Aero Elasticity
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Lecture - 7

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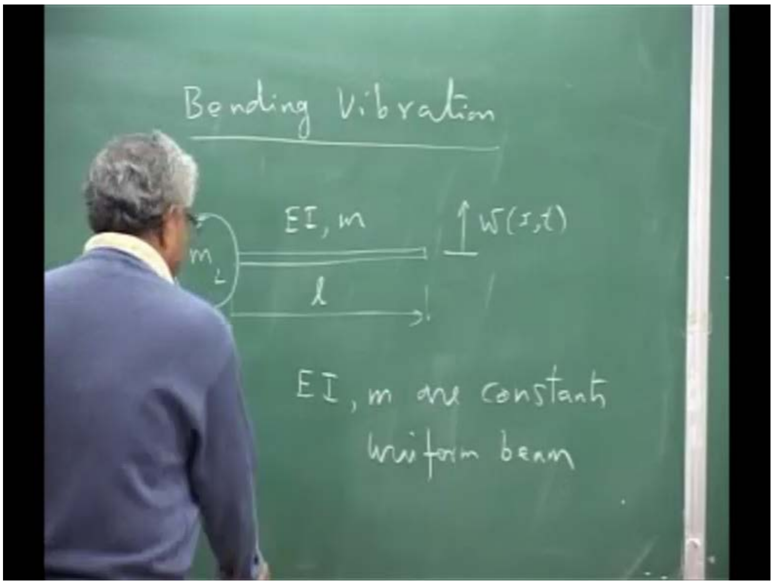

Today we will see the solution for you vibration problem. And these are exact solutions of a simplified problem, let us take this as the fuselage mass left side, and this is my wing whose length is l and E I and m are it is properties, and this is vibrating. And we have developed the equation of motion, and then we applied the separation of variable, and we got the differential Eigen value problem. So, I will write the differential Eigen value problem, along with the boundary conditions directly.

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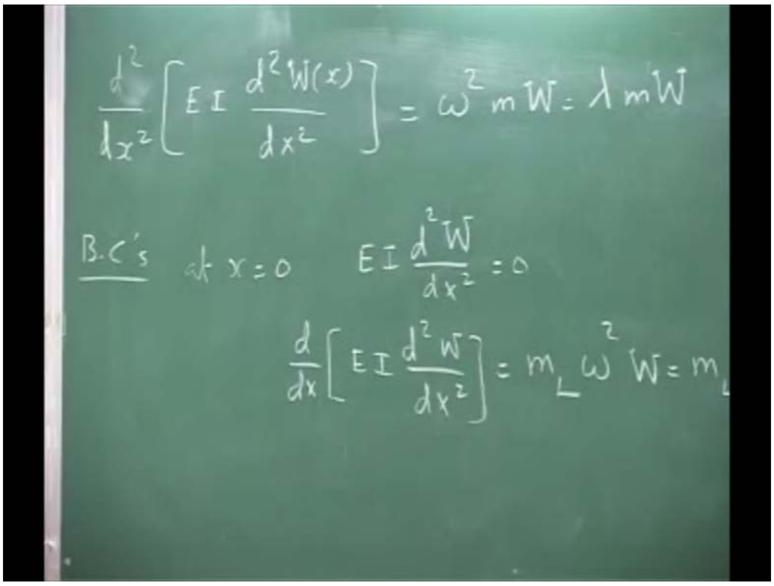

So, you will get d square over d x square E I d square W is a function of x over d x square equals omega square m W are which we wrote it as lambda m W, where m is the ((Refer Time: 01:52)) with the boundary conditions at x equals 0, we had E I d square W by d x square is 0. And then d over d x of E I d square W by d x square equals m L omega square W or you can write it as m L lambda.

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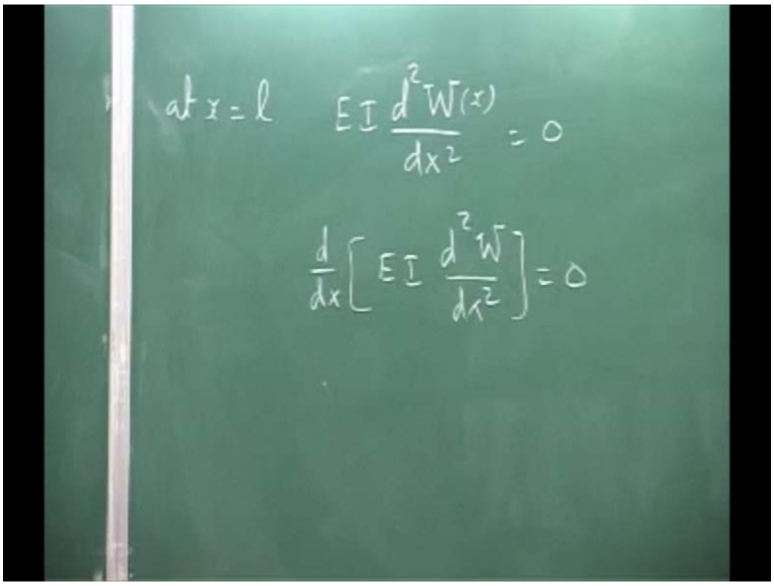

And at x equals l, we have the other two boundary conditions which are E I 0 and d over d x of E I. So, we have the four boundary condition, now how do we solve the problem let us make from approximation, the approximation is E I and m are constant, which means it is the uniform beam. When we write it as a uniform beam, then for this we can get the exact solution I erase this part.

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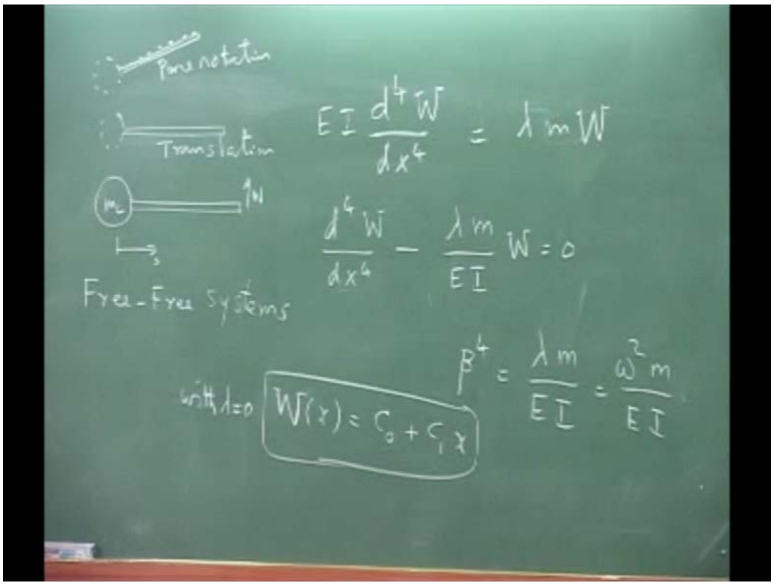

Your equation in the field it become E I d 4 W by d x 4 equals your lambda m W which you can write it as, d 4 W over d x 4 minus lambda m over E I. And this particular quantity, we are going to call this as a beta to the power 4 lambda m over E I, please understand lambda is the omega square. So, if you want you can keep that just for, now this is the constant, and this is the 4 th order ordinary differential equation.

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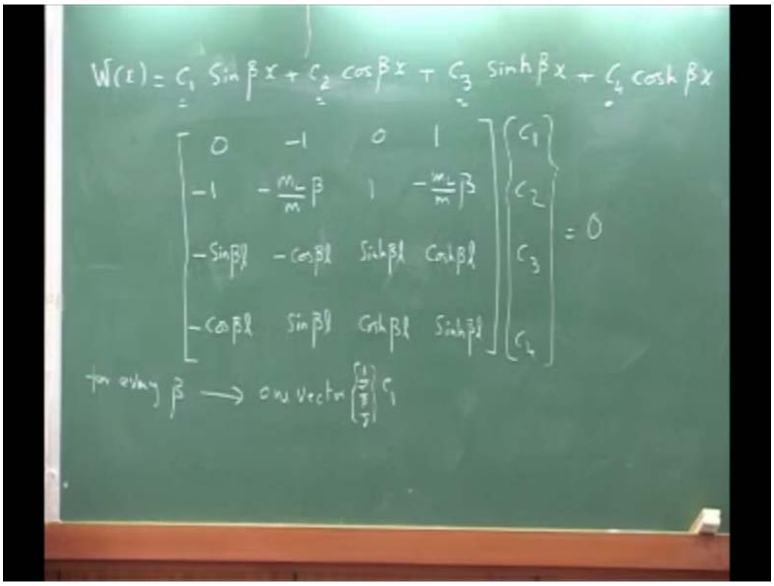

You can write the solution as W of x as c 1 sin beta x plus c 2 cosine beta x plus c 3 sin hyperbolic beta x plus c 4 cosine hyperbolic beta. Now, if you substitute these you will get this equation is exactly satisfied, now you need to find out these 4 constants c 1, c 2, c 3, c 4 those are you cannot obtain them uniquely, what you have to do is, you have to take this function, substitute in the 4 boundary condition. Then you will have 4 equations, if you bring this because W is there.

So, you bring it to the left hand side you will find that 4 simultaneous equations, which you can put them in a metrics form c 1, c 2, c 3, c 4 equals 0, I have to fill up this metrics that is all. And this will contain beta because I will finally write that because when you substitute because I do not want to differentiate everything this you can do it by yourself write the final metrics form. You have to take second derivative of that at x equal to 0 that is 0, then at x is equal to 0 this is the third derivative that it equal to this.

And then like that you set for all the 4 boundary conditions, what will happen is you will get an equation of this form, which I maybe I will write it in this place. Than I have to areas it this will become, I will write the directly the applying the boundary condition please understand, you will get 0 minus 1 0 1 minus 1 minus m l over m beta 1 minus m l over m beta and erase it, then minus sin beta l minus cosine beta l sin hyperbolic beta l and cosine hyperbolic beta l. And the last equation will be minus cosine beta l sin beta l cosine hyperbolic beta l than sin hyperbolic beta l c 1, c 2, c 3, c 4 equal to 0.

Now, this is my boundary condition equation, in this the unknowns are of course, c 1, c 2, c 3, c 4 you do not know, but you also do not know beta. Now, for this to have a non trivial solution, nontrivial means I should not get c 1, c 2, c 3, c 4, 0 is exactly it will satisfy, you want a nontrivial solution. Then you say, the determinant of this matrix is 0, and the determinant this will be a bit lengthy stuff you have to do numerically only, I will not be solving the entire thing.

Once you said the determinant 0, the only unknown is beta for what values of the determinant is 0 finite number, but they are all discrete numbers. Then corresponding to each beta, if you substitute that you can find the Eigen vector c 1, c 2, but not all of them will be arbitrary, you have to pick 1. Because, this is the 4 by 4 matrix, if I say c 1 can assume any value then I can write c 2 in terms of c 1, c 3 in terms of c 1, c 4 in terms that is a ((Refer Time: 11:13))

That means, I have my I will go back and substitute the corresponding beta values here, and the c 1, c 2, c 3. So, every beta you will have one vector which is something you can say one some value, may be alpha, beta, gamma into c 1, because I am taking c 1 as 1, all the alpha time c 1 will be c 2, beta time alpha, beta, beta everything is beta solve it does not matter, this is an independent maybe alpha bar, beta bar, gamma bar. For every value of this beta, you will get one Eigen vector, this vector is not call the Eigen you may call it eigenvector here, but actually this is your beta.

If you go and substitute you will get the corresponding omega which is the frequency of vibration of the system. You will get the frequency, when you substitute these values here, you will get the deformation that you call it as a mode shape are in the current problem it is the Eigen value and Eigen function. So, Eigen value Eigen functions I get, but these are exact because I solve the, ((Refer Time: 13:07)) but invariably I will not be able to get an E I m are not constants that part we will learn later today we will say a simplified case.

Because, there is something more in this particular problem because it is a you have only a mass and the beam, and it is free. You can also have another solution, please understand deviating another solution, where W of x equal some constant c 0 plus some c 1 x with lambda 0, please understand for the given problem if my W is of this form with lambda 0. That means, then this also satisfy not necessarily c naught has to be 0, c 1 has to be 0, these are all come constant or you may call it a naught, a 1 whatever way you want to call it.

Now, this particular they will satisfy the boundary condition also, if you see boundary conditions are second derivative or third derivative. Anyway second derivative is 0, third derivative is with lambda is also 0 this will satisfy the equation, after that the boundary condition. That means, this particular shape; that means one c naught it is a constant; that means, the entire system can move up and down uniformly with 0 this is a rigid body motion.

This you call it rigid body translation, when this entire system moves up that is this, rigid body motion 0 frequency. Now, the other one is a c 1 x, c 1 is a constant x means as you go along x that displays one is a function of x linear function that; that means, I can have a situation where, the mode shape can be like this. This is the rotation, this is translation, this is pure rotation I can add both of them; that mean this is also a rigid body mode, and this is also a rigid body mode.

The rigid body mode have 0 frequency, this is always this will come only in free, free system. Because, we have not restricted this system anywhere it is free to go up and down, along with this we have W also this is W which we said that W is this, all the point move same and there is a rotation another one. Then what is this, these are your flexible modes, this is the rigid body mode, this is the which is the function of x sin, cosine, sine hyperbolic they are flexible modes.

And the flexible modes will have a corresponding non 0 lambda, flexible modes will have non 0 lambda. And essentially your solution consists of some rigid body mode, and some flexible modes is it clear for you, now we can simplify the problem.

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Now, the question is what really happen in the actual system that is a response problem, what we say now is the differential Eigen value problem gives me a set of frequencies, a set of deformation or you may call it shape functions or Eigen functions.

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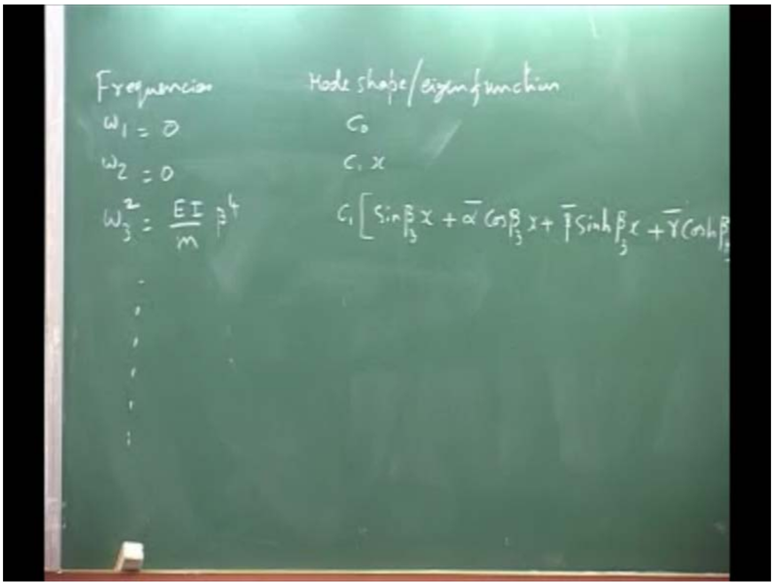

So, I have essentially a frequency one I can plot it like this, frequencies and mode shapes, which is I can call it Eigen function this is the Eigen function. First frequency I may say this value is 0, for this is a c naught everything is moving up and down, second frequency that is also 0 this is c 1 x. Because, second frequency you may be omega 1, may be omega 2 I will put it, when I go to omega 3 that is my beta I have to get it from beta 3, omega 3 means omega 3 square will be beta 3 something.

So, I can put it what is that will be omega 3 square if I take it E I over m and beta some whatever that power 4. Usually it will be always beta l they will write some value, anyway here you take it now from here, omega 3 is this value you know the beta value you can substitute you get this, corresponding to this you will have some c 1 time something sin beta x beta 3 x plus some alpha bar cosine beta 3 x plus beta bar is a another constant sin hyperbolic beta x plus alpha beta gamma bar cosine hyperbolic beta 3 x, this is my shape function.

Because, c 1 I do not know ((Refer Time: 21:16)) like here this is c naught I do not know how much it will move, but every point will move c 1 I do not know, here are also this is. Like that you can have, you can put it 3 as m and you can correspondingly this will change, now you see corresponding to each natural frequency, you have a corresponding mode shape or you can say Eigen function this as a exact solution. Even here exact I said this you have to solve by numerically.

Because, these are all sin and cosine hyperbolic functions, you have to solve them numerically. Now, what really is going to happen in the actual situation because everything is a solution, so you will go and write you are this is what is call the expansion theorem ((Refer Time: 22:23)) I erase this part, this is for just now my motion can how all these possible.

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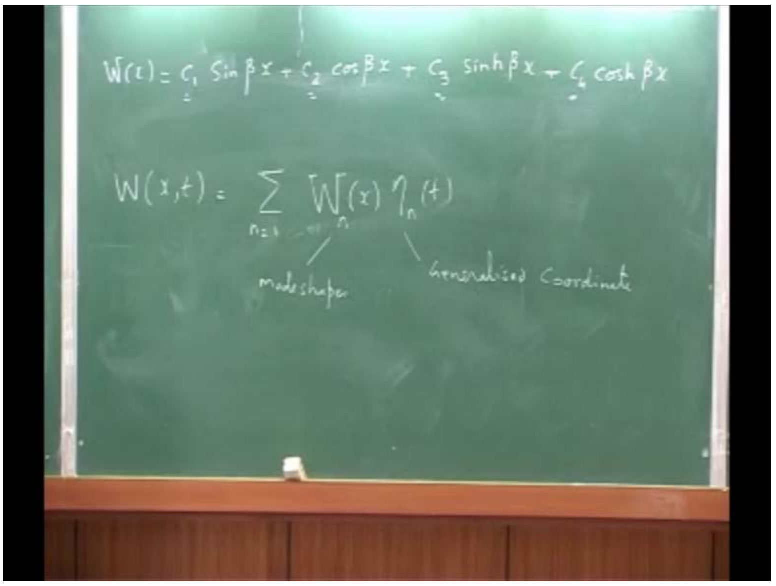

So, my W which is the function x coma t I have split them into separation of variable W x is all this functions. And I will simply write here, some W n x eta you can go up to technically it is up to infinity you have, but n running from 1 to n that is all, now you see this is my general motion. Because, everything is a solution please understand everything is a possible solution, and you know that these are Eigen functions, and they are orthogonality condition they will satisfy, you follow this is all the expansion theorem.

Basically you are expanding your general displacement of the structure, in terms of it is mode shapes or in Eigen functions. But, these are free vibration problem solution, now you have to solve for forced vibration, forced vibration means there is an external load acting, how do you solve that part we will come to that later, how do you deal you have to basically substitute this and then you will be principal and you will get now equation n. Now, I have represented my that structural vibration or structural motion, in terms of some mode shape and these are I call it now generalized coordinate.

These are my mode shapes or Eigen functions, now you see it also leads to how did I get that mode shape, I got the mode shape by solving this exactly in this particular case I was able to solve the problem. And if I do not if I cannot solve this problem, then you can always say I make a assumed mode shape; that means, this function you assume it, but what we got exactly the function, which satisfy the boundary conditions as well as the equation.

Where as in the mode shape, you have either it can satisfy the equation if may not satisfy because you do not know. But, you can at least make sure that they satisfy the geometric boundary conditions, which are admissible functions suppose if those functions satisfy, geometric as well as the poles or natural boundary condition. Then you call them a comparison functions, if will satisfy the all the boundary condition, then the equation then that is Eigen function. Now, you see we got Eigen functions for this particular problem, exact Eigen function for an approximate the problem.

Now, what I will do is the I will give just if I make this m l 0, how the mode shape will look that is a simplified problem for which you will have the solution in the book. I erase this part a simplified case further simplified that is what I am saying, well it say it will little tricky it showed reach that well. But, you will still have a rigid body mode please understand because what will happen is the flexible modes will be sending towards that value.

But, you still have rigid body mode which you cannot because I have a heavy mass that does not mean, but I can move up and down. So, you cannot remove these two unless you enforce that it is fix, it cannot move, so do not think that the solution of a cantilever beam is a limit of having m L ((Refer Time: 28:17)) you truly correct, but the flexible modes will approach that value. But, not their rigid body mode will exist as a rigid body cannot stop there, you have are clear because it is not that this is way simplified case of the other case, other problem you have to see physically what ((Refer Time: 28:44)).

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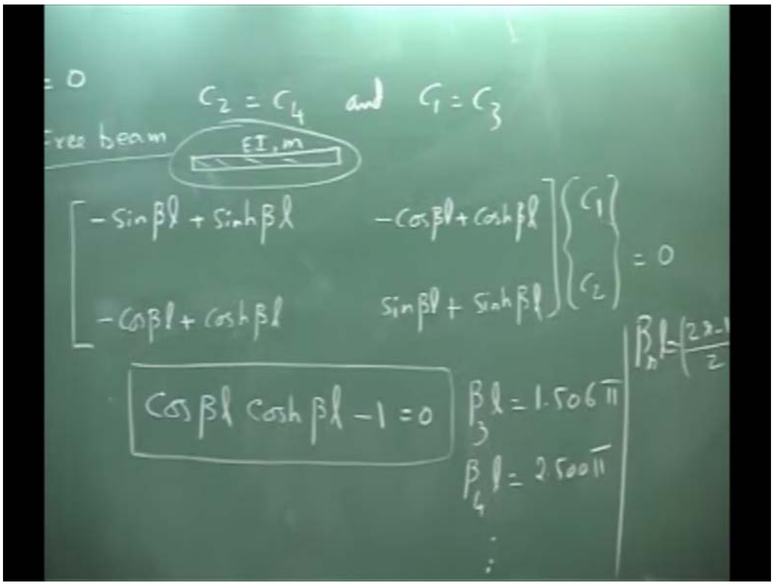

Now, let us take this very simple case m L is 0 this is like I am having a highly idealized situation of a flying wing, the entire wing is represented as a beam. Now, what are it is mode shape and frequencies anyway rigid body mode will stay, the elastic modes are the flexible mode that determinant you can simplify, and it will become simply like this. Because, when m L equal to 0 you will get c 2 equals c 4 and because c 1 equal c 3 this is the boundary condition, when you substitute the boundary condition.

Finally, you will not this type of equation, I am not writing all the steps sin hyperbolic beta l cos beta l plus cosine hyperbolic beta l minus cosine beta l plus cosine hyperbolic beta l sin beta l plus sin hyperbolic beta l multiplied by I am putting c 1, c 2 is like a simplified that. Because, c 4 is c 2, c 1 is c 3 from their boundary conditions, now for a non trivial solution that determinant have to be 0.

If I multiplied get the determinant that is your characteristic equation that will become and simplifying please understand I am not writing all the steps hyperbolic beta l minus 1 is the row, this is my characteristic equation m L is 0. Now, the solution is simply this is for a free, free beam is just this beam, how this will vibrate E I m that is all we have done this problem. Solution of this, you can get it graphically because you know that cosine beta l is a cosine function, you take it to a right hand side, 1 over cosine hyperbolic, hyperbolic is exponential function it is always decrease.

Wherever these two cross they are the solution, so I will write the solution here beta I may call it 0 is there and 2 zero's are there. So, I am calling it as a beta 3'rd value l equals 1.5065 and beta 4 l is 2.5005 and so on beta r l that is any other is approximately 2 r minus 1 over 2 into pi, and your function, this function will be like this.

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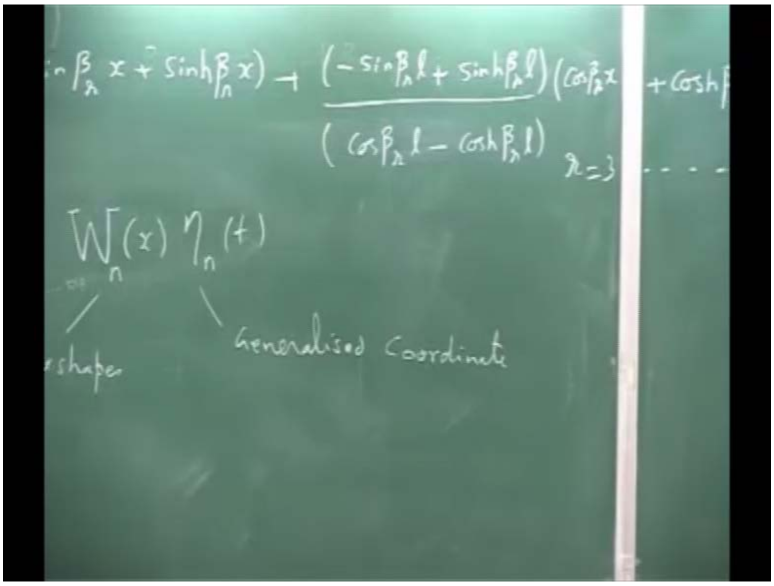

I am going to write here, sin beta r x plus sin hyperbolic beta r x plus minus sin beta r l plus sin hyperbolic over cosine beta r l minus cosine hyperbolic beta r l multiplied by cosine beta r x plus cosine hyperbolic beta r. And you are W 0, W 1 they exist W 0 is some constant c naught, and W 1 is your come c 1 x, r running from 3 onwards because here r goes from 3, 4, 5 etcetera. So, you see I now have from here the frequencies, this is my mode shape or Eigen function. And I can plot let me erase this part, just for simplicity you know just for a graphical, these are the mode shapes or Eigen function.

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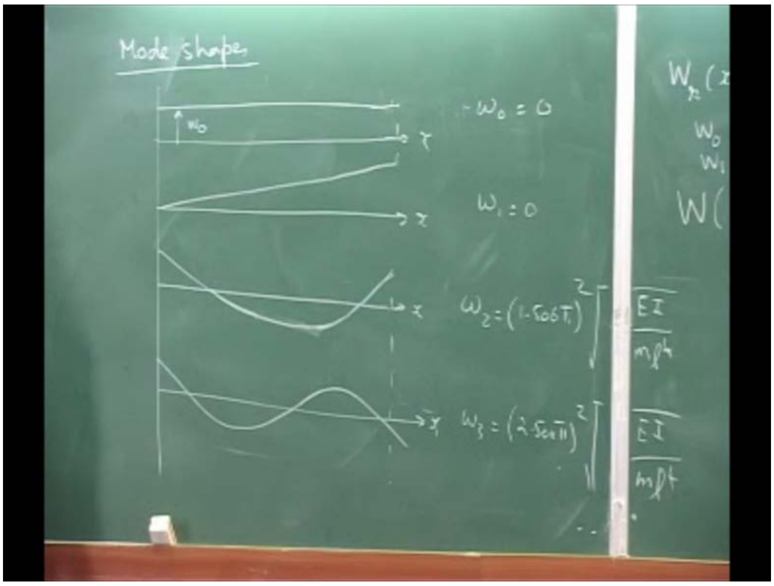

So, this is W, so this is my x, so this is constant this is W 0 that is the first mode, frequency omega 0 is 0. And the second one, I can take it like this it is a rotation c 1 x, and this is also omega 1 is 0, when I go to the third one, how it will vibrating it will vibrating this is at l this is the length l, maybe I should plot it. And for this the frequency will be 1.506 pi wholes square E I over m l 4, because I have taken 1.506 substitute that beta, and get the omega square.

And the next one will be here this is omega 3 will be 2.500 pi wholes square again square root of E I over m l 4 and so on and so forth. You can have many like that, so these 2 are rigid body modes, these are flexible modes and your general motion is this, you can substitute n I put actually starting from 1, even you can start from 0 also there is nothing run because 0 is a rigid body mode, there is one you can start from 0 or 1 that is just a index.

But, interesting thing here is in this case that beam vibrates in a symmetric fashion, symmetric in these sense both side will go up, when as when you look here this is not like that, this is an anti symmetric mode. So, you will have symmetric mode, anti symmetric mode several if you go to that a next one, fourth one you may get symmetric mode it may come down, up and then go up. So, like that you will have symmetric, anti symmetric modes shape, but if you solve in this fashion you will get exactly error the symmetric mode, anti symmetric mode all the modes what are possible.

On the other hand, if you that is why when you solve the full wing problem it is better to solve both side. If you are solving for a actual plane, where as if you restrict half you are saying, I restrict only one half of the wing, then you need to be sure what boundary condition you set in, at the midpoint.

Suppose, this is my air craft if I cut it I have a half if I want to analyze only half of the system, I will be clear what boundary condition I set in at the midpoint, because you are analyzing only one half of the system, if you set a particular type of boundary condition. Suppose I say this placement here is 0, slope is 0 right then what I will get only symmetric type of things whereas, here I allow rotation, but I do not allow this placement. So, you may have to change the boundary conditions suitably whereas, it is solve the full problem as it is, you will get everything fine this is just for a sample, I did it free, free because your aircraft is a free, free system that is why when you analyze.

Now, if you both solution then you how to change the boundary condition, no here you allow what, if I say this half I want this is going slope is 0. And then your curvature is you set it curvature this placement I allow whereas, here you have to take the other where on, some changes this placement is 0. And then I think here curvature is 0, here curvature is 0 something like that here curvature is 0, here curvature exit.

So, you have to put curvature second derivative you cannot set it 0 here, so that is how you have to be very careful what I allow, where as if you solve full problem as it is you will have everything without any problem. Because, this is just to indicate that if you take half the problem, I am just living you have to be very careful about what boundary condition you are setting it in that one hand, that will give you only that set off solution it will not give you the other solution that is the reason.

Why because they avoid that confusion I solve the full case in this is I said m L 0 and that is a full free, free beam. And now you know that I can solve any type of problem because I can have m L here, I can have m L here both wing or I can put one mark in the middle, put everything. But, boundary condition will be here and here some mark is setting here that is that will come part of your equation. Now, if you take a the most familiar problem which is the cantilever problem; that means, one in this fix I will write again this part, I erase no I thought I will just write the cantilever solution directly, maybe I erase this whole thing.

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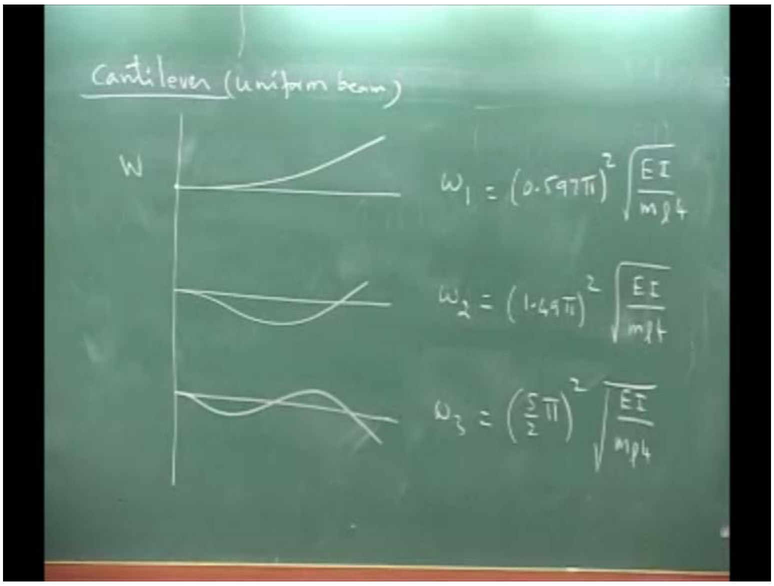

The familiar problem of cantilever beam uniform cantilever uniform, and liver uniform beam if I say now I do not have rigid body mode because displacement on the left hand is 0 slope is also 0. So, your first mode itself will start like this, and your second mode will be like this, and your third mode may go like this, so I will write the corresponding way frequencies, that is basically clamped. Here it is 0 this is W you will have omega 1 is 0.597 pi E I over m l 4. Then omega 2 you will have 1.49 pi wholes square E I over m l 4, and omega 3 is 5 over 2 pi E I like that, so on and so forth. And the interesting part for this, the characteristic equation I am writing characteristic equation, the just for a your interest.

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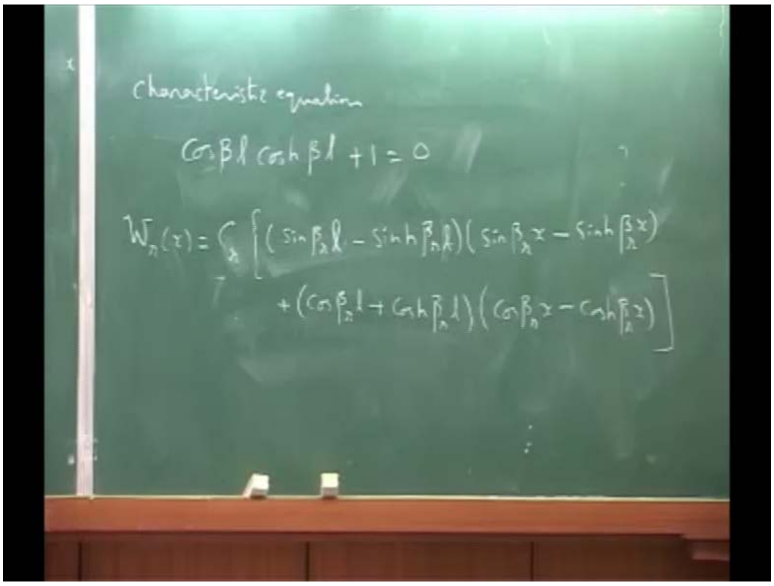

But, cantilever beam your characteristic equation which is nothing but your determinant equal to 0, you will get cosine beta l cosine hyperbolic beta l plus 1 equal to 0. This is for cantilever, in the earlier case you got minus 1 equal to 0, and your current body mode shapes are W r which is a function of x this will become again c r sin beta r L minus sin hyperbolic beta r L multiplied by sin beta r x minus sin hyperbolic beta r x, and then you also add plus cosine beta r l plus cosine hyperbolic beta r l multiplied by cosine beta r x minus cosine hyperbolic beta r.

This is the shape, please understand this particular function is this, earlier we flatten the other function. So, you find cantilever beam, but the beauty is equation is not changing what is changing in only boundary conditions are change because of a change in boundary condition, you are getting different frequencies, different shape functions or different Eigen function. So, that is the essence of this your field equation may remain the same.

But, you are boundary conditions will completely change the solution that is why it is very essential for you to understand this thoroughly. And once you understand this part, than solving any problem basically your aircraft wing is a vibrating system, and you will be able to solve all the problem associated with that, first thing you must know basics that is why this is the mention. Now, what I will do is I will take the torsion problem similar to what we did for bending, I will take a simple torsion problem right it is mode shapes because they look a little different. So, I completely erase this I hope you have understood thoroughly, now let us take that torsion just for completeness shape. Because, the procedure is identical only thing is the equations are different.

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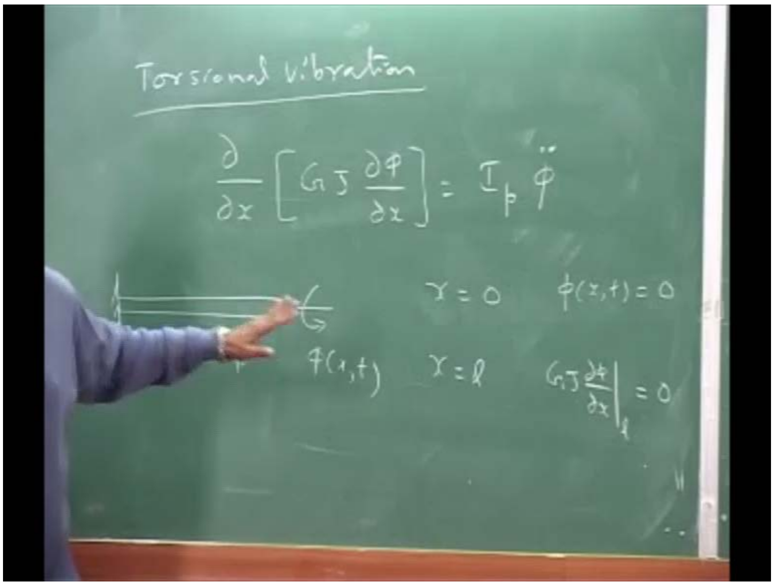

Let us take the case of again the wing is represented as a uniform beam of a torsional vibration, again free vibration equation we have this. If you look back your notes you will have this, delta p by delta x equals I p p double dot double, dot is second derivative with respect to time. And if it is a fixed this is G J and I p and this is the corresponding boundary conditions at x equal to 0, you will have free this is 0. And at x equal l this is the free and there is no moment, so you will have G J delta p by delta x at l this is 0, this is my actual boundary condition as usual again you go put the separation of variable that is P.

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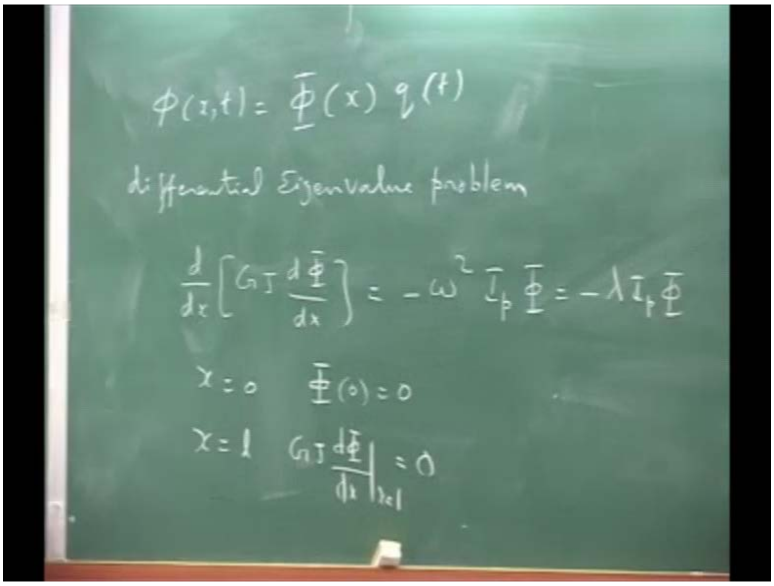

You can write it as some capital P, P which is the function of x t you put it capital P x and some q of t, substitute collect the p of t term one side, and then the p term on the other side. And you will get the differential Eigen value problem, so I am writing directly the differential Eigen value problem, this becomes d by d x of G J d x equals minus omega square I p p or which you can write minus lambda I p with the boundary condition x equal 0, p 0 is 0 and x equal l you will have G J d p by d x at x equal to 0, so that is a fixed end, this is a free end.

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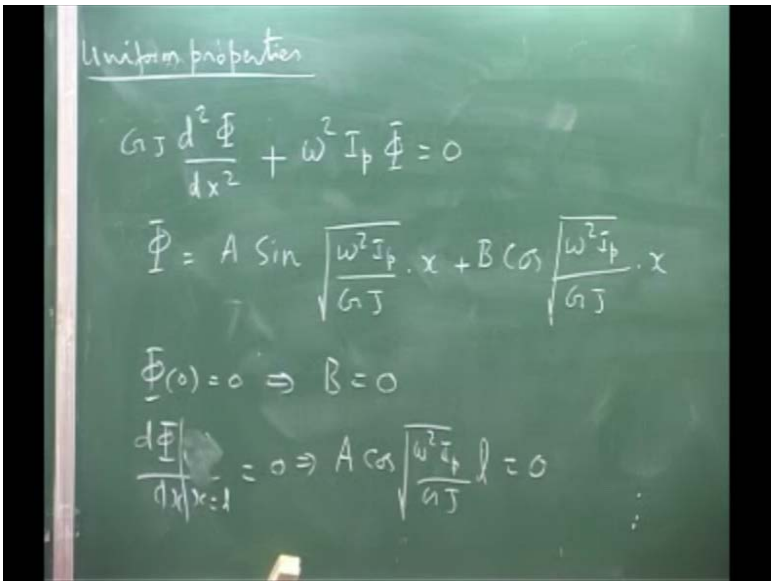

Now, if I assume G J I p everything is constant that is uniform properties, then my equation will become G J d square p by d x square plus omega square I p p equal 0 or I can write my solution as A some sin. Because, I can divide by G J this is nothing but a second order differential equation, simple vibration problem, you will write square root of omega square I p over G J into x plus B cosine square root of, now you substitute the boundary condition, you will get when p 0 that is x is 0 automatically this term goes.

So, your first boundary condition t 0 0 gives me equal 0 implies B equals 0 right, for else now you take the derivative. Because, here d p by d x d p by d x sin will become cosine that should be 0 and A is not 0, so you will get the second d d x as the x equal to l this become basically 0, implies that A cosine square root of omega square I p over G J l equal 0 that is my equation. Now, here again few indicates that if A 0 everything is 0, then for nontrivial solution you want this to be equal to some pi by 2 that is all you will have.

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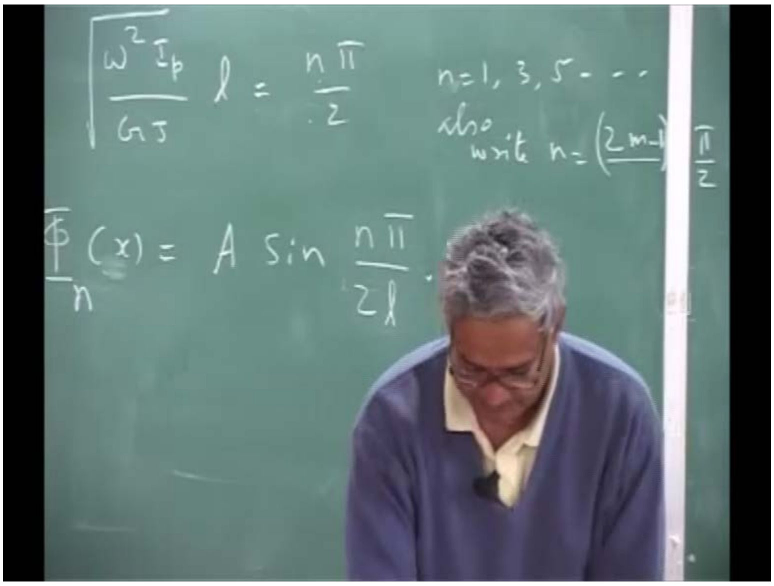

Therefore, your solution will be essentially square root of omega square I p over G J l will be n, your n running from 1, 3, 5 etcetera that's all. And your solution is that be, you can get the frequencies and the mode shape becomes pie this will become essentially any n, I will put equals some A sin you are going to put omega square this is any n n pi. That means, the n equal to 1, omega 1, omega 2, omega 3 like that you will have a lot of values.

So, I am going to put this itself as n pi by 2 l root of G J over I p sorry n pi by 2 l into x because this entire term is l you bring in that is square root of omega square I p by G J that the this term. So, you will get this is my mode shape these are my frequency, so I can plot them like a that is not a problem or like you said you can also write n equal what is that 2 m minus 1 by into pi by 2, where m runs from 1 right 2 is what is that pi by 2 2 is 3 pi by 2 you can have.

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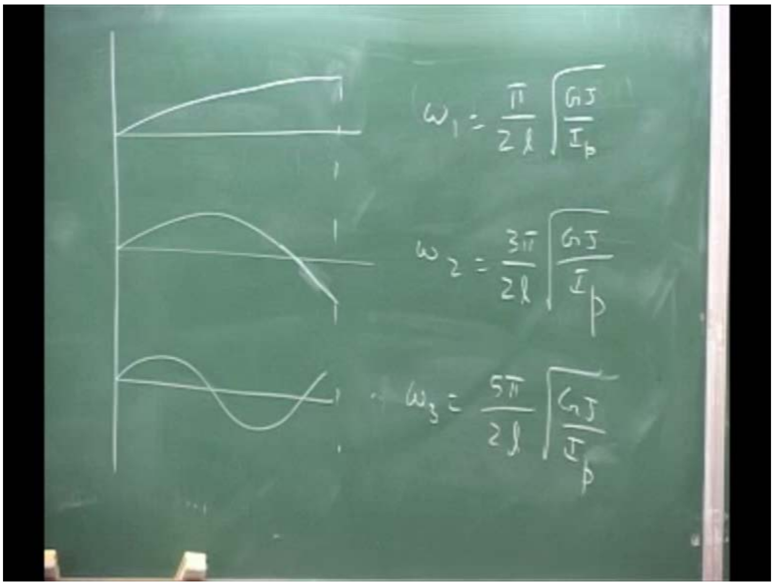

So, this is the shape and this is the what you get is now you plot, the mode shape you have sin wave this is l, and the second one will be like this and the third. So, you will have omega 1 is pi over 2 l root of G J by I p, omega 2 3 pi over 2 l root of G J over I p, and omega three is 5 pi over 2 l root of G J over I p. So, this is for the torsion problem, so you have the complete I would say simplified problem solution, which we know how to solve now.

These sense one's the equation of motion is given if it is simple there is what the key is, then you can write the solution, apply the boundary condition, get the frequencies and the mode shape. And you have plotted all, now any general motion is the summation of all the mode shape or with a generation coordination, this is for problem where you have a exact solution. So, we will learn next before I proceed further I would like to finish one small section, and then we will go to how do you get approximation. Because, you cannot get exact solution for all the problem, now your next question have how will I solve the response that I will answer first that is response of the continuous system.

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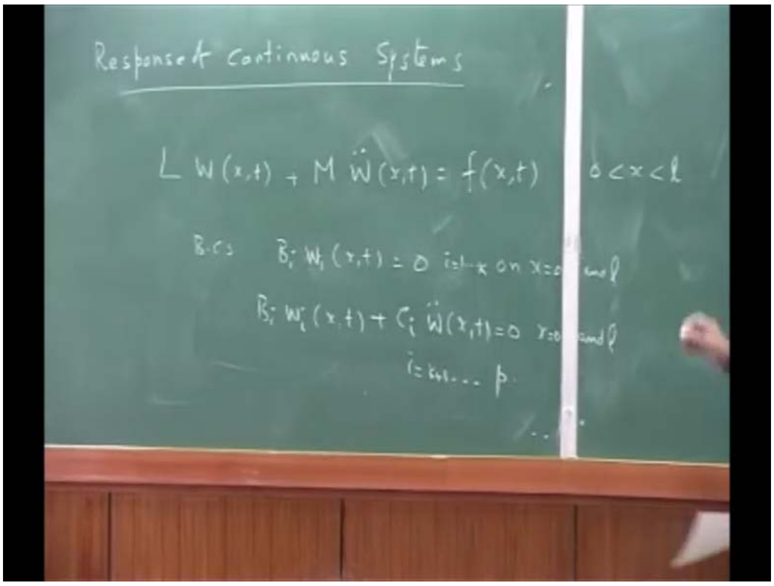

Because, you got the exact solution for this type of problem, what you do is I erase I will write the mathematical solution, then you will understand easily response of continuous system So, now our full equation of motion in operator form please understand, this is W x comma t plus something W dot x comma t equals some f of x comma t, with the boundary conditions B i W i 0 and you had B i W i plus c i w double dot x comma t on x and x equal 0 and l, this you can take i running from 1 to k, here you use I running from k plus 1 to p, and x equal to 0 and l, this is what we formulated our original problem. Now, I have obtained exact solution, assuming that I somehow obtain the exact solution; that means, Eigen values I got it.

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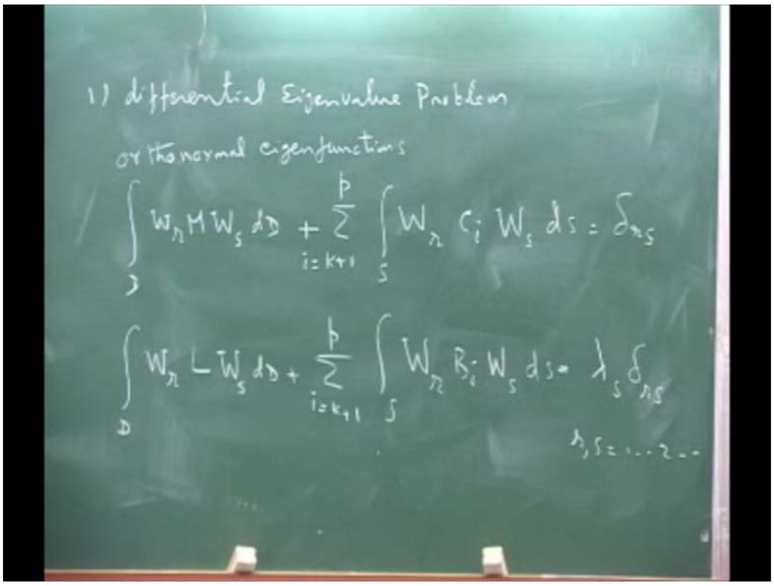

So, you immediately go back and then your first step solution you will formulate differential Eigen value problem that differential Eigen value problem I will get for take of formulate that is you first get this. And from here you will get your solution, I obtain my solution and you know one of the property of the Eigen function, this they are that orthogonality condition. So, what your that ortho normal condition I will write it now what is that over the domine minus sorry plus summation i equals k plus 1 to p.

The other these are the condition referring to orthogonality, please understand that I will explain now s B i W s d s is lambda s r and s runs from 1, 2, 3 excreta. So, please understand here I said delta r s, this is shape function, Eigen function r, Eigen function s they are orthogonal because when r is not equal to s they are 0. But, when r equal to s what you do is, you normalize it that is what is call the normalizing the Eigen function, such that this term become 1.

Otherwise it will some value, now that is why you call it ortho normal, orthogonal and also normal Eigen functions. Suppose you say, what type of normalization you can do because there are several ways normalizing it please understand, is not that there should be only one way. Like I can take the in the mode shape keep deflection is one, in the shape function I am just drawing ((Refer Time: 01:05:12)) functions for a cantilever beam I just drew like this.

I am drawing like this, what value I will give that is depend on that a r r c r values c 1, c 2, c 3 I can set c 1 as 1. But, then this is the shape when I substitute that c 1 as 1 here I may not get delta r s as unique quantity, I will have some finite value on the other hand I chose particular value for the normalizing factors, such that this is one that is another way of normalizing that is why they are called ortho normal Eigen function. Suppose if you say if I do not normalize it what will happen nothing will happen, only thing is that corresponding value will get multiplied everywhere.

So, it is not a there are various ways of normalizing the functions, one of them is this condition. Now, you see I have solve the Eigen value problem I know these are my normalizing conditions I go back and then I got all the Eigen function and I will write my solution like this.

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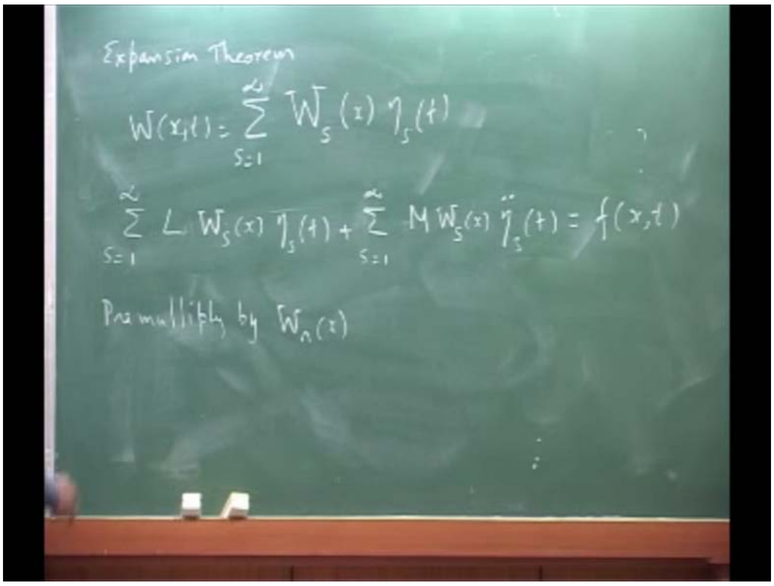

This is what expansion theorem what I am doing is, I am writing my W which is the function of x comma t as s running from 1 to infinity W of x this is x and eta s which is the function of time. This what we assume first, we split that into space and time variable now I am combining and the writing, and I got all the Eigen function, now my aim is to solve for this. I take this substitute in my equation, when I substitute in my equation and then I integrate because I have to apply the orthogonality condition.

What I will do is how many unknowns, how many generalized coordinates I have, I have in finite and you know that I have in finite Eigen function. But, I will not really solve for in finite number of equation, I will set it to a finite number usually in a vibration problems. Sometime you will be take about 7 modes, 8 modes sometimes it will may take about 20, 30, 40 modes or 50 modes, than actual aircraft analysis will take some more around 60, 70 modes some may take little bit more.

Because, they say even those the higher modes may not will contribute, this is their estimation of the aerodynamic is better if I include all the higher mention something like this. That means, you can have more number of modes, you do not go to infinity that normally in helicopter rotor blade, we take about 6 modes, 6 or 7 or 8 well that is all; that means, our entire continuous system is, now converted into only 8 degrees of freedom 10 degrees of freedom.

So, this is what is called you can call it model analysis you may call it, so the expansion theorem allows you to write this, and then this you substitute in your equation use the orthogonality condition. So, when you use the orthogonality condition what you do is you, when you substitute you will get summation s running from 1 to infinity L W s of x eta s t plus summation s 1 to infinity capital M W s x eta doubled dot s t equals f of x comma t f of x comma t is the external loading, which is your aerodynamic load.

Now, pre multiplied by W r of x and then integrate, when you do that what will happen you are putting W r here, W r here W r there. And you are integrating over the domain over the length of the wing, are over the area of the wing because if it is 2 D you will have two dimensional, this is a single that why x along the length of the wing. What will happen is W r L W s in here, W r L W s equals lambda s delta r s, it will go that side.

Similarly for the mass, some you will get this part that will go that side, then what will happen to these two terms, these two terms are c i W s into eta double dot you come here, B i or you can say B i is the function B i W s plus c i W s. In the sense W s there is a into eta s double dot that is nothing but 0 this is the boundary condition directly that is why what will happen is this terms will actually drop out.

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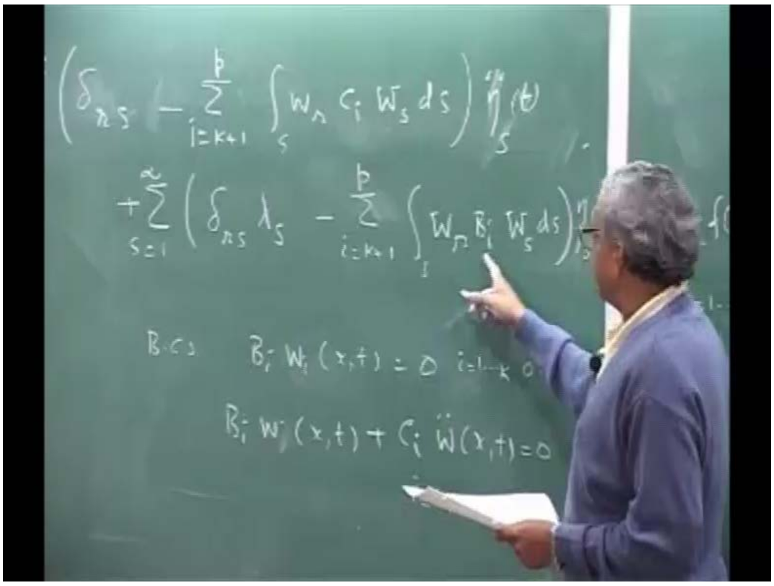

So, you substitute this as it is if you put it ((Refer Time: 1:12:22)) maybe I will erase this part, and write it here you will get like this summation s running from 1 to infinity delta r s minus summation i running from k plus 1 to p W r c i W s d s into eta doubled dot t. But, maybe I put this here eta s double dot t plus summation s running from 1 to infinity you will have delta r s lambda s minus summation i k plus 1 to p integral W r B i W s d s into eta s t equals integral W r f of x comma t d r running from 1 to infinity you will have in finite equation.

Now, you see you look at this W r is common here c i W s eta s double dot, B i W s eta s that is B i w s eta t is c i W s eta double dot. Because, this is a second derivative automatically this term will cancel leaving behind very simple equation, if you look like this, so that is why the boundary condition will come here make sure that those terms cancelled out what will you get. Because, these two terms go up and r running from 1 to infinity wherever s become r, we will find that is one; that means, x should be replace by r.

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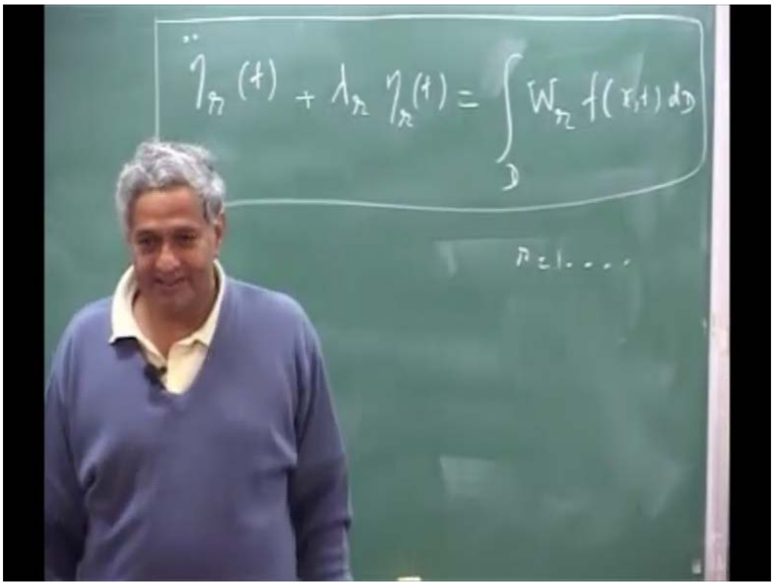

So, you will have eta r double dot t plus lambda r eta r equal integral W r f of x comma t this is my equation. And these are my is not f of x comma t is my external loading W r is the shape function or Eigen function in the arc mode multiplied that is the generalized pose for the r mode shape. So, you are running from again 1 to, so many equation now if you want the initial condition, if you say that what is my initial condition I should use for eta r in as you knows here what is the initial condition right.

You can always apply the orthogonality principle to get eta s or in otherwise what you can do is I can take this, get the solution substitute that here, and use a initial condition that is another way or else initial condition directly use. Knowing this you can always get the initial condition from the orthogonality relationship, I will not go into the details this part is supposed to have been the background when you come for the aero elastic.

Now, you got the general form how do you get the equation motion, and then how do you really go to the response solution solving this, this is simple second order differential equation each one is independent please understand, each eta r is a generalized coordinate this is independent. If it is solved only thing is do this you must know W r lambda r will come, now you see the natural frequency a corresponding mode shape for the Eigen function, they come in this in the response calculation. This is what is done in the aero elastic problem, now you know that I need to know only this that is the aerodynamic part. But, one more thing which you how to know is if I cannot get a exact solution what do you mean that part I will do next class.