Aero Elasticity
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Lecture - 5

Today, what we will do is, we will use the Hamiltonian principle to derive the equation of motion for a very simplified problem.

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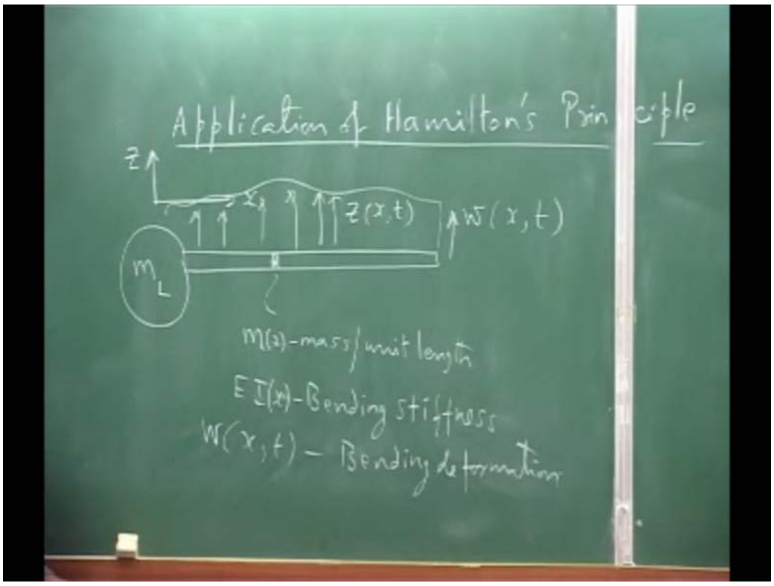

We call it application of Hamiltonian's principle, but what we will do is, we will formulate the equations for a beam like structure under bending deformation. Then we will use the same principle equation for again a torsion problem, that way you have bending torsion of beam like structure derived, which we will be using it for the later aero elastic analysis. Now, let us take, the wing is represented by a beam, I am not put it as a cantilever beam or anything like that.

You can have different situation like, I can say you can put a wing, which is the ((Refer Time: 01:49)), I can call it. I am not going to call it m left, I am only putting left, the mass which is attach to the left side of the beam. And the beam I define my coordinates system, this is my x, this is my z direction and the mass per unit length I call it lower case m, this is mass per unit length of this beam, see this can be choose like a mass, you follow.

I can now another beam drawn this side then, this may be in the middle of the beam, I can have half of it and then E I, that is the bending stiffness, these are the quantities which are defined for my beam. So, this beam is basically my wing, but they are functions of x. So, it can vary along the length of the and we define W, which is the function of x comma t, is the bending deformation, W x comma t is bending deformation. And please note m, E I, they are function of x, so they need not be constant, so if you want, you can even put like this. So, for this beam, you have to write the kinetic energy, potential energy or strain energy and then, the distributed load. If you take the distributed load, we can define that by some z of this is my external load which is acting on the wing.

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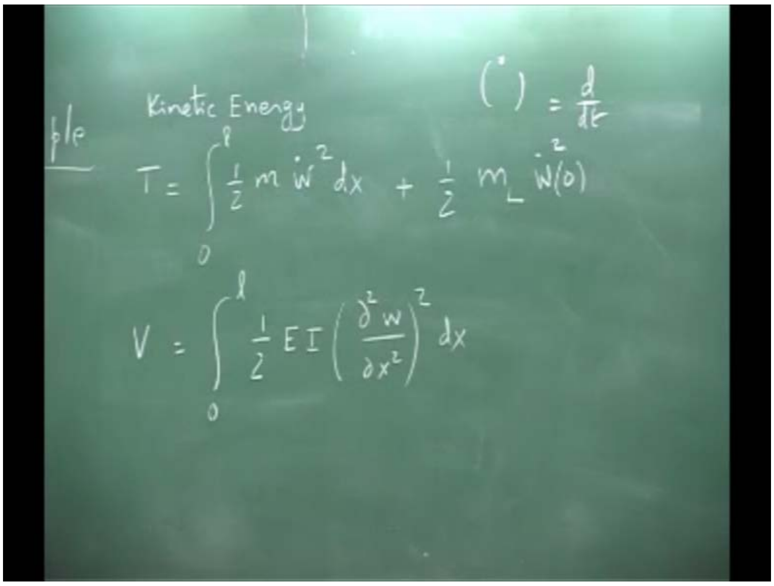

Now, let us write the kinetic energy expression, now kinetic energy T is, the length we take it as l the length of beam. So, you will have 0 to l, half m W dot square d plus I have to add the... Why I have included this is, to indicate what will be the effect, I can put one more also on this side, it does not matter, I can have any condition. Because, I am just describing for one situation then, for different situation, you can derive things for yourself, plus half m left W dot at 0, the dot is d by dt.

So, when I have this definition that means, d over dt square, there is no integration for this, because this is the heavy mass, which is acting at the left side. Then, you can have a strain energy, which is again 0 to l half, for bending strain energy you have E I, bending moment and curvature. So, you will have del W dx, because this is the curvature expression and you have a external load, external load that we will include in later, right now what we have to do is, apply Hamiltonian.

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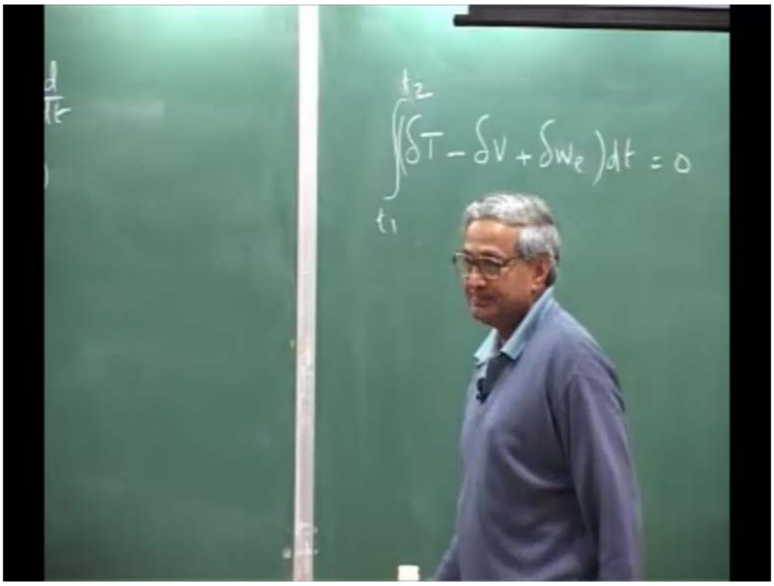

Hamiltonian principle says, integral delta T minus delta V plus delta, t 1 to t 2, so what we will do is, we will evaluate each quantity term by term. Because, otherwise the board is not sufficient for me to fill up, so what we will do is, we will evaluate first the variation of kinetic energy expression, we will write it. Then, we will write the variation of strain energy expression and then write it, later we will write the variation of external work, write. Then, we will combine all of them and then, get the final expression, so I erase this part, we will take first the kinetic energy expression.

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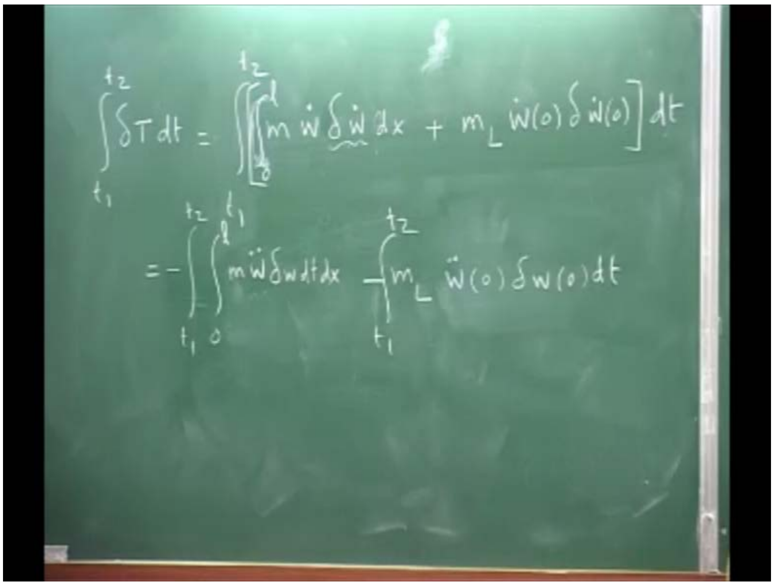

The kinetic energy expression, you will have integral t 1 to t 2 delta T dt, which is I have to take the variation of T here, the variation is t 1 to t 2 m W dot delta W dot dx. Because, this will remains as it is, plus you will have that other term m left W dot at 0, delta into, because you need to have one more integral here, please understand that here. This integral is 0 to l then, I will put a bracket here, so this inner integral is over dx, outer integral over dt.

What I will do is further, because this is the first step I will switch the order of the integration and then, integrate first with respect to t 1 to t 2. And then, if the integral 0 to l outside for this term, this term is not an integral, this is directly only t 1 to t 2 with d t. So, let us write first this part, if I integrate by parts, by parts means what I do is, delta W dot is nothing but I will write it. Delta W dot is delta of delta W by delta T, which I am writing it as delta by delta T of delta W, W is the function of x and t, that is why I put a partial derivative.

Now, what you can do is, this into dt, I can integrate by parts, so I do integration by parts, I will write here 0 to l m W dot del W in the limit t 1 to t 2. You will have a x here, I can put because this is t 1 to t 2, this integral is t 1 to t 2, I have dx also, minus integral 0 to l integral t 1 to 2 m W double dot delta W dt dx. And then, this is for the first term, second I will again do integration by parts, if I do that, that would be plus m L double dot

W dot 0 delta W at 0, that is at t 1 to t 2, minus integral t 1 to t 2 m L W double dot at 0 delta W at 0 dt.

Now, this integral is over the space, but this delta W is over the time, initial time, final time and what we say is we said that, the initial time and the final time, the variation is 0 in our Hamiltonian thing. Therefore, this term goes to 0, same is the case with, because this is variation in the initial time at location 0, this 0 is the starting point x 0, so final time was also that. So, these two integral, these two terms will become 0, leaving behind only this two term that means, whatever I have written delta T dt it is...

Now, I can erase this part and write it in very short form and that becomes much simpler for me in, so I erase this part then, put it as. Because, later it will be easy for me to substitute, minus integral, here now I have change t 1 to t 2 integral 0 to l m W double dot delta W dt dx. And then, minus integral t 1 to t 2 m L W double dot at 0, 0 is the 0 not time, zero location, delta W. If you want to put, you can put a comma t, W double dot comma, here you can put comma t. Next what we do is, we have to take the variation of a strain energy expression.

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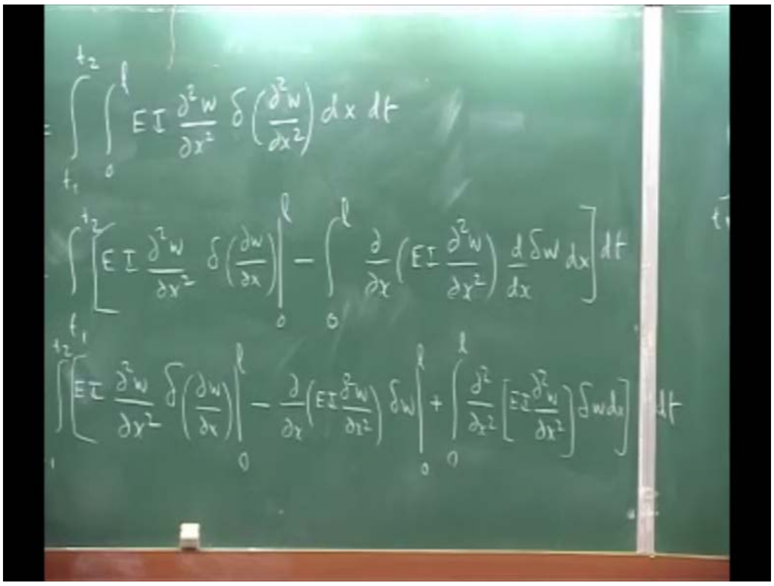

Strain energy expression if I take, here again t 1 to t 2 delta V dt t 2 0 to l E I del square W by delta and as usual, again I will take the variation and this differential, because this what we call it as a weak variation. Variation of the curvature, curvature of the variation, you can inter change both, because that wise the variation is the weak variation, small you vary, do not put a large, it a very suttle, it is not that one differential you can just, variation is different from derivative.

Now, this expression again you integrate by parts, because I will be taking this inside and this derivative, I will take it outside. Then I will have dx, I can integrates by parts twice, twice actually first time I will integrate, this will be essentially t 1 to t 2 E I. This is del square W over delta x square delta of delta W by delta x, you can write it, because this you can take it inside or outside does not matter, minus integral, here there is a 0 to l.

Because, please understand this integral I am doing it over space, here I did partial differential I did it over time, here I am doing the partial integration by parts, I am doing over space. Here, I will have 0 to l, so you can put it bracket here, delta by delta x of E I dx dt, this is the first then, again I will go and do the integration by part to the second term, because this d over dx del W is there.

So, this will be, I will write it t 1 to t 2, this term will remain as it is, E I del square W over delta x square delta of delta W delta x 0 to l minus, I will have this is integration by part again. This will be delta over delta x of E I del square W by delta x square delta W 0 to l and again minus and minus, this will become plus, plus integral 0 to l del square by delta x square E I del square W by delta x square delta W dx and then, you have dt, close this bracket. So, now you have three terms here and you got here one term, delta T, this is delta V and you have to write delta W e.

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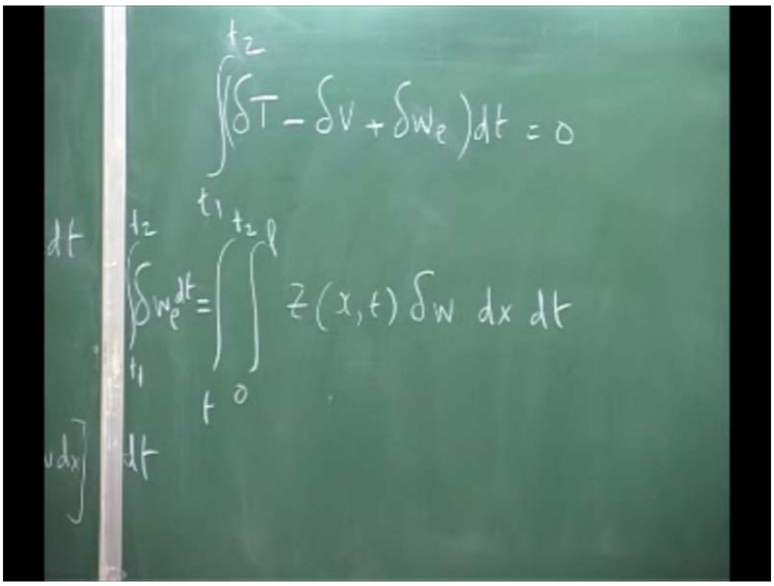

Delta W e is fairly straight forward term, which is integral 0 to l, because we said the loading is, loading does not change during the virtual displacement, this is delta W dx. And you have t 1 to t 2, this is dt, this is delta W e integral t 1 to t 2 dt, this is this expression. Now, what you have to do is, you have to combine all the terms and when you combine, I will write the full expression so that, it becomes clear.

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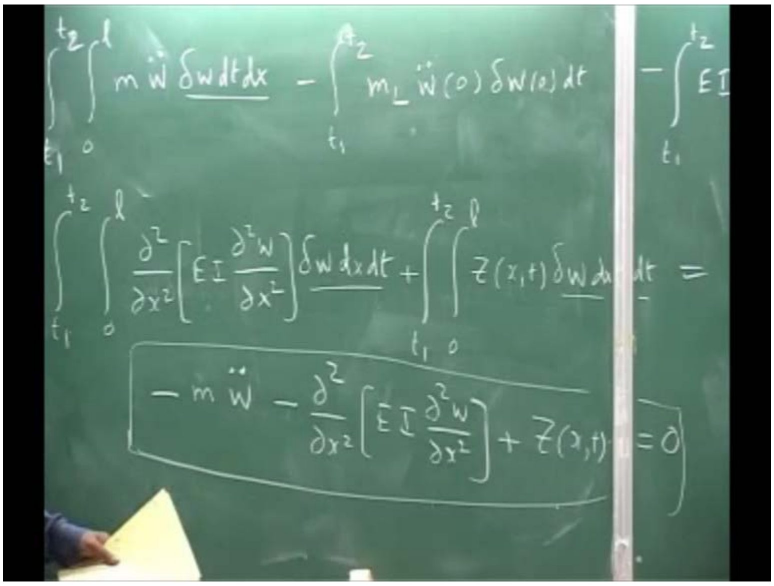

So, what is my first, maybe I will start from here, that way it is easy for you to write it, please understand, I have to start from delta T d t, that term is minus integral 0 to t 1 integral 0 to l m double dot delta W, this is the one. Then, I have this expression, minus integral t 1 to t 2 m L W double dot, this is 0 then, I have to add this, add mean I have to subtract. When I subtract, integral t 1 to t 2.

Now, I need to take this term, put it here along with this, what I am going to do is, because that t 1 to t 2 is here, I will again write t 1 to t 2 E I del square W by delta x square delta of delta W by delta x, 0 to l and this integral is over dt. So, I am having dt here, only dt and the second term, this minus and this will become plus. So, I will have plus delta by delta x, you can have a integral also, t 1 to t 2 E I del square W by delta x square delta W, again 0 to l d t.

Now, this term if you look at it, is has a t 1 to t 2 0 to l and this, so I am going to add that only here directly, I will put it just below expression that will become a minus sign. So, I am putting a minus integral, 0 to l del square by delta x square E I del square W by delta

W dx dt and then, the last term also have two integral. So, I am going to put the two integral also, that will be plus, so I am putting plus, integral t 1 to t 2, integral 0 to l z x comma t delta W dx dt, this entire term is equal to 0, because that is what the Hamiltonian principle, because substituting all equal to 0.

Now, what we have to do is, we need to collect the terms and then, write our equation, while collecting the terms, you see all of them have integral t 1 to t 2, 0 to 1, t 1 to t 2 and you have dx d t delta W, delta W is common for all these terms. Now, delta W is function of x and t, and this is the virtual displacement, virtual displacement is arbitrary. So, along the whole 0 to l, that path you can have fix a time, you can have a variation then, I can set that terms within the bracket of delta W equal to 0.

Because, this variation you say right hand side is equal to 0 then, we have, what happens to these terms and this term, because these are variation at a particular, what is that time, because this is at location 0 at time t 1 to t 2. Here, you will have again variation, location 0 and t 1 to t 2, so you will find that, these are only a specific points. Whereas, this is, I can vary over entire domain which means, if everything has to be 0, the domain part independently must be 0 then, I can have independent variation in delta W over the entire domain 0 to length l.

Therefore, you write your equation, this is combining all, you will find minus m W double dot minus del square by delta x square E I del square W by delta x square, this is the second term then, plus x comma t, this is 0 independently. In other words, this is valid everywhere, you always says 0 full in the whole domain, because delta W is arbitrary in the entire domain, so this term is z equal to 0. But now, let us look at this, these terms, this is the variation of W at location 0 that is, the left end of power x equal to 0. This is variation of my slope 0 or l that means, these two are different, whereas this term if look at it, this is delta W. One of the term is at 0, another is at l which means, these are over delta W, each one of them must be independently may be equal to 0.

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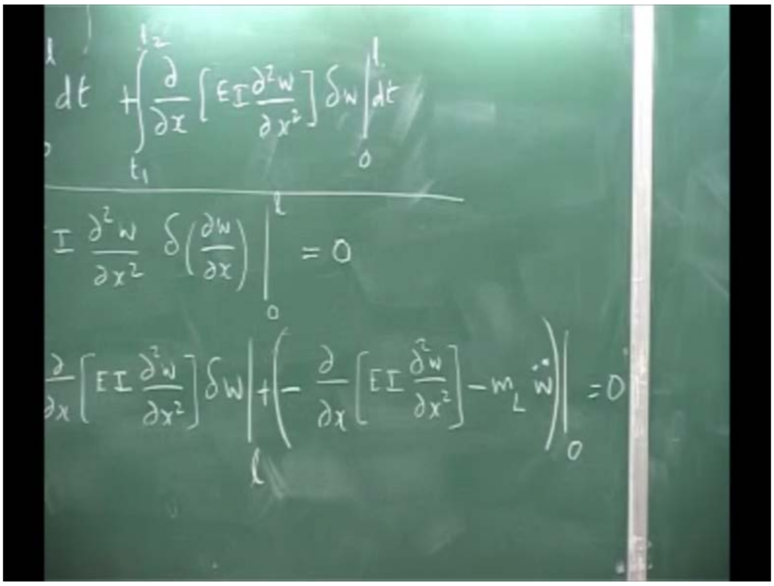

That is, let us take first this expression, this expression will have E I del square W by delta x square, variation of delta W by delta x at either 0 and at l. This is, now the question is, you will specify what type of condition I am having, that is where sit comes, that very, very important. Now, when you write this expression, this is again what you will have, you have plus delta by delta x of E I del square W by delta x square delta W. I am putting only first term l, please understand I am not putting, this is only at location l I am taking.

Then, location 0 I am writing separately, location 0 will be minus delta x E I, location 0 is also, this is location 0, so I will have minus m L W double dot, this is that location, this entire term if you want, you can put a plus sign and minus, make this whole thing as one term and location 0, this is the location l, this is the location 0. Now, let us look at it little bit carefully, what our original beam is and how we obtain this boundary conditions, because that is the key to the entire problem, this is my field equation.

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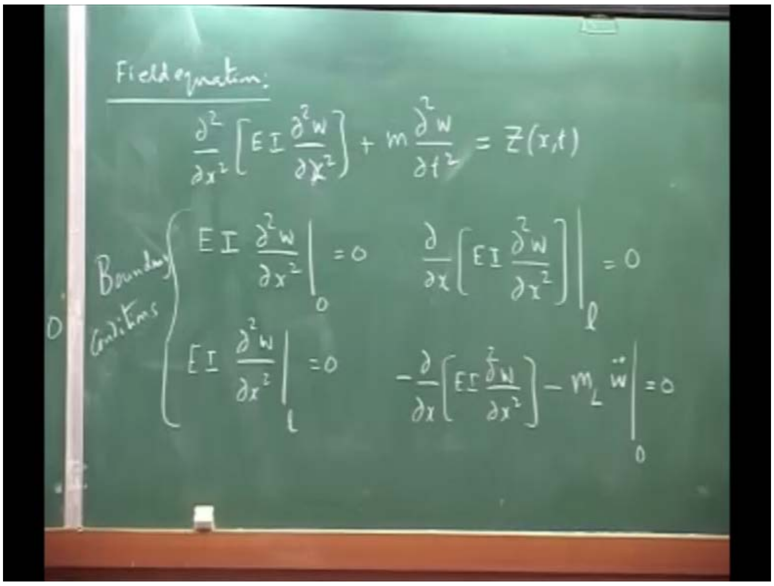

I will write my field equation, which I always specify as field, this is I can write del square by delta x square E I x square plus m double dot. If I want to represent it in partial, it in some books they will write it delta W by delta t square equals z x comma t, this is my field equation. Now, you go to the boundary condition, what the given problem, the given problem was, we put a large mass m left side and then, we add the beam.

I have not restricted the motion at the left end which means, I allow the motion, if I have that motion. Slope, I am not restricting the slope, I have having displacement, I am having slope at a left end, I am having displacement and slope at the right end also which means, only this quantity. This quantity is, I can have a variation that means, what should be 0 is, E I del square W by delta x square at 0, equals 0 as well as E I del square W by delta x square at l equals 0.

Because, l also you can have this a free end, this also free, I am not restricted anything, now let us come to this particular term. Here, I miss one term here delta W, there must be one term I missed here, I think delta W I have to put, I will write a delta, I have to put a delta W. Now, let us look at, this is at l, l means that is this center, at this end my delta W that is free, I can have a variation in this that means, only this term has to be 0, which is del over del x of E I del square W over delta x square at l is 0.

Now, let us come to this, left hand side delta W, this is 0 condition, it is free to move that means, this term must be 0, which is I am going to put minus delta over delta x E I minus m W double dot at the 0 location. Now, these are 1 2 3 4, my four boundary conditions, so I called them this as boundary conditions. Please understand one thing, here I want to emphasize that, the field equation that is, this equation will be same even if I change my boundary conditions.

Suppose, for the sake of explanation, I am saying, I have made it into fixed, there is no m L now that means, my m L is gone. Now, this is the fixed boundary condition, fixed boundary conditions when I go, I will first start with here at the 0, because this is at l, this free. So, I cannot any slope that means, this must be satisfied at l, but at 0, whether this is the condition or some other condition, at 0 you know that, you are not allowing the motion and it is actually cantilever. So, you do not allow rotation that means, you will say, slope is 0, instead of this condition, bending moment is 0, you understand.

So, your entire condition will change, slope is 0 here and at this point, so whichever is 0, when you come to this point, any way right hand end it is the same, shear force is 0. But, when you go here, you will not apply this, because m L is not there, so you do not put, you will look delta W, I do not allow displacement. So, for a cantilever beam, my boundary condition will completely change, so I will write that a boundary condition just for clarity only.

So, you find depending on the problem, the boundary condition will completely change, if it is a free free beam, you will have one other boundary condition, I can put a spring here, it will have another boundary condition, you follow. So, depending on the situation, if it is simply supported, I allow another. Because, there simply supported means, I do not allow displacement by allow rotation, you will have another boundary condition.

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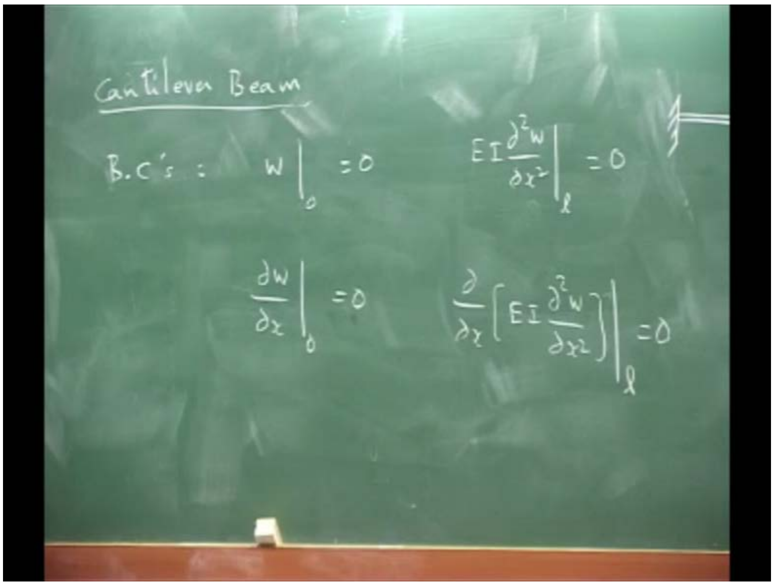

So, the interesting part is, for a cantilever beam, boundary conditions that will become displacement will be 0 that is, W at 0 is 0, delta W by delta x at 0 is 0, the slope is here. I have put restricted and then, you will have E I del square W by delta x square at l, this is 0 and delta over delta x of E I del square W by delta x square at l, 0. So, you see, these are the boundary condition for a cantilever beam, these are the boundary condition for a beam with the, this is one.

Like that, you can have free free beam, free free beam once you leave it, you are not restricting that means, the same condition will come on the left side, you understand, bending moment is 0, shear is 0. Now, the solution to the problem, that is where the next step is, because you obtain the equation, please understand Hamiltonian principle will give you the equation, which is valid for the field. And beauty of this is, it also gives you boundary conditions, boundary conditions for that given problem.

Why I choose this m here is, one mass at one of the ends I put it, because later when I derive very general principle for solution of this type of problem, if I put everything 0 0, you will find that, I will not be able to bring out the generality. Whereas, if I put a mass at any end, left end right end anywhere, I can give a general expression for one formulation. How do you get the solution for that, because this is where the, these are you can collect the boundary value problem, but these are partial differential equation, please understand.

This is the partial differential equations based on time and then, how do you go for solution. These were we learn, how to solve and these are operators, because these are differential operators. We will first to do some separation of variable, convert this problem into one ordinary differential equation. First understand, these are all PD, that is why when you derive with Hamiltonian principles without doing any approximation, later we will do use the Hamilton's principle.

But, you will get the ordinary differential equation, that is another way of deriving it, but that part I will teach later. Next, so you understand how do you get the equation of motion along boundary condition, that is very, very essential for our aero elastic problem. First what is the equation, that in this structural equation, because please understand, this as the structural part, this also has the inertia part. Now, this is the only part, which we do not has on toady, in the sense this is the external loading part, which is the aero dynamics part.

That part we will derive separately, first we assume that, that is somehow known, how do we get the equation of motion, number 1 and what are the various solution methods. Because, please understand, you cannot get a solution for this problem exactly, because exact solution are available only for very few special cases. You cannot get a exact solution for all this any kind of problem, that is why you need to learn approximate methods of obtaining the solution of the equation. Now, you want to wait a minute, I will erase this part then, we can start.

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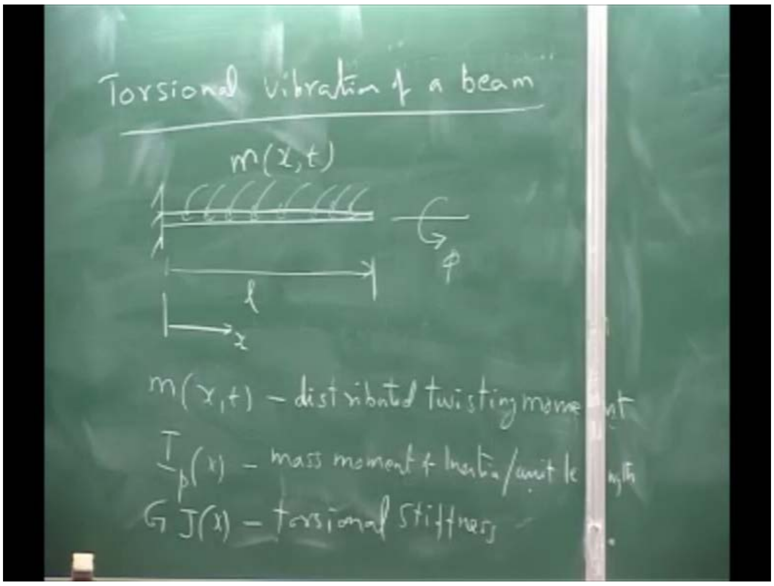

Now, we will derive the equation of motion for a torsional vibration of a beam, here again we can, that is why I said any boundary condition. You first when you describe the problem itself, that comes what type of boundary you are having, I can put a mass here, the term inertia. Let us consider a beam of length l and this is acted on by, this is my motion I am describing, torsional motion this and there is a distributed moment, which I call it m.

So, this is my x please note, here I am using the symbol m for moment not for mass, this is the distributed twisting moment. And then, I will write I p which is mass moment of inertia per unit length, I is mass moment of inertia per unit length, all are per unit length also, this is the distributed moment. And then, I need to have similar to bending stiffness, I have a minus torsional stiffness is, you call it G J, which is also ((Refer Time: 47:56)) function of x, torsional stiffness. Now, again you go back and then, apply Hamiltonian principle. You need to write the kinetic energy, you need to write the strain energy and you need to write the ((Refer Time: 48:20)) external moment.

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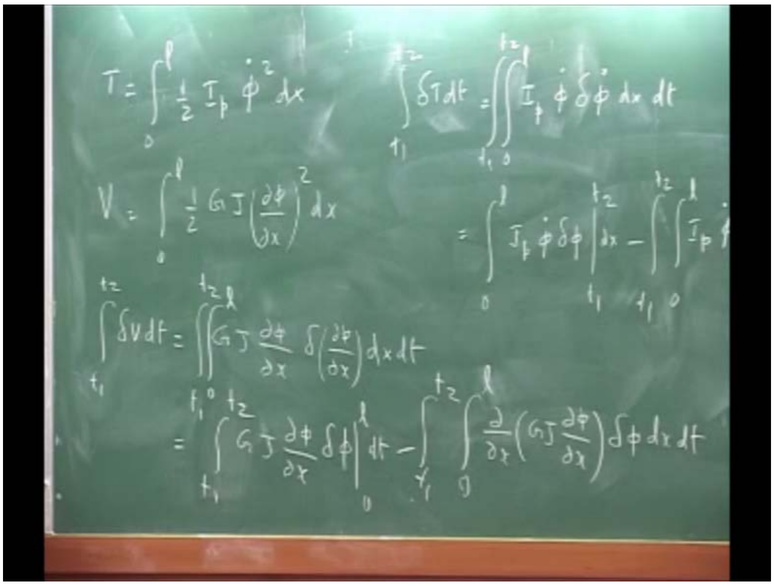

So, your kinetic energy expression T will be l half I p dx and your strain energy half G J delta p over delta x whole square, this is my sinbonaz torsion problem. I write my strain energy expression, that is why all this thing are initially under the what is the kind of sinbonaz torsion. Please understand, J is torsional constant, because we are dealing with aero foils, aero foils are not supply section. So, J is defined as the torsion constant, it is not that polar moment or something like that, do not have that confusion at all.

That is why, we write G J as the torsion constant, J is the torsion constant for the given cross section. Now, if you are given a single cell box beam, if it is the box beam, because my aero foil I can define like a box beam, single cell or multi cell, please understand. I can do it like this, I have to get the torsional constant for this, I perfection, multi cell box beams, is what you learnt in structure one course, if you have not learnt it, you better go back and then, learn that.

For simple problem, you can take a single cell box beam then, you say as though the entire wing is only thin wall, only one cell, for that you can apply the ((Refer Time: 50:48)) equations and get torsion constant. This you have to know, because how do you get the torsion constant is, you should know that. Now, let us apply the Hamiltonians principle, so first is we will write integral delta T dt t 1 t 2, I will substitute here, this will become t 1 t 2, 0 to l I p phi dot delta phi dot dx dt.

Now, I integrate by parts again, if I integrate by parts, it will become 0 to l I p phi dot delta phi t 1 t 2, there is a dx here. Then, minus integral t 1 t 2 0 to l I p, this will become phi double dot delta phi dx dt, may be I erase this part not necessary, let it be there no problem. You obtain for the delta T, similarly you go and write for delta V, you will have t 1 to t 2 delta V dt is t 1 to t 2 G J delta phi by delta x into delta of delta phi by dx dt.

Again you integrate by part, now with respect to this then, you will get the equation integral t 1 to t 2 G J delta phi over delta x delta phi 0 to l and there is a dt, you will sitting here. Then, minus integral t 1 to t 2 integral 0 to l delta over delta x of G J delta phi over delta x delta phi dx dt. Now, as usual, I will erase this part, the external twisting moment that delta W e will be just m x del phi, that is all.

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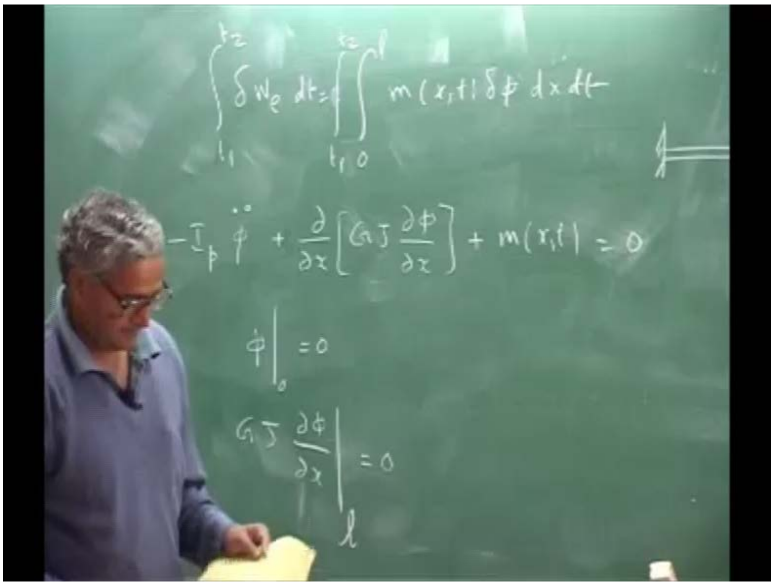

So, that you will write it as, this will be t 1 to t 2, now you combine this term, this term and this term, combine all the three with proper sign convention. Then, you will have the field equation, will be phi double dot plus delta over delta x of G J delta phi over delta x, because the first term is minus I p phi double dot that is, the minus sign and then, T minus V, so the minus and minus will be plus, plus m x comma t. This is 0, this is my field equation and then, boundary condition you go, you know that, here anyway t 1 t 2 initial, del phi has go to 0, therefore this term is out automatically, t 1 to t 2.

Whereas, when you come here the boundary condition, for the given problem we said that, the left end is fixed, fixed means, at 0 I do not allow any variation in the displacement, therefore phi is 0 at the left end. So, phi at 0 is 0, but when I go to the right side that is, the length l, at this point is a free end that means, what is 0 must be this term, which is actually G J delta phi over delta x at l equals to 0. That means, this is the fixed end, this is free end suppose, if you put a mass, that is what I am saying, if you put some disc or something like that at that end, that will have it is own inertia, that kinetic energy.

Then, that kinetic energy expression has come here then, that will come as a boundary condition. So, you will find that, depending on what you put where, the boundary condition will change. Suppose you say, I am just going a little further, you may have some problem like this, in an old aircraft, you are supporting by a some structure here, somewhere in the middle then, how will you model this. All this problem you have to think, because you are modern, today everything is a cantilever beam all the wings structure.

In old days, they put a strut and then, attach it that means, there are two ways, one is you can break it into two parts of the beam and then, match the boundary conditions there this. Or you can say, this is like a delta function, which is an external load which act at that point, additional external load. Today, you know that, most of the aircraft, they carry a what, engine, may be it does not hang down. Engine is here, engine is bounded on the wing and that mass is sitting there at concentrated mass at some particular location.

How will you treat that into account, there are two ways of doing it, one is you put it in the kinetic energy expression, use it as the delta function in your m. Please understand, here this m is mass per unit length it is going, but there is another heavy mass, which is sitting at some particular location, what happens is, at that location ((Refer Time: 59:35)) suddenly. So, either you can put it like a delta function into the whole function problem or you may have to split the beam into two parts then, put one end that mass, another end you start at the wing.

So, the problems get little confused, you cannot solve that so easily, depending on you put various thing. Now, this is where you learn the approximate method, because you cannot get very close form solution. Suppose, you have a beam, which is supported at several location, that is the multiple supports. You can apply that piece ways these equations, substitute the suitable boundary condition at every point, that is where the problem is verym, very important.

How you formulate your entire equation of motion and the boundary condition, we have taken only. Because, why I did bending and torsion, because we need that, because the wing can then, wing can twist, so that is why, bending torsion problem we have developed it. Now, the interesting part, some of interesting thing between these set of equations if you see, here the order of the equation, order in space derivative, time derivative is always inertia that is second derivative, space derivative here is 2, second order.

Whereas, in the bending problem, the order of the derivative is 4, so you having a higher order problem. Now, if you want to get a solution for general, because here I am going to describe all these problems in a general fashion, how do we do it so that, all of them are brought under one category of problems. So, for really analyzing these problem, because please understand, I have an external loading which is sitting here, moment there I have if you say lift, that lift is there, this is the moment on the wing, both are sitting at the problem.

But, how do I solve this entire problem, because I have a time derivative also, suppose if it is a static problem, only wing deformation under the load, I am not bother about the vibration. That means, this is gun and there also, this is what you had your bending of beam static equation is, this is set of terms, this term is 0. So, you find Hamiltonian principle gives you even the static problem along with boundary condition, only thing is all the partial will become total derivative, because there is no time variation brought in the problem.

Now, the question is, if you have time, because static problems I taught you once you have learnt that part, ((Refer Time: 1:03:12)) method of obtaining approximation solution. Because, you cannot get solution for any E I variation, because please remember here E I is function of x, similarly G J can be a function of x. So, when you want to solve static aeroelastic problem that is one set, when times derivative is on, you solve the ordinary differential equations.

Whereas, when you go to differential problems, these are partial differential equations, so what we do is first is, we do free vibration problem first. What are the free vibration characteristic of beam in bending and torsion, that is the first step. And then, using that solution, please understand using that particular solution, you convert them into ordinary some... Actually when you do separation of variable in the free vibration, you do it and after that, convert it into ODE, Ordinary Differential Equation.

Now, that is where, you get a free vibration problem, first you have a partial differential equation, this partial differential equation you convert term into ODE. And then, the ODE is, now you have to know how have to solve that ODE, you follow. And then, because that is not your only aim, because you need to find out what will happen to the beam when you have a external load also. Then, that is a response problem, but for studying the response, you need to first to know, what is the free vibration characteristic of the your wing or your entire aircraft.

Because, that is essential to get to the response problem, so you need to study, how do you get the free vibration solution and that is where the first PDE is converted into the ODE. Then, come I told you, what type of operators that in Sturm Liouville's theorem, what it says, if the equation is at this form, you will have solution of this type, you understand and that is where that equation is applicable. But, that is derive for only second order system, but you will find that also.

So, that application if you see boundary value problem in any book, differential book differential equation and boundary value problems, they will deal with that. You can solve, they will give a solution for second order, but you will also apply to fourth order. There are several condition, orthogonality of the that function, etcetera, those are generally proven for a most general problem.

They will not be very specific to bending problem or specific to the torsion problem, that is why what we will do is, next class we will have a general problem and then, write the equation, you will have an idea, how those are written. So, first I will just briefly give you one introduction to that, let us take this equation free vibration, free vibration means I do not have this.

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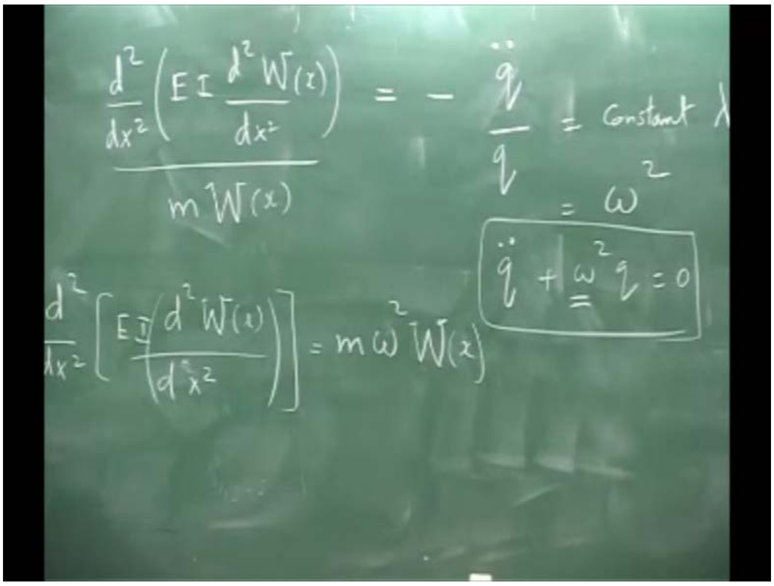

So, free vibration of a bending problem, because I just want to indicate one thing and then, free vibration of a beam. You write it, z of x t is 0, so I will have my equation E I del square W plus m is 0, of course boundary condition will keep them at there, later we will substitute that. First, I am going to assume that, my W which is the function of x comma t, I can write it as from something like this, this is what separation of variable. I am writing as a assume solution, I will go on substitute here.

When I substitute in this place what will happen, this is a all partial, now this is only a function of x that is, the function of t. So, I will have this term q t into, I will have d square by dx square please understand, into E I d square capital plus, here I will have because this is a... So, I will put m q double dot W of x 0, now what I do is, I take this terms to the right hand side and then, collect all the space dependent term on one side and then, time dependent term on the other side.

If I do that, I write it here itself to that it is easy, so I am now collecting, I keep this term, I will divide by W x, I take double dot that I will take it separately. So, you will have d square by d x square E I d square W by d, you can put equals, I am dividing by m, because please why I am taking m here, m is also function of x, that is why I brought it here and I will put this as minus q double dot over q. Now, you see this is totally space, this is totally time, if these two or equal to each other, it should be equal to some constant.

So, I will write constant and I am going to use it as, that constant as I am going to put lambda, constant lambda. Now, you know that, I will look at only this equation please understand, I am looking at this part of the equation, it says that, minus q double dot equals... Now, I can bring it this side, this is the second order differential equation in time, so this equation is q double dot plus lambda q is equal 0. Now, this will have solution, suppose you have to see, whether lambda should be positive and lambda can be negative.

Suppose, and what should be the nature of lambda that is the key, this comes from physics, you can always write q equal to some q bar e power s t then, your characteristic equations will be s square plus lambda equal 0. So, you will have characteristic equation, s square plus lambda equal 0 or s equal to this is going minus what is that, minus lambda under root, so you can put it two roots s 1 comma 2 in plus minus. Now, if lambda is positive then, it is an imaginary number, that means my roots are imaginary.

Imaginary roots means, s will be i plus minus i, which you can write it as sin and cosine function, which are oscillatory function. On the other hand, if lambda is negative then, what will happen, negative and negative this will become positive then, positive under root, you will have one of the positive roots, one negative root. But, positive root what will happen, it will take increases time that means what, your solution is going to blow up, but nature in beam or anything, is nothing, it just vibrate.

Therefore you say, my lambda has to be a positive constant, that is the reason we will now write, instead of writing lambda, this is the proof normally this is given, because what should be the lambda positive, negative, etcetera. So, we always say, instead of writing lambda, you will put it as omega square, because you know square is always positive. So, q by q dot, now you will write your equation, this is my a sin omega t, etcetera d cosine omega t, this is the harmonic function, time varying excellent solution.

But, the next question comes, what should be my omega, you say that, this is the positive number, what should be my omega, whether it is valid for all omegas, because positive number means, you can have any numbers, what is the condition. That particular solving for those omega, which satisfied my problem, that is called the differential Eigen value problem, because you are now going to this side. You are going this side, because I know this is equal to omega square, that is your differential Eigen value problem.

I can write it as d square by dx square E I d square W of x over dx square equals m omega square, this is my field, this purely a ordinary differential equation in space. Now, let us look at the boundary condition, we will go here, we have again substitute W, because we know that, at root equal to 0 moment, so q of t, that the time varying function that will go up. So, you will have E I d square W by dx square W, because these are space derivative only, this is also space.

So, wherever you will have, you will have capital W then, you come here, you will have space, there is W double dot, here you will have what is that, omega square because you have to differentiate, because time derivatives comes, q W dot will be there. But, q double dot is going to be omega square, what is that, minus omega square q. So, what will happen, that omega square term is also sitting in boundary conditions, that is why I use this particular example to indicate, this omega square can be, it is in the field equation, it is here, it can also sit in the boundary condition.

Whereas, in this boundary condition, it is not sitting, there is no omega square here, this boundary condition does not have, this not have. These are just homogenous continuous boundary condition, directly equal to 0, there is no omega square sitting in, that is why the boundary conditions are also split into two types of boundary condition. One, there is no omega square present, another one there is the omega square. Now, these two are different please understands, because when you actually solve the problem, you have to that orthogonality principle everything that has to come, otherwise several books will simply deal with only this part and a rest of them 0.

But, some books explain that, you have to take the omega square, that is why splitting the boundary condition into two parts. And this is what is, I will just generally mention the form of the equations please understand, now this is not necessary, I will just write the equation, after that we can, I think this also varies.

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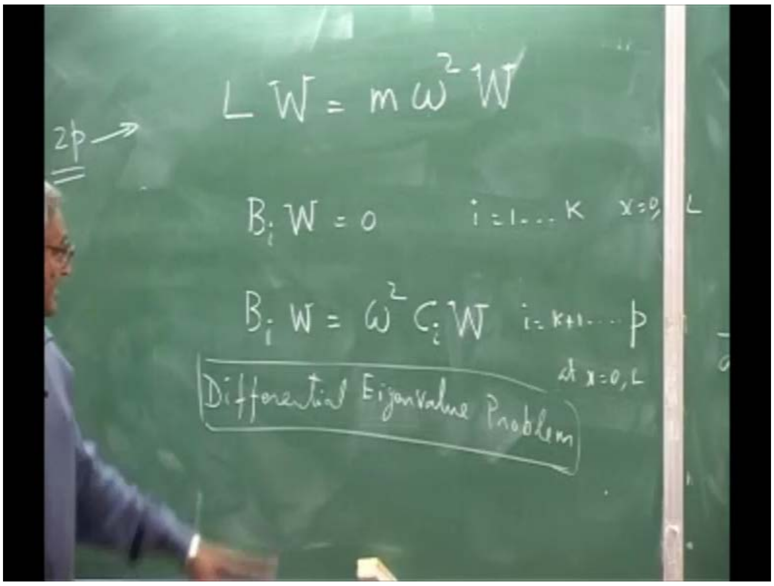

So, you will write it as L W equals sum m omega square W, with these are all function of x please understand. And then, I will write B I 0, this is one type of thing, another one is B i W equals some W, C i contains that mass. Why I have written is, this is you can have this boundary conditions of this type or omega square also can be sitting. But, how many boundary condition you will have, what is the order of differential equation, that is why we always say, this is 2 p order.

2 p, p can be 1 2, if it is 2, that is 4, if it is 1 second, that is what torsion problems and you must have that many boundary condition. So, you can split into this part, so boundary condition are of this type, some boundary condition are of this type. We can

write it as, this is i running from 1 to k and this is again i running from k plus 1 to p, but I will say at x equal to 0 and l, and at x equal to 0 and l, the 2 p, total 2 p, this is how I write my differential.

You can say, this is a linear differential boundary value problems of order 2 p that is, the linear operator, this is the linear operator, you may call this as a differential boundary value problem or differential Eigen value problem or boundary value problems, anything you can say. Because, that is the boundary value problem, now you have converted into solving omega square, that is why you may call it as a differential, you can put it even Eigen value problem, this is the general form. Now, we need to analyze what is the type of operator l is, here it is the fourth and this is where adjoint operator, self adjoint operator, all those thing will come into the picture.