Aero Elasticity
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Lecture – 24

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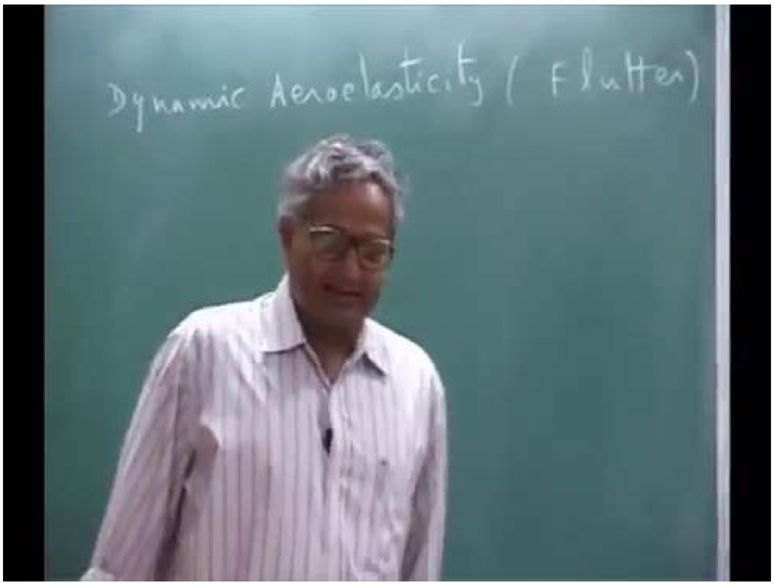

Today, we start dynamic aero elasticity, essentially flutter problem. See this is a dynamic instability, which happens in any elastic body which is in a flow, and particularly wings, control surfaces, vertical tide anything. When you have a lateral force that is like a lift on a vibrating twisting wings, at a particular speed, then it starts getting into a oscillation. And beyond some speed, the oscillations become violent and the structure will break, but below that speed any disturbance, the structure will oscillate and it will die out, that is the oscillations die out.

That means, essentially in the flutter, you need to find out what is that particular speed at which the wing will have sustained oscillation. And it is a basically our theory is a linear aerodynamic theory, because Laplace equation linear equation we have chosen. So, the study is essentially a perturbation stability analysis, because it is does not taken into large deformation analysis, because your flow is still attached, because everything is within the domain of linearity; so this is the linear you can call it stability analysis.

Now, what really happens in a body, which is immerse in a flow where it is vibrating, because there is an unsteady aerodynamic lift, which is acting as a external force and it is exciting the motion. So, there is a motion, there is a load and the load is dependent on the motion, so this is like a some kind of a close loop system. And essentially it goes to, whenever you have a damping in the structure, damping in any system we know that it damps out, the any disturbance will damp out if the energy is dissipated.

But, suppose at a particular condition the damping is 0, then what will happen is energy is not getting dissipated, so it will have a sustain. Once the damping become negative damping, negative damping in the sense it pumps energy into the system, then it will get into a violent diverging oscillation.

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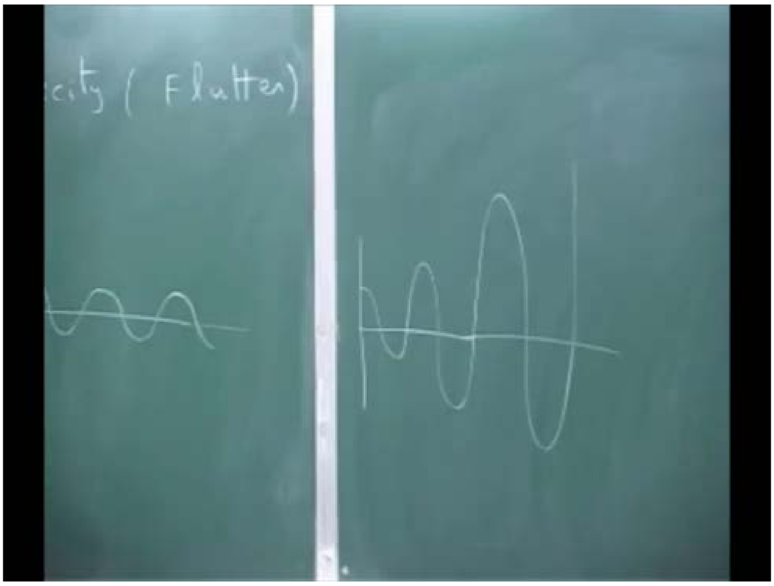

So, you can represent, like if there is a disturbance, if it dies out this is fine, it does not die out steady and then, it goes like this is a diverging. So, the analysis which we say flutter is this condition, because we basically have to find out the speed at which the wing gets into a sinusoidal motion. That means, the speed below that, the system is stable, the speed above it system becomes dynamically stable. But, when you have at the flutter, this is a flutter boundary or flutter speed you call it, you have a speed that means, the value and the frequency of oscillation, so that is the flutter frequency.

So, you always have there is a omega F, this is the flutter frequency and there is a U F which is the flutter speed, you need to determine these two. But, what happens at this time is essentially the aerodynamic damping goes to 0, and the system continues to oscillating. But, it is not easy to calculate aerodynamic damping separately and then, say oh what condition it goes to 0, because you understand there are two unknowns, one is the flutter frequency, another one is the flutter speed.

And we will directly go for that 2 degree of freedom system, you may ask another question, can I have a 1 degree of freedom flutter, 1 degree of freedom flutter means I will have only one motion that is only the pitching motion, no up and down motion. Yes, it is possible, but that can happen only in a turbine bed, there are some conditions, initially it was analyzed to understand the physics, what is really going on, then they identified that look here.

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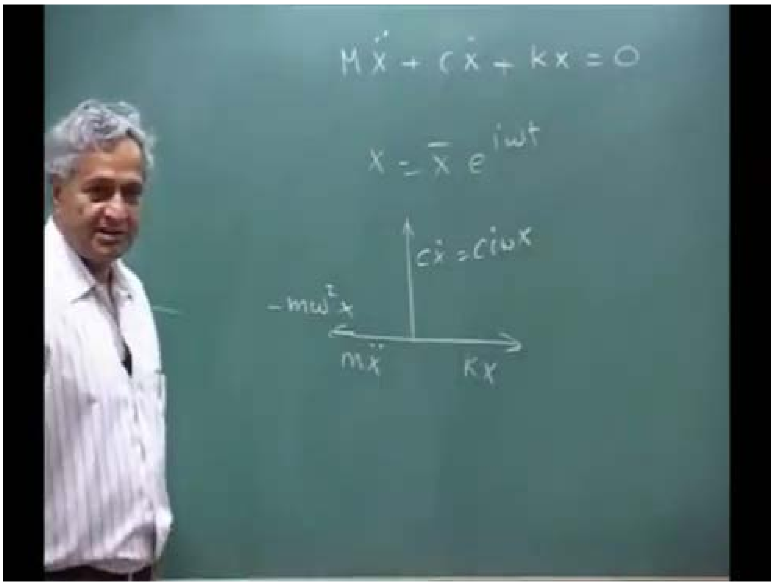

If you have a system, because this is a very familiar system for us is equal to 0, we say that in this system if x is equal to some x bar e i omega t that is oscillating. If you draw the delta Laplace, the x axis is K x, then your damping term is at 90 degree phase to the displacement and this is 180 degree phase to the displacement or you say spring portion. Because, any dot if you take i will come, i is e power i pi by 2 that means, 90, 90 you can add up.

Now, when you have the aerodynamics again you say that I know m, but my C matrix is the function of aerodynamics, K matrix is also a function of aerodynamics stiffness can be there. But, M also you can have apparent mass terms, because which is brought down last time which is a function of h double dot and alpha double dot, which is like apparent mass terms. That means, this is your simple spring mass damper system, you have identical, but only thing is you have a C of K which is sitting in your system.

But, the K is a complex number not K, K is the reduced frequency, but C of k is complex number that is the lift deficiency function is complex. So, it became the complex eigen value problem that means, you have to solve real part separately, imaginary part separately. But, here if you look at it this is the real quantity, this is C i omega x this is the complex part, this is basically minus m omega square x, my damping must go to 0.

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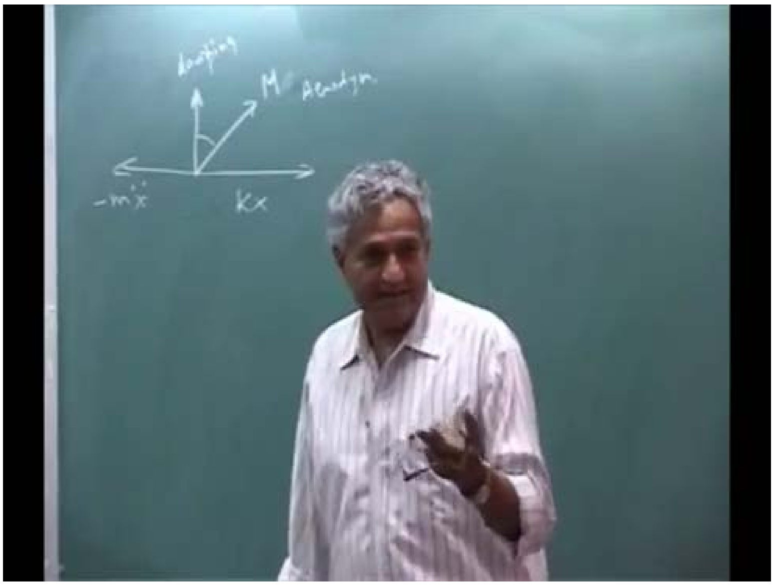

If you look at the aero elastic problem you will have a spring some K x, you will have again an inertia which is M x double dot, but you will have a some aerodynamic value, which will have it can give a spring, it can give a damping this is due to aerodynamics. Some value I do not write it, this component is the damping and the horizontal component gives you balance of these two. Now, if I want to solve a problem in which imaginary component must go to 0, and the real part must be matching the difference of the real, that is what really happens.

But it is the eigen value problem in which I have said imaginary part 0, real part 0 separately, then the condition under which these two are simultaneously satisfy will give me your omega F and a U F. But, this is a little complex problem, because you have a theory which says that, if my aerofoil is executing sinusoidal motion of this type ((Refer

Time: 10:32)), because we put h is equal to h bar e i omega t, alpha equals alpha bar e i omega t.

That means, my aerofoil is oscillating in a sinusoidal motion steady that means, my aerodynamic theory is valid only for harmonic motion, which mean it is valid only at a flutter. Because, flutter I have this portion that means, my aerodynamic theory is valid only at flutter, but I still do not know what is the frequency, what is the speed. So, this is a little tricky problem of eigen value, my theory is valid only at a flutter point, but I do not know what the flutter point is.

So, how this whole approach is evolved to solve basically omega F to F we directly go for the 2 degree of freedom problem, but only thing is physics point of view, you say that my damping should go to view. But, I will modify this in a different form my damping is 0 that means, that is the flutter point, from basic physics of spring mass damper system. If my damping is 0 it will execute harmonic motion, in the case of my flutter problem I have aerodynamic damping, I can have some structural damping also in the system.

But, put together both of them, if you add structural damping then the net must be 0, that is how the theory that is the starting point. Now, let us see how the whole, first we will write the formulation after that we will go to the approach, this is a special technique which you have to know the technique, that is all.

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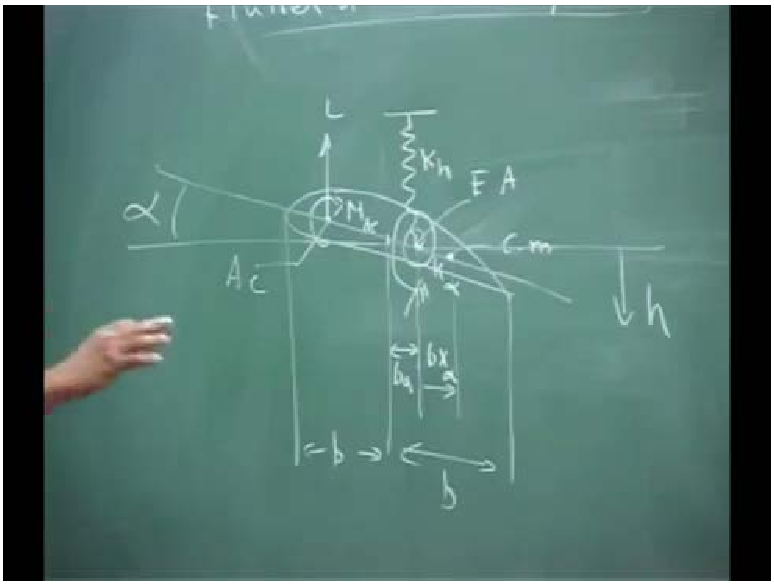

And we will learn that technique, so we will say flutter of a 2 D, is basically an oscillating aerofoil, because we have done this earlier, we have derived the equation of motion for this. This is b, b and you have your center of mass this is the elastic axis, this is K h, this is K alpha and you have this distance is b a and this distance is b x alpha, and this is aerodynamic center, and you have lift and you have a aerodynamic moment. And your degrees of freedom are two which is this is h and this is alpha, these are the h is downward displacement nose of this alpha. Now, we have derived the equation of motion earlier, so Ii will directly go and write that equation.

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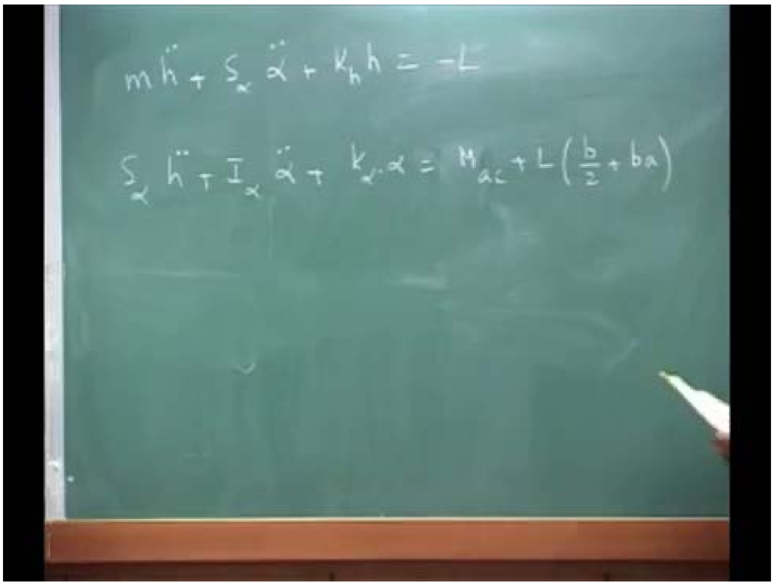

This is m h double dot S alpha alpha double dot plus K h h equals my lift which is of minus L, because lift is upwards, this equation we have derived earlier, because I am not going to again sit and derive. Then S alpha h double dot plus I alpha alpha double dot plus K alpha alpha equals moment M a c, M a c plus lift into, because M a c is moment about aerodynamic center lift into this distance that is b by 2 plus b a, earlier we derived the moment about electric axis only, so that way it is fine.

Now, you have the expressions for lift and moment given, and that expression I wrote to you which is in symbolic form, I am going to write that in symbolic form, symbolic form how was it written.

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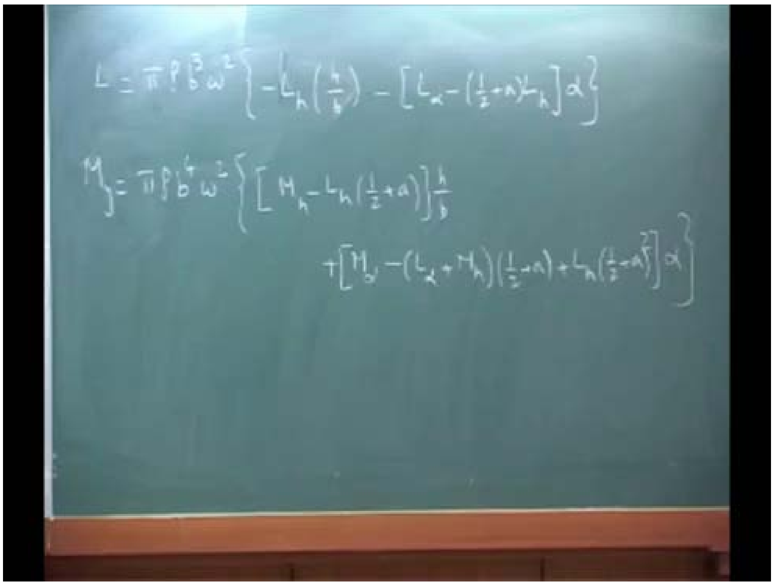

Because, let us first right your lift was written as what was that, lift was written as pi rho b cube omega square minus L h h over b minus L alpha minus half plus a L h alpha, this is how we have. And that L h L alpha I defined earlier, similarly my aero dynamic moment was written as this entire term, I am writing it as M y, so M y was given as pi rho b 4 omega square M h minus L h half plus a h over b, then plus M alpha minus L alpha plus M h half plus a plus L h half plus a whole square alpha bracket close. Because, this is the reason I just convert h over b alpha h over b alpha rest of them, now here I go and first thing I do is I substitute, because I said that my aerofoil is oscillating in a harmonic.

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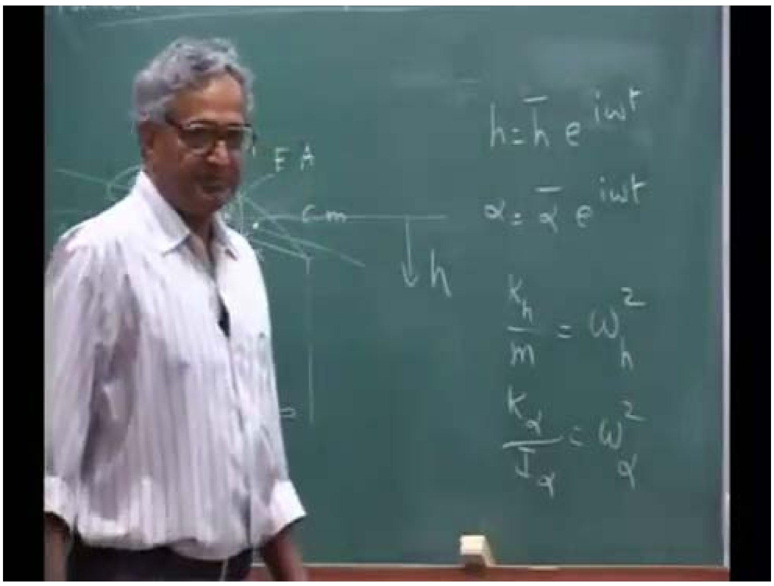

H is equal to alpha equals alpha bar omega t and then, I can write these also K h over m that is basically the mass of the aerofoil, this I am writing it as omega h square. And similarly K alpha over I alpha is omega alpha square, because this is you may say bending frequency, this is torsion frequency.

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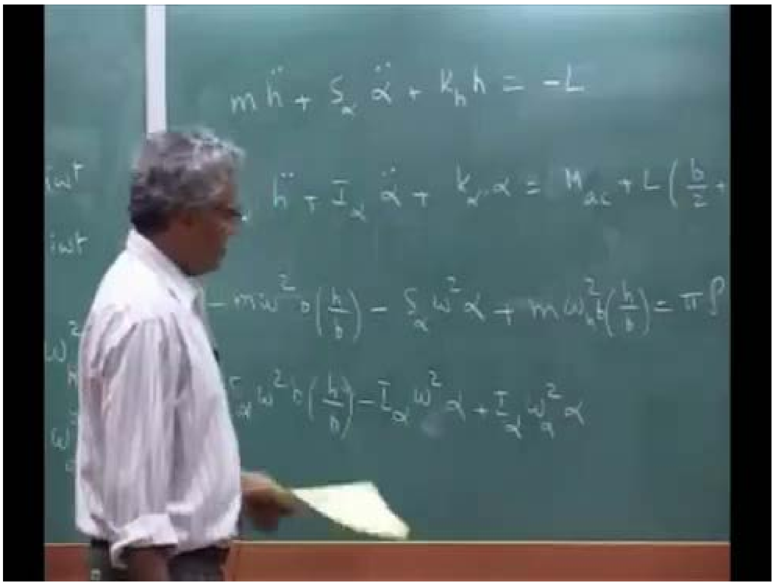

Now, you substitute this S alpha M b x alpha all those things, because the definition remains the same there is no change. So, you go back substitute this here and then, put your L on that side, M y on this side, now that in your equation all the terms or functions of either h or alpha. There is no other term sitting there, it will become a matrix with h over b and alpha equal to 0, so I will write that part and non dimensionalize it, because there are lot of non dimensional things come.

So, let us take the first equation which will be minus m omega square, because m is there, I will put b into h over b, because just putting non dimensional this equation I am writing it, plus here this is S alpha omega square alpha, this term is K h is m omega h square. So, I will write m omega h square h over b, this is equal to my of the lift, so I will write that as pi rho b cube omega square L h h over b plus L alpha minus half plus a L h alpha, this is my first equation.

Then, I go and write my second equation, that will be S alpha again minus S alpha omega square b h over b, that is this term minus I alpha omega alpha square alpha, this is plus, this is I omega square alpha I alpha omega alpha square alpha, this is omega square alpha, this is omega alpha square alpha, because this is alpha double dot.

Student: ((Refer Time: 22:24))

Where, yeah you have to have put a b there, there is a b, now this is equal to that entire term that is pi rho b 4 omega square M h minus L h half plus a h over b plus M alpha minus L alpha plus M h into half plus a plus L h half plus a whole square alpha. So, this is my entire equation these two, let me erase this part ((Refer Time: 23:38)), now what you do is you divide by pi rho b cube omega square first equation, second equation pi rho b 4 omega square. When you divide the first term will become, because omega square b omega square 1 b will go out, we will have pi rho b square.

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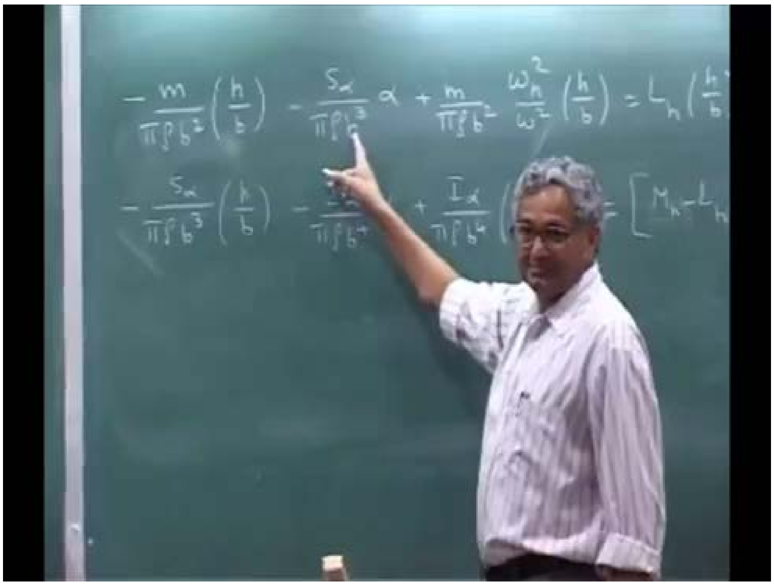

So, you will have minus m over pi, this is s alpha over pi rho b cube alpha, this will be again m over pi rho b square omega omega square h over b is equal to L h h over b plus L alpha, then half plus a L h alpha. Similarly, the second equation will become minus S alpha over pi rho b cube h over b minus I alpha over pi rho b 4 alpha plus I alpha over pi rho b 4 omega alpha over omega whole square alpha equals, you have the rest of the terms M h minus L h half plus a h over b; then plus M alpha minus L alpha plus M h half plus a plus L h half plus a whole square alpha..

Let me erase this ((Refer Time: 26:25)) these equations, now you see these are my two equations, now I introduce some non dimensional quantities. Let me write the non dimensional quantity, maybe here mu m over pi rho b square this is mass ratio, that is mass of the aerofoil divided by this is mass of air, basically a cylinder with b as the radius. This is mass ratio, then you write S alpha over m b, because S alpha is what m b x alpha, so this is x alpha this is static what is that, let us take this term what is that, m b is you look at that second term.

This term will become S alpha is what m b x alpha m

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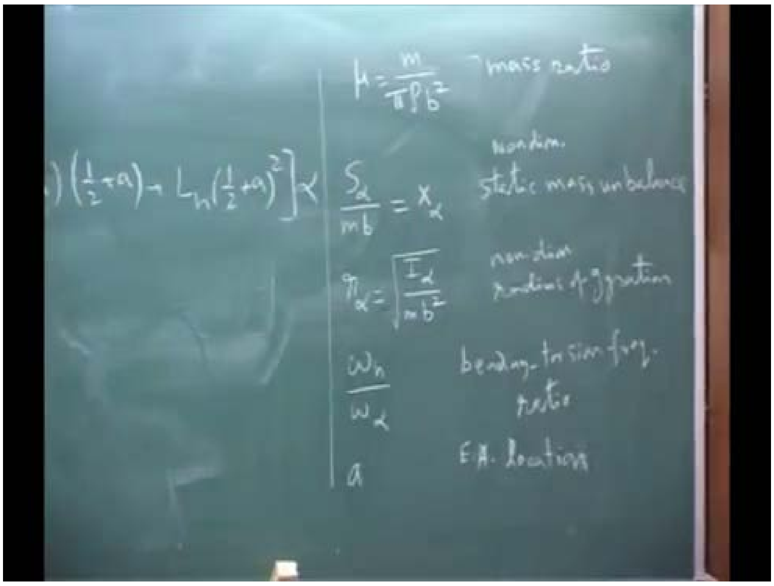

So, let us look at that second term only, you will find that that will be S alpha over pi rho b cube which is over this is mu, this is x alpha b and b. So, that is why we introduce this type of non dimensional quantity, this is static mass unbalance non dimensional, this is non dimensional with respect to b. Then r alpha, this is square root of I alpha over m b square, because I alpha is how we have defined

Student: ((Refer Time: 29:23))

M r alpha b whole square, because you look back we defined like this m, so this is again a non dimensional number, so this is again non dimensional, radius of gyration. Then omega h over omega alpha, this is bending torsion frequency ratio, then the last is a which is axis location, basically elastic axis location from the midpoint. Now, we have five non dimensional quantities, because please remember L alpha L h everything is defined; they are known, now you go back write down our use this non dimensional thing put it in a matrix form.

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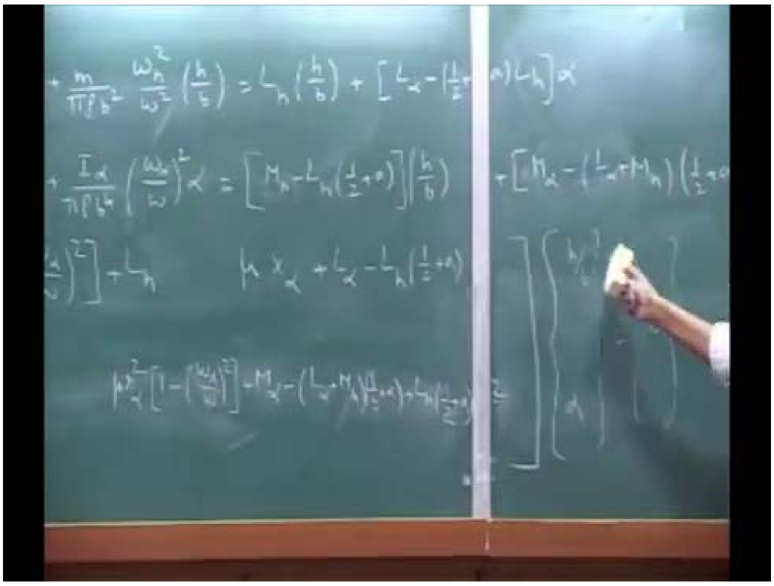

The matrix form will look like mu 1 minus, because what I do is this, I will change it omega h over omega alpha omega alpha over omega that is how I convert this term, I will have one minus omega h over omega alpha whole square into omega alpha over omega whole square plus L h plus, because I am taking this term to that side. So, this will be mu h over b is constant that I am taking it as the outside matrix multiplication, then you will have this minus sign that is what this term is. Then you will have mu, where is your mu ((Refer Time: 32:39)) this is mu, not this term this term, mu into

Student: X alpha

X alpha, when you take it to the left hand that will be plus, so you will have plus L alpha minus L h into half plus a and I am putting a matrix h over b alpha is 0. Now, here when I come to this term, first is h over b this is mu x alpha this term, I will have this term plus M h minus L h half plus a that is my first term, and I have to take all the other terms ((Refer Time: 33:43)) this term and this term, this will be mu r alpha square into 1 minus omega alpha by omega whole square these two.

Because, omega alpha by omega square is this term, this is mu or alpha square, then the rest of the term plus M alpha minus L alpha plus M h into half plus a plus L h half plus a whole square that is all. So, now this is my full equation, let me erase this part ((Refer Time: 34:38)) now this is my actual complex eigen value problem.

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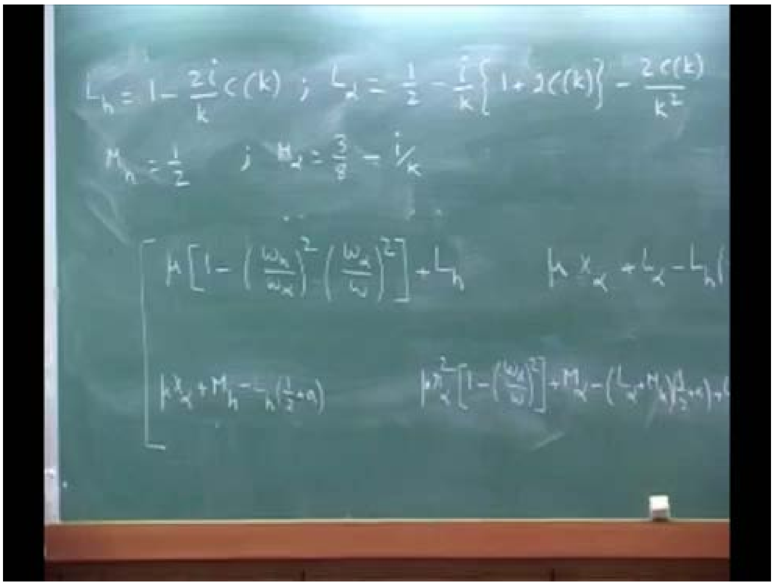

Because, if you want I will write down the, because that L h is what 1 minus

Student: ((Refer Time: 35:11))

2 i over C of k and then, L alpha half minus i over k 1 plus 2 C of k minus 2, then your M h is half and M alpha is 3 over 8 minus i over k. Please make sure that this is you have checked it or you have not checked it, because I would like you to check that, if you last, because I told you verify that what I wrote is or because you have to substitute and then, get that L h. Now, this is my equation, this is a complex equation, because L h C of k, C of k is a complex number, now how will I solve my flutter problem, and that damping has to be 0.

How do I know what is damping, it is not easy, it is a difficult problem that is why the approach that is adopted is, because that I have to get what omega I do not know, that is the flutter omega alpha please remember, all those quantities are known quantities. Because, you that mass of the aerofoil x alpha, wherever in the C g location r alpha you will know, omega h by omega alpha, a elastic axis locator that means, these quantities if you are given the aerofoil section.

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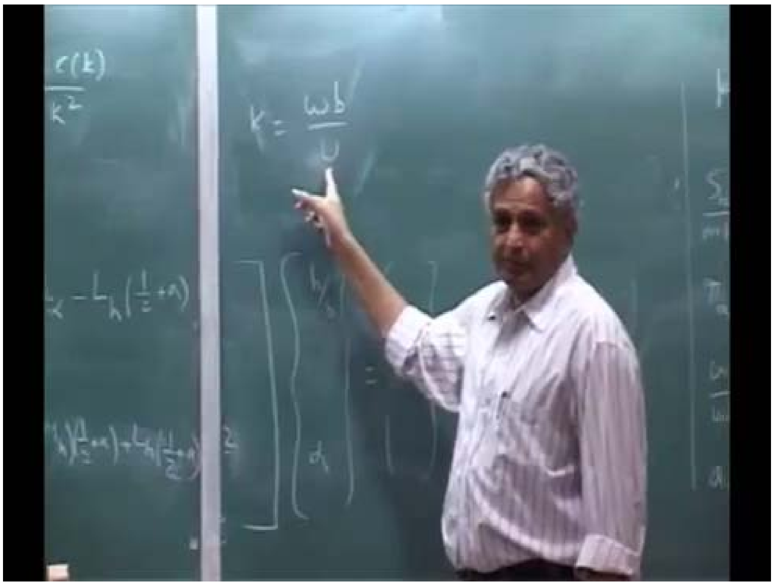

Now, I do not know k, because k is omega b by U this contains two unknowns, omega and U, omega is there essentially I need to find out what is omega, what is U which is the flutter frequency and the flutter speed. For which the real part and imaginary part everything must be 0, this become the typical task, therefore because if you start with any value of k, because you have to get that value of k, at which everything is beautifully set the real part, imaginary part all are satisfied.

Then what are the approach that was adopted is, that is why this is a special, there are two techniques I will mention one of them first, then the second one is fairly simple, this is called the I will erase here, it is called the V g method V g or U g any name you can give. Now, how do we solve this problem, what was done is I will first describe the approach, later I will give some effect of various things.

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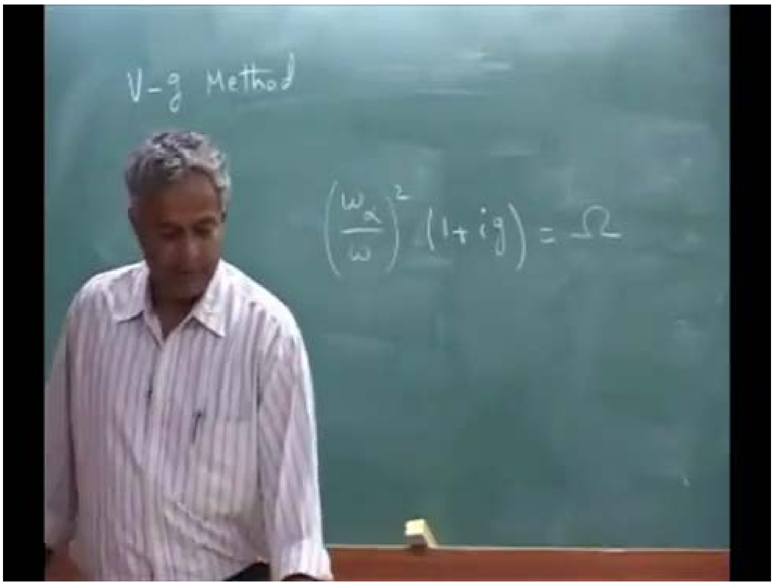

You now write is equal to some, I will put it into 1 plus, what I have done is omega alpha over omega whole square into some quantity, this is a complex number complex I g equals capital omega I am saying. Now, because how why if you want to know a little bit we can go back to the, what is that g, this is like I am adding a fictitious damping. And I will go replace in my equation, wherever alpha over omega square is sitting there, I am going to write this omega that is here and here, I will write capital omega which means I am introducing two unknowns, now one is this another one is this.

Because, I do not know both of them, why I introduce I will come to that, because you know that my equation is 1 where g is 0, only when g is 0 I get my flutter equation, this is the correct equation, but I am adding an I g, I g is a complex damping. Basically it is a damping which comes as a complex number, that is a not complex damping, it is a damping I have put it as a complex damping. Now, that if I assume a K, because my aerodynamic coefficients are known if I am given the value of K, K is omega b by U.

So, what I will do is, I will go start my equation I assume a value of K say 0.1, 0.2, 0.3, 0.4 something like that, then the moment I assume the value of K I know all these L h, L alpha everything is known now. That means, I can substitute everything in this eigen value problem, except omega alpha over omega plus into 1 plus I g, that I do not know. So, what I am going to do is, I am going to replace this term as omega and omega, this is an unknown, this is an eigen value problem, because I know everything else, but only thing is this is a complex eigen value problem.

I am having a omega square, an algebraic equation only thing is the coefficients are all complex number, here you do not go and do real and imaginary part.

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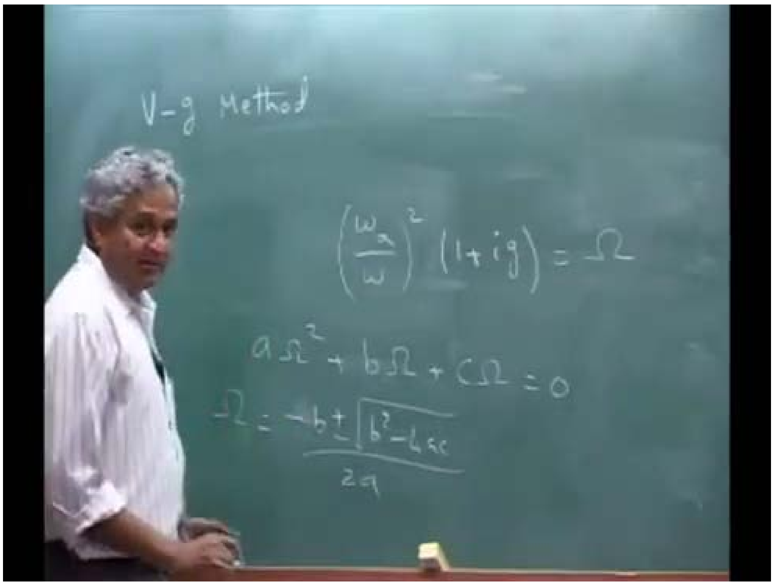

You just solve the complex eigen value problem directly, because you may have, say a omega square plus b omega plus some C omega equals 0, because this is omega square only, this into this is the omega square. Now, a, b, c are all complex numbers, that omega is also a complex number, so I will directly go and solve omega equal put it plus or minus, but minus b plus or minus root of b square minus 4 a c over 2 a. So, I will have two roots omega 1, omega 2, but they are complex numbers, because you just directly substitute the complex number, I will have two omegas which are complex number.

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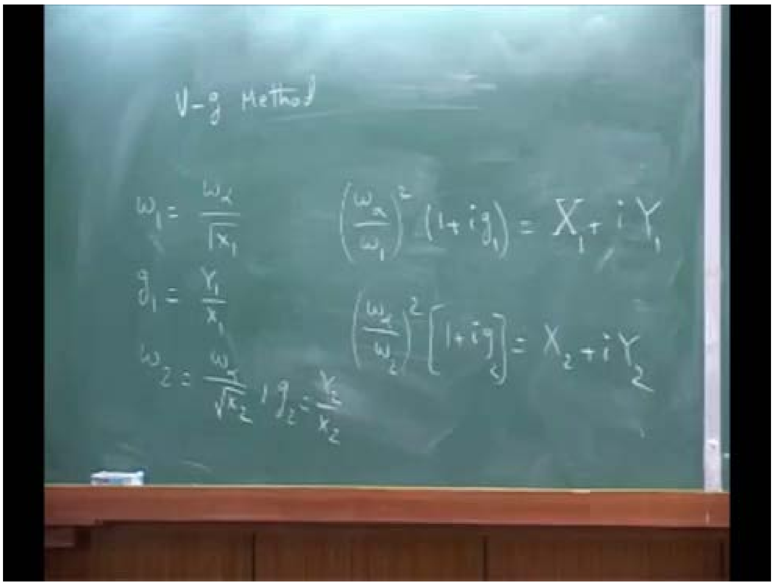

Now, that complex number I am going to write it as, X plus i Y first one X 1, Y 1, then I can have X 2, Y 2, so I will X 1 Y 1, the second one will be, so I can put it as another root will be that means, I am going to call omega 1, omega 2, g 1, g 2. I know X 1 is what, omega 1 is omega alpha by root X 1, and g 1 is what Y 1 over X 1, similarly I get omega 2 g 2, so I will have omega 2 is omega alpha over root X 2 and g 2 becomes Y 2 by X 2. Now, I have omega 1, omega 2, g 1, g 2 what I do is I go and then, plot the values.

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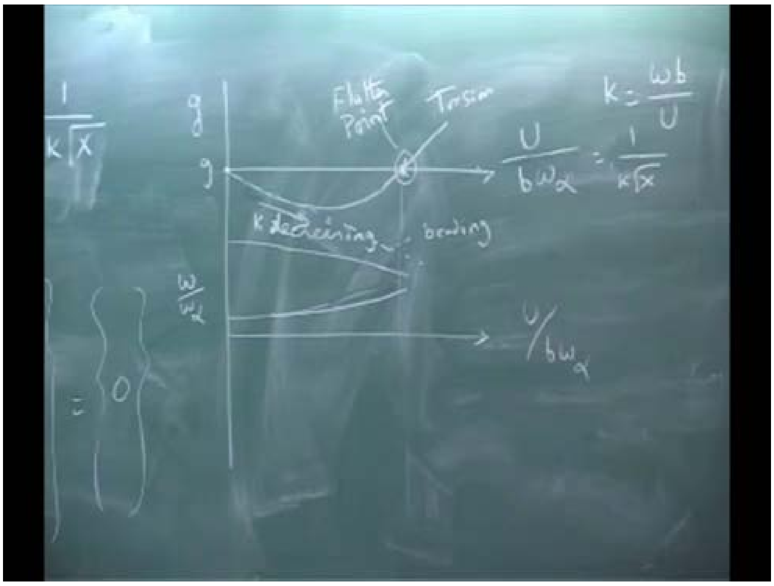

One is the plot of g, theoretically that is what is required, another one is the plot of omega I will just mention that, another is the plot of omega over omega alpha, that is omega 1, because I am plotting non dimensional. Now, the x axis what I keep is, what is k, k is omega b by U, now can I write this, let me write this here what is 1 over, that is U over b omega, this is what U over b omega alpha, and what is omega alpha over omega that is square root of X.

So, I will put it square root of x no, this is square root of X that means, in my x axis I can plot U by b omega alpha, x axis I am plotting U by b omega alpha, because I know omega alpha I know b, because that is given in the system. U I have to get it from, because that what is k, k is what omega b by U, that is why I am getting this value from here. I know the value of k, I know the value of X for each root, because you have to remember I get two roots X 1 and X 2.

X two represent omega alpha by omega 2, X 1 represent omega alpha by omega 1 that means, I can get X 1 and X 2 put the value root k, I know my x point y point I get g. So, what I will do is, I will start up usually you will find the curve will go like this, one curve will go like this, another curve may go like this. This is torsion, this is bending, because please note you are plotting g versus which is 1 by k root x, you are plotting this, the two values of g you will plot. Similarly, you plot omega by omega alpha, again U by b omega alpha you can plot this will go like this, and that may come like this.

And ((Refer Time: 51:45)) this point where you make the value of g have gone to 0 even in any one root, this root will not go to 0, this root one the value of g gone to 0, represent that this omega the g is 0. That means, directly omega alpha by omega, that become that is my eigen value of the flutter point, so this is my flutter point, that is why this diagram is called V g diagram, V g or U g. But, you have to plot it you have to solve the problem then only you will know, otherwise it is not that easy.

And this is for 2 degrees of freedom I have shown, but if you take an actual wing you may have more degrees of freedom, then you plot all the curves. But, the point any one of the curve crosses the 0, because this is g equal to 0 line, you will find that it hits the flutter point. Now, when U equal to 0, why I put g 0 because U equal to 0, there is no aerodynamics, then you said all the aerodynamic 0 this is nothing but...

Student: Spring mass vibration

Spring mass vibrating system, you will have two values of eigen values which is corresponding to two eigen values and g is anyway 0, that is why it starts from 0, now this is your flutter. And once you know this value this crossing point, you will immediately know u alpha b omega that is U f, so this value you know you can plot the flutter speed, so flutter speed you will know and that is the flutter frequency, which you get there.

Now, this is what is done by, but usually you will find k reduced frequency will decrease, k will decrease normally what you have to do is you plot this diagram for different values of k. But, generally this is what happens, but sometimes the nature of the curve of g will be a little complicated, it will not be like a such a beautiful thing, which I have shown here, sometimes that is why they say whenever you get g positive, then you know that that is a flutter.

That means, your system beyond this your speed is going to or your system is going to diverge, below that speed your system is stable, but sometimes in certain peculiar cases, the curve may do like this. In the sense when you are solving the roots, it is a little bit tricky problem, so whether you take this speed or this speed or that speed what will be a flutter.

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These are some peculiar cases, but normally they go in the, so this is your V g analysis of flutter is that clear, because this is you have to do one problem, and using the unsteady aerodynamic theory. And this technique is followed widely in industry, because whenever they say V g method, it means that this is what they are doing, may be for a wing, but here we are lying for a 2 D section. But, you can write the wing problem into a bending and torsion, single degree of freedom in bending, 1 degree of freedom in torsion, you can have 2 degrees of freedom of the wing.

You take first two torsion and the bending frequency, you can formulate the same problem, and you will have a bending torsion flutter problem. Now, I will just briefly give you some effect of various parameters on the flutter, just for see one is I know there is a flutter. Now, how do you avoid the speed I get a flutter speed, for every wing every glinting surface, they calculate flutter speed, you have to calculate that. Once you know this is my flutter speed, provincially do not go near that speed that is all, that is why in flights regulation they will say you must be 1.25 times, below.

Your maximum speed will be 1.25 times less than that other one flutter speed or divergent speed or control to any of these. But, control reversal effect will always be there effectiveness of control surface will always be there it will decrease. So, you do not reach that point, suppose you design the wing you find that it is low I want to do something, you want to modify the design that go and decrease your torsional stiffness that means, omega alpha make it higher that is one.

Another one is you shift your mass center, that is you shift your x alpha, mass center towards the leading edge, you can also apply I told you, that lift itself can be written as in the finite state model. Then you have a time domain aerodynamic model that also you can use here, then you will have in addition to this W 1 delta 1, last class I told you get that x, they will be additional state variable. And you will have corresponding equation for that and then, you add those equations also and can solve, do you understand this is how the flutter problem is solved for a meaning.

Now, we take the next topic, which in will give you very brief introduction that is all I will not go into the details of the problem, I will just briefly describe what is the problem, what are the key aspects and then, I will give the reference for that. So, which you can look at it you can learn, because it is not complicated, the next topic is the panel flutter.

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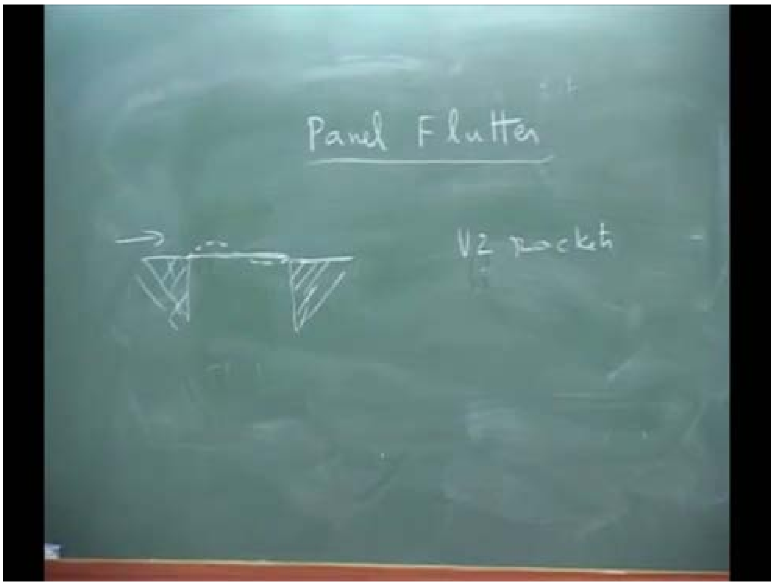

What is it mean by panel flutter, see till now we said that if I draw an aerofoil this is a panel, the panel does not deform or if I have something like this, if I have a panel the flow is going over it. If it starts deforming like this, that is a card wise deformation like your fluttering of a flag, what happens the flag is changing it is shape, it just deforming the shape itself is changing, till now we said our aerofoil panel, aerofoil retains it is shape; only thing is it can rotate it can bend, but its surface cannot deform.

Now, you talk about the fish type of, here we talk about the surface itself deforms, then what type of problem we will have, this is what is first talked about it. If my surface is also moving, then it is a complicated problem that is why that problem you do not talk about flow fast an aerofoil, what you do is you take a panel. This is the panel and the panel is fixed on a, because between two supports, now the panel can uniform, if it is a very thin thing you can have deformation, because that when wind blows, if you have even, if you have a some other Shamiyana or anything, what it does it does lateral vibration.

Now, those thing at subsonic speed for it happen, it should be very thin, but for the aerospace application there is nothing like that thin material, we do not use that, therefore it does not happen in subsonic cases. Actually the phenomena was observed in the aerospace line, I am not talking about the flag fluttering or a Shamiyana you put it then wind blows and then, the whole thing can go up and down; you can even have a static divergence type of a problem.

Now, the first time the panel flutter was observed in, it was in German V 2 rockets, because the panel started vibrating, but it is flow passed only one sided, it is an external flow, internally it is a whatever pressure you can take it as an atmospheric pressure, it is not like an aerofoil, in the sense the flow is on both sides. So, here normally you say what is the you have to get the pressure difference, inside pressure you already know, you say that is p infinity, but what is the surface pressure.

And this was observed in normal aeronautical thing for the thickness of the panel, it happens at the supersonic speeds, that is why they say it is a supersonic panel flutter. So, 60's people were solving this problem of panel flutter substantially, but then supersonic speed, so they started using unsteady aerotic theory, associated for supersonic. And we found out, you remember we derived piston theory, you can use piston theory to get the aero dynamic force on a surface.

And then solve, but the problem is a panel problem, it is not a wing type of problem, so the whole study went into what kind of boundary condition you can have. Then in a panel under thermal stress, when you have constraint like this, if your temperature changes the stresses will get developed, axial stresses. Because, of the end fixes, then the panel can deform due to thermal loading, and when it deforms your surface is changing now that can also another cost.

So, your surface deformation thermal show, how the problem was treated is I will just draw a simple diagram and then, I will basically give you a few introductory thing. Then, you take a, now you have to know panel equations, your structure is not a wing it is a panel or you talk about plate problems.

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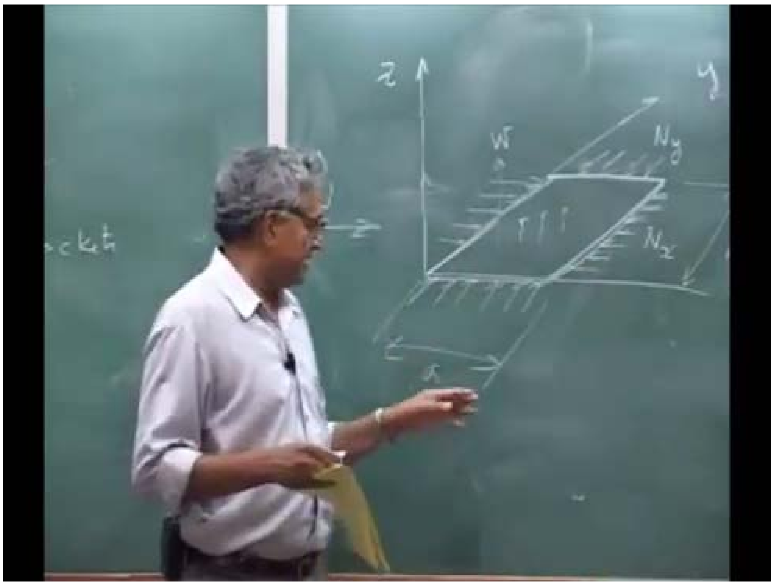

So, the problem was this is x, y, this is z you put a panel which is acted on by and a panel deforms in w direction, you will have pressure everywhere, these N x, N y are the compressive stress, you can say due to the surface, a boundary condition, Now, this is the problem, you can now start deriving the equation for a, you can have curved panel, you can have flat panel and then, the pressure this is the flow which is coming over it, not under it, because this is completely covered and the dimensions a b.

So, the problem that was considered is basically a plate problem, so initially you take my plate is flat under the action of N x, N y, what will be my deformation. Because, this is like you have to talk about for small deformation, the governing equations I will write the governing equation for this.

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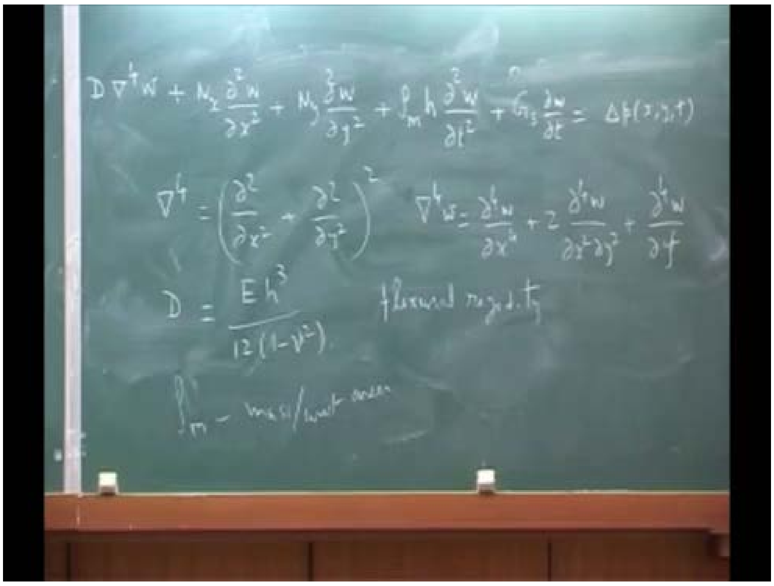

D del to the power 4 w, w is a function of x, y and time, this is the plate equation actually N x del square w by delta x square N y del square w by delta y square plus, this is the mass density, h is the thickness of the plate del square w by delta t square. And if you want to add some structural damping, they will put a G of S also, they will put a G of S delta w by delta t this is some damping term, this is there is a pressure delta p. And what is del power 4 this is nothing but del square by delta x square plus del square by delta y square whole square.

Or in other words, del power w implies del to the power w by delta x 4 plus 2 del 4 w by delta x square delta y square plus del 4 w over delta y 4. And then D, D is called the flexural rigidity E h cube over twelve into 1 minus nu square, nu is the Poisson ratio this is flexural rigidity. And then rho m is mass density, mass means is that the area you can say mass per unit thickness, because h is the thickness mass per unit, you can say unit area, not unit area that h is the mass of that.

Now, this is the plate equation, because I do not know you have done any plate theory or not, since you have not done plate theory, unless you do that plate theory, you will not know, now this is what the starting point is. But, delta p that is the pressure differential between inside, outside, you would supersonic, so this is where they use piston theory. But, again there were lot of approximations that was used, I will just briefly give you the kind of approximations which people have used and you will know that, that is due to piston theory only. Because, piston theory what we had you remember piston theory that is the high frequency approximation.

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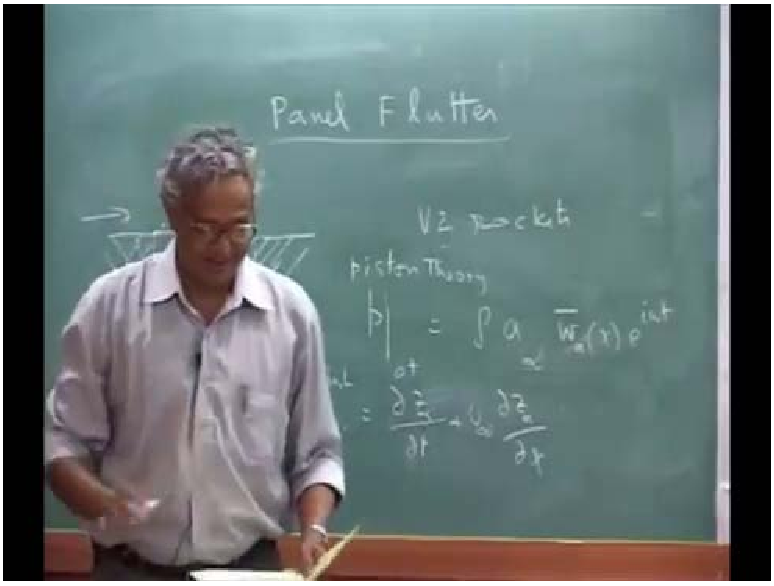

Piston theory says p some 0 plus, this is due to piston theory, which we derived earlier high frequency of the supersonic, we had rho a infinity w bar a at x, w bar into if it is oscillating this is w bar e i omega t that is all. But, w bar e i omega t from the, because w a is what w bar a e i omega t, which is delta z aerofoil delta t plus U infinity delta delta x, right you got this expression. This is see w is the velocity and this is the surface z a is the displacement at that point, now in z a you substitute w of the plate deflection at that point, here that w is plate, here this w is the velocity.

So, you should not get a confusion over these two, now this is where they use the piston theory, then there where lot of different approximations. So, I will just give you couple of equations which people have used, and that will just give you some idea.

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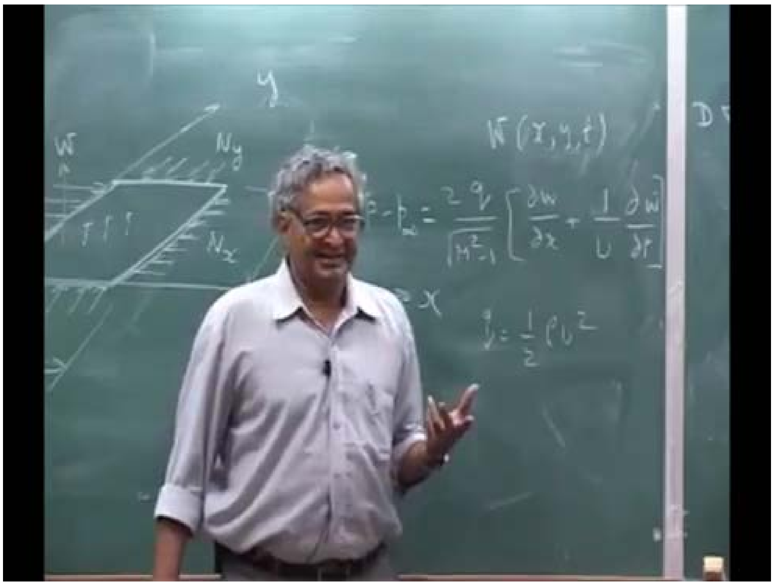

What they did was p minus p infinity is, this is from square root of m square minus 1 into delta w by delta x plus 1 over U delta w by delta t. Because, q is half rho or they have different expression also, sometimes they have put one more term here for different mach numbers, that was derived from another approximation. That is why you will find in the literature on plate theory, I will give just the reference, different expressions they are using, but you use sometimes people neglect this term also.

Because, this represents the instantaneous, what is this delta w by delta t is the velocity at that point divided by U that is the local angle of attack, this is local slope, so you add both of them. Now, this is there were modifications to this type of this expression there were considerable modifications there q, therefore it is use different. Now, and the 60's I will give the reference now, there where lot of studies which were performed on this panel flutter problem.

What is just, here I will give two key references, then from there you can drag down, even now some publications in fluids and structures, they write about sample flutter for composite materials.

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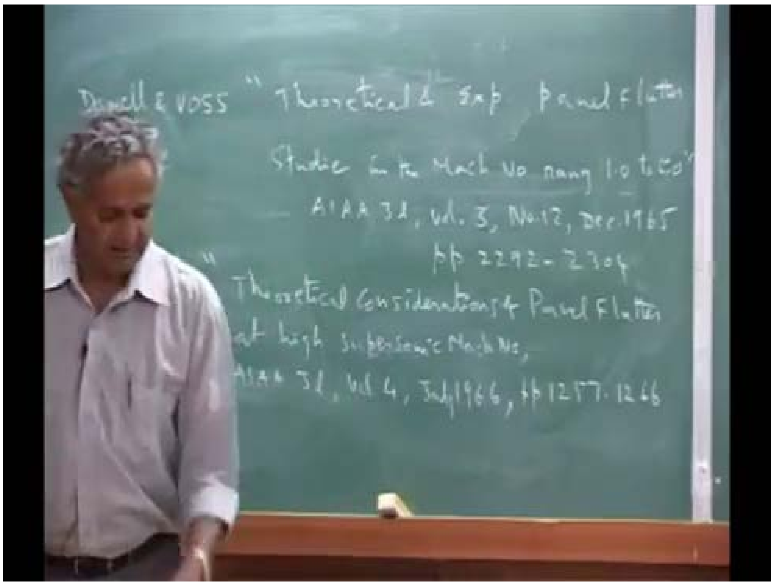

So, this is by Dowell and this is theoretical and experimental, panel, flutter studies in the mach number range 1.0 to 5.0, this is A I A A journal volume 3 number 12, December 1965, and page number 2292-2304 this is one paper. And another paper is, this is by Dugundji, because there are few people and then, similarly you do not they all did theoretical considerations of panel flutter at high supersonic mach number, this is also A I double a journal volume four this is July 1966, this date 1257 1266.

See these two references, there pretty much they tell you, because there is a nothing difficult about the problem, because you are not exposed to panel equations, how they are obtained degree, this is the panel equation under suppressive load. Or you can say on this side and then, there is a pressure that pressure expression is given here of course, they have different please note here, there is another possible. They will have some factor multiply, some M square minus I think I will just give that, because this is from an approximation, that is why you will suddenly find what is this different people are using M square minus 2 over M square minus 1.

This term will be sitting here along with p minus p infinity, p infinity is now pressure inside is p infinity the top is, so this is the delta p at any point simply substitute, this is in terms of w, w, w, w. Now, what this equation they have solved for different a by b and then convert the condition under which it will have flutter, these are non dimensionalize and things like that, because I know the procedure. Since we have not done plate problems no point is going into in that, but this is just to give you an idea that, but one of the simplest thing is to avoid panel flutter thickness.

So, the thickness is the little bit more than panel flutter, but then earlier it was all metallic structure, then they said can we eliminate panel flutter whether they composites. So, now, with a composite structure you can have that then, lot of publications came in that, see one is the mathematical approach to get the flutter speed. Another one is depending on the choice of material can I postpone my, basically the flutter speed if I postpone, then it will be fine.

So, these are studies once the composite structure came, then visa there is one another paper like visa, I thought these are old classical 65, 66 paper, but Y. C. Phung also has done on panel flutter, so in the 60's there were lot of studies. Now, also you will find you do the Google search you will find panel flutter, recently 2002, 4 some publications are coming which refers to panel problems. And I think with this just the brief note on panel flutter, there are other types of flutter problems also, I will just one is non-linear flutter, but that implies it is actually a non-linear problem. All this we have done linear theory, non-linear flutter you can have a stall flutter, stall flutter is the sense, your aerofoil goes into stall and then, again it comes back get attached.

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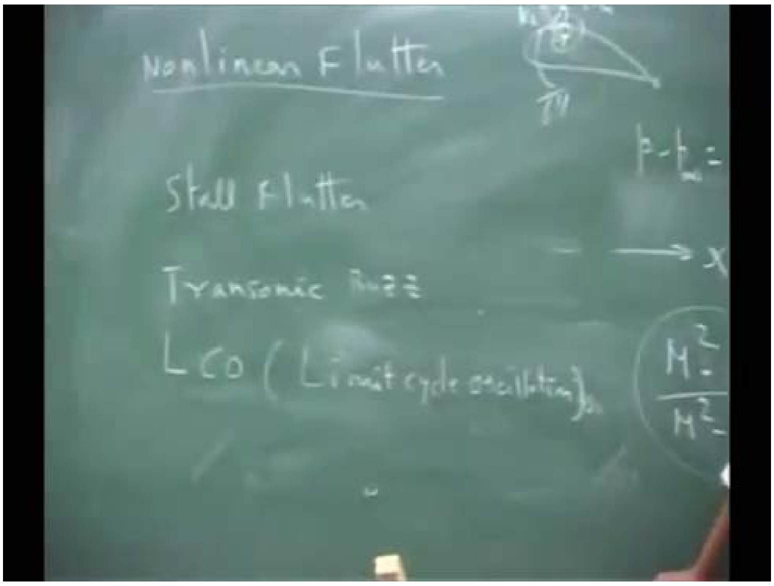

So, these are all, then there is a one is stall, another one is transonic buzz, but even another problem it will, see you will find these all in the late 90’s, I would say suddenly people started copying LCO, that is Limit Cycle Oscillation. See what this is limit cycle of this means, it happen in some of the I think started vibrating, but it did not flutter in the sense, flutter is it has to completely get into a unstable. But, it did not become, unstable means it will break the amplitude will keep on increasing only, but in the limit cycle oscillation, what happened is the amplitude reached a stage.

And then, it started only within it oscillated continuously, then what was the problem, so now people started getting to non-linear effects, because the moment some large amplitude come, the non-linear effects of a problem come. Even in your own this type of can I have a limit cycle oscillation, in the sense it will not go out of bounds, but it will continue to oscillation that means, you can have non-linearity from two sources, one from the aerodynamics, another one from structure itself.

Structural non-linearity means, I can put this spring k h, k alpha not linear spring, they are non-linear springs, then I can assume some non-linear analyze the flutter problem, then show that beyond that peak well, it does not blow up, but it continues to oscillate. And another one is aerodynamic non-linearity you can have the solve, because if the amplitude goes a little more, then what will happen is it will start having some kind of a bounded motion bounded, but continuously this will lead to lot of fatigue life.

Because, fatigue damage will completely very severe, because you are continuously vibrating, now we have done once, that is the one of the PhD students did, but there are lot of studies on this. Now, that LCO then stall flutter of course, helicopter plate we know that it goes into stall and comes out of the stall and things like that. We use this basically the stall model for this and then, we said that that can be a kinetic motion only.

In the sense kinetic motion means, there will be all sort of frequencies coming into the picture, but it is the deterministic problem, it is not a random problem, everything is deterministic, but it is non-linear problem. So, non-linear problem have their own, what do you call phenomena, like suddenly you started seeing bifurcation in the sense, you will expect one type of motion, suddenly it can go this way or this way. Then you can have more frequencies coming into the picture and you can have kinetic motion, so these are all in the non-linear domain.

But, there it is essentially the research group, whatever there are working on they get into the this is all such problems, but transonic is a non-linear problem, that is different. Here stall, you have to have a stall model, because it is a flow is attached detached, all over theory is attached flow, potential flow, small disturbance everything. So, you find the field is also growing in different areas, now another one is micro stuff, whatever we going and rather the Reynolds number or whatever is stuff different zone altogether.

Ad they have the viscous, the fluid viscosity is more important and they have their own theories development, but still they use only Theoderson theory and other things, Theoderson theory is what it is a potential flow, there is a no viscosity or anything. But, they will make statement would finally, use Theoderson theory or even

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Yes, that is all they will use that, and then solve them out.