Aero Elasticity
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Lecture – 9

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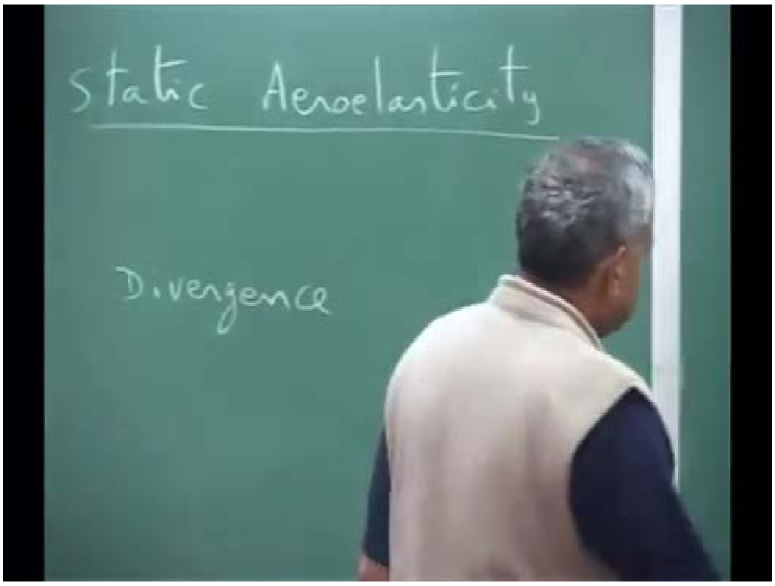

We will again start the static Aero Elasticity, from the basics very simple problem. We will consider the divergence of a 2 d section.

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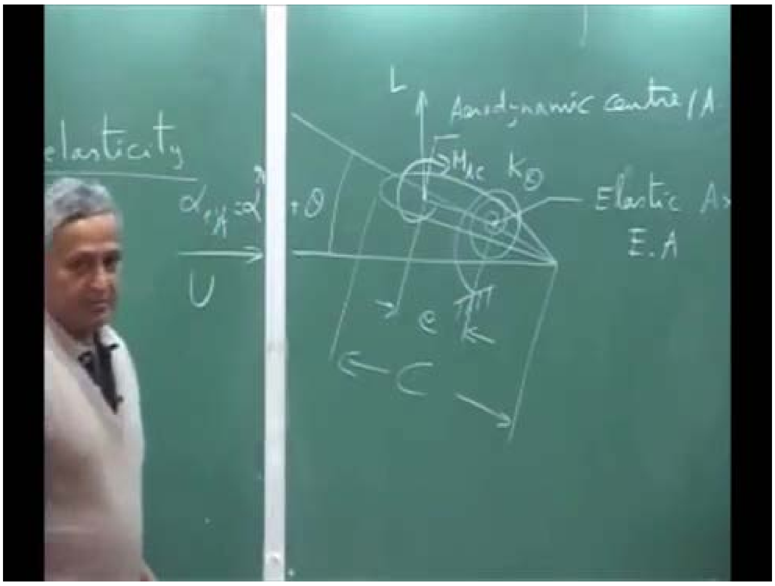

And we will consider an aerofoil, whose the fractional stiffness of the wing is idealized as a torsional spring, with the spring constant k theta. And the this spring is at the location, which is the elastic axis, I will write elastic of the wing, because this is the 2 d model. And then we have the aerodynamic centre, we call it this is E A this is A C and the wing chord take it as C and then the offset between the aerodynamic centre and the elastic axis, aerodynamic centre, elastic axis that, you call it as e and if the aerodynamic centre is ahead of elastic axis is positive, so that is a notation.

And then we will take this is at a angle of attack, which is the un coming flow is U, the angle of attack we call it alpha effective is some alpha rigid plus theta. What I mean by alpha rigid plus theta is, alpha rigid is the initial angle of attack which you have set the 2 d section, but due to flow, due to elastic deformation, the angle of attack has changed through an angle theta. So, the net angle of attack is alpha plus theta, now what you do is you write down the equilibrium in torsion, so for that your aerodynamic load acting on the aerofoil is the 25 percent pad you take, this is my lift. And take the moment like this, this is MAC, so I am taking counter clock wise movement last.

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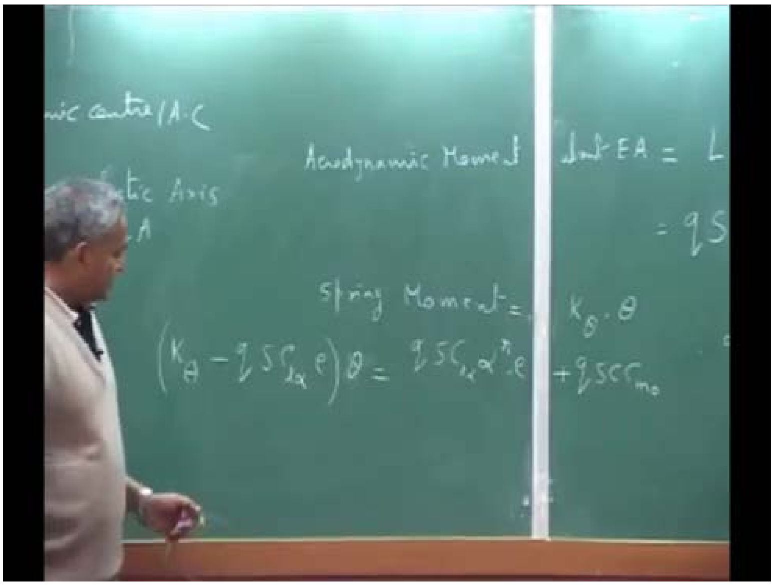

Now, you will have your aerodynamic moment, I will call it aerodynamic moment about elastic axis about E A, now the expression for lift in subsonic, we will take it as all subsonic incompressible flow. So, we can write it as this is dynamic pressure into area, which I take C into some depth, instead you can always call it as C into the depth as the area. And then C l alpha C lift coefficient, but we will write the lift coefficient as C l alpha times angle of attack, which is the effective angle of attack, which is alpha r plus theta into e.

This is the lift and this is the offset distance plus aerodynamic moment you will have q again area and one more C will come, because of the units and then you will have a C m 0. And the very fact that you define aerodynamic moment is, it is independent of the lift portion that means, it is independent of the angle of attack. Now, this moment is counter clock wise acting about this point ((Refer Time: 06:00)), now what you do is, you have to because of this moment this spring is deforming, the spring will give a counter moment.

The spring moment will be or you can say the spring moment, this is essentially or you call it as elastic moment, ((Refer Time: 06:32)) this is aerodynamic moment, this is elastic moment this is nothing but k theta into theta this is the spring moment. That means, since equilibrium they must be equal, because the as per the diagram lift and moment will give you clockwise moment, this will give you counter clock wise moment. So, when you add them actually you put a minus sign or in other words, you say that net satisfying the torsional equilibrium that is the key, because you are saying that my wing is in equilibrium, static equilibrium. You equate both of them, when you equate both of them you will have q S C l alpha into alpha r plus zeta into e plus S C C m naught equals k theta theta.

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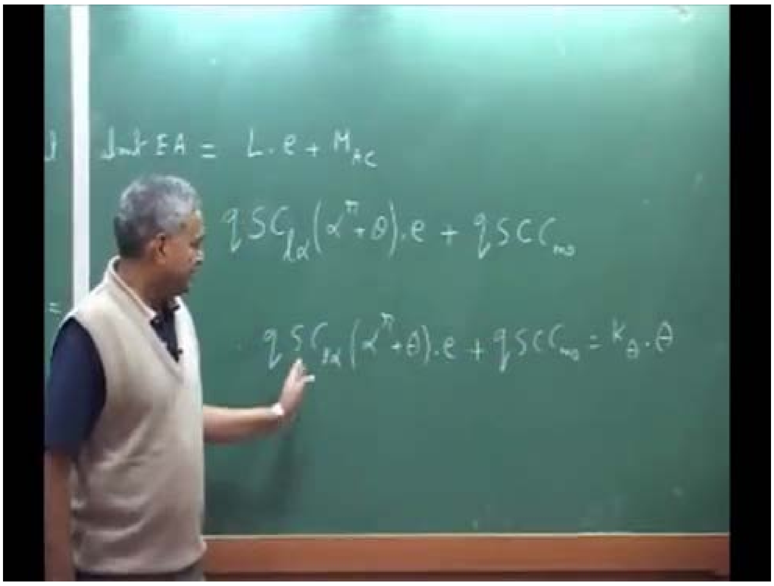

Now, what you do is, you take the theta term to the right hand side, and write it like this, you will have ((Refer Time: 08:04)) k theta minus q S C l alpha e into theta equals you will have q S C l alpha alpha r into your e plus q S C C m naught. Now, basically I want to solve for what is the elastic deformation, now if I solve this, I will get my elastic deformation as this expression I erase this.

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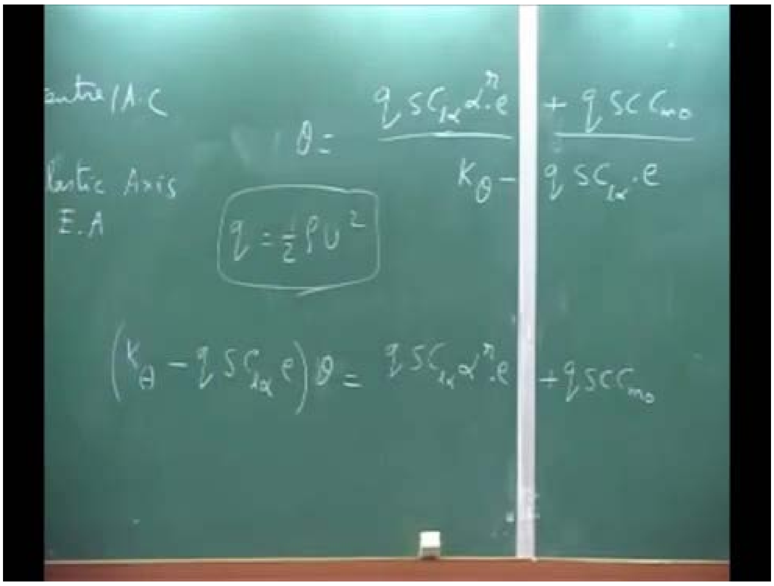

So, you will get theta equals q S C l alpha alpha r into e plus q S C C m naught over k theta minus q S C l alpha into e, now we have to just analyze only this ((Refer Time: 09:41)). When you look at a what are the variables and what are constant, the only variable is the q, which is your dynamic pressure which you define as half rho U square. Whereas, all the other part it is fixed, because your area there is a characteristics of the aerofoil, and your rigid angle of attack this offset, C m naught everything is fix, e is fix.

Now, as you increase your q, then what will happen this, this term is positive, so as you increase your q slowly this term is increasing and then at one point it can become equal to that, when that becomes equal your theta is infinity, which means a condition theta is infinity means my wing is completely fixed.

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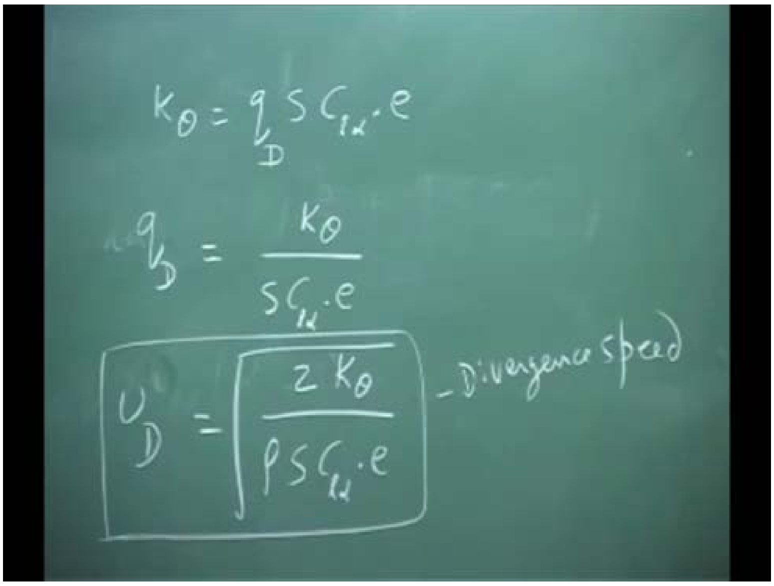

Now, the condition when k theta equals q S C l alpha into e or in other words, you write this is the q D, q D is the diversion dynamic pressure at which this very good, you will have k theta over S C l alpha into e. Or you can write U D square is half rho, so you will get 2 k theta over rho S C l alpha, this is the diversion speed you can take the under root, if you want you can define U D will get under root, this is my divergence speed. Now, you see the diversion speed is the function of course, the passional stiffness density of air just as you go at a, and area this is C l alpha, the e is another the offset between elastic axis and aerodynamic.

Aerodynamic centre sub-sonic if you say, it is the quarter chord, now elastic axis is the ((Refer Time: 12:54)), now the question is you have to play the elastic axis properly, in your aerofoil design, given their aerofoil section. How do you design my structure such that, the elastic axis comes close, because e is small what will happen, that diversion speed is increases. But, it e is negative that means, elastic axis is ahead of the aerodynamic centre, then diversion speed is no imaginary that means, the wing cannot divert, there is no diversion speed.

But, in normal wing design, it is not possible to put the elastic axis ahead of the aerodynamic centre, usually you will find that may be someone around 40 percent. Elastic axis will be around 40 percent of the wing, but in the case of the road triplet it is structurally design to bring this here, right at 25 percent curve, in the road triplet design.

So, there you have to do a lot of interaction to see how where I put the structural property, so this is in aerofoil I want to this ship do here. Now, this is where you should have studied the structural one, how do I identify the location of the land axis number 1.

Number 2 is by change doing what structural modification, I can shift them and these are all involved structure, you can have 2 cell, 3 cell box wing, you have to know that. And it now you know the diversion speed, what we have done is essentially we assumed first, because there is an elastic deformation, and that angle of attack exchange. Therefore, my lift force is changing and the other hand, usually what people will do is, if I do not consider the elastic deformation in the estimation of my aero load, that is like the region aerofoil.

If I do not consider, then what will happen my ((Refer Time: 16:01)) alpha, this is like in wind tunnel test, wind tunnel test what you do, you put it as different angle of attack of regional model, and measures the lift the moment etcetera drag, here we are neglecting the drag portion basically. So, at that condition the moment equation, that is what we have doing is, we are neglecting the elastic deformation in the estimation of aero load.

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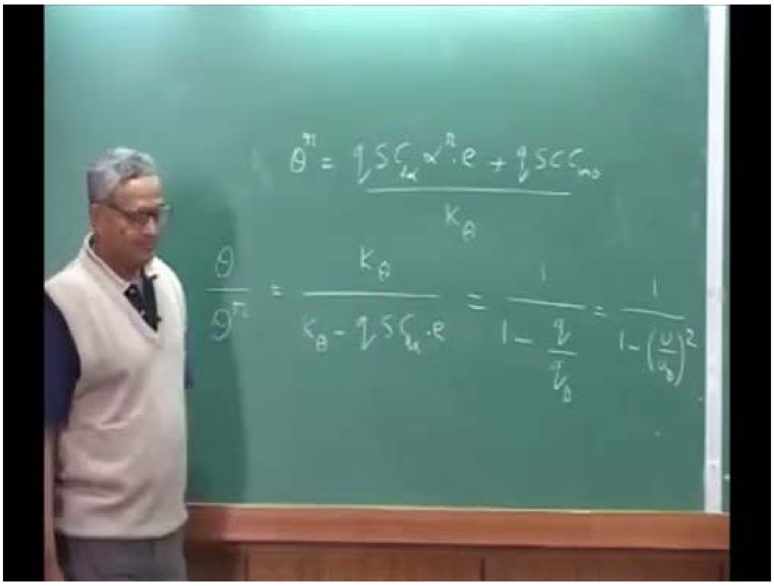

If we neglect that our moment equation will be k theta theta equal basically you are q S C l alpha alpha r into e plus, you will have q S C m right that means, I say that this is with an assumption, that my aerofoil is rigid. So, I write to symbol r just for differentiate between that theta and this theta, now if I calculate what is the error between that theta which is defined there, and the theta which I am writing it here. Here, this theta are will become just simply divided by k theta, this if I calculate theta common expression, this is going to be what will be the expression, just write that.

That will be k theta and this writing theta over theta r will become over q S e, which I can write it as q is any dynamic pressure, but I have defined my q D as k theta over S C l alpha e. So, I am going to replace, I divided by k theta everywhere, so it will be 1 over one minus q D or in other words, it is 1 over 1 minus U over U diversion square. ((Refer Time: 19:06)) I erase this drawing, now you have a relation like this, theta which is evaluated, taking into consideration that elastic deformation in the aerodynamic loads.

Theta r is I do not care, I just say this is my angle of attack, initially I said this is the moment and then I get it as though it is rigid. If I plot this as the function of U over U D that means, dynamic pressure with has as the ratio with respect to the diversion.

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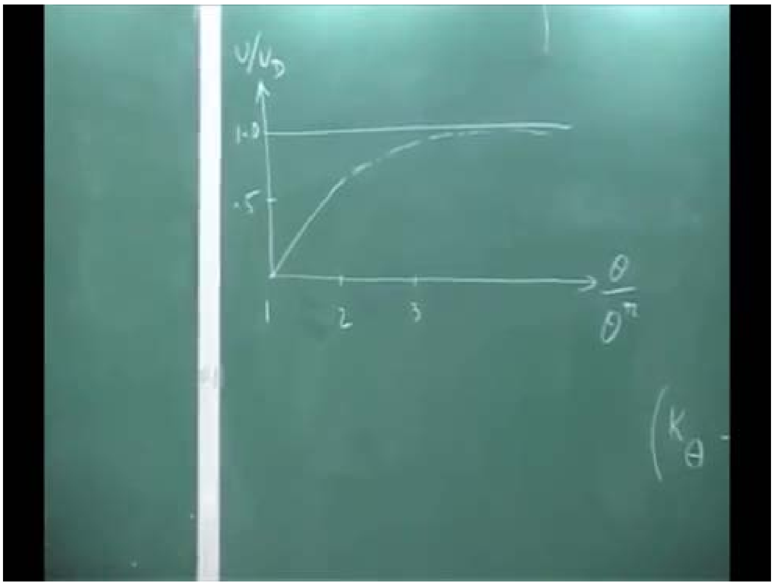

So, I will have if I draw this diagram, because this is a very interesting diagram just to see y take, because U over U D maximum 1, because after that at 1 is wing a strait and here you take, when U over U D is actually U is very small when is 0. You will find that, you start with 1, maybe 2, maybe 3 etcetera you go, now if you plot this ((Refer Time: 20:46)) it will be like this, here I will take it as 0.5 this is 1.0. That means, when this velocity is very small, I have to get the diagram something like this and then it will go maybe as...

What you will find is even when U over U D please understand, if it is half the divergence speed, then ((Refer Time: 21:35)) this is 0.5, 0.5 square is 0.25, 1 minus 0.25 is 0.75 that is 3 over 4, it will become 4 over 3. 1.33 times you will have the rigid that means, about 33 percent more than what you predict with this approximation, whereas if it becomes this is 50 percent, if it is 60, 70 as you go higher you will find that this is going to become little high.

That means, the error you make is going to be very large, why it is necessary with you need to take this, it is not to calculate your diversion speed. Because, structurally you are twisting it, more than what you predict by this circumstance which means, the stresses which you calculate using this not be correct ,you need to do a aero elastic analysis to really predict what is the correct value of the stress. Then only you can design your structure, that is why in the aero craft structural analysis, initially in the preliminary design, you will not be bothered about elastic deformation and other things, you will make you are design.

But, after that you have to do and aero elastic calculation, see whether the stresses are within the limit, otherwise your predict in even half the diversion speed, you have 30 percent increase in your twist, that is at the half. If it is goes more, then you may be 2, 3, 4 times it may defend going up, then as the result you find that this is very essential for you to take into account aero elastic deformation. Or you can say aero elastic it is as the topic is very important in the design of your aerospace part, that is the key.

Otherwise, we do not have a bother about that, that is why we say the wing is flexible, it is only when the structure is flexible the study of aero elastic stave, is the very dominant role. Today in the design the called it multidisciplinary optimization, in which aero elastic constrained have brought in the right at the design face, preliminary design face itself, then only you are structure can be optimally design. So, this is as far as the simple diversion is concern, next we do the aileron reversal I gain very simple approach, once you understand that what do we mean by the diversion and aileron reversal or control effectiveness, then you can proceed further with your actual three D.

We will take the wing this is only 2 D model, next we will take the aileron reversal, using a similar 2 D model only, is it clear; that I want to you understand that, this is the very simple problem, I keep this part here ((Refer Time: 25:37)), because I may need it.

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We take aileron reversal, as usual we take the 2 D aerofoil, but with the control surface and here, you have the oncoming velocity U alpha effective is alpha r plus theta. And you are lift is aerodynamic centre, you have a lift and you have a MAC, and this distance is as usual we have the e, and this is my you can take it this is my chord. Please understand initially that arise 0, now this is the control surface, you say that the aerofoil is kept at a angle of attack alpha r, and with a velocity U.

And U is small you can take it small, but small in size is small what it is not the diversion speed, please understand it is flying at a speed much lower than you divergences, that is essential. Now, what you do with the control surface, controls surface essentially you operate to change your lift, if you are flying at the same speed, if you want to turn what you do, you fly your aileron and you change your aerodynamic lift. So, please understand in this case, it is only the change you are looking at not the actual lift that is the essential, you are looking at the change due to beta.

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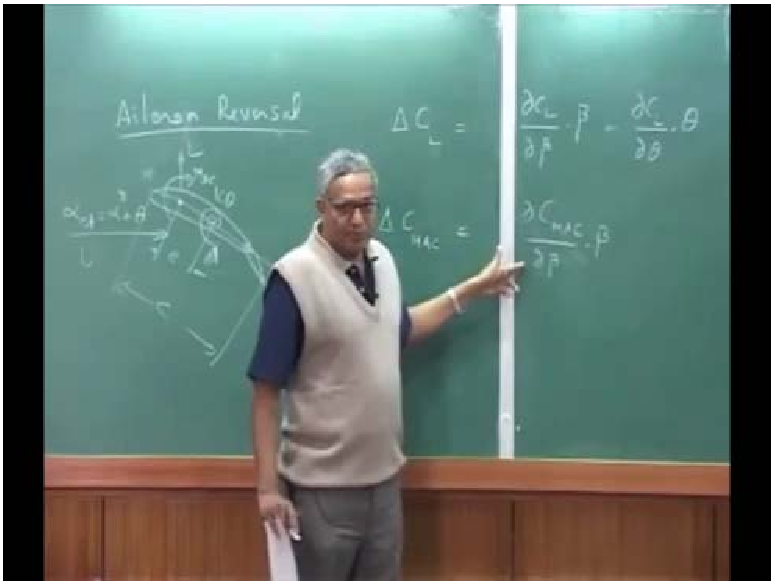

So, I am going to write my delta C L that means, please understand this is the change in my lift coefficient, I am putting it delta C L, if your multiply by dynamic pressure area etcetera, you will get the change in lift itself. This is I am writing it has delta C L over delta beta beta that is number 1, but because of the change in lift, there is a change in the theta also, because theta means a angle of attack. Because, of the change is there is a twist, so I am going to write is this as plus into theta and then you will have your change in moment also delta C M, this is you will have delta C MAC over delta beta into beta only, I do not put theta.

Because, my moment about the aerodynamic centre is basically independent of the angle of attack, that is why I do not put theta here please understand beta is change in chamber, theta is change in angle of attack. Now, that this is delta C L over delta beta is change in lift due to unit deflection in ((Refer Time: 30:54)) this is change in due to unit change in the angle of, this is C L alpha basically. And this is change in moment due to a unit deflection is beta, now what happened is again you have to balance this, because this is the in static equilibrium. So, you have to balance the change in moments, now it is not actual moment change, it is the changing moment aerodynamic moment.

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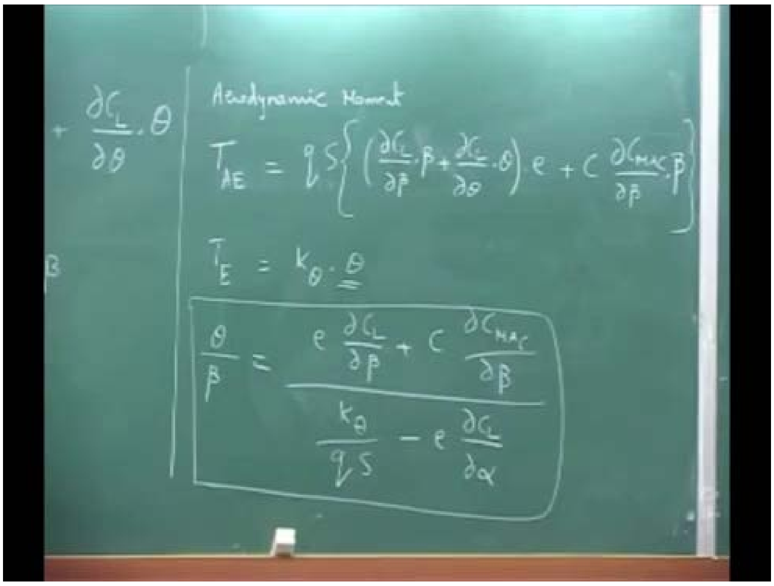

Aerodynamic moment is basically you can called it aerodynamic star q dynamic pressure into your lift coefficient, that will be basically delta C L over delta beta time beta plus chord delta C L over delta theta theta into half set plus chord delta C MAC over delta beta times beta, this is the aerodynamic moment. And your elastic moment as usual this is going to be T elastic is k theta into theta only, and this two are equal for equilibrium, T elastic.

Now, you equate both and then you collect the terms of theta on one side, and leave the beta terms on the other side, if you do that what will happen is I will write that expression, you will get directly I am putting it, theta over beta equals e delta C L over delta beta plus C delta C MAC over delta beta divided by k theta over q S minus e delta C L over delta r, you check it. What we have done is, we have just taken, the change in lift and the change in moment due to control in the controls surface.

And then static equilibrium aerodynamic moment we calculated about the elastic axis, please understand this is about elastic axis and this is anyway k theta into theta, you equate both of them. And you got a now relation between the angle of attack change, please understand to a deflation of the control surface, so if I know the control surface deflection, I know what is my angle of attack change theta. That means, I have an expression now theta in terms of beta, delta C L is my change in left due to a deflection is beta.

Now, I can go and substitute theta, in terms of beta in ((Refer Time: 35:27)) this expression, if I do that I will have change in lift coefficient, due to a deflection in only beta. So, if I do that I can erase this part and write the so please understand, substituting for theta in terms of beta in this expression, and I will get like this.

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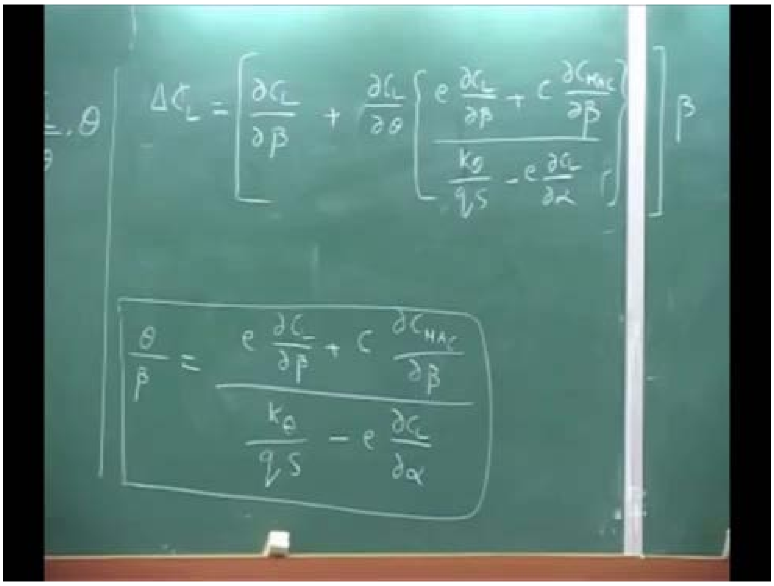

Equals delta C L over delta beta, that is this term plus you will have into this entire expression e delta C L over delta beta plus C C MAC over delta divided by q S, 1 minute this is a q S into there is one q s only, q s into minus e delta C L over delta. And this entire expression is multiplied to beta that means, now I have an expression which says, if I have a deflection in the controls surface beta, what is my change in lift. And what I have to do is essentially is, here I have to do some rearrangement jugglery is basically some algebra, or algebra I erase this part ((Refer Time: 37:45)). Because, now this is what is the key thing which we have to analyze, you put them as a numerator and denominator etcetera, simplified this expression I erase this part.

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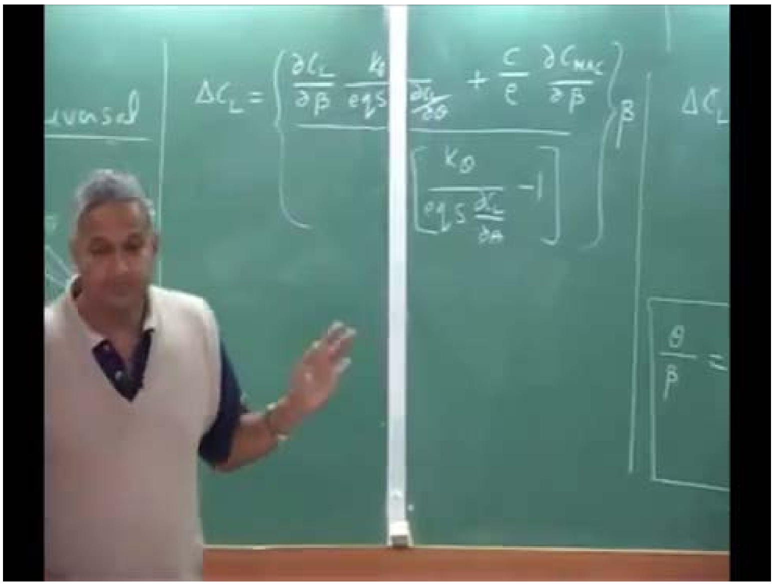

You will have your delta C L becomes I am simplifying this please understand, this is delta C L over delta beta k theta over, because you can simplify this e q S delta C L over delta theta. That delta theta or alpha, what is that this is alpha I am using, am I using alpha or theta, theta know I think I ((Refer Time: 38:52)) let me put theta only, so do not change to, I am sorry about the change in symbol. And then I will have what plus C over e delta C MAC over delta beta divided by k theta over e q S delta C L over delta theta beta.

Please understand, I have essentially taken, because you see delta C L over delta beta, if I take this term here into e that will cancel out with this, leaving behind only delta C L delta theta this ((Refer Time: 40:02)) come. And that delta C L by delta beta, I will actually eliminate here, in the denominator I have taken this term that is why that will come out and then put it in the denominator I am doing, so this is just an algebra you have to do, simplification I would request that you do it yourself.

Now, what we look at it is, suppose the numerator is 0 then what happens, in the numerator is 0 whatever I change beta, it is not going to change my lift here, but is it possible, that is a first question you will ask. You know that delta C L by delta beta yes it is, but moment coefficient is a negative quantity, this is positive k theta is your stiffness, this positive e positive, q is positive, s is positive, this is positive, this quantity is negative point, this is the aerofoil.

Now, as you keep increasing your q, what happened this term is decreasing, at one point you can make this equal to 0 and that is the control reversal speed. In other word you say no matter at the speed, whatever I do beta nothing is going to change my delta C L is 0, you may have lift please understand that is why lift is not 0, the change in lift is 0, so that particular condition.

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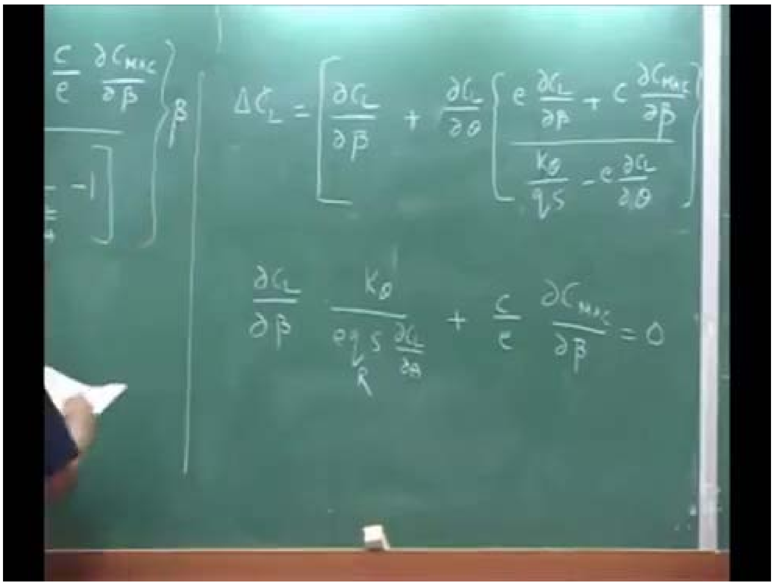

Now, I will keep this term as it is that we write here, so you will get delta C L over delta beta into k theta over e q S delta C L over delta theta plus C over e delta C MAC over delta beta equal 0. If you take q, this is condition when q reversal I am putting it, that other q you can have some value, now I simplify that is take it to the right hand side and get only q R.

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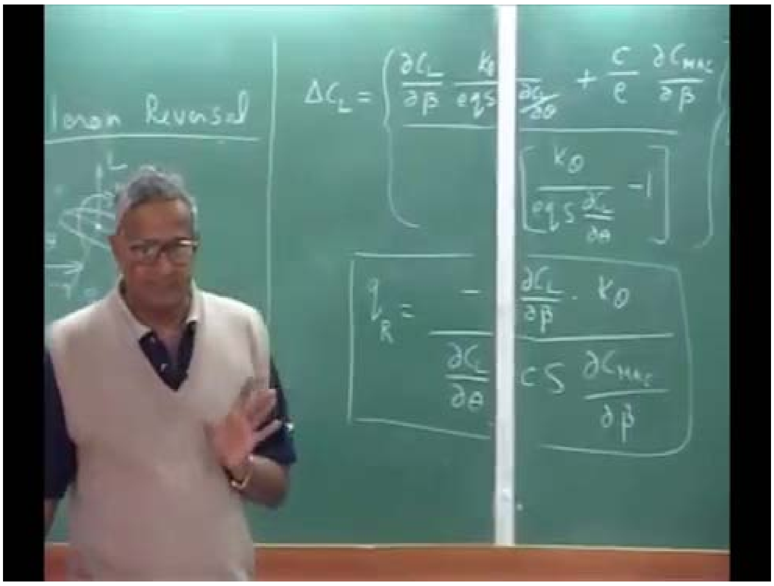

So, I will write the q R expression q reversal is minus delta C L over delta beta into k theta over, because what I have done is I took it that side, I multiplied by q and divided that here that is all. Delta C L over delta theta C S delta C MAC over delta, this is my q R. And please understand ((Refer Time: 44:27)) e cancelled out, so the interesting thing which complete out this is, the offset is no meaning, in the sense is does not matter. Your control reversal speed or aileron reversal speed is independent of the offset, between the elastic axis and the aerodynamic centre, ((Refer Time: 44:59)) write here that is why I wanted to put it in one place.

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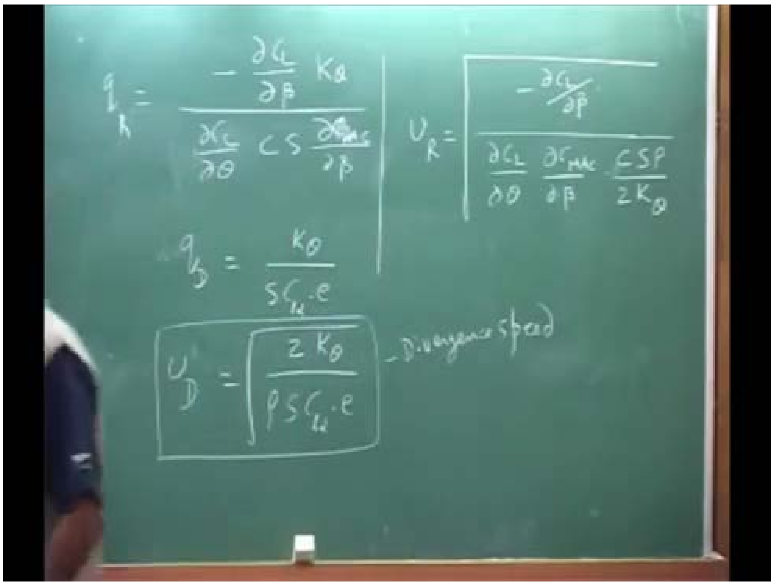

I can write q R as minus there is beta over delta C L over delta theta, theta is basically C L alpha, system is C L alpha, C S C MAC is over delta. And similarly, you can define you are U R this will be square root of minus, and I again simplifying these delta theta delta C MAC over beta into you will have C S, and I have to have rho over 2 k, C S rho 2 k theta. So, this is my control reversal or aileron reversal speed, so you have now which you have defined control reversal speed.

Now, suppose again this is a interesting algebra I will put it more and more algebra, rather than much of it is essentially it again, if I say that my aerofoil is rigid. Then what will I say, like into went analysis you make the entire unit of the wood and then only changed the control; the angle is not going to change, only thing is you are lift will change with respect beta only.

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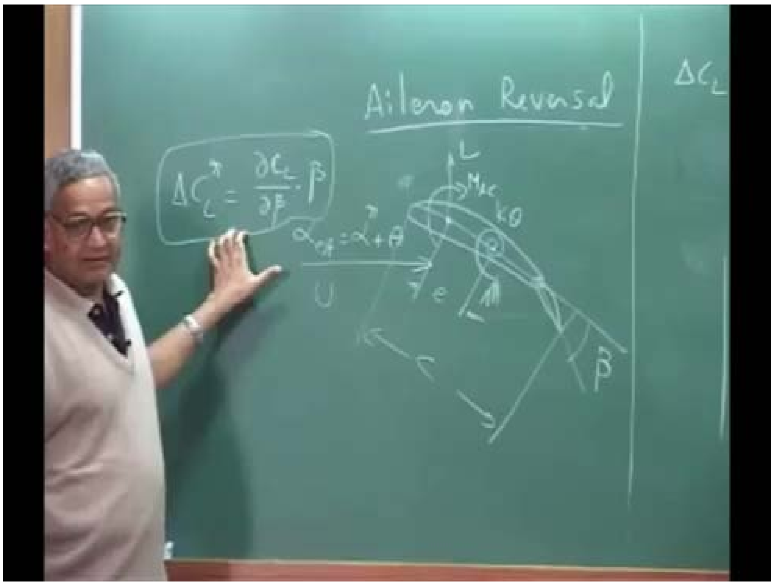

The second effect of that twist that happens, so you will be writing is like this if I say it is rigid you will write it only like this into beta. You will not use any other way, because of the change in lift, there is change in angle of attack and that change in angle of attack has also change in my lift. So, you see this is with respect to an actual wing, where there is a deformation, fractional deformation whereas, this is an having where I do not consider the torsional differential.

Now, if I take the ratio of delta C L over delta C r and you can again plot some nice curve, but that particular simplification, I will write the simplify expression, it is like an exercise which you do it yourself, to come up with that expression. I am not going to do the algebra part, please understand this part you do it yourself, but you have all these expression, so you must use it properly.

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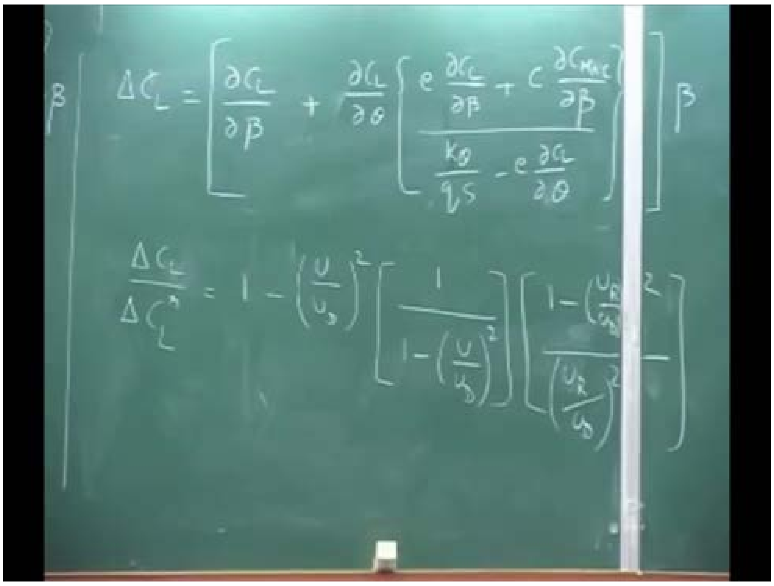

I am writing it as delta C L over delta C L r is in a very compact form one1 minus U over U diversion square 1 over 1 minus U over U diversion whole square 1 minus U reversal over U diversion square over U reversal over U diversion whole square. What I have done, I have ((Refer Time: 50:23)) this expression delta C L and I have delta C L r, what I have done is, I took the ratio and then I have formulated this, this is the very need expression.

But, this is required from algebra, I have that with me, but I would like to give it you as a exercise, one more homework you take it, try to get this expression from there. Now, let us analyze this particular diagram, not diagram this particular expression, which we have given here, erase this part ((Refer Time: 51:22)). So, I would request you to try this out completely, if we take a plot, because you see here U is any speed, U diversion is given there, U reversal is given there.

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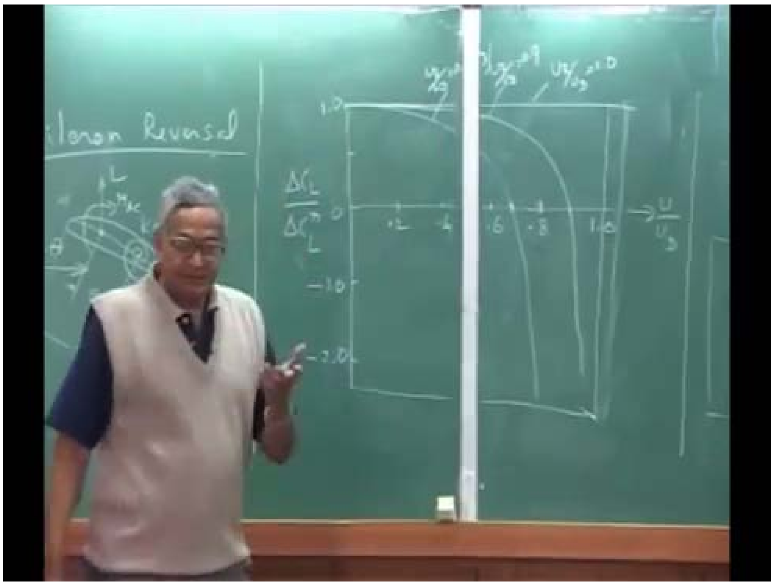

Now, I am going to make a plot of the carve, which is this is delta C L over delta C L r this is U over U diversion, this is the 1.0 minus 1.0 and minus 2.0 etcetera, you can have 0.2, 0.4, 0.8, 1.0. Now, you see when you reversal speed is equal to U diversion speed, then what will happened this may this what that means, U r over U D is 1.0, this is 1.0, so that goes up. That means, delta C L over delta C L r is, fine it is what that means, there is no difference between diversions, and the control reversal it really does not matter.

But, my delta C L is, which will be ((Refer Time: 53:50)), which is there I make a flexible wing approximately, my delta C L over delta C L r is always 1, provided I keep my reversal speed and diversion speed same. Suppose, reversal speed is 0.9 of U R over U D that will be something like this, I am taking it like this here 0.9, so I will draw the diagram it will go like this. This is U D 0.9, you see what I have done, when U R over U D is 0.9, then what will happen then U, because here is the U over U D when U over U D becomes 0.9, then automatically this and this they will be 1.

Because, ((Refer Time: 55:16)) this will cancel with that, this will cancel with that living behind 1, 1 minus 1 is 0, so I will get 0 here, in the delta C L over delta C L r I will get my reversal speed, that is what declined is, reversal speed is 0.9 of U D. So, the curve will come like this, similarly if you the 0.7 is here, the carve will come like this, this is U R over U D is 0.7. Now, the key think you have to consider is, what is my change in lift even if ((Refer Time: 56:15)), because you do not fly the reversal speed, you do not fly again diversion speed, you will always fly lower than that.

But, how much of effective control am getting, because delta C L over delta C L r I will get 1, I get 1 that is very good, but if I reversal speed is less than the diversion speed, I have to find even at, if I fly in this zone at a much lower speed than the diversion speed, and reversal speed, I am getting a less effectiveness of the aileron, because of the wing deformation. So, this region you are actually losing you are effectiveness, this is what effectiveness is, this is basically the reversal and control reversal, control effectiveness.

You may say I want my wing, the aircraft the role at these ray that means, it may require if you want initiate, if you want role acceleration, you want so much of to know the inertia of the aircraft, you want so much moment to be generator. So, you will say deflects my wing through some angle, but what will happen is when you deflects is, because of the deformation of the wing, what you get as the control moment it will be much less than what you really incendiary that means, your effectiveness is reduce. Now, you can see what is the condition you can have both U D and U R equal, that is the best if you can make reversal speed and divergence speed equal for your design, that is very good.

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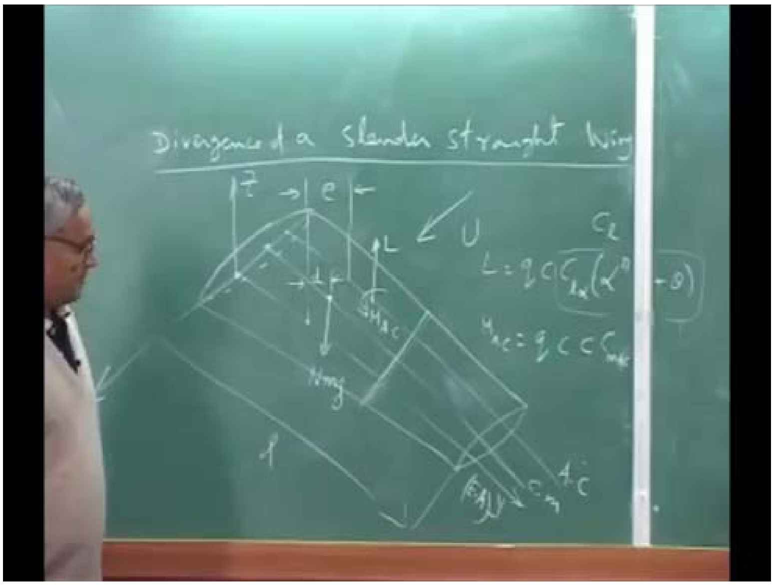

I would like to have it the this is at divergences of a slender straight wing, you can take it large aspect ratio wing, so we will now draw wing and this is my x axis, and this is my z axis, and I take my y axis like this. And I am going to now define couple of points, this is my please note elastic axis E A, my y axis is an elastic axis, this is my aerodynamic centre line, and there is also a mass centre line. Now, this is my centre of mach line and as usual we defined the half set between the elastic axis, and the aerodynamic centre that is by e, and this axis we called it d that is the mass centre, and the elastic axis.

Now, essentially this is my slender wing and making this assumption that my elastic axis is a straight line, straight line this is it can go, but it can be double no problem, stay for everything. But, they all fall, I am just saying it for each of writing it, now for this and this is in a wind speed, so the oncoming wind as the velocity U and please understand this is the straight wing. Therefore, you are lift is acting here and there is a moment acting, I am putting in this lift and moment, moment aboard aerodynamic centre.

But, this gravity is acting down, and if the wing is now flying in a load factor condition, you can take the load factor has N, which is given the level flying N is 1. If it is doing a memoir some other higher load factor, so you always take this as some load factor time m g, m is master unit wing. You are going to write the only traction equation for this long straight wing, the assumption you make couple of a assumption, one is this axis is perpendicular this axis, y axis is perpendicular to my... Root section and it is a straight line I am taking it like this, second one is incomparable flow and third is I am assuming strip theory.

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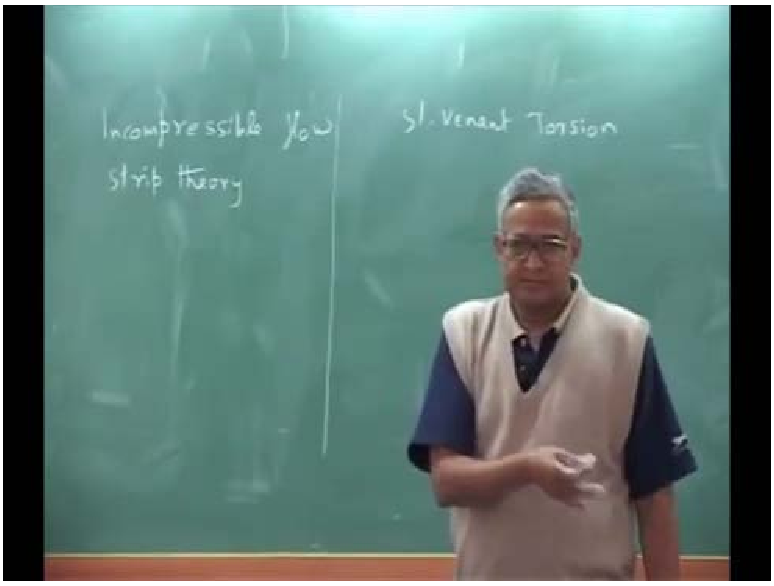

So, we will say incomparable, because I am using the simple incomparable flow and then I apply strip theory that means, every section I am taking it as the two-dimensional, I am not taking any 3 D effect and etcetera. And then the structure is the straight line there is no sweep back that is very important, that is why straight wing I have make assumption. Later we will do what you have to do for straight wing, and there is no other deformation in the plane of, because it can band, due to drag board it can do that.

But, we are not taking all those addiction to account, and thirdly is every chord wise segment you take this ((Refer Time: 1:04:36)) and this line you take it as L, every chord wise segment is bridged, in the sense it cannot bend or anything like that, it can only twist this is simple St venant tortion theory. So, you apply when an fraction and for this you know the equilibrium equation fraction, for which we developed in the first section.

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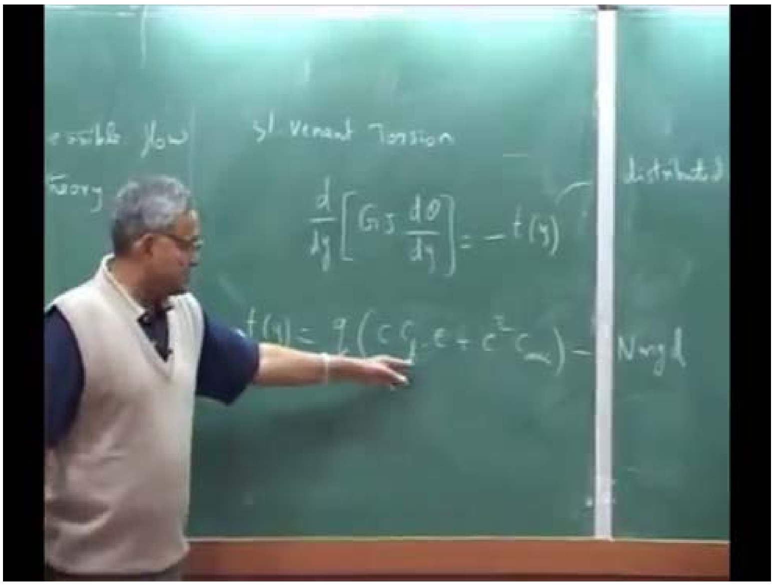

That equation becomes d y g j equal minus, I am putting please understand this is my distributed fraction moment, and this is a starting problem, so in the sense there is no vibration part, you do not know inertia, but the gravity is having this effect. Now, you have to wait what is my t y distributed torsional moment, you write it as a torsional moment about what, now you say I am writing this is about twisting about the elastic axis. This is the twist about elastic axis, therefore I should take moment all the moment about the elastic axis keeping d e as a axis.

Now, if you write will have q which is the dynamic pressure, C because this is per unit line q C C L into e plus you will have a C square C MAC minus N m g into d, because lift is q, this is lift more unit span now. That is why it is a not x, this is q C C L alpha times whatever you can put it alpha r plus theta you can put it like this, which is basically this is my C L, this term is C L. And then your moment is q C and you have one more C, that is why it is C square C m not or C MAC I will put it C m aerodynamic centre.

And of course, this is the load factor which will be minus sign, because this is due to the gravity, which will give a nose down, this is the nose up. Now, what you have to do is I have to go substitute this here, and I will now you are C L, because you say initially my aircraft wing is kept at a angle of attack alpha r. Please understand now, all these quantity you have to understand, e is this offset that e, d, chord all can be functions of y they need not be constant throughout, unless it is an assumption, this is a straight plan know, so that plan form wing. Whereas, here everything can be wearing supposed the aero foil itself is very C L alpha is wearing it is in the same aero foil throughout, but for the we will write the generally equation; but then when we solve we will simplify the problem and then solve it as the whereas, for the uniform being.

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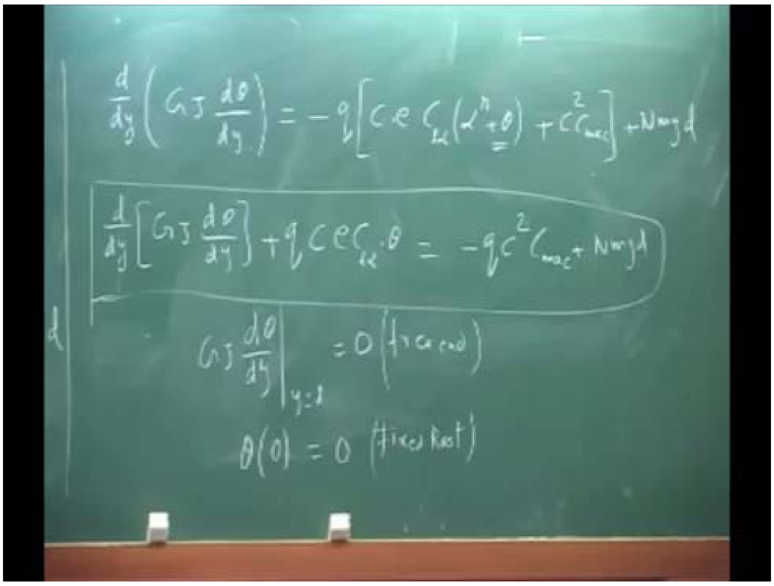

Now, let us the write the equation, the equation with the boundary condition because this is the deformation of the wing d theta over d y equals we have put it minus q c into e while C L is actually C L alpha when alpha r plus theta plus you have q is taken now. So, c square C MAC and minus and minus will become plus n m g d, but what you c is that the term is an the right side you take it to the left side.

And your equation will become d over d y of g j d theta over d y plus q c e c l alpha into theta equal minus q c square c m a c plus m a g d with the boundary condition you will write, because the free end free end is free, so I will have moment is zero. So, I will put the theta over d y are you can say g j at y equal to l is 0 that is the free end and then had the root it is fixed is attached to the fuse light I am saying the that is fix.

So, theta at zero is zero this is the fixed end fixated root now this is my equation what you have to do is you have to solve this equation. So, this a way that minus minus q c e c l alpha alpha, please note down at that term. So, you have that is my equation I need to solve the yes q c q c e c l alpha alpha, how do we solve that part we will do the next class.