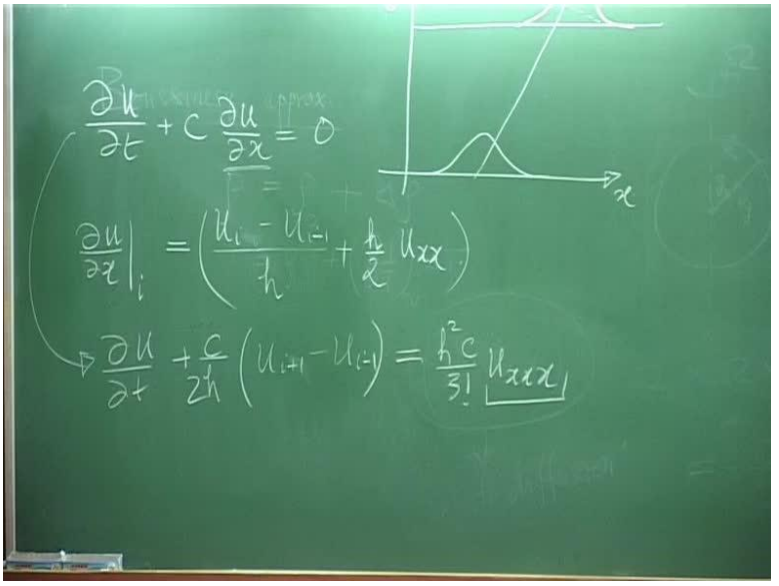

So, we will get what? Subtracting, so basically this, so I will get h square by 3 factorial, so what would that do? (( )) That would give you dispersion; you see, this is an odd added derivative. So, even though you are not attenuating the solution or amplifying the solution, you can actually get some things which you do not want.

So, if I have a compact solitary wave like this, I am trying to compute with this methodology. If I try to compute it over a long time, even though there is no attenuation, I would see there would be dispersion. So, you are now getting little bit of flavor of computing. As I always say jokingly, that computing is like a poor man's blanket, in you try to cover your head the leg comes out and vice versa.

So, you got to know what you need to cover at what point in time, so that is a very very important issue about computing. You cannot say, I will give you a method which will do everything for you. If anybody tells you, be very suspicious that there is no such things, there are no free lunches anywhere, you would not get it.

So, now, basically I just wanted to make you aware of what are the ways we actually adopt methods. So, for example, I told you that even trying to solve this equation we have to be extremely cautious in what we are doing.

We cannot just simply say, like what people do most of the time computing? It is like a cook book recipe, you just pick up this method from there, that method from there - put them together and compute, sometime it works and sometime it does not; so, we should not do it.

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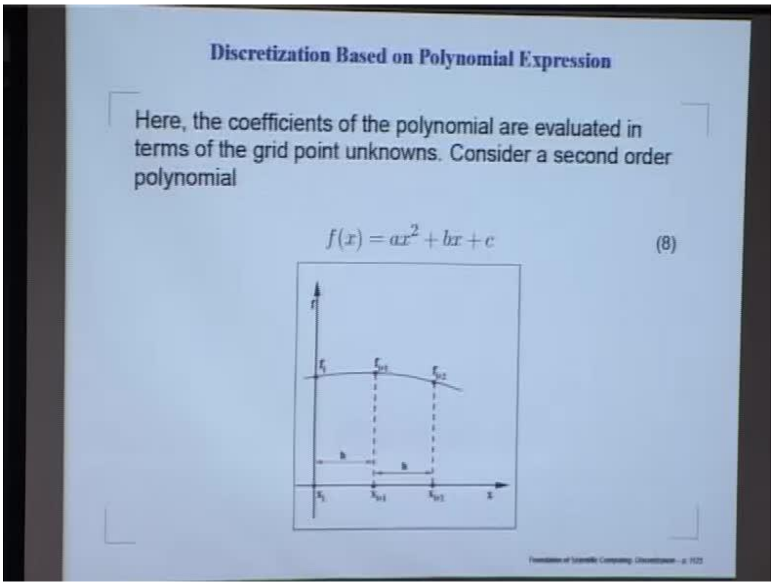

There is alternative ways of discretization, it is something which we may like to do. For example, the methodologies we talked about needed the existence of a grid point around the point that you are locating it.

Sometimes, it may so happen that you want the derivative, let us say at a terminal part like, what we have called here at x1, sorry, x i and the value is f i. So, if I want to calculate the derivative there, the first derivative or the second derivative, I do not have nodes on the left. So, what I could do is, I could try to fit a polynomial and then try to fix the coefficients of this polynomial by the function values at the neighboring nodes, but here the trick is we are doing one sided manner. I am taking all the information from one, the side of the point in question.

So, that is something like what we have talked about - the forward or backward different there, we have done it for in a one sided manner, either take it from the right or take it from the left.

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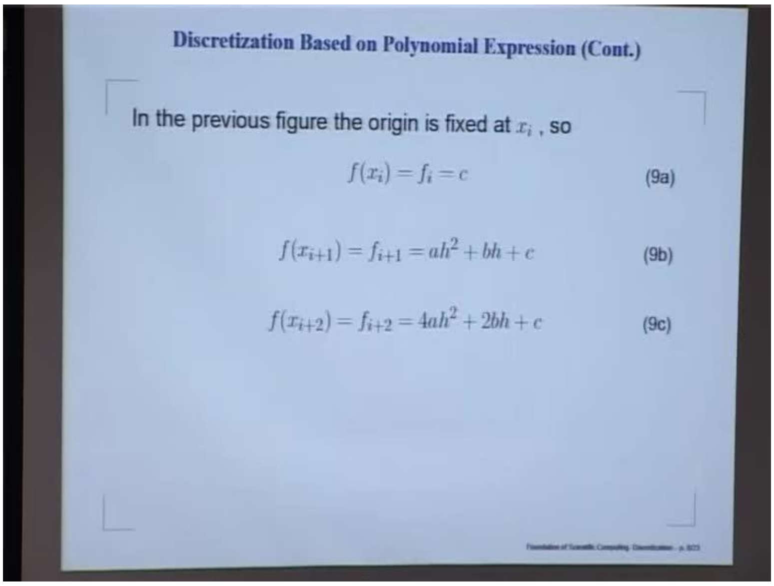

So, suppose, I fit in a quadratic here and of course you need three points to get this three coefficient a b c; that is what you do, satisfy this polynomial at three points - f i, f i plus 1 and f i plus 2 and solve for a b c and this is what you get.

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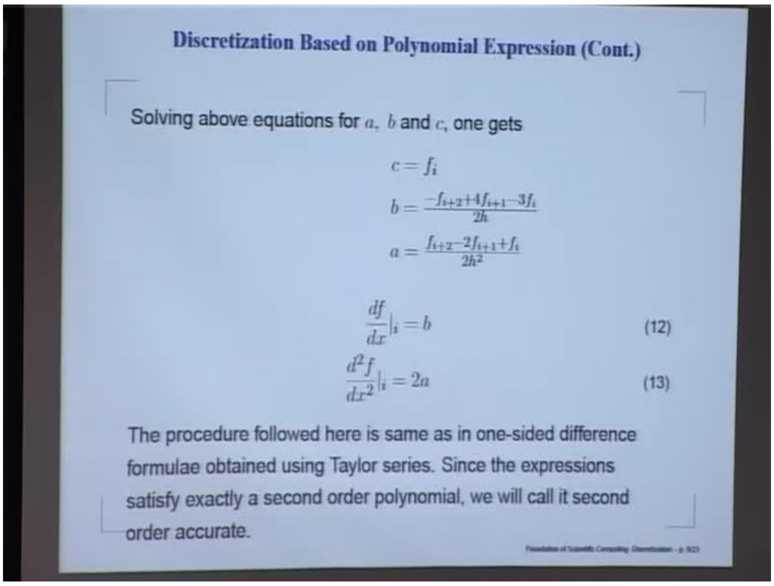

Now, it is very easy for you to understand that if you are trying to find out the derivative at x equal to 0 because we have written a x square plus b x plus c. So, the first derivative is simply b because we are evaluating at x equal to 0, the second derivative at x equals to 0 will be 2a; so, we can actually make use of this.

So, I think, I will stop here. I will finish this in the following lecture and then we will go to the usual way of solving differential equations for parabolic elliptic equations and so on so forth.