Aero Elasticity
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Lecture – 4

Last class, we saw the constraints, suppose if we write the general constraint equation.

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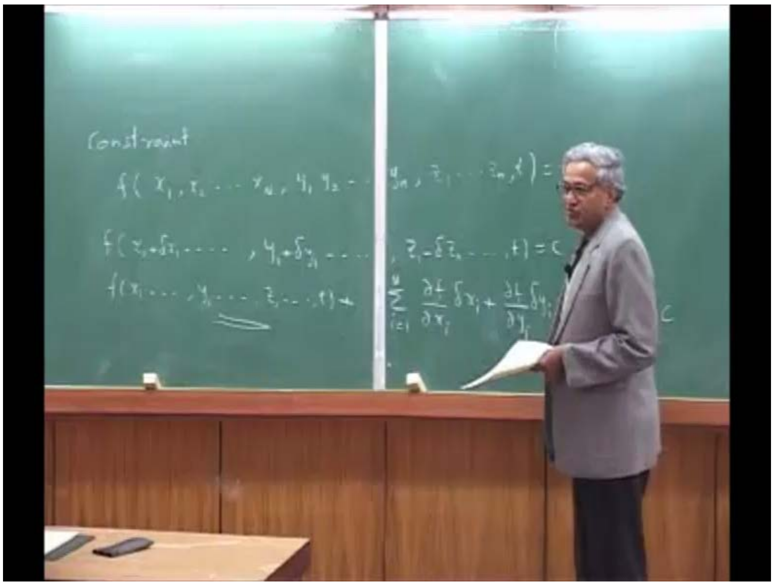

Constraint as a function of the coordinate of y 1, y 2, y n comma z 1, z n comma t is some constant, is the polynomial constraint. Now, when you have a virtual displacement, because we need to find out what type of the virtual displacement you can give, because they must be consistent with the constant relationship. If you give a virtual displacement, the same relation is going to become like this, delta x 1, so on and so forth comma y 1 plus delta y 1 comma z 1 plus delta z 1 and this is the key, at time is kept same.

You do not give a virtual increment to time, because what we are saying is, at the same time, if there is varied position, what that constraint will look like, so the time is frozen and you give a virtual displacement. Now, you can expand this by tailor series and keep it as a first order term, it will become f of x 1 comma y 1 z 1 comma t, plus you will have the summation i running from 1 to N delta f by delta x i delta x i plus delta f by delta y i delta y i plus delta z i delta z i.

And only first order, because virtual displacement is a small displacement, so higher order term we are neglecting. This is essentially C, because even in the virtual displacement, the constraint relation does not change. Now you see, this term is same as the first term, this must be 0.

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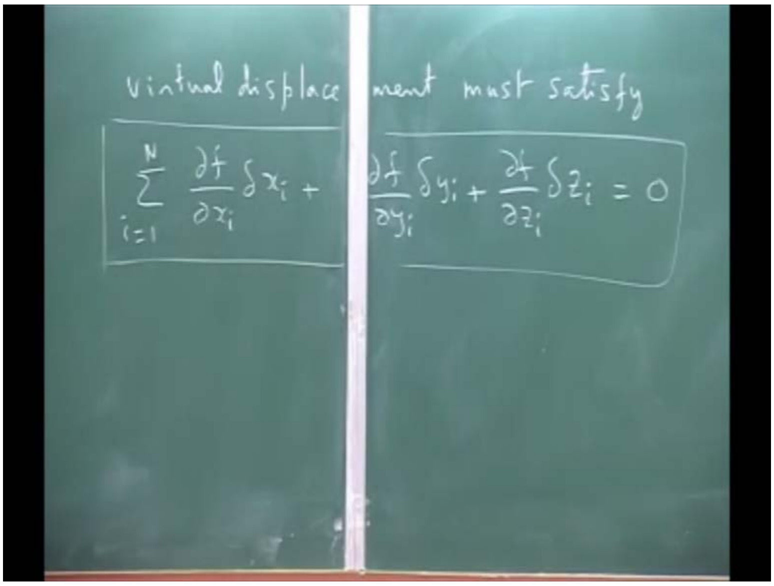

So, the virtual displacement must satisfy, so I will write that, must satisfy the relation summation i running from 1 to N delta f by z i delta z i, so this is the relationship. Now, I will just show by small example, what that relation really mean, it is a very simple example, I just want to show that so that, you can have the understanding of, what we mean by this relation.

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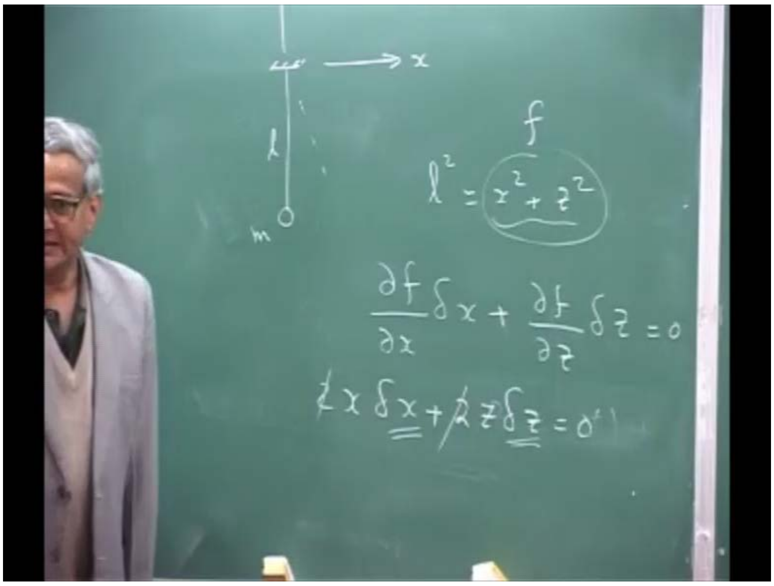

We will take the very simple problem, where we had the mass like a pendulum, this length is l and l is constant. So, if you say, this is my x and this is my z, any position for this mass, your constrain relation l square is, because the length is constant, this is the constraint which is similar to f of x 1 to x n then z. Now, if you vary, let us apply that, because our function is this, this will be delta f over delta x, because there is only one variable, so you will have delta x into delta x plus f over delta z, this is 0. Now, substitute this is our f, if you substitute delta x by delta x, this will become 2 x del x plus 2 z del z equal 0, 2 2 cancel out. You now know, I cannot give independently any virtual displacement if I treat one of them as independent, this is the dependent on that.

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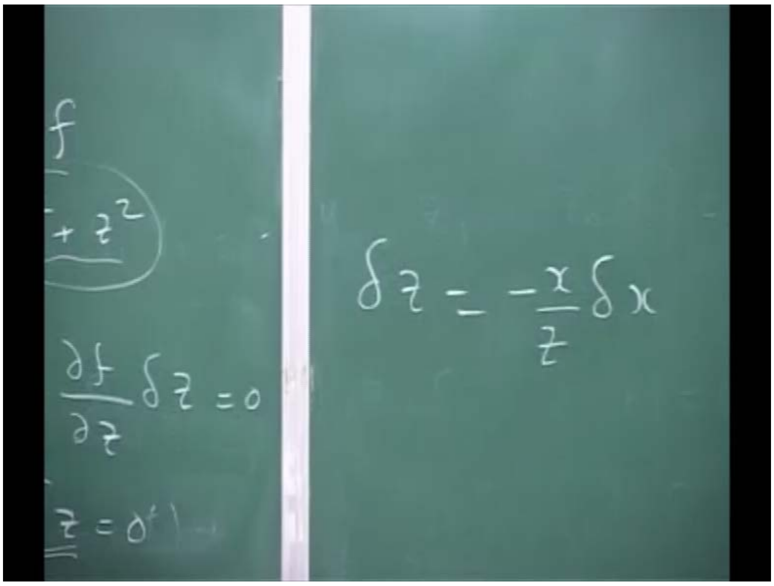

So, you may call it my delta z is minus x over z delta x, but it really means is, suppose if you take this particle, because this length is constraint, it can only move this way. It cannot go anyway which means, this, this, this is x displacement, this is z displacement, this angle is theta. So, this angle is theta, this angle is theta, you know theta is x by z, you can tan theta with the minus sign, because it is negative. So, you will find that relations essentially tells, I can give a virtual displacement only along the direction, which is perpendicular to the stream, that is all.

That is what, this relationship tells you, because you that, this will be del x, this will be del z, they cannot be arbitrary, because you can give only along this, that is why x over z del x, that relation. So, if you complete this triangle, you will know that, if I give del x, what should be my del z, that is all. So, essentially the constrain is imbedded in this relationship, now this is the basic, because this is always you must understand, when you give a virtual displacement, it must be consistent with the constrains and that has a relation, virtual displacement must satisfy that equation. Now, let us go to the situation of a energy virtual work done.

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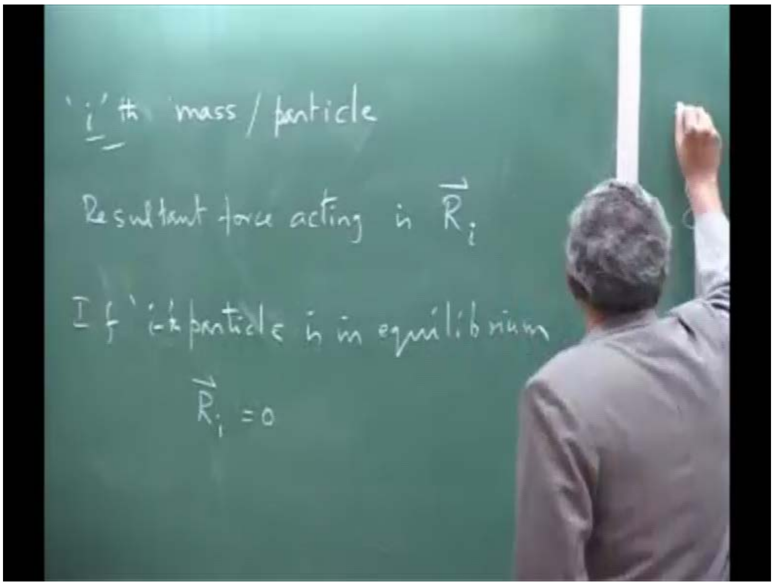

Let us take one particle, one i th particle, i th mass or i th particle anything, because we are going to have n particles. So, we can say i th mass or i th particle and the resultant force acting on this i th particle is, i call it R i. If this is in equilibrium, if i th particle is in equilibrium then, R i is 0, so R i automatically becomes 0.

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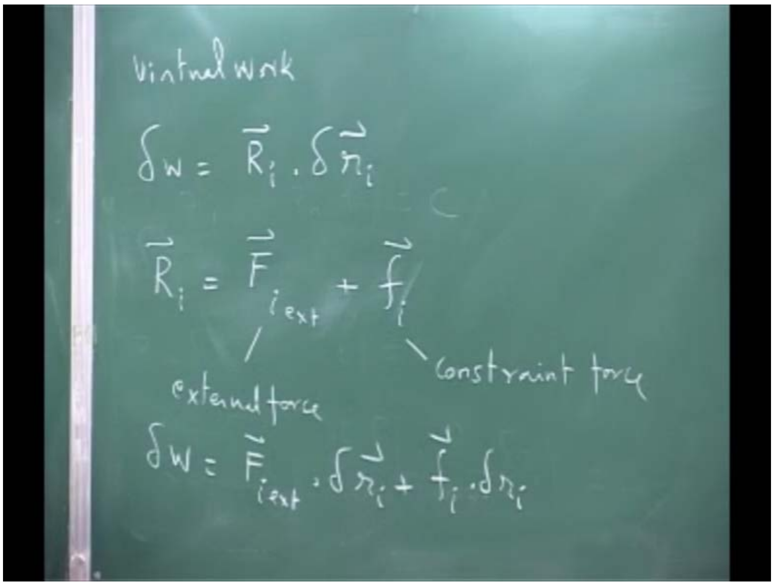

And the virtual work done, in moving this particle virtual work, I am writing virtual work, delta W will be R i dot, this is the virtual work and this is 0. But, now you go and then look at, how is my R i, what are the contributions. R i will have a an external load, external force and the constraint force, because you can have, like the simple pendulum case, there can be a gravity which is external load and then, the tension that is, the constraint force.

So, you will have, this R i can be split into F i which is external plus some another lower case f i, which is constant force, this is external force. Now, if I substitute this here, I will have del W as F i external dot delta R i plus f i dot delta r i. And you know that, if it is equilibrium, virtual work is 0, this must be 0 equilibrium, but you know that, this term constraint force cannot do any work.

Because, you can only have virtual displacement, which must be consistent with the constraint that means, the virtual displacement has to be normal to the constant force, therefore this will not do any work. So, this term is 0 and what will come out as the virtual work principle now is delta W is F i dot delta r i equal to 0. F i dot, F i is not 0 please understand, whereas here resultant force R i is 0, because the body is in equilibrium. Whereas here, please understand F i is not 0, what is 0 is, the virtual work done by the external force.

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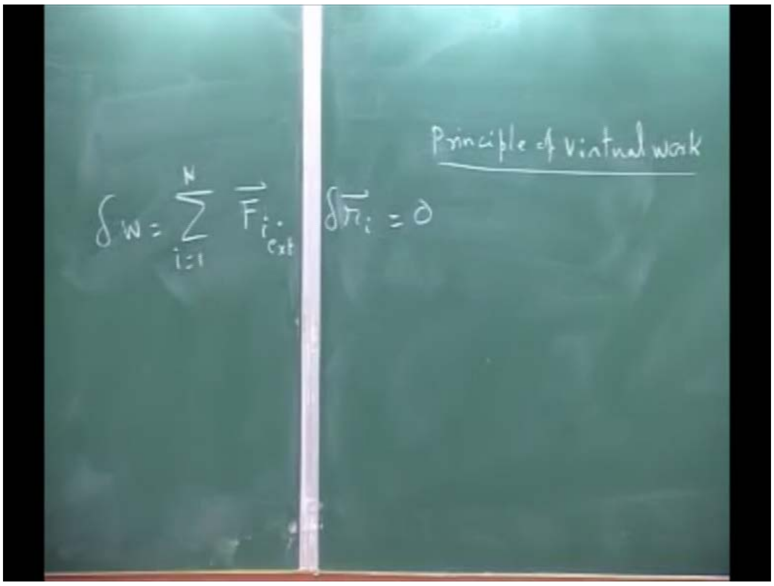

Now, if you have several particles that is, we said that, there are N particles, if you have N particles, you will have delta W, you just put the summation, summation same relationship hold. So, you will have F i dot delta r i equal 0, i running from 1 to N, this is my principle of virtual work for static equilibrium. Now, comes your, I will write it, this is the principle of virtual work, suppose we have a situation of dynamics, we are going through dynamics, what happens in dynamics, the particle is acted on by...

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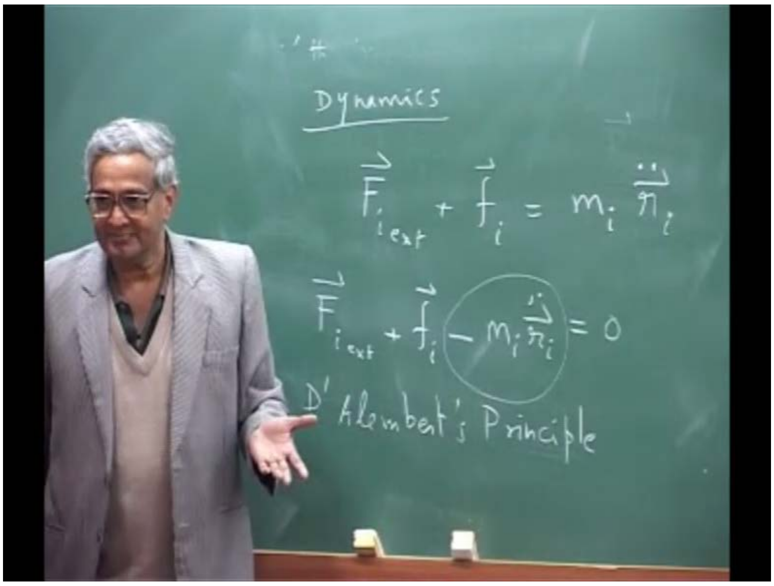

We have the particle is acted on by, this is the dynamic situation, so I am writing dynamics, you will have an external load F i external, possibly if you want, I can use the word external also. It is not a constraint force F i, but most of the book simply say, F i that delta r i is 0. F i external plus you will have constraint force and the particle is moving. So, you will have m i, mass of that particle is m i with r i double dot, double dot is second derivative with respect to time in mass time acceleration for the particle.

Now, you see this is where the D' Alembert's principle comes, this equation is converted into a similar to static equilibrium equation. In the sense, you bring this term to the left hand side, left hand side if you bring, external f i minus m i 0. So now, this particular term, I am calling it as an inertia force and this is called the, this particular thing, transferring m i r i double dot to the left hand side and then, calling it as a force. As though it is a additional force, this is called the D' Alembert's principle, call it as a force.

Now, summation of all the force is equal to, this is what we said r i equal to 0 like a static equilibrium. So, this principle is called the D' Alembert's principle, it looks like very simple, but is very meaning in that. What I am doing is the dynamic situation, I am converting into an equivalent static situation by calling this term as the inertia force. And now, I simply go and apply my principle of virtual work, where summation of R i dot delta r i is 0.

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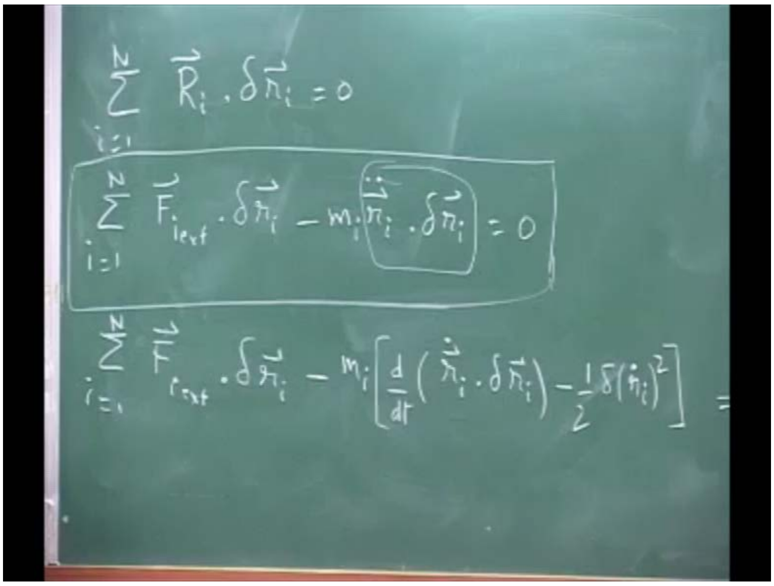

So, I will have what, we had this, what we wrote for static situation i running from 1 to all the particles. I will go and substitute this term there, you know that constraint forces will not do any work, therefore what is left is, summation i running from 1 to N, F i external dot r i minus m i r i double dot delta r i is 0. Because, F i does not do any work, now for this particular term, can be replaced in this fashion.

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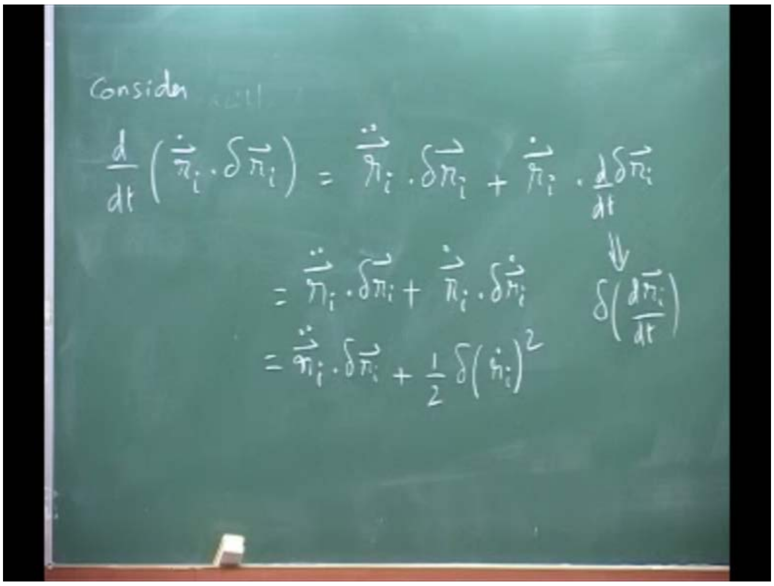

So, what you do is, this is just a small algebra here and you can just replace this term, suppose if you consider d over d t of r i dot dot delta r i. I am just taking the derivative of this term, this is going to be d t of, let me write it, I think it is better to write, this is first r i double dot dot delta r i then, plus r i dot dot of d over d t of delta r i. Please understand, here I am going to make the time derivate of the variation is same as variation of the time derivative.

That is why, the varied part is a small slight change, so that is important thing, you can have a different type of a situation, where these two need not be sent. But, here I am making that assumption that, this term I can write it as delta of r dot, d r by d t that is, this will be right, I am writing it like this, that is why they varied. So, delta of velocity, so this term I am going to write it like this, r i double dot dot delta r i plus r i dot dot of delta r i dot.

This is you can write it as delta of r i dot r i which is nothing but, r i dot, basically square, velocity square, this is the magnitude, you can take it, I will leave it as it is, r i dot square, velocity square. Now, what you do is, you have this expression, r i double dot delta r i, you have here r i double dot delta r i. So, bring that term to left hand side and write this itself as ((Refer Time: 23:47)) summation i running from 1 to N F i external dot delta r i then, I am going to subtract that term, that will minus of m i.

I will have two terms please understand, d by d t of r i dot dot delta r i minus half delta r i dot square, this is scalar and this is equal to 0, because this is equal to 0. I am just replacing this term by this term, I will take it to the left hand side and then... Now, if you look at the individual terms, F i external dot delta r i that is nothing but, the virtual work done by the external forces on all the particles.

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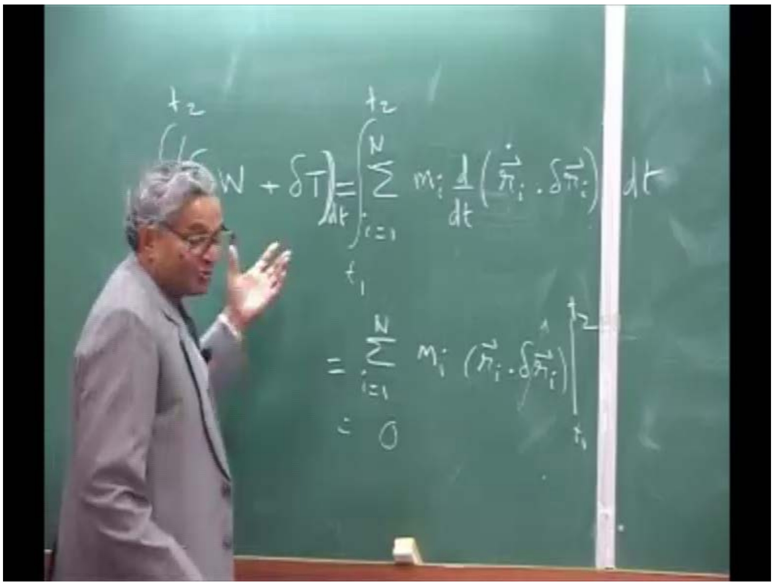

So, I can simply write delta W, virtual work done by external loads then, you go here, minus m i 1 over 2 r i dot, r i dot is velocity, half m v square velocity, that is kinetic energy. So, I have the variation of the kinetic energy, which this is the minus and minus, so that will become plus delta T then, I can transfer this term to the right hand side. When I transfer it, I will have equals summation i running from 1 to N m i d over d t of r i dot dot delta r i.

Now, I erase everything so that, we will discuss little bit about this particular relationship. So, you see what we have done is, the virtual work for dynamics or dynamical system you can say, under dynamic condition is converted into this form.

Now, in this form, it is not useful immediately, so what is done is, this is where Hamilton's principle comes into picture, because dynamic is motion with respect to time.

So, what he said is, he took a internal of this quantity with respect to time, when you integral of this quantity with respect to time is, you put t 1 to t 2 and this all also becomes t 1 to t 2. Whereas, t 1 is the starting time, t 2 is the final time and then, put a d t, please understand you have to put a d t, because you are integrating, so you have to put a d t. So, I am basically evaluating the scalar quantity, this is work done, please understand this is work, this is kinetic energy.

So, these are all scalar quantity, the integration of this, basically the scalar quantity over the time t 1 to t 2 and then, I am taking the variation of that, but what is that variation I am talking about.

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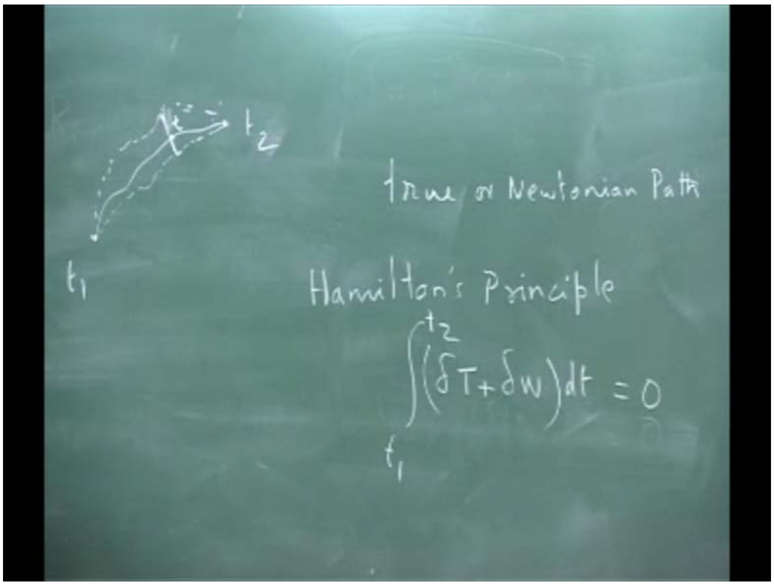

This is basically assuming that the particle, that the system of the particle, they move from system of, this is the time t 1, this is the time t 2, initial time, final time. The system goes along some particular path, if you have, at every time you will have some particular position, that is you call it configuration of the system at that particular time, which is basically described by your generalized coordinates. Generalized constrains are there, you remove the constraints, you have the generalized coordinate of the system.

Every time you have the generalized coordinate and then, this system moves along some path, this path is, in nature it moves and this is called actually true path or Newtonian path. This is what is happening, but if you have a varied path that means, some path which is different that means, time is frozen, the path can be, this is at the same time. I can have this path or I can have that path, that is why time frozen, but it is at a different position.

That is why, in the constraint equation, time is frozen, virtual work when you take, time is frozen. Now, you say that, why the system has to follow that particular path, instead of this path, because among all possible, you have to go from one time to another time in the configuration test motion. It follows some path, that is what we call it as Newtonian path, because that follows Newton's law, F is equal to m a, everything follows. You can write that the evolution of the motion of the particle or the system itself.

But here, if you have the slightly different path, any various possible paths are available but then, what this says is, the variation of this quantity integrated from t 1 to t 2, this is the extremum or minimum, that is what we will show that. What we have done is the starting point is known, because there is no variation in the starting point, I am not put and end point also, I assume it is known. So, there is no variation in the starting point, there is no variation in the end point, that is it.

In between, you can go several path, among those various path, the system chooses the particular path, for which this become an extremum, that is what we will show. Now, what happens to the right hand term, right hand side this term, because we said that, the variation at t 1 and t 2 is 0, because there is no variation in the path at t 1 and t 2. So, what you do is, this integral, the right hand side become essentially summation i 1 to N m i, because this is d t, d t will go differential of this, it will be r i dot delta r i at t 1 and t 2.

Delta r i at t 1 and t 2 is 0, so this term there is no variation at starting point and the end point, therefore this term becomes 0 and your Hamilton, this is what Hamilton's principle is, Hamilton's principle t 1 to t 2 variation of T plus variation of W d t is 0, this is my Hamilton's principle. Now, I can slightly modify this, now you have obtained, but the beauty of this is, this is purely a scalar quantity. We are not talking about vector field, because Newton’s law is actually force and acceleration relationship, it is a vector quantity, this is scalar.

Now, you see the nature, the beauty is, this gives you exactly the same result as what Newton's law gives, because this principle is like in a dynamical system. If a particle or system moves from one configuration, one time to another time moving, it chooses a particular path, for which the variation of kinetic energy and the external term is 0 and this path turns out to be same as the Newton’s law path. Now, you can write this, because delta W is work done by external forces.

I can split it into two combinations, because I can have conservative force, please note this I can have conservative force or non conservative force, because you can say gravity conservative. Because, the definition for the conservative is, what is the definition, the work done, it is independent of the path and the work done only on the initial and final position, that is the conservative force. Non conservative force, no, it is a function of path like a friction, friction is a function aero dynamic flows, you can takes it as a conservative thing.

So, conservative force, the work done in conservative force is independent of the path and here the work done you can write it as, it is actually a change in the potential. So, that in this case, this will be minus del V or del U, whereas V is the potential, we call it potential energy. For conservative force only we can write, for this it will stay as it is, non conservative it will be just, work done will be just as it is. You do not do any potential type, work done will be simply delta W non conservative.

Now, I am splitting the delta W itself into two part, work done by conservative force, work done by non conservative force. So, I will put it this as delta V plus delta W non conservative. Now, I substitute here then, I will have my V, usually this is written as extended Hamilton’s principle, delta T minus V plus W non conservative, t 2 delta t is 0.

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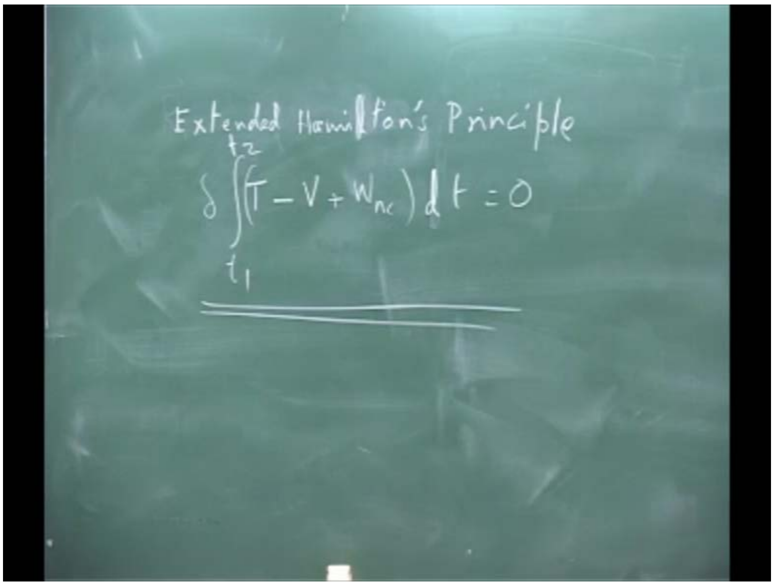

So, this is my Hamilton function, now you can write, so you see, this principle in the broader class of calculus of variation. So, it falls into that, because this is like a function ((Refer Time: 39:25)). See, this is the function, kinetic energy, potential energy and work done by the non conservative force. Now, this is integrated from some between two points, here it is time and the variation of that is this. So, in normally, you know maxima minima, if you are given a function, if you want to find out what is the maximum or minimum, what do you do.

You take that first derivate, second is equal to 0 and then, solve for what points it is 0 and then, if you want maximum or minimum, you go to the second derivative, that is for a one function. Here, this the integrated value, a function is integrated and each one is the function of a, it is a path, there are several paths, so these are functions of another functions, so they call it functional. That means, all these are functions of path, you are integrating over the two time limit, in which path if I go, I will get every path.

You go along the various path and then, you will find that, the variation of this, first variation 0, this is why this is called the extremum they will say. Because, you do not say minima or maxima, because you do not go on calculate the second derivate, that is why this falls under the principle of calculus of variations or variational calculus you can call it, because that is a very general mathematical premises. Now, we can derive the Lagrange’s equation from the Hamilton.

From Hamilton's principle or we call it extended Hamilton, extended is otherwise what they will do is, if you n c is 0 then, they will say, this T plus W equal to U, that is it under equilibrium, you will split into that, you write it like this. Now, for the deformable structure, you know that, kinetic energy you can get for any point, strain energy you can get, virtual work done by external load, so get that, you will write the equation. But, please understand, this is not going to give you solution, it will give you only the equation of motion. We will do in the next class, how we apply this for our problem, how do we get the equations, etcetera, that we will do it as an example.

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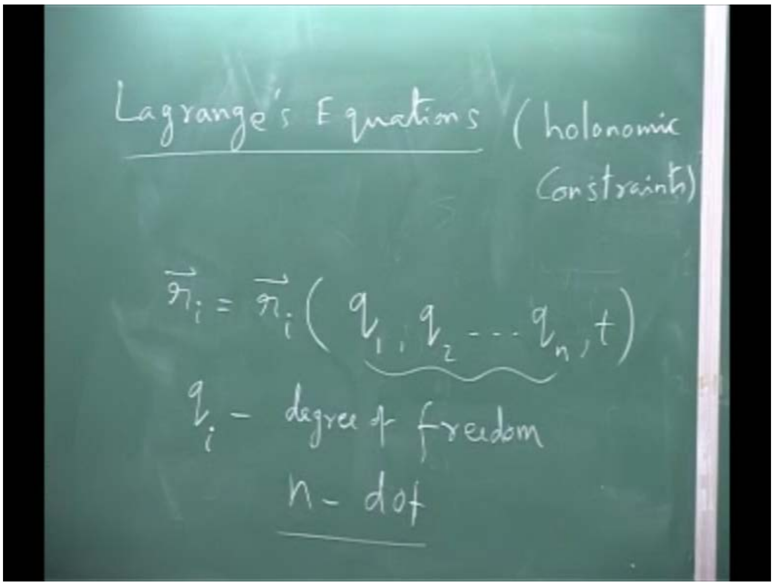

Now, let us do the Lagrange's equation, so that Lagrange's equation is, we have holonomic constrain. Please note, we have only this that means, what is my position vector of any particle r i, position vector is now written in terms of generalized coordinates. Because, holonomic constrains are there, I am not going to put x 1, x 2, x 3, because they are all the coordinates. But, there are constrains that means, number of independent degrees of freedom, which are called the generalized coordinates, they are less, because of the constrain.

So, I am going to call those independent coordinates or independent, basically they are degrees of freedom. I am going to use the symbol q 1, q 2, q n comma t, where q i these are degrees of freedom. Degree of freedom that means, there are n degrees of freedom, n d o f, so n d o f, this is the n degree of freedom system, independent coordinates you can call it, independent actually, that is what the configuration comes.

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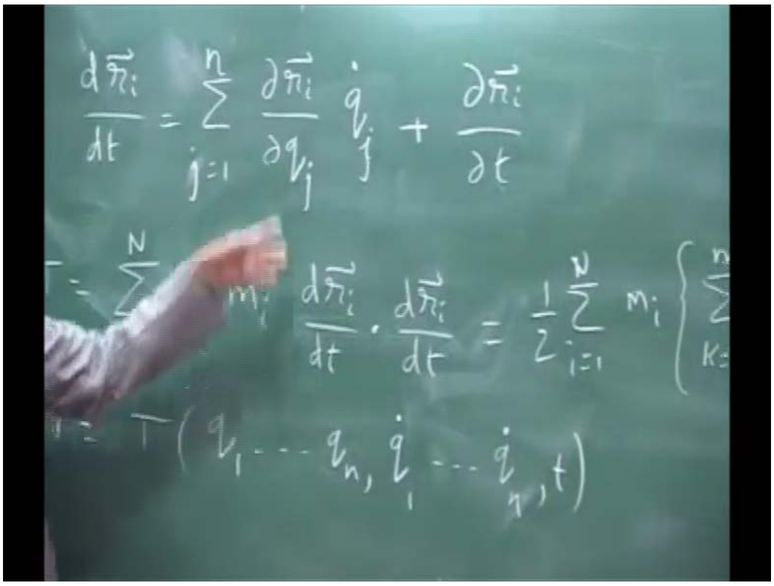

Now, I erase this part, you calculate the velocity, velocity will be what, time derivative with respect to time, time derivative basically, derivative respect to time. You will have summation i running from 1 to lower case n, delta may be I will change the symbol. Because, this is r i know, I will put it here j, q j, where j is this, into delta q j by delta t, which is actually q j dot, plus you will have...

This is my velocity expression, now I have to get the kinetic energy expression, so the kinetic energy will be T is, I am going to have a summation here of all the particles, that will be you can take it i running from 1 to capital N, 1 to capital N half m i d r i by over d t dot product of... Now, if I substitute this here, please understand one thing, I am going to substitute this, this will be like a square term.

So, I will have something like this, I can write it has half summation i running from 1 to capital N m i, open a bracket summation, I will have k running from 1 to lower case n, summation s running from 1 to lower case n delta r i over delta q k delta r i over delta q s dot product with q k dot q s dot, this is one term. Then, you will have, because I have to square it, what will happen, this whole thing square means, this term square plus 2 into this into this term, plus this term square.

So, I can write it as 2 delta r i over delta T summation, I can use any symbol k running from 1 to n delta r i by delta q k q k dot plus delta T dot, this is the long expression. Now, why do we write this long expression is, we always say kinetic energy is a function of velocity that means, the velocity is the first derivative of time, but here you note that, r i are functions of generalized coordinate. Please understand, they are functions of generalized coordinate, therefore when I have this term delta r i by delta q k, these are the functions of generalize coordinate.

I also have a time derivative of a generalized coordinate you understand, therefore your kinetic energy expression when you write it, it is a function of, T is a function or I will write T as the function of q 1 to q n comma q 1 dot to q n dot comma t. Please understand, kinetic energy is the function of generalized coordinate and also it is time derivative. So, that is why, this is very important to understand, because normally we will think kinetic energy means, it is only the function of velocity square.

But then, when you are writing it in it is generalized coordinate form, it will be function of generalized coordinate as well as it is time derivative. That is the reason, why we write all this long expression for you to understand. Now, we do not need this long expression, we will erase that part, we directly go for the... Because, you know that, potential energy is a function of position that means, generalize coordinate that means, q 1, q 2, q 3, q n, so I will write this part again.

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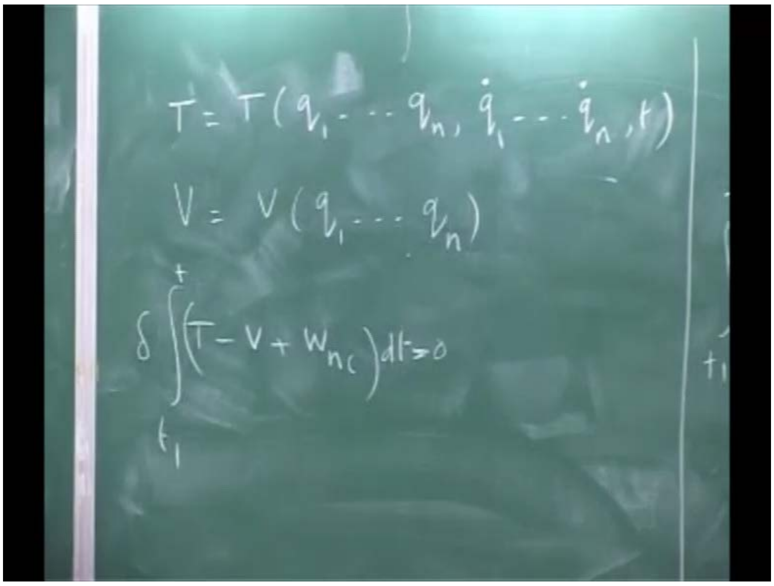

So, my kinetic energy is the function of q 1 to q n comma q 1 dot to q n dot comma t and this is a function of only q n position, because that is what the potential energy. Potential energy is only a function of position, now what you do is, you have to go and substitute these expressions in the Hamilton's principle and integrate them independently. So, we will do step by step one after the other, if you take the first term, because the extended Hamilton principle is, delta of t 1 t 2 T minus V plus n c d t 0. So, I have to get first term then, the second term then, the last term, that variation.

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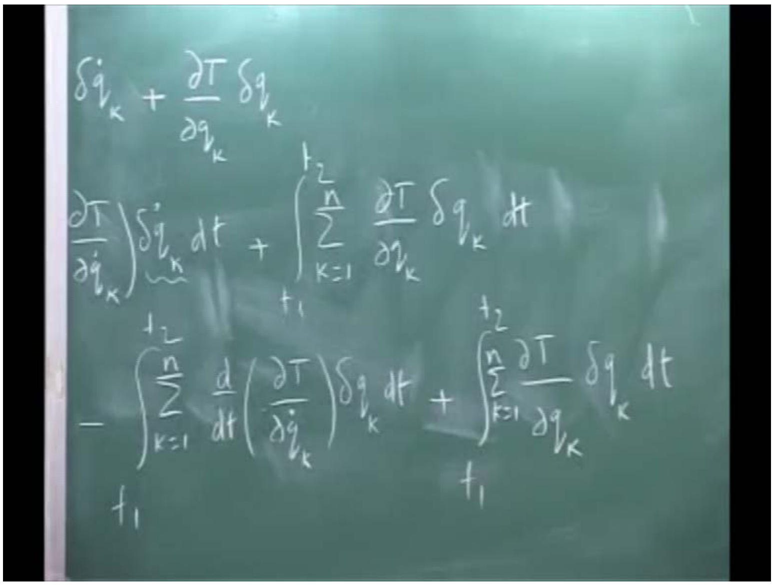

So, what we will do is, we will take the first term now and that is, we know that, delta T, because it is a function of q k q dot k. So, I can write it as summation k running from 1 to n delta T by delta q k dot plus and then, you have to integrate. Integrate means, integral delta T d t t 1 to t 2, I am going to put this, this will be integral, you can have summation inside or outside does not matter, 1 to n t 1 to t 2 delta T over delta q k dot delta q k dot d t.

I am writing it in two terms, plus integral t 1 to t 2 again summation ((Refer Time: 54:36)) k running from 1 to n again delta T over delta q k delta q k d t. Now, what you do is, this first integral, because this is delta by delta q k dot delta q k dot d t. This term you can write it as d by d t of delta q k and then, you can use the u d v kind of a expression u d v and then, integrate by part. When you integrate by parts, you will get this that is, delta T over delta q k dot.

Because, your summation is still there, delta q k t 1 to t 2 minus, because k is this 1 to n, minus integral t 1 to t 2, you will have still the summation k running from 1 to n. Since this is integrated by parts, it will become d over d t of delta T over delta q k dot delta q k d t plus, this term will remain as it is, summation k running from 1 to n delta T over delta q k delta q k dt. Now, what you have done is, you say that, this is delta q k t 1 to t 2, because there is no variation in the initial and final time in the configuration, therefore this term goes to 0.

So, what I have done is, I have taken this first term in this delta T d t, I have written it in that form. Now, we can write the and this itself maybe come here and then, we come here. Now, variation in potential energy, that is delta V, because we said that, potential is only a function of the position.

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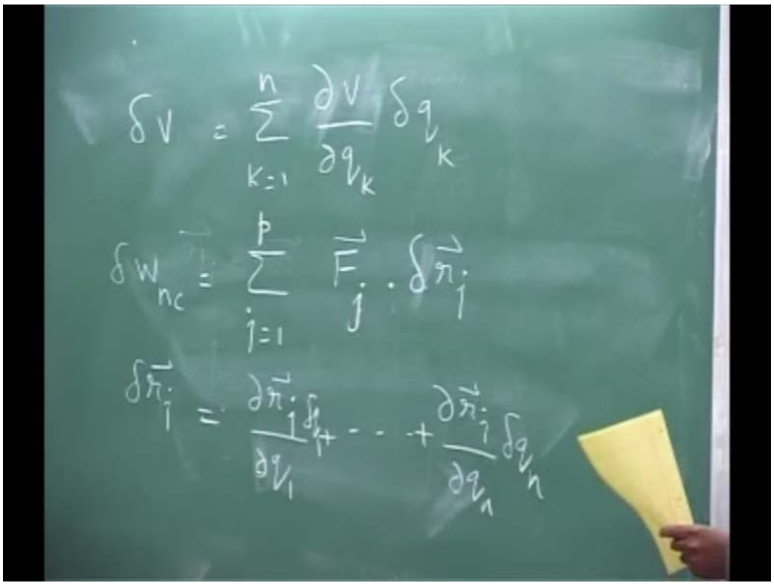

So, you will have, this will be summation, again k running from 1 to n delta V over delta q k and then, we need to know delta W n c. Delta W n c that is, the virtual work done by the non conservative forces, you know that non conserve forces we have written it as, you may have say some p non conservative forces, I am going to call that non conservative force as ((Refer Time: 59:04)) F j. Now, d r i, I have not this d, I am sorry about that, this is delta r j. This I have to write it in terms of generalize coordinates, which will be delta r j will be delta r j over delta q 1. So, 1 plus delta r j over delta q n, I have to put delta q 1 here and delta q n here, what you do is, you take it, put it back here.

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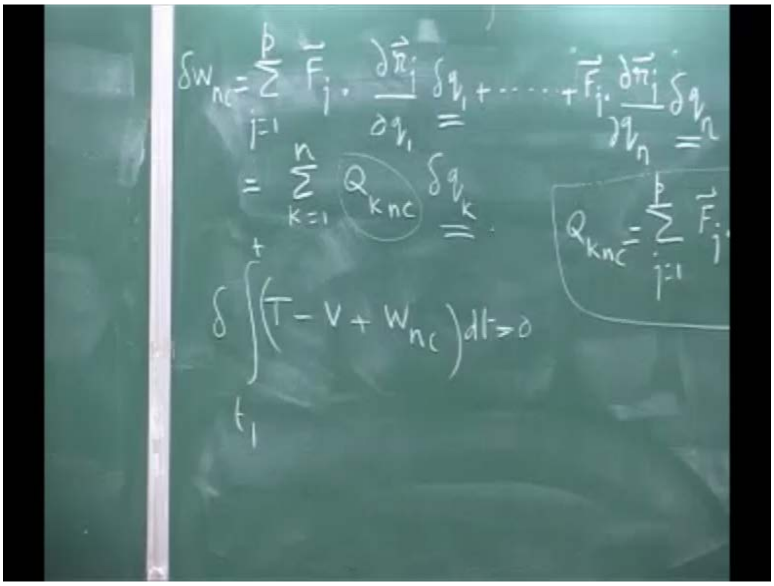

And then, collect the terms, may be I erase this part here so that, your delta W n c is going to become j running from 1 to p, you will have F j dot, every term will be added. Now, that will be delta r j by delta q 1 delta q 1 plus so on, plus you will have F j dot delta r q n. That means, you have these n terms are there, I can write this as in a modified form, I will write it as summation k running from 1 to n Q k n c delta q k, because I am summing now with delta q, rather than in the j.

Now, what is this factor, this is the generalized force, this is the generalize virtual displacement of the generalize coordinate. So, Q k n c will be, I can write it here, Q k n c is summation j running from 1 to p F j dot delta r j over delta q k, this is my generalized non conservative force. So, essentially what I have to do is, I have to, because this is my equation, Hamilton's principle. So, I have delta T minus delta V plus delta W n c, I have to add then, this term and then, this term and I have d t. In every term you will see, delta q k, delta q k, delta q k, delta q k and d t is common.

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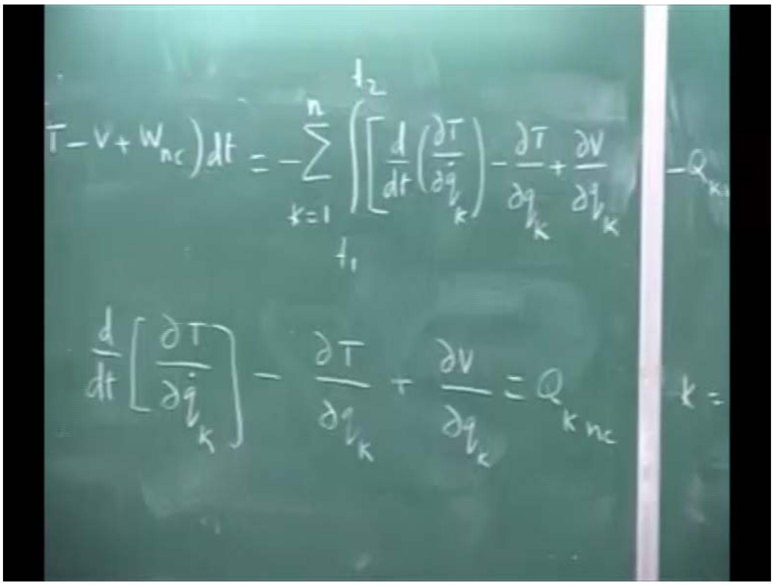

So, you can write it as one general expression, one long one, I will write that part so that, that becomes simple that, this is the final variation of t 1 to t 2 T minus V plus is minus k running from 1 to n, because I am taking minus, because delta T there is the minus sign, that is why I have taken the minus sign out. I will have this integral t 1 to t 2, I am opening the bracket, I will write d over d t of delta T over delta q k dot and then, the second term minus delta T over delta q k.

Then, I have to add the potential plus delta V over delta, because this minus sign is going to become a plus sign, because I have taken the minus outside, delta q k. And then, that n c term, n c this will be a minus sign, it will come, because minus is out. So, I will write this as minus Q k n c delta q k d t is 0, now you say that, since my q ks are generalized coordinate, each one is independent. So, I can vary that arbitrarily that means, the integrate must be 0 and that is my Lagrange's equation.

So, Lagrange's equation is d over d t delta T by delta q k dot minus delta T by delta q k plus delta V by delta q k equals Q k n c and k running from 1 to n. Please understand, you have n equation, because k is n, now this is one form of, this is actually you can use it as it is, there is absolutely no problem, but some time you define T minus V as Lagrangian.

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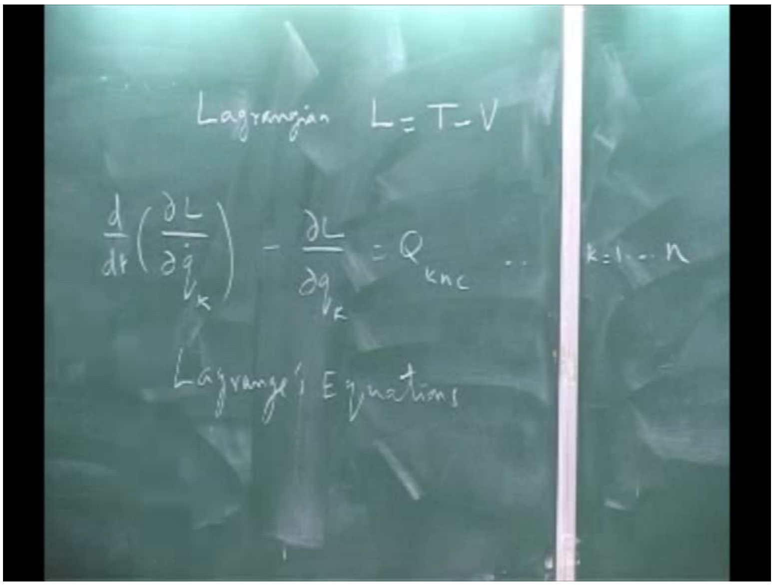

I erase this section, because the Lagrangian is defined as Lagrangian, you can say L is T minus V then, what will happen. This term is delta by delta q k delta by delta q k, so you can write is as minus of T minus V, T minus V is L. But, here you know that, potential is the independent of the velocity term, velocity means the time derivate term. Therefore, even if you add here, it does not matter, so what they will write is, d over d t delta L over delta q k dot minus q k, this is the Lagrange equations.

Please note, you have k running from 1 to n, these are the Lagrange's equation and L is T minus V. Now, you know even the Hamilton's principle, I am just giving you, Hamilton's principle was written as delta of integral. If there are no non conservative force then, what will happen, no non conservative force and this is Lagrangian. So, it can be written as delta L d t t 1 to t 2, this is called the principle of least action, they call it, all are variation calculus.

If you see the math, if you study the various calculus, they will write for general function, here we are only writing kinetic energy and potential energy and some external non conservative force. This split everything into energy expression, whereas variation calculus, they will just write some F function and then, they say, there are several interesting problems in math, I will not get into that. Now, we have learnt two things, Lagrange's equation, Hamilton principle, what we will do in next class is, how do we use this to develop our equation of motion.

Please understand, these equations will not give you the result, solution will not be given, they will give only the equation of motion. So, do not think that, because Lagrangian equation if I derive, I have the solution no, solution you have to solve later. That is why in the aero elastic problem, first you get the equation then, you look at, how do I get the solution. So, these are the two steps, we have the structural problem, but linked with, because here I have developed the equation with particles and like a dynamic.

What after we deform the whole structure, you can develop the same thing please understand, but that you have to go for deformable bodies, but you will get identical Hamilton's principle and Lagrangian equations are same. So, whether it is for a deformable body or for a rigid body does not matter, you can use it anywhere. Only thing is formulation, how you develop it to this form, here I used a rigid body dynamic.

You can also have, otherwise it is like a reputation of same formulation starting from deformable body, you go here the rigid body, because anyway D' Alembert's principle is used. In the deformable body, you use the cache boundary condition and equilibrium equation in terms of stresses and finally, you will get the same formulation. There you write instead of potential, you write strain energy, that is why the derivation and the formulation is same, Lagrange's equation, Hamilton's principle. And these are extensively useful in developing the equation of motion for aero elastic problem.