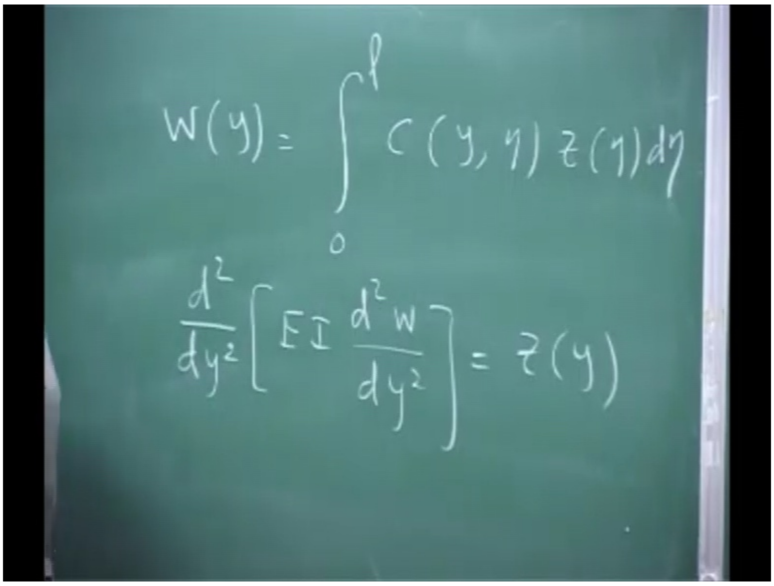
Aero Elasticity
Prof. C.V. Venkatesan
Department of Aerospace Engineering
Indian institute of Technology, Kanpur

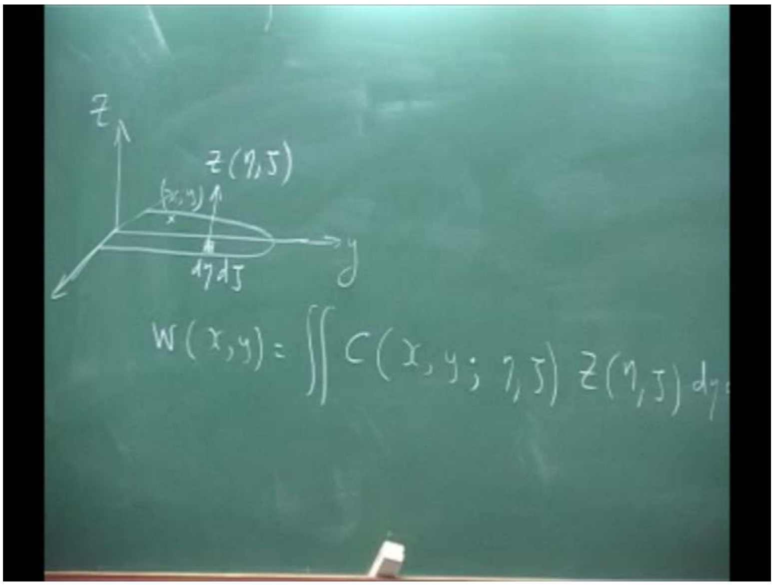
Lecture – 3

(Refer Slide Time: 00:25)



Last class, we saw the evaluation of deformation by integral methods. In the sense w of y we put it as y eta zee eta d eta, this is a integral approach and this is the solution of basically the deformation for a given loading condition. But, you can write the differential equation of deformation, which we wrote it as beam bending equation, very simplified. We wrote E I this is a differential equation form, now E I is a constant you can take it out and you can solve the forth order differential equation, apply the boundary condition and then you can get the w. Now, similarly this is a one dimensional problem in the sense we said w is only a function of y, suppose in the case of wing.

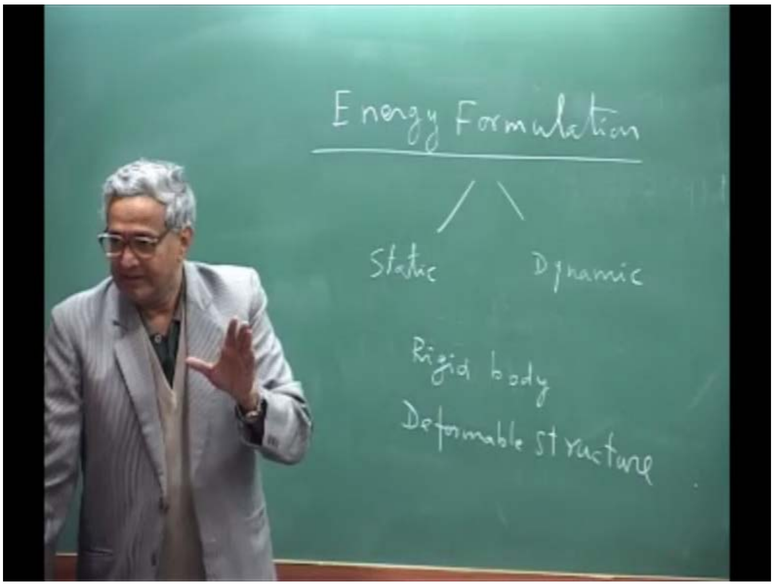
(Refer Slide Time: 01:49)



If you draw the wing such as approximately this will be like a, this is your x axis, y axis this is my z axis. So, the wing is in a plane and any small element you can have lift force or the pressure, this is a two dimensional problem; in this case you can write similarly like, you say any x y w at any x y it will be a area integral you can put a double integral and your influence coefficient will be x comma y, you can have eta zeta, zee eta zeta, d eta d zeta.

So, this is like the deformation at x and y coordinates due to a loading at eta zeta and elementally area is d theta d zeta. So, you can have this is the d eta d zeta, this is zee at eta zeta, so deformation at any point you can take x comma y; now, this is all in the integral approach you have to get. There is another approach of getting the deformation which is I will erased this part which is the energy formulation.

(Refer Slide Time: 03:59)



So, I will write the, we can say energy formulation see this is another approach to solving the structural problem. But then you may ask why do we have to use this, you can have always have force balance like what in the earlier case, we balance vertical force moment and then you write your equation moment equation, that is based on Newton law.

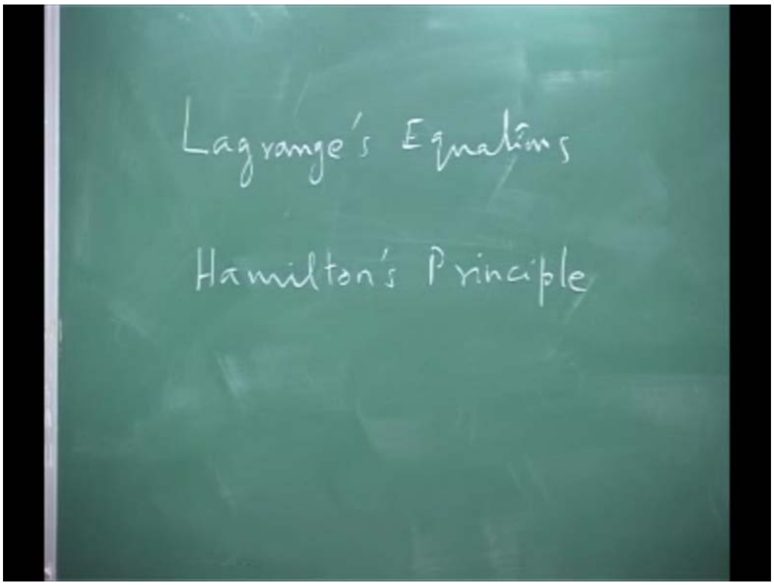
Energy approach you are dealing with scalar quantities then energy is the scalar whereas, in the other approach you deal with vector quantity which means this is coordinate independent that is number 1. Number 2, the advantages of q c energy formulation is that it will give you the differential equation and it will also give you the boundary conditions properly. Whereas in the force formulation like Newton's law, if you put at tip match like misail or anything on the tip of the wing, what is the boundary conditions you have to impose their these thing you have to be very clear.

Whereas, the energy formulation when you follow there not only you will get the field equation, you will get the boundary condition also as a part it. But, please remember that this is not going to give the solution it will give you the equation, another approach is in the energy formulation when you want to get approximate solution. The condition which are imposed on the choice of functions is less stringent, I would put it that way less stringent.

But, all whatever I have said in words you will get know as we proceed further because with example we will solve then you will understand, this is the reason why energy formulation is quite attractive. Now, the question is energy formulation you can do for static problem, you can do for dynamic problem, both you can have static, you can have dynamic situations. But, we will go through the approach first we will do the static and then we do the dynamic later.

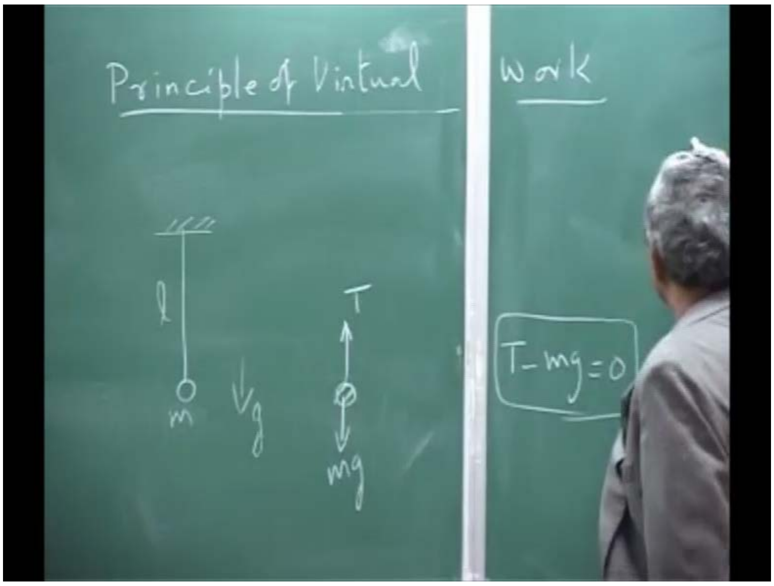
And, if you look at the different books, some books will deal with their same static dynamic problem for rigid body. And in another approach you can say deformable body, you can to the same thing deformable body also, so you can have rigid body and deformable structure. Please, understand you can have this formulation for this and this but if you the approach that is adopted to get the final equation for rigid body is one part, when we go to deformable body it is in a different form. But, both will lead you to the same final form of the equation, which is like you may call it as the Lagrange equation or Hamilton principle. We will come to those two later because these are very important.

(Refer Slide Time: 08:10)



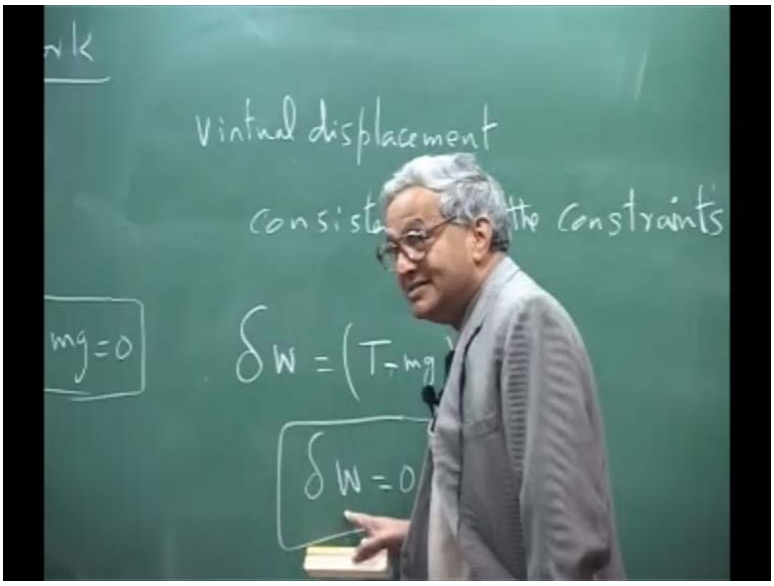
The Lagrange equation, Hamilton's principle, we can a apply this for dynamic situations, but they are derived from all these are obtained from essentially principle of virtual work. The energy formulation if you see it will start with the principle of virtual work.

(Refer Slide Time: 09:03)



What do you mean by virtual work, we will do with a example case very simplistic situations, so that you get an idea of what do you mean by virtual work and how that is applied to get our energy formulation. Suppose we will take very simple case which is like a string with the mass m, this is the sting length l it is in static equilibrium and under the action of gravity. That means, what is that if you draw the free body diagram of that mass you will have m g and you have some tension T it is in equilibrium. So, T minus m g you say is 0 this is what you write, now in the virtual work what you do is in the equilibrium position. If you give a virtual displacement please, understand it is not an actual displacement it is a virtual displacement.

(Refer Slide Time: 10:58)



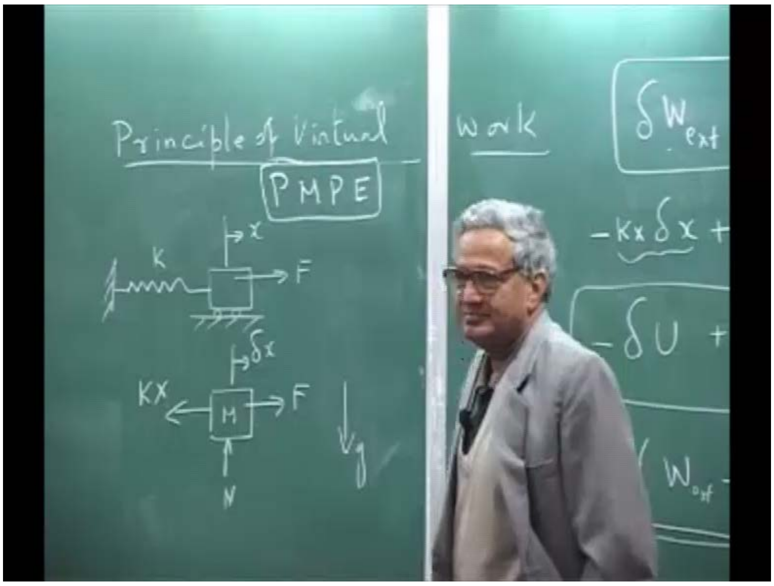
Now, what is the virtual displacement you can give please, understand the virtual displacement must be consistent with the constrained in the system that is important. This must be consistent with the constraints in the system, now what do you consistent here we said that link is constant that means, it cannot extend we cannot give a virtual displacement in the vertical direction, you can only give a virtual displacement in this direction, you can give a virtual direction in that direction that is all about.

But, then you know that if I calculate the work done, work done means the virtual work done because the forces are actual forces and I am doing a virtual work which I call it delta because this Is the symbol which I use for virtual delta w. Now, what happens for this system it is t minus m g is the force that is acting on the system. But, I cannot give a virtual displacement in this direction because length is fixed the static situation that means, this is out I can give a virtual but this displacement virtual displacement were is ninety degree to the forth therefore, the work done.

In this case it is nothing but t minus m g into delta may be I call this as delta y but this is at 90 degree therefore, the dark product goes to 0 very simple if this goes to 0 then t is equal to m g this is then you say what is that it is very obvious situation. Now we take a slightly different, so the principle of virtual work is basically what for static equilibrium of the rigid body, you say then delta w static equilibrium delta w, w is all the forces that act on the system.

Now, I am going to slightly multiple the example and then change this itself into a different format. Let us this is only a simple example, now we take another examples which you are all familiar with which is like.

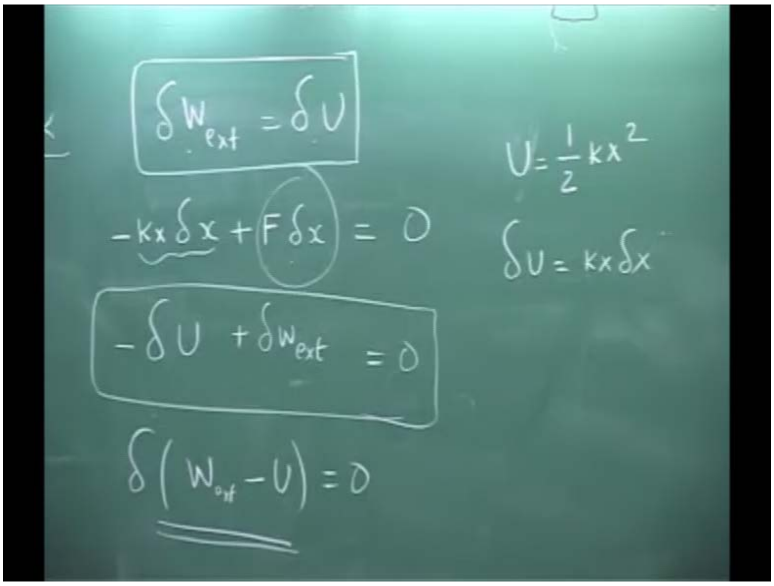
(Refer Slide Time: 14:48)



I have a spring m r and this spring is k and you have applied a force f and it as found through a distance x it has already found through a distance f. So, you will from static equilibrium f is the external force. And let me put it this way, I think I will go here on the mass you have vertically k x, which is acting in this direction there is a force which is acting of course, there is a normal which is a ground direction reaction that may balancing with the gravity.

Now, in this problem what is my virtual work principle how it will look like, now I know the forces acting on that body and it is constraint to move only along the x access, only in the horizontally direction. This can move only it cannot go vertically therefore, the n and g they cannot do any work, but it can move in a horizontal direction if it move horizontally with a virtual displacement of delta x.

(Refer Slide Time: 16:35)



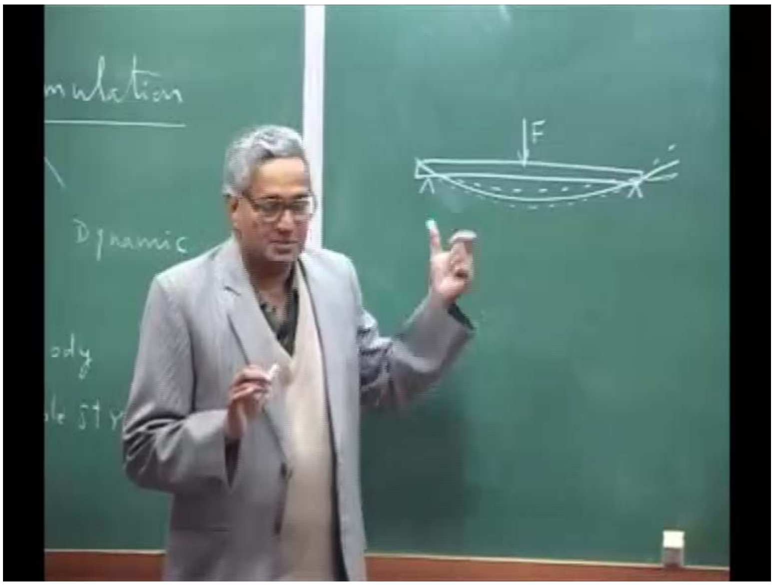
Now, what will you do you will have minus k x delta x that is this force doing displacement through a virtual displacement which is delta x, this also does f delta x. Now as per our thing this is 0, but now what I have done is I am writing this force into this as I am going to write delta w, this is the virtual work done by the external force I am going to call the spring force as an inertia force.

Now, in the spring force you know that energy of a spring is k x square, right if I have a variation in the energy which I will write it as work, what delta u over a that is variation in the sense the variation in strain energy due to a virtual displacement. This will be you do the same thing, it will become delta x and that is this one. So, I can write this is minus delta u, now you see what is my virtual work principle is simply I am saying delta of w external equals delta u this is one form of writing.

Another form is you can write it as delta minus u, this is another form of writing this is like delta you take it as something like, a first derivative even though it not first exactly first derivative please, understand it is virtual displacement. This is a variation due to a virtual displacement and first variation is 0. This is a function first variation 0 means, first derivative when you can take it is equal to 0, it is an maximum or minimum right.

So, you called it generally if you say this is the principle of minimum potential energy and you define this as potential energy of the system. So, you say principle of virtual work from here you get the principle of minimum potential p m p e principle of minimum potential energy. This is the principle of virtual work that is that is delta w external equal delta u for static problem. Now, I just want to I will not derive, but I am going to ask you to think in the sense suppose I am having see I have taken a simple problem like this simple example very simple examples.

(Refer Slide Time: 20:46)



Let us take another simple example, which is like a beam it is supported at from some two ends. I am Appling the some f here external force. Now how do I write the virtual word because you say the reaction forces are here and they are constraint taken cannot move you understand. So, that means the virtual work done by this 0 by the virtual work done by this is there it is not 0 and that is the value, which is the delta w external which goes and gets stored in the variation, in the strain energy of the deformed structure.

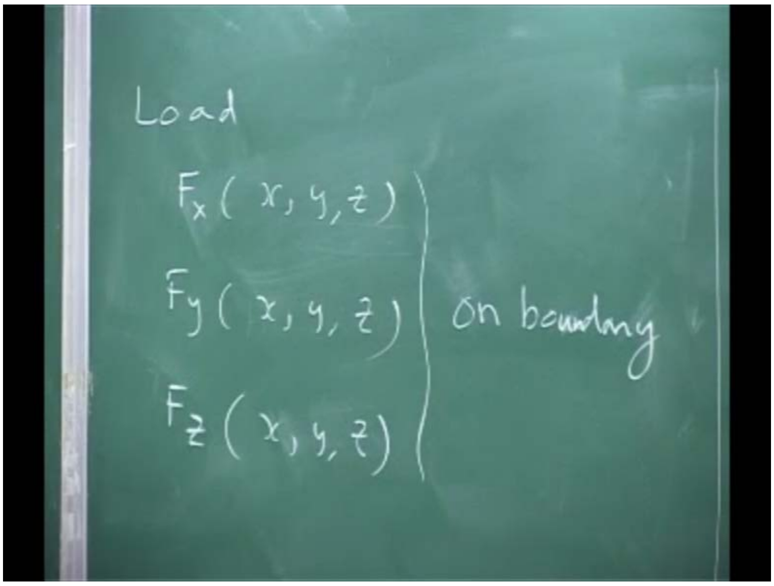
Now, how it deform that is what we have to calculate, so the essentially principle goes like this. When you apply load the structure deforms to some shape or you can call it some configuration, that configuration you may say I will just draw by single line it may be like this some, but it is one. When I apply that f it deforms always in the same configuration, why it chooses that particular configuration why cannot it choose slightly different slightly different.

Any other varied configuration, if you follow why it chooses that particular deformation if I call it paternal or configuration or anything among all possible configuration. Because, you have like this also and you can define like this also some another line there are several ways, but you choose only one. So, essentially what you are saying is it chooses that particular configuration for which variation is 0, first variation is 0 then it is like minimization problem actually maximum minimum that way people do not call it.

Even though, the principle of minimum potential energy is mentioned actually in a dynamical system, they write as x minimum can be minimum or maximum. it will do not go derive the second derivative, but it is mentioned as minimum potential principle of minimum potential energy. Now you have got an idea how virtual work principle is used for structural analysis, now let us say that is this is understand static the same principle is extended to dynamics.

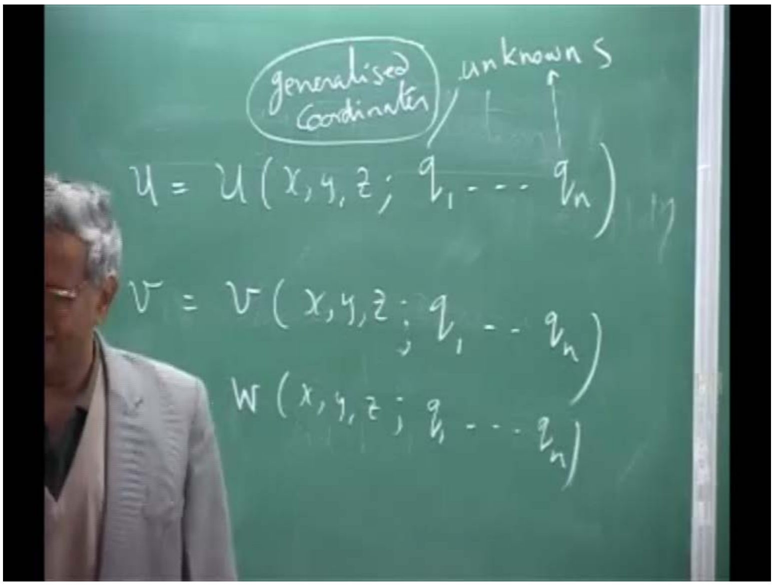
But, that we will do later first we will take the static part and I get and I will do one example problem. How this principle is use in getting the deformation of a wing, we can say an approximate, now you write the deformable structure what happen you are applying a load.

(Refer Slide Time: 24:59)



Let us say you applied load at several point on the boundary you will apply, f x which is the function of x comma y comma z, f y some other and f zee these are the portion acting on boundary of the structure and they are going to do the virtual work.

(Refer Slide Time: 25:55)



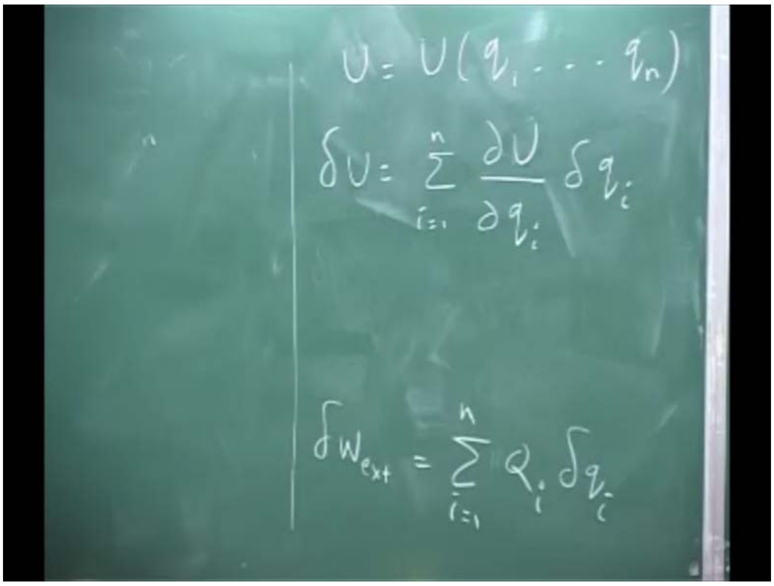
So, what you do is every point in the structure under goes because of the application of load they undergo a deformation. The actual deformation you may call it u v w this is the actual deformation because of the application of the load and this is a you may say a function of because x comma y comma z. Because, this is the point x y z this is an identification and how this point has moved after the application of the load.

Now, that new the deformation pattern or you may call it deformation configuration, you are going to refer something in terms of several functions no one functions those functions satisfy the boundary conditions. So, you may have the co efficient of those functions are basically the unknown. So, you represent them by q n these are my unknowns, we may say later I will show how this is apply.

Similarly, you will have v which is a function of which is actually again the identification point, again it will be and w that is also q 1 to q n, you may call it these are the unknown these are generalized coordinates you may call it by any name that is unknowns are generalized coordinates, you may call it this actual displacement I am representing like this.

Now, what is the virtual work done by the virtual work will be only by the surface portion, it cannot be by the internal. Because, external w external if you have body force will do the work, but here we neglect the body force hundred percent we will take the external load, this is on the boundary.

(Refer Slide Time: 28:31)



I will erase this and we will write our delta w external this is the surface integral, f x delta u f z delta w into d surface, this is what we have. Now, we need to see how delta u delta v delta w we can write it in terms of these what we assumed, you can write it as maybe I will erase here you are delta u. The variation in u is due to variation in every 1 of this generalized coordinates.

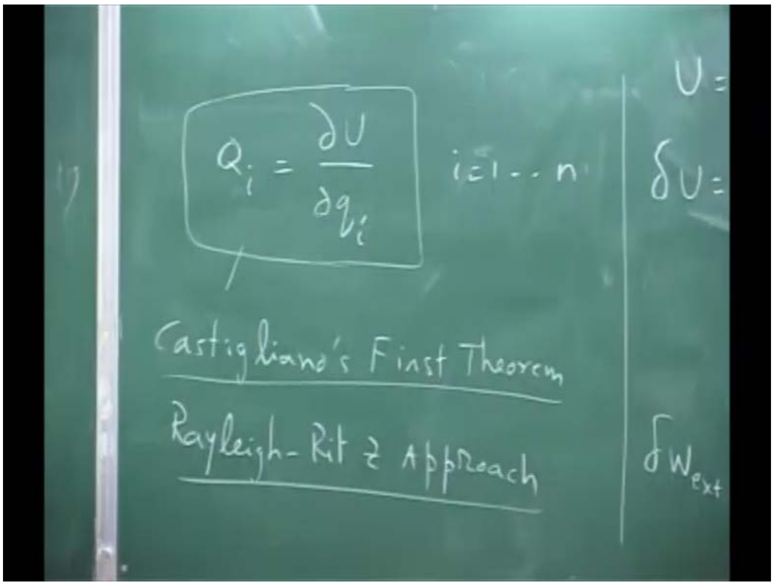
So, you will write it as delta u by delta q 1 del q 1 plus, so on, so on, so on q by and which can be written as summation delta q I. I running from n similarly you can write for u you can write for w and take this and substitute here and what you will get, you will get delta w external is integral s is there, you going put a summation. I running from 1 to n f x delta u over delta u I plus f y delta v over delta q I, f z delta w over delta q I and I put a bracket here, del u I and you have a d s surface.

Now, what you can do is theses are virtual displacement each virtual displacement is each is a independent coordinates. So, each you can give independently the virtual displacement, so you can take the integral inside and write it as summation I running from 1 to n integral you can put it f x delta u by delta q I plus f y delta v by delta q I plus f zee delta w by delta q I d s into delta q I, these only at the surface.

Now, this entire expression you can write it as delta w external is you will write, it in this summation q I into small I running from 1 square. This is my generalized force, this is my variation in the generalized displacement how do I get the generalized force q I, q I is nothing, but this integral. Now, let me erase this portion you have to get, now expression for u. So, let me erase this.

Because, here also it is very simple because u, v, w are actual displacement, so virtual displacement are functions of these. So, your strain energy is going to be a function of q 1 q 2 q 3 q n. So, you can simply write, u which is the function of u q 1 to q n and del u will be you will have summation delta, u sorry capital U delta u by delta q I del q I. I running from 1 to n.

(Refer Slide Time: 34:45)



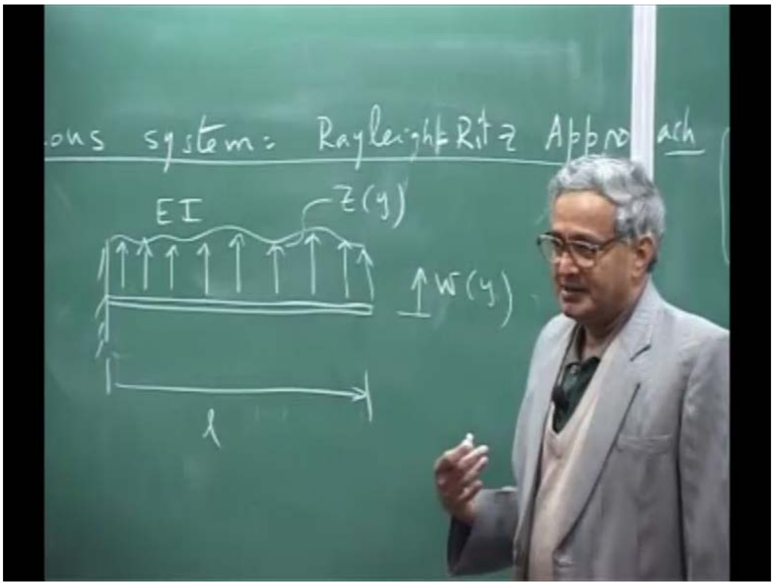
Now, what you are doing is you are equating these two, when you equate these two summation I running from 1 to n. Now what you will have is q I becomes, delta u by delta I running from 1 to n. This is my equation basically, you will find that this will become one multi break equation for a static problem, the bending problem. We will show that and this particular this is called Castiglione's Firstly, it is called Castiglione's first theorem.

Now, we have put n only up to n suppose if this number goes to infinitely beyond actually, n keeps on increasing can be shown that you will get the exact result for the deformation. It will approach the exact result and this particular of using this technique with finite time that is called the Rayleigh-Ritz approach, but please understand this what we will use see this, we obtained from the virtual work.

Please, remember this equation this virtual work principle it is not balancing, the energy please understand it is not energy balance you have to know. Because, you can have a non conservative force anything you can have this is basically about equilibrium point or equilibrium deform configuration. Any variation about the equilibrium that is what 0 that is why this is a minimum or minimization approach, actually when you transfer to dynamic situation that is a actually also called principle of least action, it comes from calculation of variation or you can variation calculus.

So, the dynamical system is somewhat is exactly similar to that that is why variation of calculation. Hamilton principle exactly that first, lets that is what dynamic system, so please understand this is what we will use for our static problems energy approach you want to get, the deformation of a wing how do I solve. So, now we take one example problem very simple example problem and you know, how we are obtaining the solution to the wing deformation please, understand earlier we had the integral formulation.

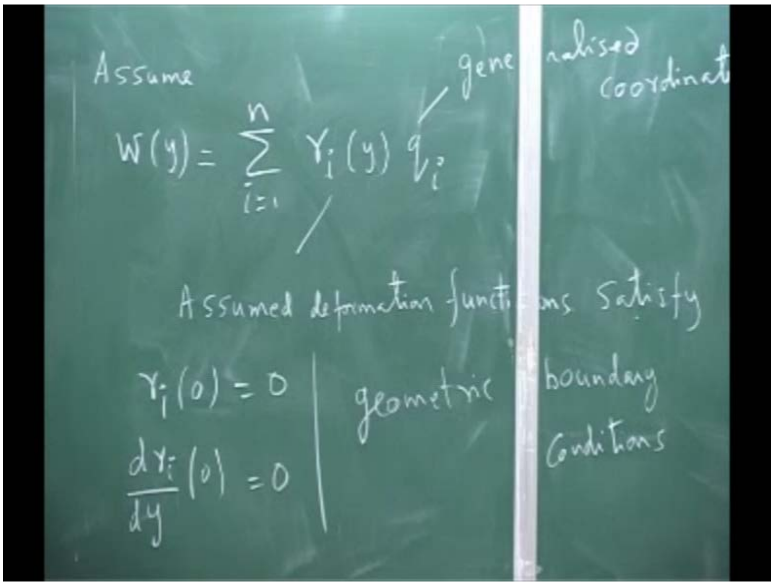
(Refer Slide Time: 38:28)



Now, we are going to have the energy formulation what we will do is we will applied to a continuous system, that is our in this case this is a Rayleigh Ritz approach principle of virtual work applied to continuous system. Our problem is very simple, what we have used earlier we are going to represent our wing like a canteen liver beam, of length l and the properties are you may say, E I is the property of the bending stiffness system. This is acted by again some external loading which we call it as delta y.

Now, we want to know how do we get the deformation of the stream and let us write the deformation, this is w which is the function of y. Now, the first thing what we do is before we go and then start writing everything, what we want to obtained we want to obtained the deformation.

(Refer Slide Time: 40:23)



So, I am going to assume that my W Y is approximate, I am approximating it please understand with the gamma I which is the function of y multiplied by q I. Now, you will be able to understand I wrote u v w, in this case I am not interested in u v and other thing only, the vertical deformation that is y w I am interested. If you are interested in every other deformation then we have to write correspondingly, we are interested in only bending deformation.

That is why W as a function of a generalized coordinate, this is my generalized coordinate, now what are gamma I, gamma I are my assumed deformation shape. So, you may call it these are my assumed deformation, you may call it deformation function. But, the important thing is this is your choice what is the assumed deformation function what is the condition, is there any condition.

Because, you have know that they any virtual displacement you give there must be consist with a constrains, my constraint here is it is fixed here that means, when I have fixed boundary condition. I cannot allow displacement 0 and slope is zero. That means, I must choose those functions where gamma I is 0 then, d gamma I over d y at 0, it is also

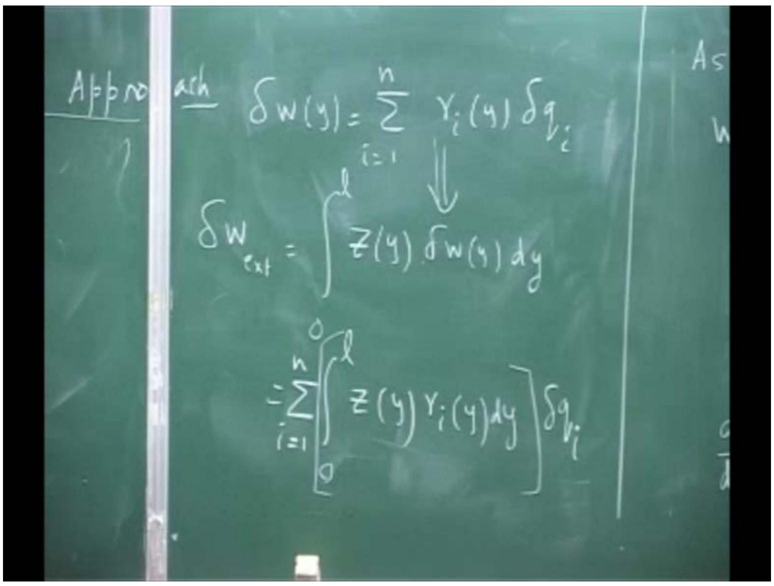
0. We can derivative y changing where who said that it is not changing, y second derivative is what Moment, if the moment is 0 what the root.

So, that is why these are geometric boundary conditions, I will write it as geometric boundary condition. So, please understand my assumed deformation function must satisfy geometric boundary condition. So, they I will write it as satisfy geometric boundary condition that is the condition we should have, suppose if you say no I do not do that than your error will be more in your result you will get a result please, understand in this you will always get some number.

But, how good is a number is a question because you are seeing approximate solution that is why the choice of the functions is very important. Because, in vibrations they are also called admissible functions admissible, what are the function which you will accept or admit. So, you may call it admissible functions gamma I, I may put it admissible function, please understand both mean the same thing admissible functions are assumed deformation functions, satisfy geometric boundary condition, actually admissible function is one which satisfy the boundary condition.

So, terminology is admissible function all right, now what you do is you have chosen deformation sorry assume a deformation shape. So, I erase this part write down the external work, virtual work and the strain energy part and then apply this equation you will get your solution or you will get the complete set of algebraic equations.

(Refer Slide Time: 46:25)

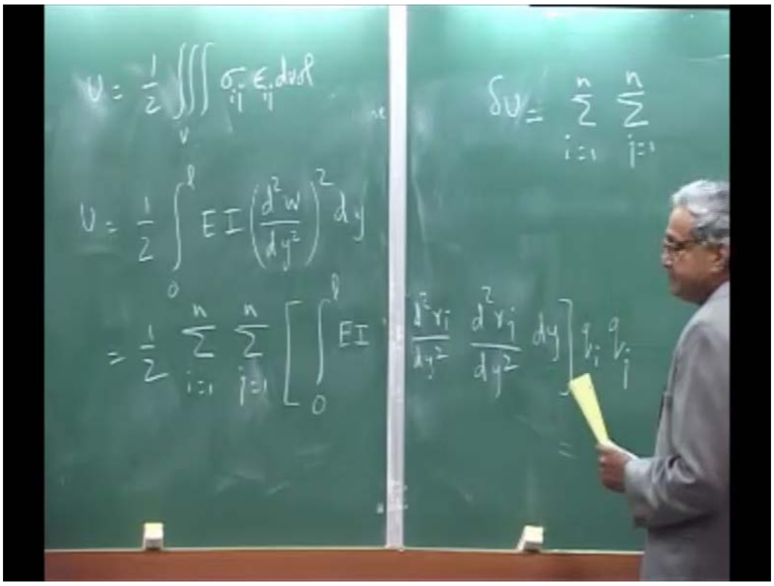


So, let us write the what is the virtual displacement delta w y is nothing, but summation I running from 1 to n, gamma I delta q I this is the virtual displacement. Now, delta w external is 0 to l, f z sorry not f z that is z y delta w y, so I can write it delta w I d y you can put it. Now, you substitute delta y to this expression here and then write it as 0 to l, I am putting summation outside I running from 1 to n zee of y gamma I y d y into delta u I.

So, please understand I have substituted here and I have take up the summation outside and put the integrals, now, this is my generalize force got it that is the q I. So, you see its very interesting what is generalized force is nothing, but the actual force multiple by your assumed shape function or admissible function. Every function you take you multiply and you will integrate.

Because, you integrate over the whole thing, so this is my generalized force, so your actually actual force multiply by the assume shape function. Now, similarly you have to write the strain energy, now we got external delta w. So, please note that this w is different from this w, do not have a confusion over the w this is deformation and this is the work.

(Refer Slide Time: 49:20)



Now, what is the strain energy in a beam because the general expression for strain energy you will always write integral half sigma f p pylon, that in a deform whole structure you always write like this. This is a volume integral sigma f pylon this is your general expression for strain energy, now your solving a bending problem you have to substitute for f pylon, what is the theory you have used we are using if you say Euler Bernoulli beam theory.

If the beam cross section is some shape then you will say how I represent my f pylon and then that f pylon, you substitute here and then you know stress is related to strain. Because, this has all the components sigma f pylon I wrote basically it has x x f pylon x x I a everything, that whole summation is there all right. So, if you want you can put it actually I j I j something like this you can have everything.

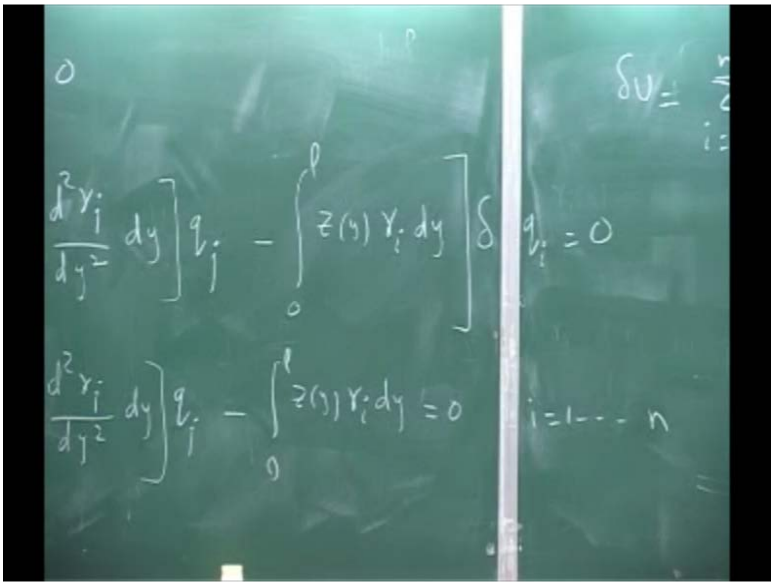
But, in this case you will have only normal strain f pylon x x and you substitute you get the sigma, which is stress strained relationship and where your material property, will come in after that you integrate and when you integrate you will finally get. I hope you know this part otherwise, I will write it half you will right zero to l.

Because, cross s section integral you will be taken E I d square w by d y square whole square d y, this is what you will have please understand for the beam bending problem, with the Euler Bernoulli assumption all those things are that, that is why I am not this is you are familiar right. So, that is why I am not deriving all things steps. So, this is the strain energy of the beam under bending deformation.

Now, what you do you go and substitute this expression here, now you see an interesting part comes in here you have to take the variation. Now, when I substitute this because we are assume that w of y is summation gamma I y q I gamma I u I, I need to take second derivatives of this function please understand. Because, though I mentioned that it must satisfies the geometric boundary condition, another condition which comes is it should be differentiable in this case two time at least.

So, that is also equally important, but you will see now I am going to substitute this function here and then this is a square. So, this will be actually I**.** So, you can put it I j two in this and sum it up over both I and j, you will write half summation I running from 1 to n summation j running from 1 to n integral 0 to l E I d square gamma I over d y square d square gamma j over d y square d y into q I q j. Now, please understand this is a two times differentiable all right later I will tell you what is the essence of this.

(Refer Slide Time: 54:18)



Now, you have to take the variation when you take the variation this will become delta u becomes, summation I running from 1 to n, summation j running from 1 to n integral. Because, q I q j actually it is square term there will be two terms will come, so will happen is the two terms will cancel out with this two really expand everything. That is why the two I am not putting here, you will have 0 to l E I d square gamma j by d y square d y, I will put this in bracket and then I will put the q j and then I will add another bracket I will put it delta q I.

So, this integral you can evaluate that is not a problem because you know E I, now please understand in this formulation I do not need to assume, E I to be constant or anything like that, E I can be varying function obtusely no problem. So, E I only thing is you need to integrate either you can use numerical integration, if you have close form you can get the close form integration.

Now, we have to equate delta w external and this one and then since delta q one are arbitrary, I will write that part then you will be able to see this is the final. So, you're going to use this delta u minus is 0, now I have my expression for delta u I, got my expression for delta w external I have to write.

So, this I am writing summation I running from 1 to n, I am putting a opening a big bracket summation j running from 1 to n integral 0 to l, E I d square gamma I over d y square d square gamma j over d y square d y into q j minus integral 0 to l zee y gamma I d y delta u I is equal to 0.

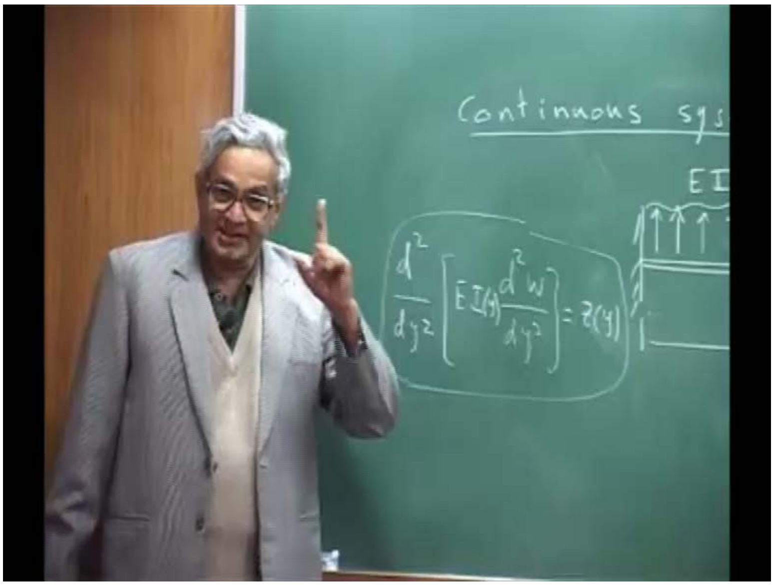
Now, you say my delta q I because q I are the generalized coordinate and I can vary each one of them independently therefore, these are independent that means, the term that is getting multiplied to that, said that u I must be equal to 0 and that is give you your equation, which is summation j running from 1 to n integral 0 to l, E I d square gamma I by d y square d square gamma j by d y square d y q j minus integral 0 to l zee y gamma I d y equal 0 and this is I running from 1 to n.

Because, you will have n equations and this is a algebra equation, now because you know E I, E I can be a function of the wing full it can vary. Because, like a wing I is changing if it is assuming uniform material, material is same and d gamma I by d y square, d gamma j that is a second derivative you multiply you integrate. But, only thing is I j you have to keep repeating it, but it is a symmetric then I become j, j become I you know It Is the same thing.

Now, this looks like some kind of a stiffness matrices see because I can put it in a matrices form. Because, k every element is this q is there this is my capital q which is in the generalized form, now I can solve for q, but please understand I am only solving for the generalized coordinates, after that I have to go and substitute to get the deformation is summation gamma I y q I you follow.

Now, this is just an algebraic equation and k is symmetric now you can obtained the deformation of a beam. Now, there this is a very powerful technique for obtaining and approximate method Rayleigh Ritz approach you can also do it for dynamic situation. But, just to derivate the little bit you know bending equation, last class we derived which was d square by d y square E I.

(Refer Slide Time: 1:01:13)



We wrote like this then this is a forth order differential equation we need four boundary conditions, please understand and the function w. Whatever you assume has to be differentiable, how many times no here it is differentiable by four times here twice, so you see that is the attractiveness of this. Because, exact solution if you want that is the only approximate solution in the limit intending to the very large, it approaches the exact solution.

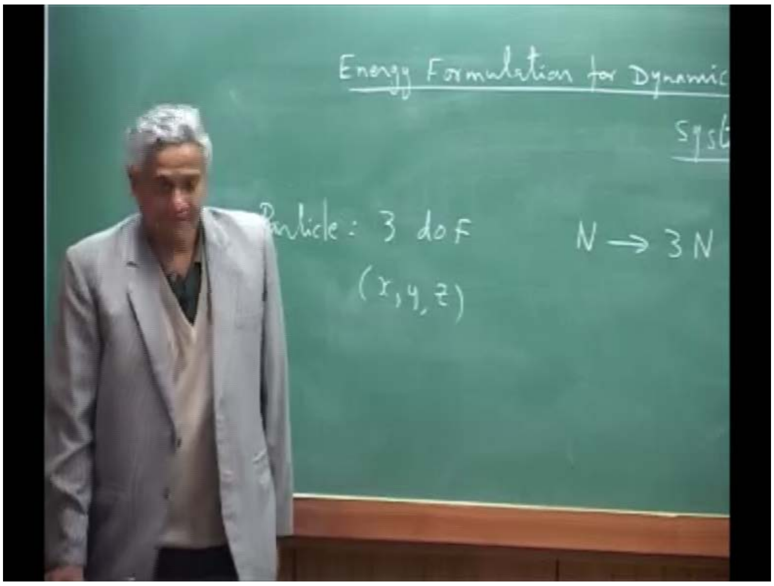
But, here I am getting exact solution for it is not easy it is a very difficult task, where as you can go ahead and do approximate solution and normally. What you do is you will take more number of gamma I and you show converge, how your deformation is converge with the increase in the number of assumed shape only.

So, the exact solution is the one, which should satisfy four boundary conditions four means two on this side two on that side, these two on that side is only the post boundary conditions, you will have the share force bending moment boundary condition, here you have displacement boundary condition, they must be satisfied and in addition it should satisfied the differential equation.

Now, you see that is the exact solution whereas, here what we have done is we are not bother about the post boundary condition at all. We simply use geometric boundary condition like displacement on slope 0 and it is also differential only lower order. So, that is the attractiveness of the energy method, you want to get approximate solution for a comply.

Because, not only you can have exact solution please understand any general wing structure you cannot get exact solution, you can always you have to go for approximate solution only and this method is very attractive and this is what you will be using in these things. I will give one more assignment which I will send it you, which will be related to this particular problem this is energy formulation for dynamical system.

(Refer Slide Time: 1:04:34)



Now, like I said earlier you can derive this for deformable structure or you can derive for rigid bodies. If you look at the classical mechanical book they will starts with rigid body particles. So, they will get the Lagrange equation and Hamilton principles everything will come here. So, what I do is I will follow that approach.

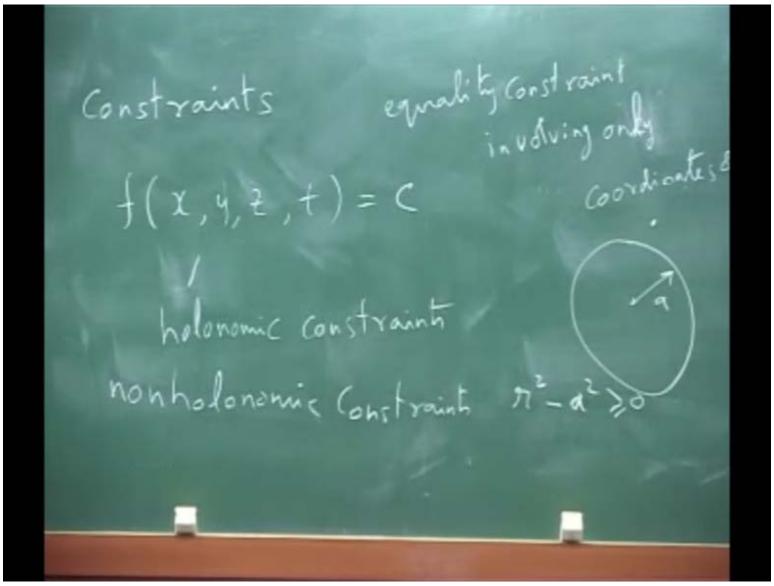
But, the final result is applicable even for flexible deformable structure, if you follow deformable structure it is identical, only thing you have to take the boundary condition and etcetera external work done by the various things, apply the boundary condition equation of equilibrium everything will be applied and then you will finally, arrive at a same Hamilton principle.

So, what we will do is I will do for the rigid body the first thing is in rigid body, we have constraints what do we mean by constraints. If we have one free particle we can have 3 degree of freedom particle x y z, you can have two you can have two into three six suppose, if you have n particle you will have 3 n degrees of freedom. Each one is independent if they are suppose, not all of them are independent they cannot move each one freely there are constraints in the system.

Constraints in the system in the sense what type of constraints, will relate to the coordinate of each point. Because, 3 degree of freedom when I say x comma y comma zee the particle is moving; that means, x is changing with time, y is changing with time, z in changing with time, if I have n particle again all the particles x with change with time.

But, if you say like a rigid body when you talk about if you take two particle the distance between them is always a constant that means, what you are doing you are putting constraint not that everything is free to go as they want. But, you have to constraints between the particles, that means these are constraint equations they say.

(Refer Slide Time: 1:08:00)



Constraints or you may say constraints, but what type constraints they can be because I can write it constraint equations can be x comma y comma zee. Because, time is also there with time they can evolve this can be equal to a constant, time can be explicit time need not be explicit, but this is a equality constraints involving.

Please, understand only coordinates that all and time, not there derivatives or anything like that this is the only these type of constraints are called holonomic. That is equality constraints any constraint which is written in this form is a holonomic constraint, you can have another type of constraint where suppose, a particle is inside a box it should not go out that means, its position is constraint within the box that means, it is a inequality it can be inside or it can be on the boundary it can be outside.

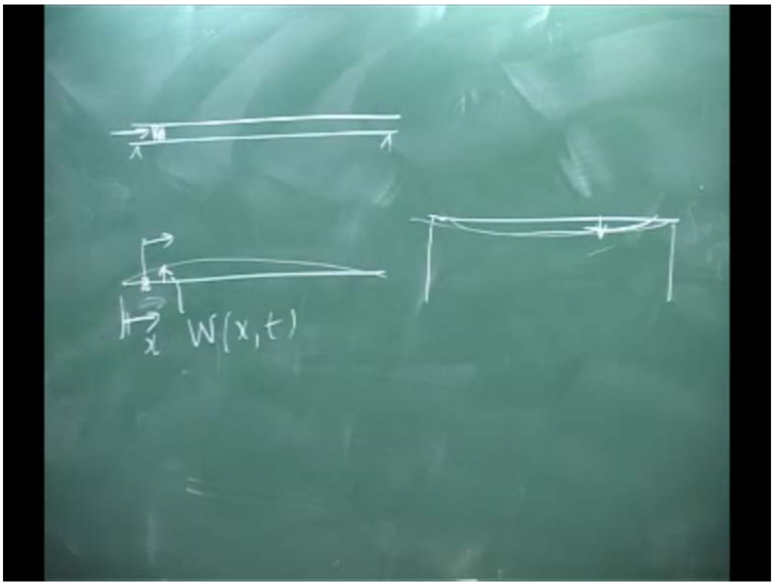
Now, these type of constraints are non holonomic constraints that is not equality, but it can be greater than or less than or whatever type, they are non holonomic see one example which they say is a particle must be r, which should always be within some bound that means, you can say that you can if it is a sphere of radius a and you want it should be always r, should can be less than suppose, you can say it can be always outside greater than 0, it can only come and I then particle can never come inside.

Now, you can have derivative constraints, but normally in coordinates because all are the position in dynamics. Its described with generalized coordinates for one particle 3 degree of freedom x y z. So, the constraints will be only in terms of these, but you can have another type of constraint, where differential will come the differential is one of the example that is given in rolling disk on a crown and in that, the constraint it cannot go up and distance traveled is actually velocity.

Because, it is rolling without slipping, so if you put all those condition then v is a omega, now it is a velocity constraint velocity is actually derivative of the displacement. Those types of constraints are also nonholonomic because they it cannot be they are not consider as holonomic constraints treating them is different, you have to treat them separately only holonomic constraints of this type can be easily consider. But, actually the validity of the whole approach is not restricted.

Because, of this please understand it is still many situation, it is applicable actually the whole theory started with the I think a particle automatic level vibration, some things like that even suppose, a bead which is sliding in a string and the string is moving. Now these type of constraints cannot be written in this fashion a bead which is sliding, but the one of the problem bead sliding is similar to I will say this problem, I thought I will let you know I want to due to derive later.

(Refer Slide Time: 01:13:19)



That is a, this is also aero elastic problem you have a pipe and fluid is going in, here you treat the fluid as the fluid mass. I am approximating this whole as this is a something and this is mass is moving, but this type also can bend in this situation. How will you write equation motion further beam or pipe like your if you take a bridge, the rails are like a our pillar you have a pillar this is a railway track on a railway track some mass is going that means, what is happen.

Since, bridge will bend and this is going down this will actually have even though, this is moving this way. The bridge is bending you will have bending load coming in this problem is the little axial flow, we always problem very similar you can derive the equation. I would like you to try just assuming that this is a fixed point starting point this is a particle, the particle is moving to the right on a string.

But, the string is having a displacement W, which is a function of x comma t this is I will take it as, sorry this is x and this particle is having its own velocity. Now, you write what is the acceleration of this mass in vertical line, it is the expression then you will find this is very similar this equation directly you can use, a flow through a pipe and if the velocity of the flow increase, the pipe will start getting into some vibration these are all there in a nuclear reactor where the cooling thing.

Because, it will start that is always also one of the aero elastic problem, that the flow creating. This another aero elastic of course that is the flattering of flagon, that also when the wind flow is flowing, that is also a aero elastic problem all these are, but modeling each one of them required different approaches. Now, let us take this as a holonomic constraints and what we will do is, how this constraints are represented.

When you give a virtual displacements what happen because when you have constraints the number of independent degree of freedom will get reduced, by how many constraints you have. So, you will finally, you will have only 3 N minus c this is the number of constraints, not constant this is a number of holonomic constraints, you may say you can take number of holonomic. This is a final degree of freedom and these are independent this is my degrees of freedom of the system.