Aero Elasticity

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Lecture – 26

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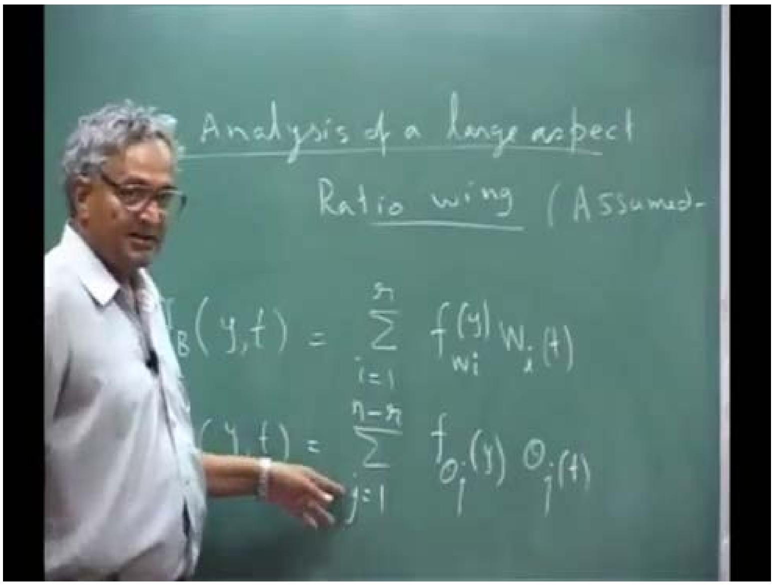

Learn how to do flutter analysis of a wing of a large aspect ratio wing. But, this is also called, you do by assumed mode method, see if you are given a wing, because you had the wing data, you first calculate the bending torsion frequencies, that is the natural frequencies of the system. Along with that, you get the natural that mode shapes, every frequency has a mode shape. Now, that is the free vibration problem, once you have, that is why in a industry, they take the wing, they do a FEM analysis, to get the natural frequency and mode shapes.

Once you have that particular mode shape, now you go ahead, because for our problem we are taking large aspect ratio, because we are splitting bending torsion kind of separator, sometimes both can be incoupled, couple mode. Now, how we did flutter, if you have this type of a situation, where you have to deal with a wing structure. So, the first thing is, what you do is, you assume your bending deformation, which is first i running from 1 to r, I will explain what this is that is, I draw a picture, this is my wing and I have my elastic axis, this is my y axis.

So, I can solve the bending problem, now these are the mode shapes, which are functions of y, I take first r bending modes, similarly I will have torsion about the elastic axis, I will have torsional modes. So, I will call that as theta t, which is the function of y t, which is again some summation, here you will have, i may be you start from or you can call it i or j it does not matter, j running from 1 to some n minus r, f theta j y and theta j t. That is, total number of modes I am considering is n, because here r, here n minus r, but some of them bending modes, some of them torsional modes. These are obtained from, this is what is assumed mode and you can obtain this from using Rayleigh Ritz principles, get the natural mode shapes.

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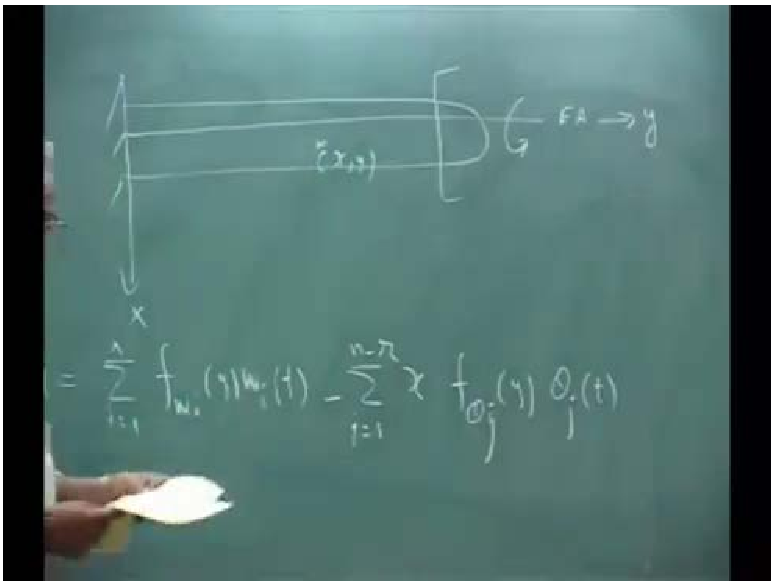

Now, what you do is, you say this is my wing ((Refer Time: 04:33)) the deformation at any point, any point means, this is my X, that is Y, this is Z. So, this point has, because all are flat plate, because I am not really looking at the profile, you will write W which is a function of x comma y comma t, W is the vertical displacement at this point, because you will have all this, contribution everything is participating simultaneously. So, you will have summation i running from 1 to r, f. W i y W i t and then, you draw the cross section, your cross section will look, it may be…

Your elastic axis is here and you are referring your X from the elastic axis, this is and then, this is your W B and this angle is your theta t, this is your U on coming flow. Now, when I take this, I will have minus, I will put that total j running from 1 to n minus r x into f theta j theta j. Now, you see using this expression I said that, my wing is going to deform in this fashion, my generalized coordinates are W i and theta j. Now, what I have to do, I have to go and calculate the strain energy and the kinetic energy and then, the external load and I can apply directly the Lagrange's equation.

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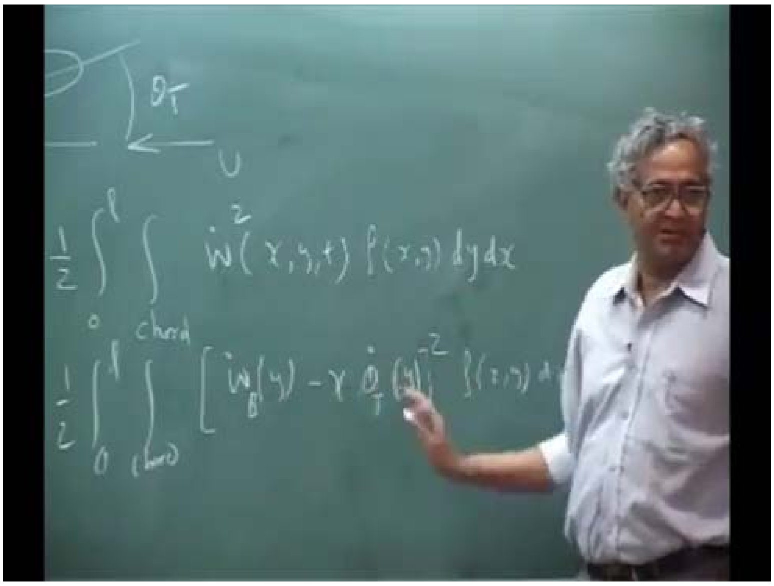

So, if I write my kinetic energy expression, T is half you will have integral 0 to l then, integral over chord, this you will have W dot x comma y comma t whole square rho, this is my chord. So, I can have, so I multiply by thickness, I am basically throwing it out, so you will have d y d x. So, this is the density or basically mass per unit area you can say, this is mass per unit area, this is the area, this is the... And now, you substitute that whole thing and when you substitute, you will have this is nothing but W B dot, because this is what, this is W B y comma t minus x theta t, that is what this expression is W.

Now, I take this, put a dot substitute there, I will have a half 0 to l chord, you will have W dot of minus x theta dot d whole square then, rho x y d x d y you can take. See, the chord integration is along the x, so I can split it integral along the chord separately. When I do that, integral rho d x that becomes the mass per unit length and then, x into rho d x, that is the mass offset and x square rho d x, mass moment of inertia per unit length.

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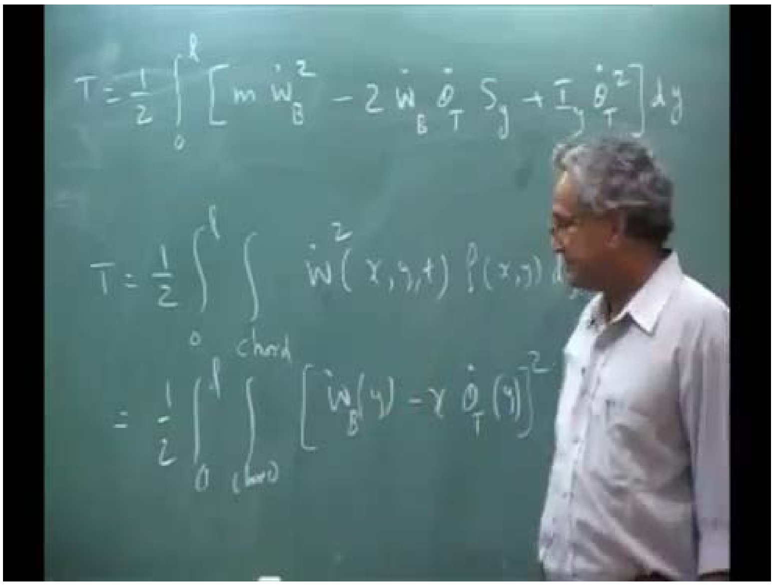

So, I will write my, may be I erase this part, the kinetic energy expression will be half 0 to l m W dot B square minus 2 W dot B theta dot T into S, this is the static m into offset distance S y about the y axis that is why, y axis is your elastic axis then, plus I y theta dot T square d y. So, you know this is the mass per unit length, this is the static moment, this is the mass moment of inertia, all per unit length.

Now, I can go ahead, substitute this full expression in this and you will have what, because I am squaring everything. So, you will have product of every term, because every term will have product, so this will be my full kinetic energy equation.

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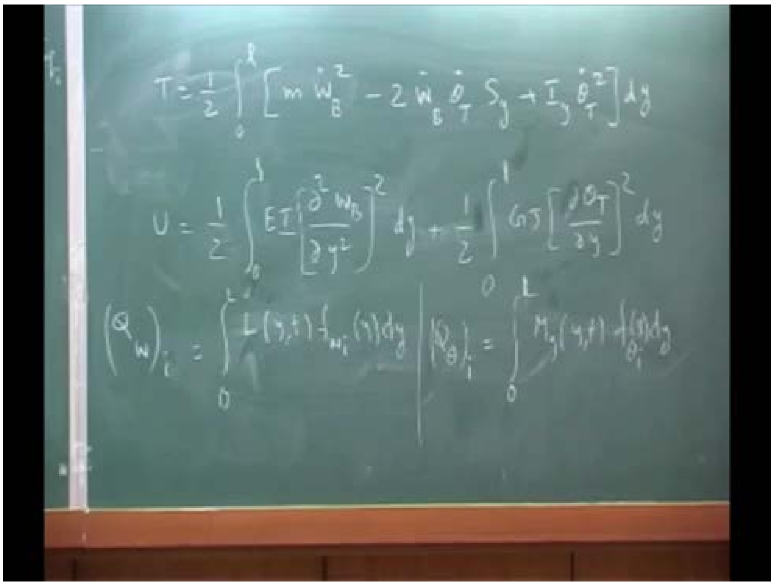

Now, when I go and write my bending strain energy, later I will do for ((Refer Time: 11:19)), it is easy to write then, strain energy will be, you will have U, you will have strain energy due to bending that is, half E I del square W B over delta y square whole square d y plus half 0 to l, this is 0 to l, this is G J delta theta T by delta y whole square d y, this is my strain energy expression.

Now, I have to get the external virtual works, basically virtual work will be due to the lift force, which is acting at the quarter chord if it is a subsonic, into the displacement at that point and a moment, because that will also twist. So, you have the virtual work due to lift and moment, and since you have taken elastic axis as the reference, you will directly take the your Q virtual work, not virtual work I am saying this is the generalized force, because you know that, delta W external, you will write it as summation Q i delta q i i running from 1 to n.

Now, these are the generalized force corresponding to the generalized coordinate, I have several generalized coordinates W i 1 to r, theta j 1 to n minus r. You have to do the work done by that that is, the generalized force, generalized force finally you will know that, it is nothing but the lift force into the mode shape, that is the assumed mode. So, you will get Q W for the corresponding to the i th value, that will be 0 to l lift force y comma t f W i y d y, this is my generalized force corresponding to i th.

And similarly, you will have Q theta i corresponds to, you will have moment, please note this moment is about the elastic axis into f theta i d y, this is the moment about the elastic axis. Now, I have all the expressions, you can go ahead and then, apply Langrage's equation, get the equation of motion. And if you take only, because it is easy to get that for one mode, that is why I will write the expression for one assumed mode for bending, one assumed mode for torsion. Because, that becomes simpler for me to write, otherwise you can have as many modes as you want.

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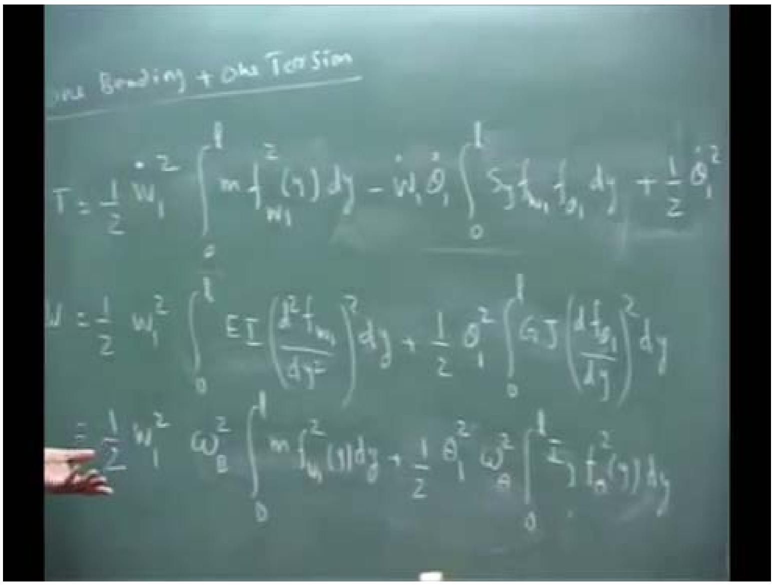

Now, assuming we have only one mode then, what will be our entire expression one mode, one bending mode plus one torsion. Your kinetic energy will be half, instead of bending I am taking W, because you will get W 1, only one mode is there. So, you will have W B, when I substitute this is nothing but f W 1, so I can put just W 1 dot square 0 to l m f W 1 d y minus W 1 dot theta 1 dot 0 to l S y f W 1 f theta 1 d y plus half theta 1 dot square integral 0 to l I y f theta 1 square y, that is all.

This is my, because single mode and then my strain energy, half W 1 square 0 to l E I, now strain energy is delta W by delta y, I am putting this is only a function of y, this is not a function of y, that is why I have taken it outside. This will be d square f W 1 by d y square whole square d y plus half theta 1 square integral 0 to l G J d f theta 1 over d y. And of course your Q, that will be just only one term, f W 1 and then, f theta 1 and y, but you know the expression for lift and moment from Theodorsen's theory given at every section.

So, you have to write that expression of lift in that place and the expression of moment you have to put it, which will have c of k please understand and that is the complex thing. So, you need to substitute that lift and moment expression here, now there are some slight changes that are made. In the sense, I can write my from previbration problem, you know that, my maximum kinetic energy is maximum potential energy. That is why, this particular thing you can write it, because W is the time dependent generalized coordinate and these are strain, ((Refer Time: 19:25)) this is strain energy, this is kinetic energy.

I can write, it is like you know that, I just write that ((Refer Time: 19:34)) equation that, M X double dot plus k X equal to f of t, but you know that, k is that is the natural frequency. That means, I am replacing my k, mass times omega n square, omega n square is the natural frequency in that particular mode. So, one of the ways people write it, you write this, replace this expression in the energy form, in terms of m omega square, so this will be written as half W 1 square.

So, omega you can say, if you want to put bending, you can put bending, square into 0 to l m f square W 1 y d y, this you can write it as one half theta 1 square omega theta square 0 to l, you will write I y f theta. You simply write the strain energy in terms of the kinetic energy, but with the omega, it is a standard. Otherwise if you do not want to do it, you just carry it as it is, there is absolutely no problem, you are just replacing this, that is all, the strain energy formed by the mass.

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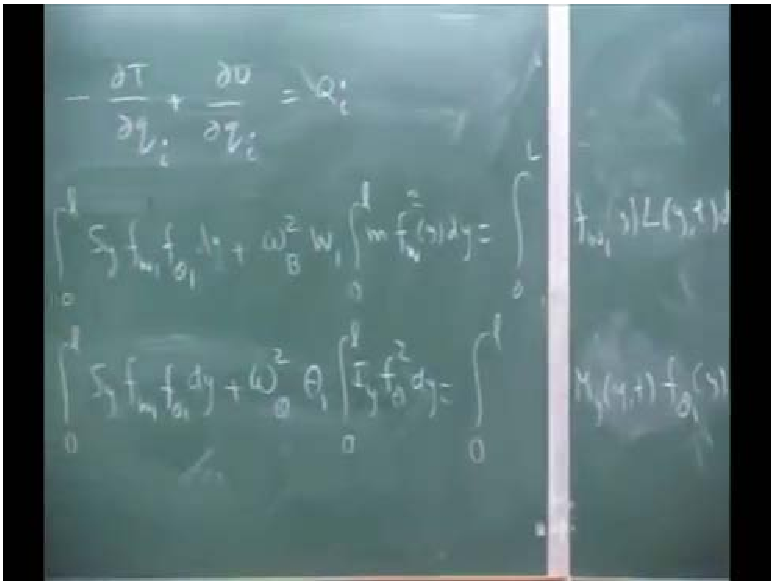

Now, you can write your equation of motion, substitute in Langrage's equation and then, get the equations, because you know Langrage's equation. Now, let me erase this clear, because Langrage's equation is d over d t of delta T by delta q dot then, because here T is not a function of q, because only q dot it is there, that is why the term will go off

Finally, you will have only, let me write that minus you put a ((Refer Time: 22:40)) i, i, i, this will be q i, this is my Langrage's equation.

Now, apply this, you will have one equation for W and another one for theta, so you have two equations, those two equations will be I will write that. So, you will have W 1 double dot integral 0 to l m f W y square d y minus theta 1 double dot 0 to l S y f W 1 f theta 1 d y plus omega bending square, W 1 integral 0 to l m f W square y d y equals your external force. Or I would say the generalized force corresponding to bending equation, that is nothing but lift f W.

So you have 0 to l f W y lift y comma t d y, this is my bending equation. Or I would call it, this is the equation corresponding to the generalized coordinate W 1, but it is coupled with theta please understand then, you will have the q 2, q 2 will be theta 1. So, you will have theta 1 double dot 0 to l I y f theta 1 square d y minus W double dot 1 integral 0 to l S y f W 1 f theta 1 d y plus omega theta square theta 1 integral 0 to l I y f theta square d y equals integral 0 to l, you will have the moment expression. So, that is the M y y comma t f theta 1 y d y, this is W 1, so you see now I have two ordinary differential equations.

Because, these integrals you can evaluate, because you know the mass per unit length of the wing, this is the assumed mode, simply multiply get this term, this is mass offset, product of these two and this is anyway the same expression as the first one. And you lift expression is what you have to write from the other one, because you know we have used, because I am just writing it for convenience.

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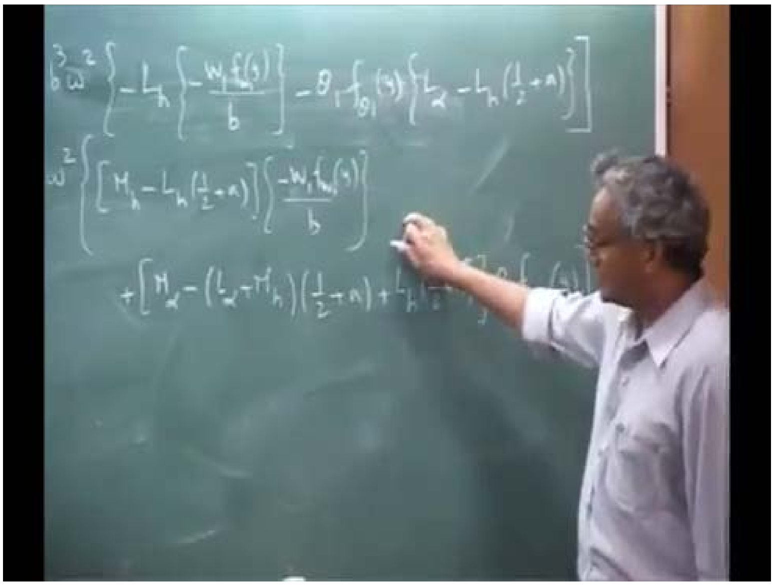

Because, you have to be careful, how the lift earlier was defined and now, because you say W is displacement in the positive G, whereas our lift was obtaining from h, which is downward. So, I will write the expression for lift per unit length, this is pi rho b cube omega square minus L h, here you will put minus W 1 f y b, because lift is a function of y. W 1 is a generalized coordinate and you put a minus sign, because this is h, you took it downward positive, here you are taking W, that is why this minus sign.

Then, minus theta 1 f theta, this is I should use f W 1, theta 1 f theta 1 y into L alpha minus L h half plus a, this is my lift expression. Now, when you go to moment, M y y of t, you will have pi rho b to the power 4 omega square, you will again use M h minus L h half plus a into same, minus W 1 f W 1 y over b. Then, you will have the other term, you will have plus M alpha minus L alpha plus M h into half plus a plus L h half plus a whole square into theta 1 f theta 1 y.

Now, you see this expression, you will go and put here, multiplied by a f theta 1 y, integrate it and what you will have, W 1 theta 1, theta 1 W 1, on the left hand side also you have W 1 theta 1. Now, this is identical to airfoil problem, only thing is everything becomes an integral, get the integral first, evaluate the integrals, put them. Then, you collect all the terms, you can always substitute that, now I am assuming W equals, maybe I erase this part.

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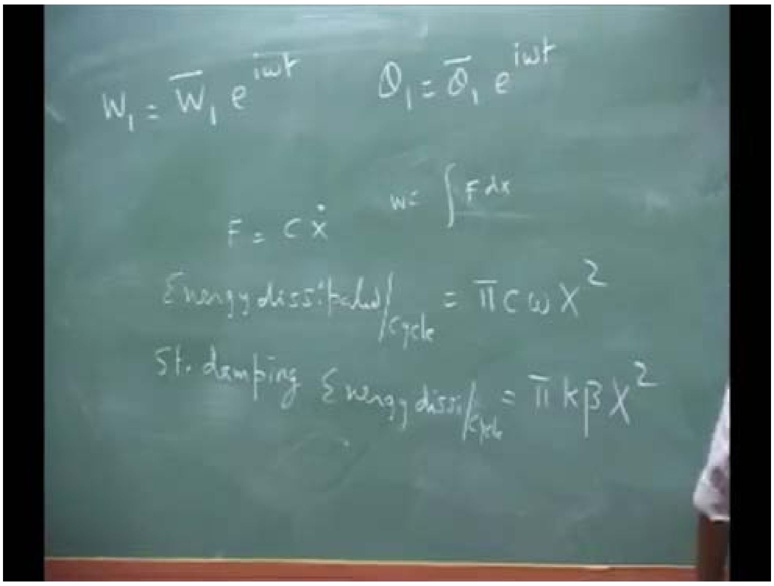

Because, you have to assume, I am assuming my W 1 is e power i omega t and theta 1 is that means, I am assuming harmonic motion, this is what you have assumed even there. Substitute that then, you are going to have a equation in, e power i omega t will cancel out everywhere, you will have W 1 theta 1 in all these, W 1 bar actually you can say W 1 bar. It will be an Eigen value problem and that is exactly what we have in the 2 D airfoil, so this problem after substituting that and then, making this assumption, it will become a complex Eigen value problem.

Now, you apply v g method or v k method, because v g method what you do, you first assume the value of k, get the c of k, substitute here then, solve for the Eigen values. Basically, it will be a complex equation Eigen values, you get x plus i y, I explained to you last time then, correspondingly you get the g, you plot. Whenever the g goes to 0, that is your flutter point. Suppose, if you have a structural damping, because this is one thing structural damping means, because usually you have structure, the material inside drops when it vibrates.

That introduces a little bit of damping in the structure, that is what is called structural damping, but I hope you all know the different between viscous damping and structural damping, both are damping. There are certain suttle differences, I will briefly describe that, because that part you have to know, what is the structural damping, what is a viscous damping, viscous damping is what, f is some c X dot that you will write, energy dissipated per cycle of the viscous damper is actually pi c omega X square.

Because, energy dissipated is at f d x that is, integral f d x over one cycle, this is your W over one cycle, f is c x dot. Now, you assume X equal to X sin omega t or X cos omega t, you substitute, you integral over one period, this is what you will get. That means, the energy dissipated is a function of amplitude square and is the function of frequency, pi is it comes out of the integration constant, c is the damping constant.

In the case of structural damping, this is called structural damping or hysteretic damping, that is the another name, the energy dissipated, it is the experimental observation, energy dissipated per cycle is, we write it as pi k beta X square, what the structural damping, they found the energy dissipated is a function of amplitude square, but it is independent of the frequency of oscillation. Now, if you have a damping, which has this characteristic, that the energy dissipated per cycle is independent of omega then, what is the type of damping, that is called the complex damping. What they say is, complex stiffness term, how that comes about is, I will just mention that part.

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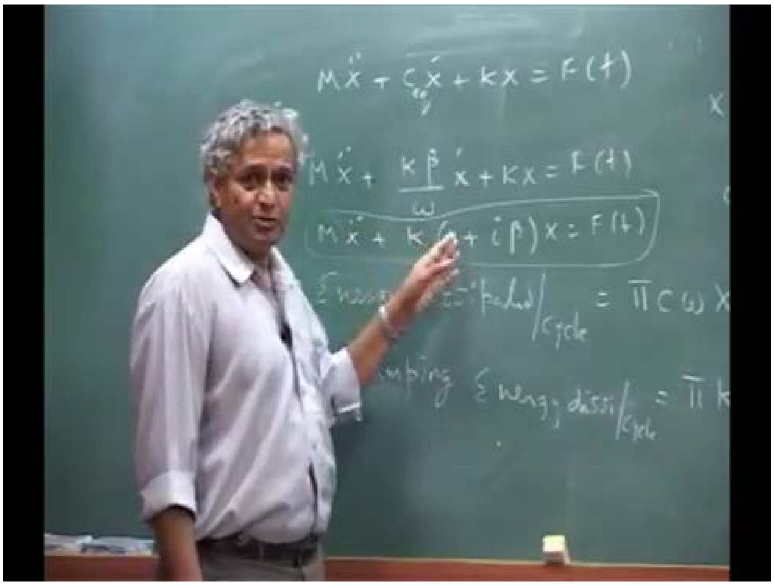

See, your standard equation is M X double dot plus c X dot plus k X equals f of t, now if I have a different damping, different type of damping, what I normally do is, I write c equivalent. When I write c equivalent, here I equate both of them, I will find out what is the equivalent viscous damper corresponding to structural damping. So, I will get what C e q will be, what k omega, so I substitute here, that will become M X double dot plus k beta over omega f of t.

But, X is I assume it is a harmonic motion only, all these things are for harmonic motion, so if I assume X equal to X bar E I omega t, X dot will be i omega X. So, what will happen, omega will cancel out, I will get k into 1 plus i beta X, because I am substituting that, so this term is now complex stiffness. So, what it means is, the energy are in this particular case, what they will say, the damping force is proportional to displacement.

See, c X dot, when you write c X dot damping for viscous damper, damping force is proportional to velocity. Whereas, if you talk structural damping, they will say it is the damping force is proportional to displacement, but it is 90 degree phase to displacement, that is why that phase comes here i. But, it is proportional to displacement and this term is called the complex stiffness term. Now, you will understand complex stiffness, k over m if I take it omega square, omega square into 1 plus i beta, this is what we have done in the flutter formula, v g method. We simply added a some structural fictitious, but your structure itself can have a damping then, that you can add it.

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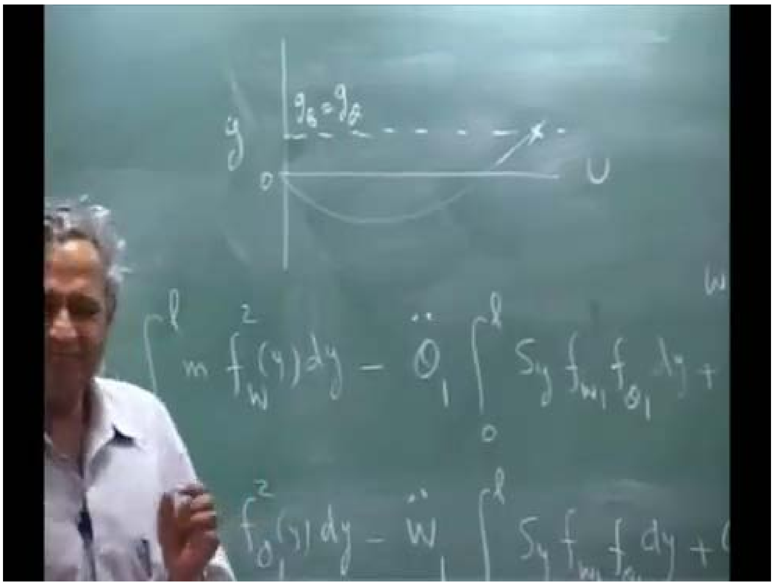

You simply go here omega square, simply multiply by 1 plus i g B, that is all that means, I have included a structural damping for this. Similarly, I can put for omega theta square, I put 1 i g theta, but normally in structural damping you say, bending does not have something separate, torsion does they will say, all are same, they use the same. Now, if you have 0 damping in your v g diagram, wherever it crosses this 0 point g, you will say that is a flutter.

Suppose, if you have a inherent structural damping, you will draw that line in your, this is the g, this is that U b over, the line will go like this, this is the 0. If you already have some damping, you extrapolate, you say this is your flutter point, because you already have a positive damping in the structure, that positive should become 0. So, that is how, so this becomes if you do not have any other, this is your g, that g B equals may be g theta, this is that usually these are very small value of damping, 0.03.

See, if it is 2 3 percent, it is very large, g theta is actually you know 2 3 percent is very large damping, sometimes you will get 0.5 percent, 0.5 percent means 0.005, the value of that g. If you get 3 4 percent, you will get a very high damping, that is the large value of damping, it is difficult to provide that much damping, that is why people say, composite material, we can have some constraint layer in between. So that, when it vibrates, I have some kind of flexible, which we dissipate.

Now, multi functional materials, these are all now current research people are doing so that, I can have a good damping also built into my structure, because normal metallic structure, the structural damping will be of the order of 0.2 percent to 0.3 percent, at the most 0.5. Now, have you understood this part, you can also apply, I told you that lift itself can be written as in the finite state model then, you have a time domain aerodynamic model, that also you can use here.

Then, you will have in addition to this W 1 theta 1, last class I told you, you get that X states, they will be additional state variable and you will have corresponding equation for that. And then, you add those equations also and then solve, this is how the flutter problem is solved for a wing. Now, we take the next topic, which I will give you very brief introduction that is all, I will not go into the details of the problem.

I will just briefly describe, what is the problem, what are the key aspects and then, I will give the reference for that, so which you can look at it, you can learn, because it is not complicated.

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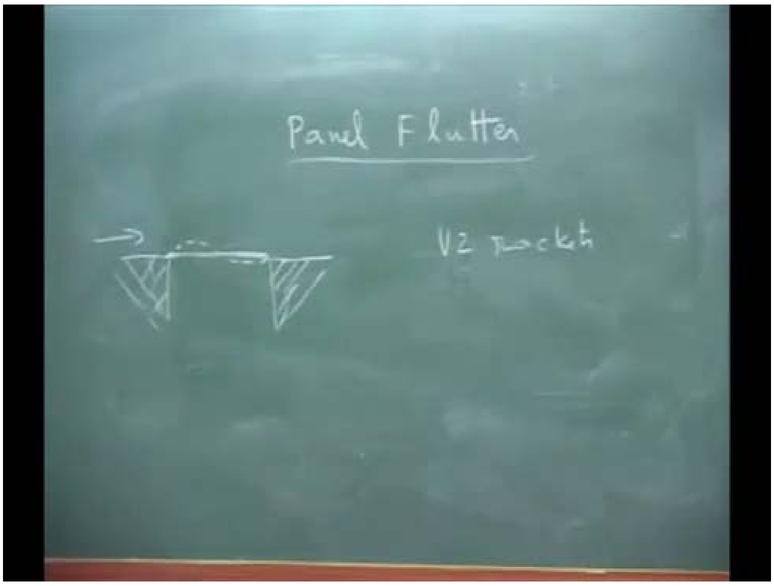

The next topic is the panel flutter, what is it mean by panel flutter, see till now we said that, if I draw an airfoil, this is a panel, the panel does not deform. Or if I have something like this, if I have a panel, the flow is going over it, if it starts deforming like this, that is the chord wise deformation like your fluttering of a flag, what happens. The flag is changing it is shape, it is just deforming, the shape itself is changing. Till now, we said out airfoil panel, airfoil retains it is shape, only thing is, it can rotate, it can bend, but it is surface cannot deform.

Now, you talk about the fish type of you know, here we talk about the surface itself deforms then, what type of problem we will have, this is what is first talked about it. If my surface is also moving then, it is a complicated problem, that is why that problem you do not talk about flow fast an aerofoil, what you do is, you take a panel. This is the panel and the panel is fixed on a, because between two supports, now the panel can be uniform.

If it is a very thin thing, you can have deformation, because you know that when wind blows, even if you have a some other shamiyana or anything what it does, it does lateral vibration. Now, those thing at subsonic speed, for it happen it should be very thin, but for the aerospace application, there is nothing like that thin material, we do not use that. Therefore, it does not happen in subsonic cases, actually the phenomena was observed in the aerospace line. I am not talking about the flag fluttering or a shamiyana you put it then, wind blows and then, the whole thing can go up and down, you can even have a static divergence type of a problem. Now, the first time the panel flutter was observed in, it was in German V 2 rocket, because the panel started vibrating, but it is flow passed only one side please understand, it is an external flow.

Internally, it is a whatever pressure initially you can take it as an atmospheric pressure, it is not like an aerofoil, in the sense the flow is on both sides. So here, normally you say what is the, you have to get the pressure difference, inside pressure you already know, you say that is p infinity. But, what is the surface pressure and this was observed in normal for aeronautical thing for the thickness of the panel, it happens at the supersonic speed, that is why they say it is a supersonic panel flutter.

So, 1960s people were solving this problem of panel flutter substantially, but then supersonic speed, so they started using unsteady erotic theory associated for supersonic. And we found out, you remember we derived piston theory, you can use piston theory to get the aero dynamic force on the surface and then solve, but the problem is a panel problem, it is not a wing type of problem. So, the whole study went into what kind of boundary condition we can have then, in a panel you know that, under thermal stress, please understand.

When you have constraint like this, if your temperature changes, the stresses will get developed, axial stresses, because of the end fixes then, the panel can deform due to thermal loading and when it deforms, your surface is changing. Now, that can also another cost, so your surface deformation thermal show, how the problem was treated is, I will just draw a simple diagram and then, I will basically give you a few introductory thing. Then, you take a now please understand, you have to know panel equations, your structure is not a wing, it is a panel or you talk about plate problems.

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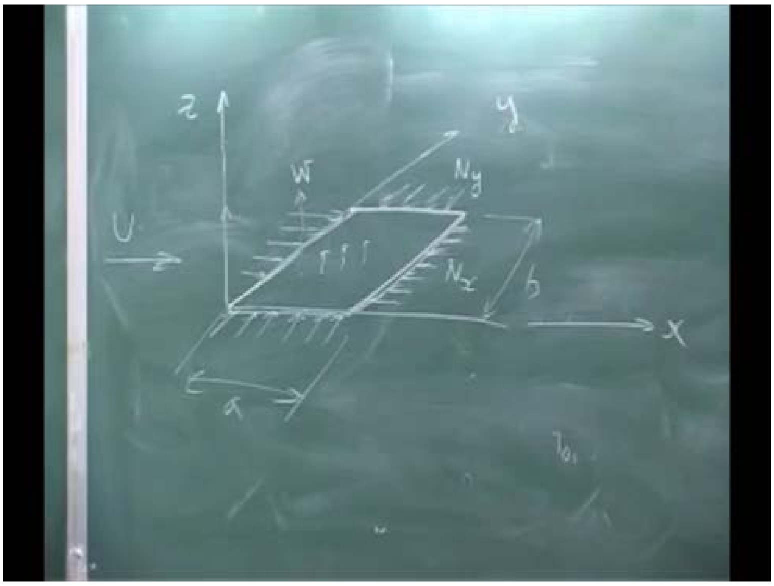

So, the problem was, this is x, y, this is z, you put a panel, which is acted on by and the panel deforms in W direction. You will have pressure everywhere, these N x N y are the compressive stress, you can say due to the surface, a boundary condition. Now, this is the problem, you can now start deriving the equation for a, you can have curved panel you can have flat panel and then, the pressure this is the flow, which is coming over it, not under it, because this is completely covered and the dimensions a, b.

So, the problem that was considered is basically a plate problem, so initially you take my plate is flat. Under the action of N x N y, what will be my deformation, because this is like your, you have to talk about for small deformation, the governing equations.

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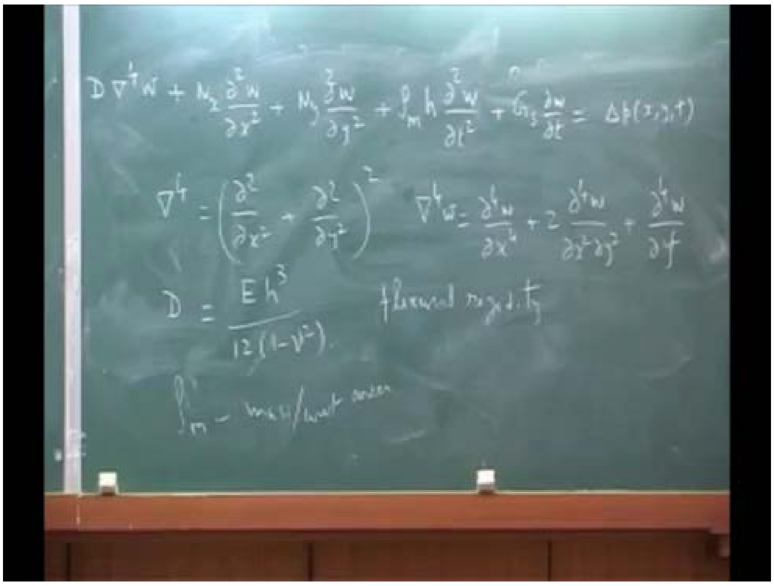

I will write the governing equation for this, D del to the power 4 W, W is a function of x y and time. Please note, W is a function of x comma y comma t, this is the plate equation actually N x del square W by delta X square N y del square W by delta y square plus, this is the mass density, h is the thickness of the plate, del square W by delta t square. And if you want to add some structural damping, they will put a G of s also, delta W by delta t, this is some damping term, this is there is a pressure delta p, and what is del square, this is del power 4 is nothing but del square by delta X square plus del square by delta y square whole square. Or in other words, del power W implies, del to the power 4 W by delta X 4 plus 2 del 4 W by delta X square delta y square plus del 4 W over delta y 4. And then D, D is called the flexural rigidity E h cube over 12 into 1 minus nu square, nu is the Poisson ratio, this is flexural rigidity. And then, rho m is mass density.

Mass means is that the area, you can say mass per unit thickness, because h is the thickness, mass per unit, you can say unit area, not unit, unit area, that h is the mass of that. Now, this is the plate equation, because I am not deriving, I do not know you have done any plate theory or not. Since you have not done plate theory, unless you do the plate theory, you will not know. Now, this is what the starting point is, but delta p that is, the pressure differential between inside outside due to supersonic flow.

So, this is where they use piston theory, but again there were lot of approximations that was used. I will just briefly give you the kind of approximations, which people have used and you will know that, that is due to piston theory only. Because, piston theory what we had, you remember piston theory that is, the high frequency approximation.

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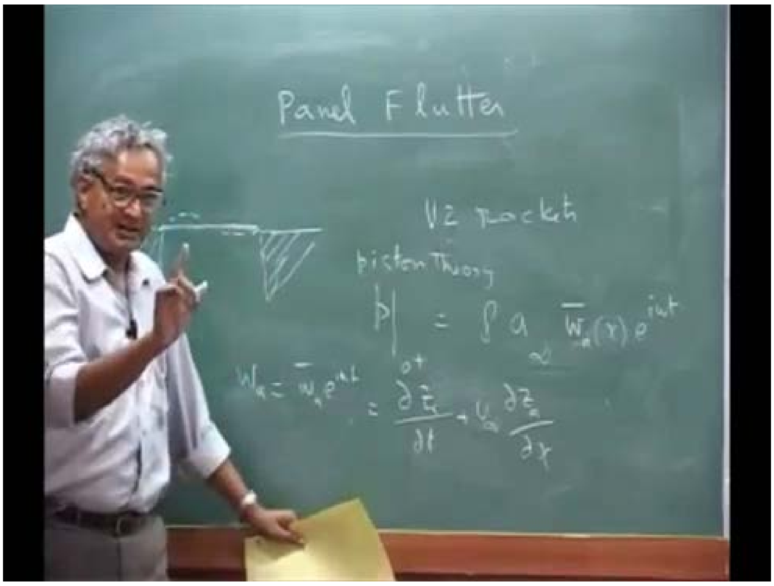

Piston theory says, p some 0 plus, this is due to piston theory, which we derived earlier, high frequency of the supersonic. We had rho a infinity W bar a at X W bar a is, if it is oscillating, this is W bar e i omega t, that is all. But, W bar e i omega t you know from the, because W a is what, W bar a e i omega t, which is delta z aerofoil delta t plus U infinity delta X. You got this expression, see W is the velocity and this is the surface, z a is the displacement at that point.

Now, in z a you substitute W of the plate deflection at that point, here that W is plate, here this W is the velocity. So, please understand, you should not get a confusion over these two. Now, this is where they use the piston theory then, there where lot of different different approximations. So, I will just give you couple of equations, which people have used and that will just give you some idea.

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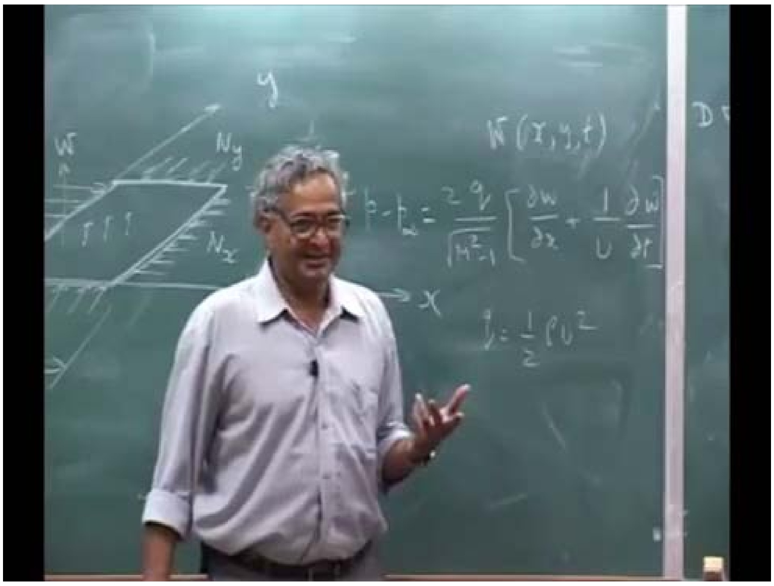

What they did was, p minus p infinity is, this is from square root of M square minus 1 into delta W by delta X plus 1 over U delta W by delta t, because you know q is half rho or they have different expression also. Sometimes, they have put one more term here for different mach numbers, that was derived from another approximation, that is why you will find in the literature on plate theory, I will give just the reference, different expressions they are using.

But, sometimes people neglect this term also, because this represents the instantaneous, what is this, delta W by delta t is the velocity at that point, divided by U that is, the local angle of attack, this is local slope. So, you add both of them, now there were modifications to this type of this expression, there were considerable modifications there, few airfoils use different. Now, and in the 1960s, I will give the reference now, there were lot of studies, which were performed on this panel flutter problem. Here, I will give two key references then, from there, you can drag down, even now some publications in fluids and structures, they write about sample flutter for composite materials.

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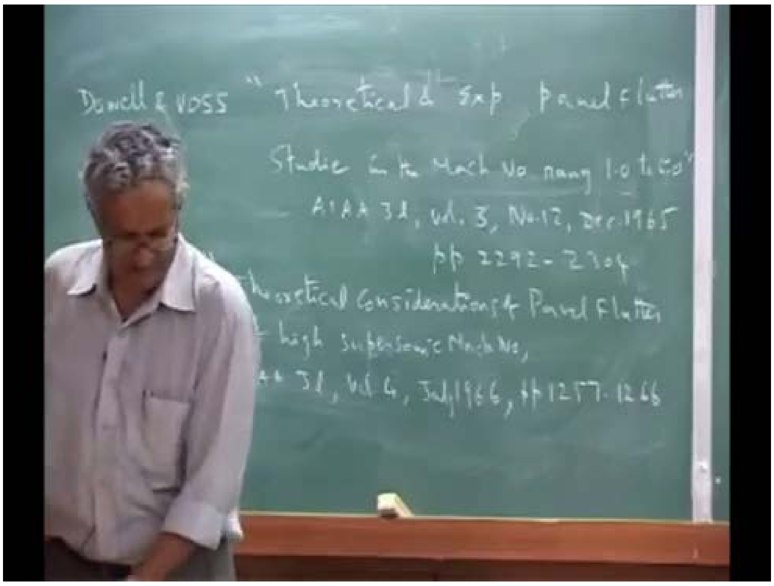

So, this is by Dowell and this is theoretical and experimental panel flutter studies in the mach number range 1.0 to 5.0, this is AIAA journal volume 5, no not volume 5, volume 3 number 12, December 1965 and page number 2292 to 2304, this is one paper. And another paper is, this is by Dugundji, because there are few people and then, similarly you do not, they all did theoretical considerations of panel flutter at high supersonic mach number.

This is also AIAA journal volume 4, this is July 1966, this page 1257 to 1266, see these two references, pretty much they tell you, because there is a nothing difficult about the problem. Because, you are not exposed to panel equations, how they are obtained degree, this is the panel equation under suppressive load or you can say on this side and then, there is a pressure, but the pressure expression is given here. Of course, they have different, please note here there is another possible, they will have some factor multiply, some M square minus.

I think I will just give that, because this is from an approximation, that is why you will suddenly find, what is this different people are using M square minus 2 over M square minus 1, this term will be sitting here along with p minus p infinity p infinity is... Now, you know pressure inside is p infinity, the top is, so this is the delta p at any point, simply substitute this is in terms of W, you know W, W, W. Now, what this equation, they have solved for different different a by b and then, convert the condition, under which it will have flutter.

These are non dimensionalized and things like that, because I know the procedure, since we have not done plate problems, no point is going into in that. But this is just to give you an idea that, but one of the simplest thing is, to avoid panel flutter, increase thickness. So, if the thickness is the little bit more than panel flutter but then, earlier it was all metallic structure then, they said, can we eliminate panel flutter with composites. So now, with a composite structure, you can have that then, lot of publications came in that.

See one is the mathematical approach to get the flutter speed, another one is depending on the choice of material, can I postpone my, basically the flutter speed, if I postpone then, it will be fine. So, these are studies once the composite structure came then, ((Refer Time: 01:04:33)) there is one another paper like. I thought these are old classical 1965 1966 paper, but Y. C. Phung also has done on panel flutter.

So, in the 1960s, there were lot of studies, now also you will find, you do the Google search, you will find panel flutter recently 2002 2004 some publications are coming, which refers to panel problems. And I think with this, just the brief note on panel flutter, there are other types of flutter problems also, one is non linear flutter ((Refer Time: 01:05:22)). But, that implies it is actually a non linear problem, all this we have done linear theory, non linear flutter you can have a stall flutter.

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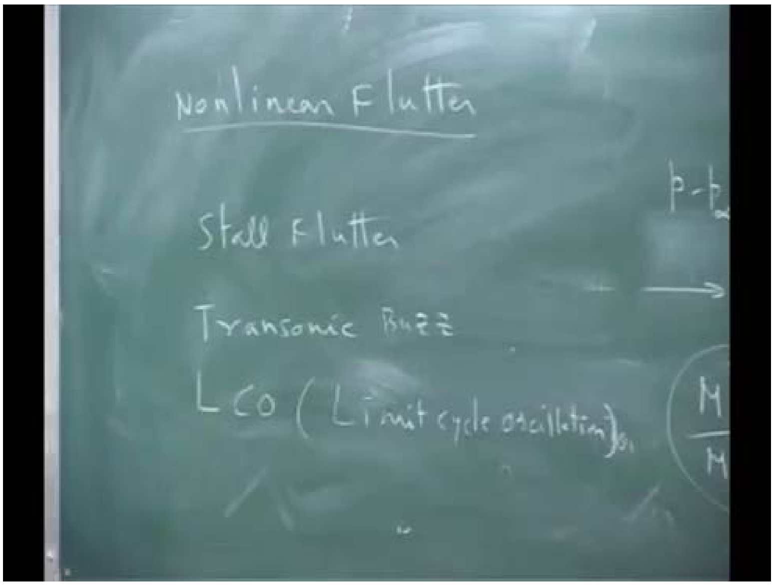

Stall flutter in the sense, your aerofoil goes into stall and then, again it comes back, get attached. So, these are all then, there is a one is stall, another one is transonic buzz, but even another problem it will tell you. See you will find, these all in the late 1990s, I would say suddenly people started copying LCO that is, Limit Cycle Oscillation. See, what this is limit cycle of this means, it happen in some of the I think ((Refer Time: 66:42)) started vibrating, but it did not flutter in the sense, flutter is it has to completely get into a unstable.

But, it did not become, unstable means it will break, the amplitude will keep on increasing only, but in the limit cycle oscillation what happened is, the amplitude reached a stage and then, it started only within it oscillated continuously then, what was the problem. So now, people started getting into non linear effects, because the moment some large amplitude come, the non linear effects of a problem comes. Even in your own this type of , can I have a limit cycle oscillation in the sense, it will not go out of bounds, but it will continue to oscillate.

That means, you can have non linearity from two sources, one from the aerodynamics, another one from structure itself. Structural non linearity means, I can put this spring k h, k alpha, not linear spring, they are non linear springs then, I can assume some non linear, analyze the flutter problem then, show that, beyond that peak well, it does not blow up, but it continues to oscillate. And another one is aerodynamic non linearity, you cannot solve, because if the amplitude goes a little more then, what will happen is, it will start having some kind of a bounded motion, bounded, but continuously.

This will lead to lot of fatigue life, because fatigue damage will completely very, very severe, because you are continuously vibrating. Now, we have done once that is, the one of the Ph.D. students did, but there are lot of studies on this. Now, that tells you then, stall flutter of course, helicopter plate we know that, it goes into stall and comes out of the stall and things like that. We use this basically the stall model for this and then, we said that, that can be a kinetic motion only, in the sense kinetic motion means, there will be all sort of frequencies coming into the picture.

But, it is the deterministic problem please understand, it is not a random problem, everything is deterministic, but it is non linear problem. So, non linear problem have their own, what do you call phenomena like suddenly you started seeing bifurcation in the sense, you will expect one type of motion, suddenly it can go this way or this way. Then, you can have more frequencies coming into the picture and you can have kinetic motion.

So, these are all in the non linear domain, but there it is essentially the research group, whatever there are working on, they get into that, they solve such problems. But, transonic is a non linear problem, that is different, here stall you have to have a stall model, because it is a flow is attached, detached, all over theory is attached flow, potential flow, mal disturbance, everything. So, you find the field is also growing in different areas.

Now, another one is micro stuff whatever we going and where the Reynolds number or whatever is stuff at different zone altogether and they have the viscous, the fluid viscosity is more important and they have their own theories developed. But, still they use only Theodorsen’s theory and other things, Theodorsen’s theory is what, it is a potential flow. There is no viscosity or anything, but they will make statement would finally use Theodorsen’s theory or even...

Student: ((Refer Time: 01:11:15))

Yes, that is all, they will use that and then solve them out.