Aero Elasticity

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Lecture - 23

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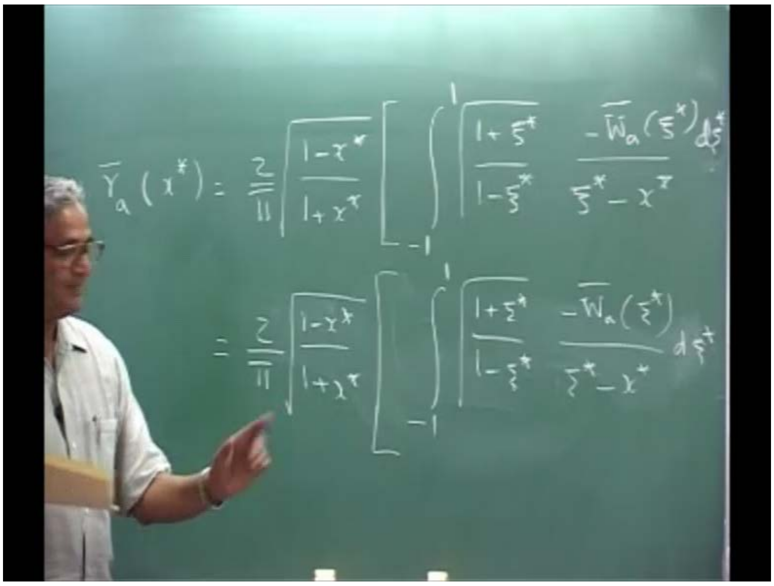

We gamma bar a x star equal 2 over pi root of 1 minus x star over 1 plus x star, and open the bracket 1 minus 1 to 1 under root 1 plus psi star 1 minus psi star minus w bar psi star over psi star minus x star, this is the one terms. Then you have the other term, plus 1 1 plus psi star 1 minus psi star you open a bracket, i k omega bar over 2 pi 1 to infinity e to the power minus i k lambda over psi star lambda d lambda bracket close, 1 by minus x star you can have here you can put it, then you can also put here d ((Refer Time: 01:54)), this is my complete relationship between the circulation and the velocity. Now, you know the relationship between this is gamma bar a x star d x star, this is essentially if we wrote minus b by this is gamma bar a d x over b, because this the non dimensional, this b which is omega bar e to the power minus i k, this is the relationship if we got.

Because the relationship between omega bar and the integral of the surface, earlier we defined that, now you see in this omega bar is sitting and here gamma a is there, so that means, that omega bar is related to this gamma a. So, what is done is you again take an integration minus 1 to plus 1 of this entire take, and the left hand side will become omega bar this, and you have omega bar on that side, bring the omega bar on one side.

And then write an expression for omega bar, now that part actually what is happening, what was done is this integral itself is multiply, this integral, because you can do the integration and over this variables ((Refer Time: 014:16)). And this term please understood, because I am skipping the steps of representing this integral by this term, that is basically this will become minus i k omega bar over 2 pi. And one to infinity pi root up lambda plus 1 over lambda minus 1 e to the power minus i k lambda over lambda minus x star d lambda.

Now, please understand let me write the full expressions, I am writing this 1 plus x star I open the bracket this remains as it is, so I am just putting it minus w bar a delta x star d psi star, ((Refer Time: 05:51)) and this term comes here and we have a bracket close. Essentially this entire term is represent like this after integration, now I mentioned that this is, if you integrate over the length of the airfoil, then that is again related to omega bar. So, I can take omega bar on that side and then I will do some integration of this and the final expression for I will write it the omega bar, which comes out.

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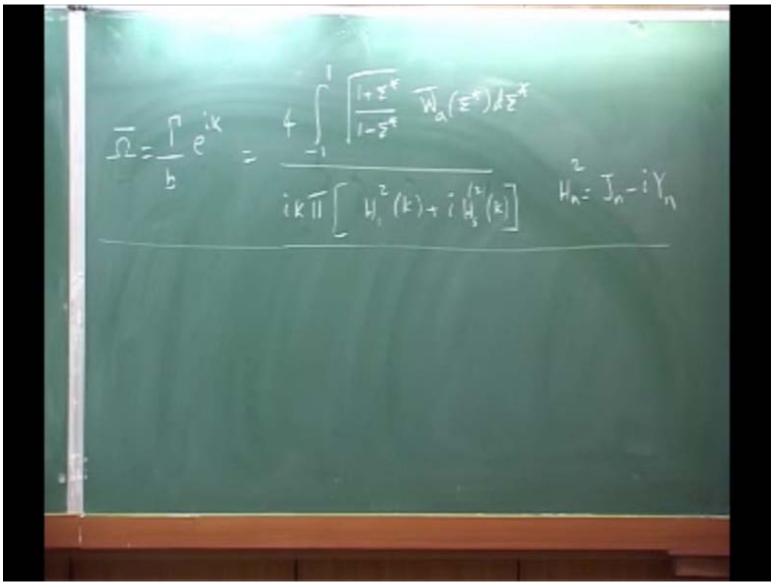

Because this is gamma over b e to the power of i k, this become 4 minus 1 to plus 1 1 plus 1 minus psi star w bar a psi star d divided by i k pi H 1 2 k plus i H 0 2, where I told you H n 2 is nothing but J n minus i Y n, these are bezel functions of first kind and second kind of order kind. Now, you see this is obtained after integrating the whole thing, now I have an expression for omega bar, what I do is I take this expression put it back here that means, I have gamma x star is fully available.

Then I will go and find out the expression for the pressure, the pressure expression we know that b u is b minus let me write it here. But, if you want to know how this integral came, this itself is another expression, maybe I write these are all very interesting thing, see this is what the integral is.

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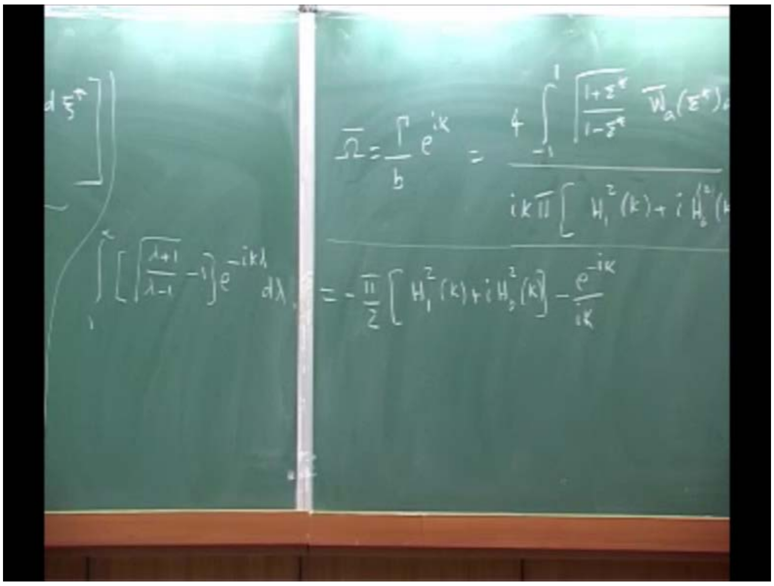

At that integral 1 to infinitely root of lambda plus 1 over lambda minus 1 minus 1 e to the power i k lambda d lambda, if you have the integral like this, this is actually equal to minus pi over 2 H 1 2 a plus I H 0 2 k minus e to the power minus i k over i, that is this. Because, that 1 e to the power minus i k d lambda that is basically equal to this, so this is what your, these are all table of integrals; now that is how you get this coming here, please understand. Now, once I have the omega bar I can substitute I can set gamma a, now I go back let me erase this whole thing, because the...

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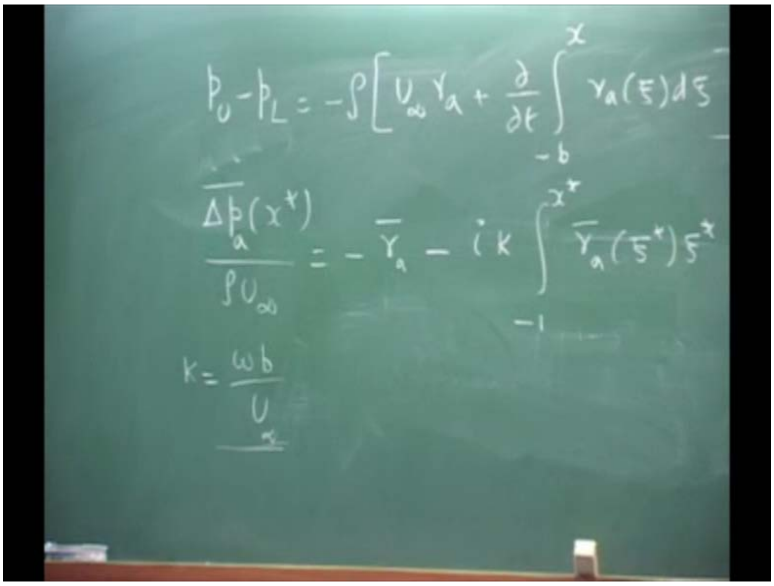

And write the pressure now, the pressure difference expression, because the pressure difference expression become p upper minus p lower, this is minus rho of u infinitely because we are looking at the airfoil. Because, you know that 2 U infinity U, 2 U is gamma a, then plus delta over delta t of that phi, that we are got it as minus b x gamma a this is what we have, I have a expression for gamma a substitute here, substitute here. But, only thing is what I am now assuming is my gamma a is osculating, so that is how I get gamma a bar e power I omega t.

So, this d by d t I am put it as in terms of ((Refer Time: 11.05)) and these are all pressure also I will say pressure is fluctuating pressure, so I basically assume that harmonic motion eliminate the time. So, when I eliminate the time this will become delta pressure on the airfoil x star I put a bar, because this is osculating sinusoidally e power I omega t over rho U infinitely, this is become minus gamma a bar, because a bar e power I omega t. This will become minus i k minus 1 to x star gamma bar a, because what I am doing is I am dividing by U infinitely, so I will have a U infinitely will come.

But, what is my k omega b by U infinitely, and d by d t will give me i omega one U infinitely will go, and this is I am putting it as over b integral, so b will come in the numerator. So, that is why I get the i k, but now I just want to the, this is called the reduce frequency, which I explain, I thought I will just briefly give you the explanation here. There are two ways of omega is 2 pi by t that is k is 2 pi over t is the period of osculation of the airfoil b over U infinity.

Now, you can look at it in two different ways, if I take t into U infinitely that is in one osculation, what is the distance travelled, any disturbance that is given in terms of... If you take it 2 b that is card length which means, the distance traveled in one osculation is in the denominator b is this, if I we take 1 b this is essentially 1 over the distance traveled in terms of number of card length. Because, I can invert it this way, I can even put it this way, this is distance travelled in one osculation, how many card length the disturbances travelled in one osculation, and it is in the denominated.

Now, if k is very small that means what, the distance travelled is very large that means, my disturbance has done very far away, when k is small; on the other hand, I can also viewing in a different fraction, that is I take this ((Refer Time: 14:52)) b by u infinity gives me the times to travelled a semi card length. Or I can say take the 2 b inside, the time for the disturbance to travel one card length, if that time is much smaller than this period of osculation, then my k is small, because my period of osculation t is very large, and the time to travel is very small over one card.

So, there is the two way of looking at how the disturbance travel, that is why this is a some kind of non dimensional thing reduce frequency, when reduce frequency is very small essentially mean the disturbance which is at the staling edge the vortex it has gone back very fast, far away that what is utilize. Now, you have a expression for gamma a which is given the earlier, I go back substitute the gamma a here and then I get delta p a, now this expression is a... Finally, you have to get the I am writing it in one particular form, that is what the representation of delta p a I will just briefly give you the expression, I am skipping the intermediate integration steps.

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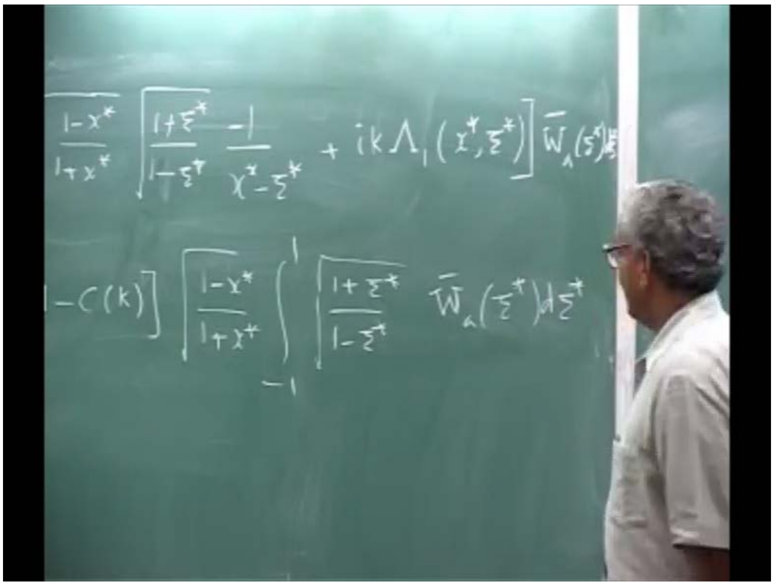

This is what it will becomes, delta p a bar x star over rho u infinity becomes 2 over pi minus 1 to plus 1 1 minus x star over 1 plus x star root of 1 minus psi star, this is 1 minus psi star 1 over, the minus sin x star minus psi star plus i k I am using a new symbol, some lambda 1, I will give the expression for that lambda 1 into w bar a psi star b psi star, this is not enough, I have one more term. This is one term, this bracket starts again, then another term is minus 2 over pi 1 minus C of k root of 1 minus x star over 1 plus x star minus 1 to plus 1 root of 1 plus psi star 1 minus psi star w bar a star b.

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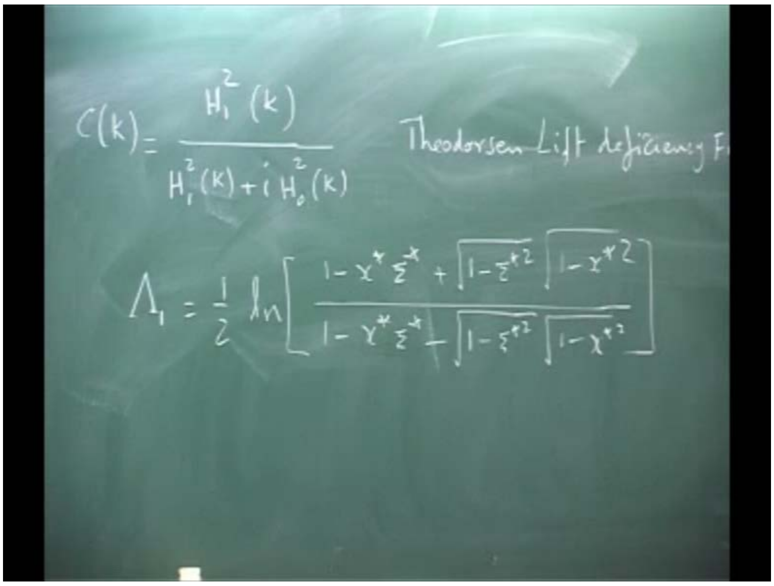

Where C of k please understand, this C of k lambda 1 I have to give what they is, this is H 1 2 over k divided by H 1 to k plus i H naught 2 k, this is called Theodorson lift deficiency functions. And then your lambda 1, because you need know what is that lambda 1 is, because that lambda 1 here I have a expression for that lambda 1 is half l n 1 minus x star psi star plus root of 1 minus x star square root of 1 minus x star square over 1 minus x star psi star minus root of 1 minus x star square root of 1 minus x star square.

So, please understand, this is how the entire equation become, now you may be wondering what is that l n, how did it come up, you have to do the integration. It is all integral minus 1 to plus 1 of various forms and I have those written, but these steps are not mentioned exactly in the despite of actually and half ((Refer Time: 20:29)) elastic book, the overall procedure is there with some intermediate keys steps. Like this expression will be there, now you know the pressure, pressure is related to in this flow at the flow on the w a on the airfoil. And the integration is minus 1 to plus 1 that means, the card length, now you have to write the expression for w bar a. That expression is I will just write it, then you simply integrate the...

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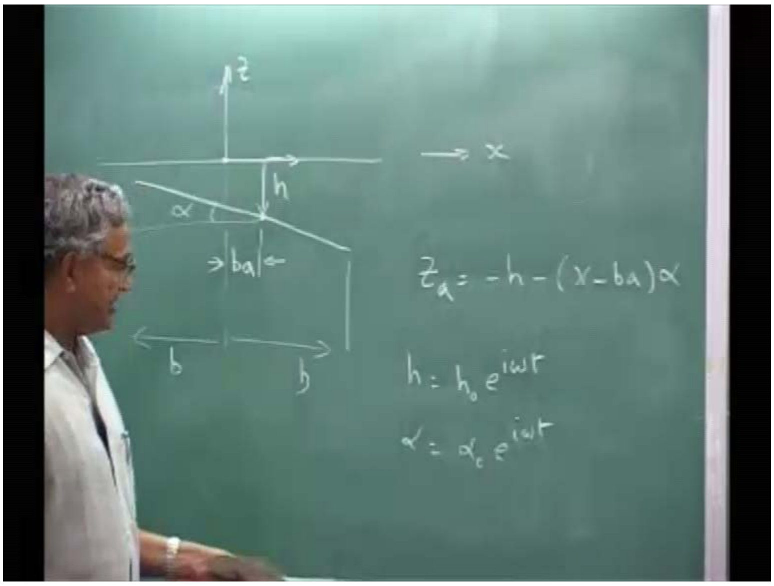

So, let me erase this part ((Refer Time: 21:17)), you now have this is your reference line and your z axis is here this is my x axis, and your airfoil is represented by a line that line is like, this is my elastic axis which comes down by a distance H. And this is my alpha and this distance is b, this distances is b and this is b, this is my airfoil, which I am representing by a line. Now, what is my z a, z a is minus h minus, because your x is refer from here x minus b a, so x minus b a alpha and this is the harmonic motion, that is h is equal to h naught e i omega t alpha equals alpha naught e i omega t. You can write h naught or alpha bar or h bar, now what will happen z bar a, because you can write substitute here z a, but what is your w a.

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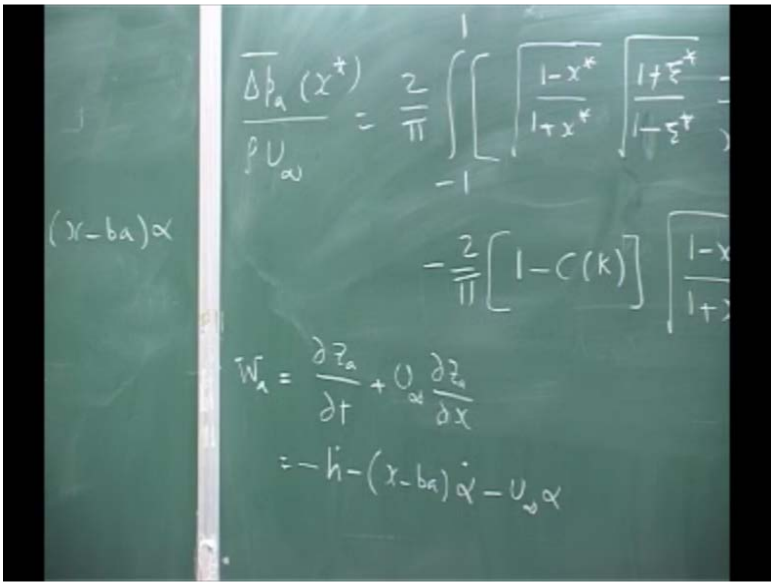

What is w a, w a is delta what, delta t plus U infinitely.

Student: ((Refer Time: 23:24))

Del x, I can substitute here then what will happen I will get w a is put a dot, so I will get when I take a derivative this will become minus h dot minus x minus b a alpha dot and then I have to take derivative with respect x if I take this is U infinitely minus alpha. So, I will have and now if I substitute this, that dot is going to become i omega, so this is also going to become i omega.

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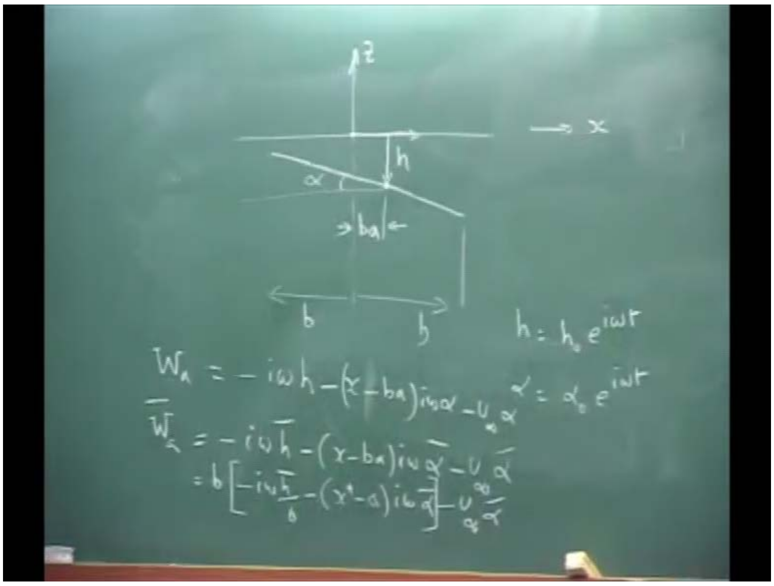

So, I will have my expression let me write it here, that is my w a becomes h dot is what I will put it a, I can minus i omega h minus b a i omega alpha minus u infinitely alpha. Now, if I want w bar a you put a bar everywhere, which is minus i omega h bar minus x b a i omega alpha bar minus U infinitely alpha bar. Now, if I want to monumentalized I have to divided by b that means, I will divided everything by b and this is w bar a, as a function of you can have x, if I non dimensional x.

This will become what, I can take out the b outside and minus i omega h bar over b, here I will put minus x star minus a, this will become i omega alpha bar, because x star is x over b and here you will get minus, because that is a what, we have a U infinitely because this is an x differential with respect to x x. So, what you get this term is over may be I put here b is over minus, because b is for this term this is kept outside that means, I have find w a as a function of x star. And I want basically that w a at the function of psi star, and I have that expression I go back I substitute here.

Now, I am having delta p, which is what p bar U minus p bar L that means, upper minus lower, if I want lift I will put a minus integral minus 1 to plus 1. So, my lift become I do not want to erase that part, let me I erase this is not necessary ((Refer Time: 27:59)), I erase this.

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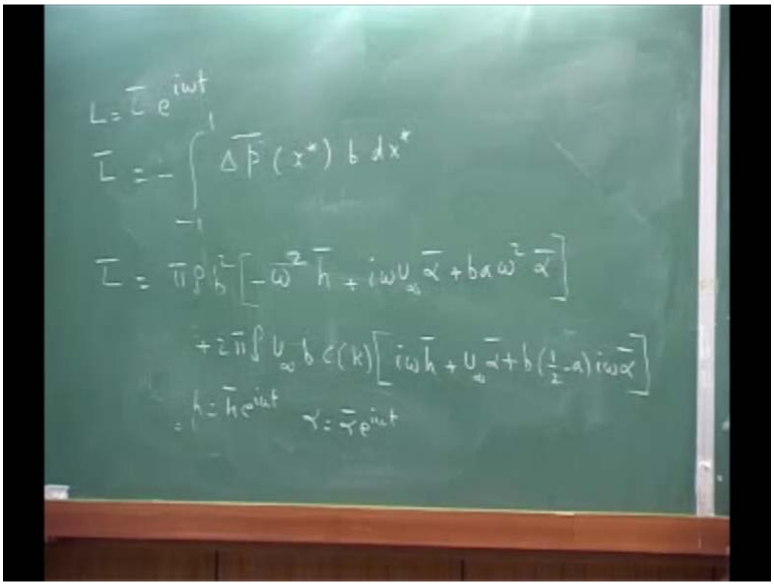

My lift I am writing it as L bar e I omega t, because lift is also osculating with the harmonic motion, therefore if I put in that my L bar is nothing but minus 1 to plus 1 delta p x star b. Because, this is d x that is why d is there follow, you have the full expression of delta p x star, take this put it here integrates with w bar a the expression which I have given. If you do that, because that integration is the messy I will give the final expression.

The final expression L bar will be, because I am getting L bar later I will converted into L itself, you will see I will get pi rho b square minus omega square I will have h bar. h bar is the amplitude of the motion, h bar e i omega t are which is h not what we have used, plus I omega u infinitely alpha bar plus b a omega square alpha bar this is the first term. Then I will have the second term 2 pi rho U infinitely b C f k i omega h bar plus U infinity alpha bar plus b into half minus a i omega alpha bar.

And that h is equal to, I am writing h bar e i omega t, alpha is alpha bar e i omega t, just some what should I exchange x naught to... Please understand this is bar, now if I multiply e i omega t this ((Refer Time: 30:44)), e i omega t on that side, then I will get L as a function of time, now I erase this part and I write my lift expression.

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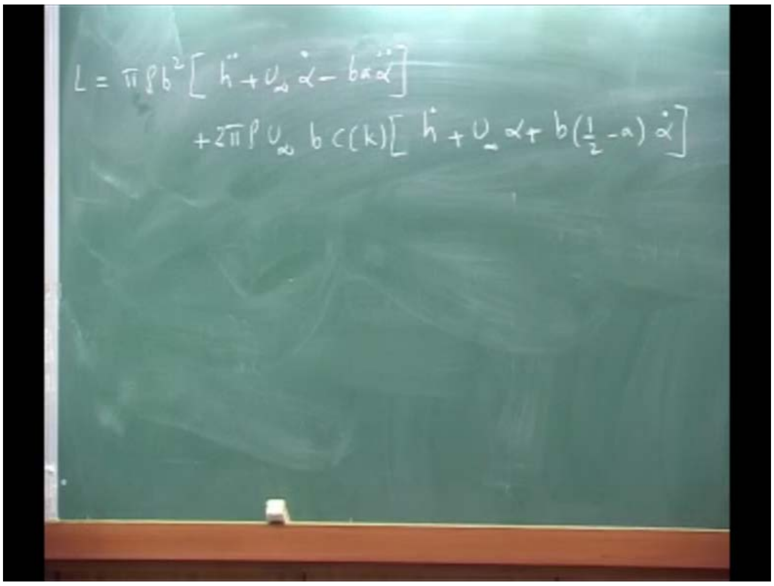

I will my lift becomes, please understand if take e i omega t, if I take a second derivative of h, that is h is double dot will be minus omega square h, because h is h bar e i omega t. So, this because of this minus omega square, I am going to put it as h double dot, this will be alpha dot that is how you get the in times, and that expression will be pi rho d square h double dot plus U infinitely alpha dot minus d a alpha double dot. Then plus 2 pi rho U infinitely b C of k h dot plus U infinitely alpha plus b half minus a alpha dot, this is my lift expression, lift. Now, you see it is a very interesting thing I am putting everything is dot, dot is time derivative, but this k is frequency parameters, k omega b by U by infinitely.

Now, I still do not know what is the omega, this is an expression which says that, if my airfoil is executing a harmonic motion with a frequency omega, this is my expression. And C of k is given by this ((Refer Time: 33:06)) and it is the complex number please understand, C of k is not a real number it is a complex number, this is Theodorson lift deficiency function. Now, you look at the moment expression, moment will be because you need to get nose up moment.

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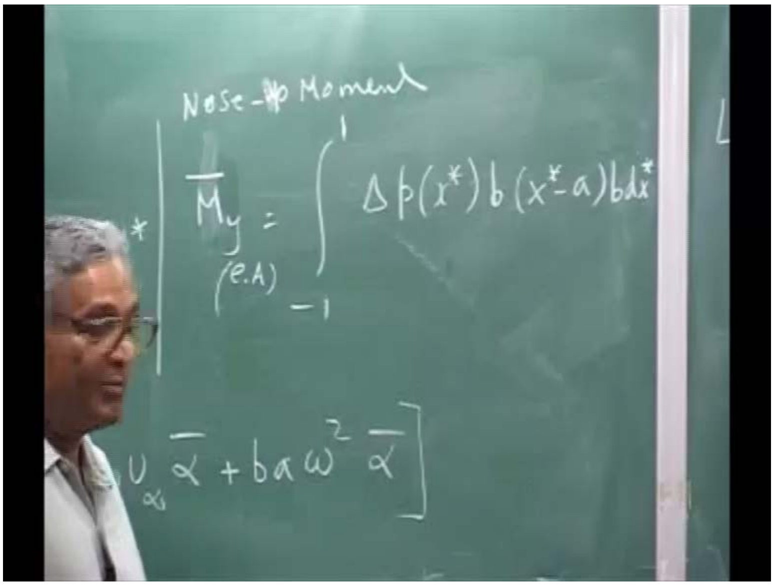

So, I am writing nose up moment that is M y bar again bar, bar is M y is, M y into a omega t, so I will put again minus 1 to plus 1 upper minus lower that is what my expression is. So, I will have delta p b into x star minus a that is the distance into b d x star, please understand I am taking moment about what point, about the point which is having the, this is a about which point.

Student: elasticity

Elasticity axis that is about b a, because we describe like this, this is the point, this is b a and my x goes here, so x minus b a, so pressure upper minus that is p U minus p lower will be down, the term into x minus b a into the nose up moment, please understand this is always a nose up moment.

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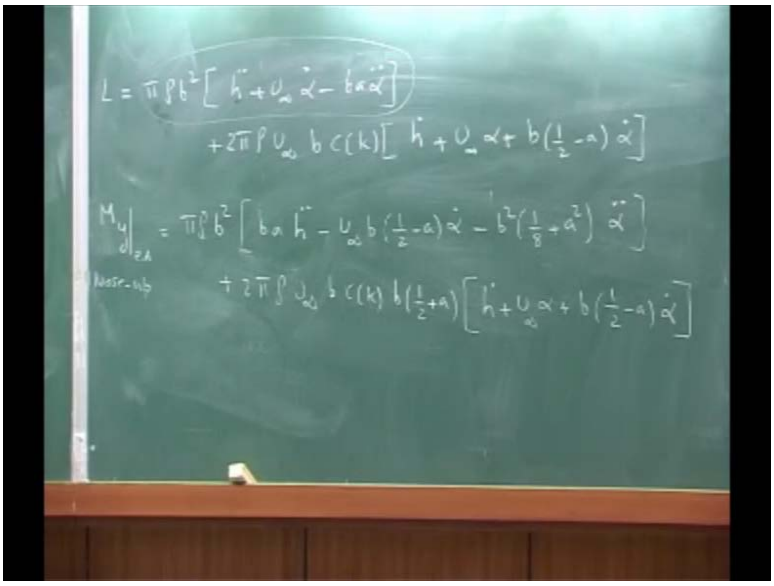

Now, again I substitute the delta p that long expression to the complete integration, and I will get my M y, M y about you want elastic excess, you want to know nose up fine everything is there, but normally nobody write all these things. This expression becomes I will write it, pi rho b square b a h double dot minus U infinitely b half minus a alpha dot minus b square 1 over 8 plus a square alpha double dot, this is the first term. And the second term is plus 2 pi rho U infinitely b C of k b into half plus a into h dot plus U infinitely alpha plus b into half minus a alpha dot; now you got the lift momentum equation.

Now, the key some of the, now we discuss this you see the second term, I am just going one by one, the second term here, the second term here only they are dependent on C of k. The C of k terms are only h dot that is dot velocity dependent, this term is double dot second derivative is there, second derivative is basically acceleration, acceleration of the motion that is why this term are called, because pi rho b square what is that, if you look at it this the airfoil, I erase this part may be that side also I will erase we will discuss only this.

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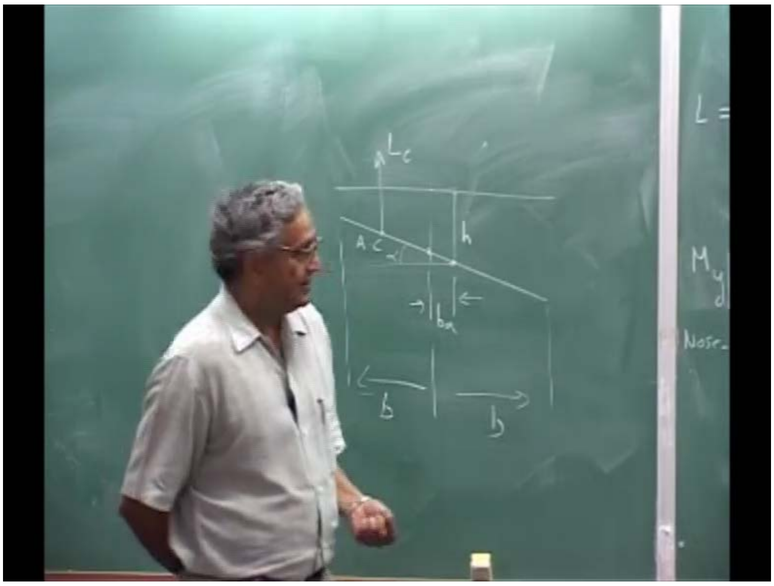

This is my airfoil and this is my b and if I draw a circle pi b square becomes a circle, rho is the density of L that means, this is the mass of L only like that cylindrical thing. This mass into expression, this is called ((Refer Time: 38:38)) apprehend mass term or apprehend mass effects or they call it effects. Because, what is mean this when I am moving, the air is not straight I carry the air with me, I have the move that means, my mass looks as though I have increase my mass of the airfoil. I have added mass to the moving system, this is the apprehend mass term.

Now, this term which is C of k, k is this is the theta from deficiency function, this represent the wake effects, wake means whatever it is going. Now, that is why this particular term is called a circulatory term or this is also called non circulatory term. That means, one is circulation due to the wave effect, another one is not due to wave, and I will briefly give you the C of k, before here one more this. Now, if you see this aerofoil, we have said that this is my aero foil, this is my b ((Refer Time: 40:25)) and this is my b a and this is from here b, from here b, this is h and this is up.

Now, if you look at this particular term this is the lift and this is the moment, you see everything is same accept b into half plus a that means, what I am saying is my circulatory lift is actually acting at see b by 2 is here, this is b by 2, this is aero dynamic center b by 2 plus a. That means, this is my what L c, L c is circulatory L n c is non circulatory, if you want to call this as non circulatory term, this is the circulatory term.

Now, if you look at the order of terms normally this pi rho b square, b usually small and rho is del C of f, there is del C of f.

Since, this is one order less than this, because this square, this term is normally neglected, but it is not necessary you have to neglected, you can have it in your formulation, you do not have any problem. But, in the earlier days people neglect that non circulatory terms, only keeps circulating, now we will slowly develop one by one. Now, you see the circulatory lift that, then where is the non circulatory lift is acting that if you look at this to this, you will find different terms, you will be acting at different point, if you want to take moment.

Because, you have taken moment about this point that means, the non circulate term you will have different terms act at some locations, that is why what people do is, my lift is acting here, ((Refer Time: 43:14)) this is my moment on the unsteady aero dynamics on the aerofoil from there. So, this is the unsteady leave, unsteady moment, if you really want to know what each term how it acts you put the corresponding this, you see that alpha dot U infinity alpha dot term is b half minus a that mean it is acting, somewhere here. That lift is acting this way that why it is going down, that is why it has different location, now C of k is Theodorson lift deficiency functions.

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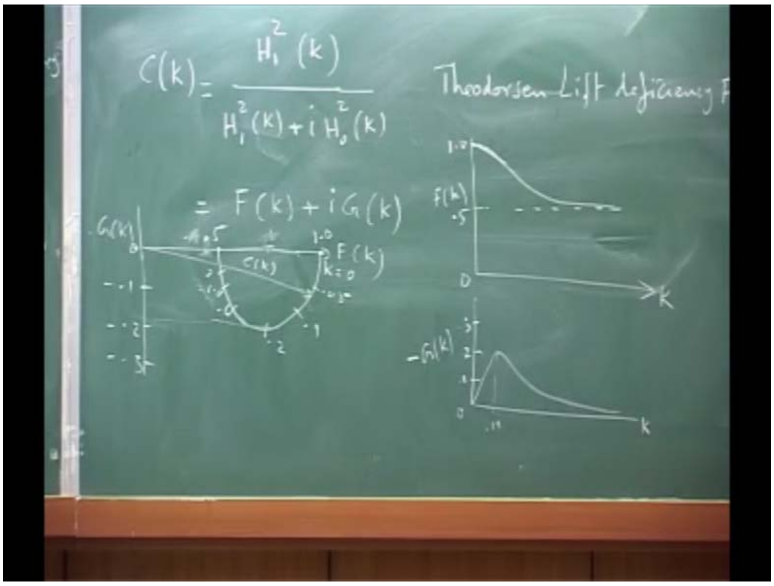

It is normally represent as some real part plus i G of k some imaginary part set of this number, this is how you will given table of data that will be specified, now how they vary with respect to k, this is F of k and here is the k reduce frequency, this is a 0.501. And similarly, you will have here a this is 0.1, this is 0.2, and point 0.3 etcetera, this is G F of k start from 1 assume vertically reaches fine pipe, G of k starts from 0 it increases and then decreases.

It has a peak somewhere around, this is around 0.19 or 0.2 or something like that, somewhere around that 0.1, 0.2 this may be around 0.2 something like. So, F of k the real part when k is 0, k is very small it is almost 1, you can take it I think it start like this and decreases. And G of k goes like this, it is also represented in it another and please note this is the 0.1, 0.2 something, now I give this actually it is minus, so I have put it this is minus G of k.

Now, there is another form of representing, so take a minus sign there that is G of k and this is F of k this is different representation, 0.1, 0.2, with the minus sign and this minus 0.3 and here you take it as 0.5 and you take it as 1. This curve a maybe I write little bigger, because 0.5 1.6, so I can draw a it come like this, this is around the parameter k, k is 0 and you can have 0.05, 0.1 and here is the round 0.2 and what about 0.6, about 1 and then 2 and then this is the variation of k. So, C of k you can have any value, this is C of k, the same curve represent in the different part, real part, imagining part.

Now, what it the means of this C f k, C f k is a complex number, what is it is say and it is magnitude please note it is less than 1 as it comes down magnitude comes down, what you have is if my airfoil is osculating C of k reduces the magnitude of my length number 1. And then ((Refer Time: 48:20)) this is the motion h alpha alpha, this is the motion of the aerofoil, this is the complex number that means, my lift lacks behind the motion I have a motion there is a lift. So, it is like input, output; input is the motion of aerofoil, output is the lift, now in between the complex number, which represent my transfer function line.

The magnitude reduces and there is the face differences, and this face is responsible for plug up, because it can get into, it start extracting energy from the air and it will start vibrating and then if you extract more than the wing will break. So, whether the damping it is actually directly related to the damping, if the damping is positive, means I am anticipating energy, then automatically it will get convergence motion I give a disturbance it will divert down, but if the damping is negative that mean, if extract energy means if I give a disturbance it will start diverting.

Now, in the flutter or whatever people say is, the point where I have sustained osculating, that is the boundary, that boundary is what we are assuming that means, k we all assume in our entire formulation, the airfoil is exhibiting harmonic motion. Now, you know that this is I can do, if omega varies us it is frequency domain I have, if I want in time domain I can convert it into time domain, but I am having one frequency parameter sitting on the time domain, now this is where all the complicity how we solve that part.

That part we will learn in next class, ((Refer Time: 50:43)) what next 1 or 2 lectures will be on how to solve this problem with this. And suppose this is the incompressible flow, please understand this is valid, this was proven excellent, but if you go to compressibility region, how the people connection because the, only look at the circulatory part of that curve this is 2 pi, 2 pi is taken C L alpha lift on so. So, simply you extend, if you want compressible effect, mark number effect you will write C l m.

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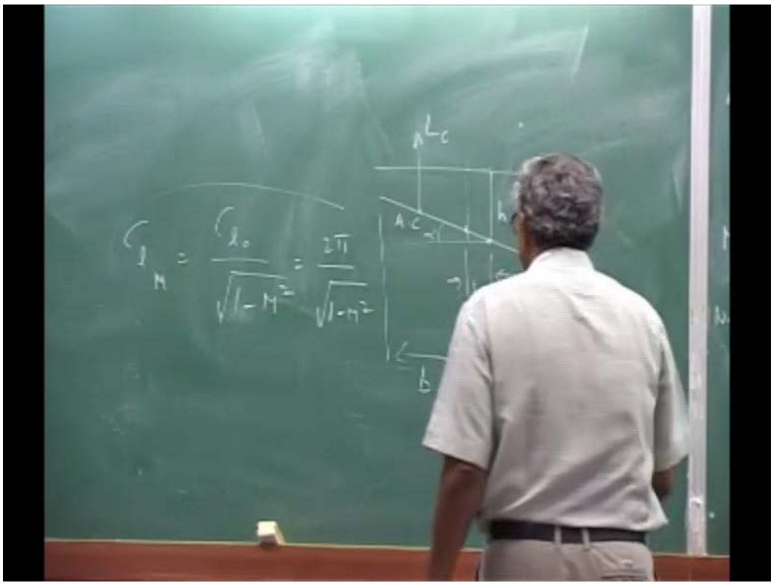

What is that C l at any mark number is C l 1 minus, simply convert this is 2 pi, so simply put 2 pi over, this is the correction you make for compressible effects. But, even though if you want to solve like I told you earlier that purer transform you have to do and then get, that the right solution, but if you want of immediate correction go head and change this 2 pi compressibility. Now, I have done analysis or as free description, so we have couple of thing C f k is a complex number, you will be given the table, you do not go calculate all this number, I have the calculated develop column of F k, G k for various value of k, you will given the table.

If you want you can and read that one, any C of k and this is called the Theodorson lift deficiency function, because the lift is getting reduced, C of k is less than 1, except when k is 0, it is value is 1, And then it gives a face difference number 1 magnitude reduction, face , it is introduces, then if you want include compressible as an approximate way, you can do this correction. Otherwise, you have to do the full compressibility formulation, which was I told you that various integral that double x method and other methods. Now, let us do few key things, key in the sense I would like to mention, let us keep this part as it is look at the approximate formulation.

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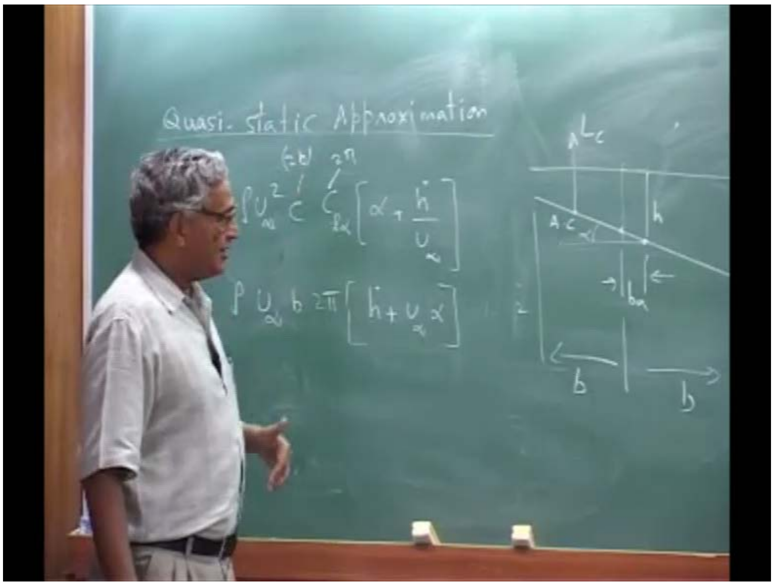

One is we call it quasi static approximation, quasi static approximation what I said is, I look at the instantaneous angle of attack at every time, and I am writing my left as dynamic pressure what is that, lift up that is half rho, this is lift per unit length C, dynamic pressure card into C l alpha C l. Now, I am writing my C l alpha times instantaneous angle of attack is alpha plus, this is my alpha effective this is what you used earlier, alpha plus h dot over U infinite. Now, you expand it substitute C l alpha into 2 pi this is 2 pi, now what will happen and C is 2 p, now what will happen this we rho U infinity, one of the U infinity you go take it inside, and this is.

Student: b

b there is a 2 pi, you will have h dot plus

Student: U alpha

U infinity alpha, this is what you have taken as your lift, quasi static you look at that lift expression here C of k into del 1 h dot U infinity alpha, but this is done, you have not taken the alpha dot come. You simply missed, because you do not have that expression, this is the quasi static and then you will say my lift is acting at maybe quarter card, I can take moment b into half plus a, that you need this expression without this. But, you do not have this two terms, there is no apparent mass, but you this is called quasi static assumption.

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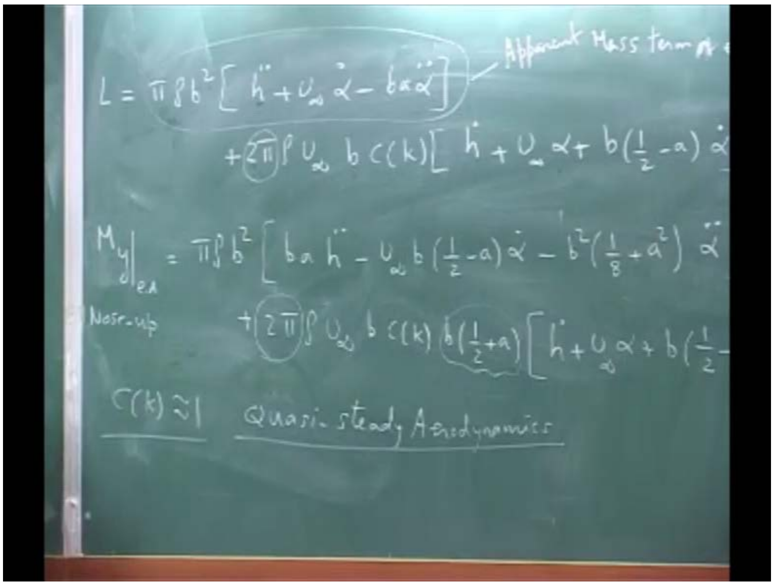

Now, you go and then say I can have a slightly better approximation, knowing the value of how C of k various that is if I set C of k is 1, then this is quasi steady aerodynamics. Now, you see if I said C of k 1 which mean, the parameter k basically represent the wake effects, though it is the reduced frequency, C of k represent the effect of wake I am basically neglecting the effect of wake, I just say I am not bothering the way, so I put C of k 1. But, then will you include apparent match, if you want you include there is no problem, but usually because this is the h double dot alpha double dot.

But, is done is different people adopted different approximation, either you can take it as no problem, but set C of k 1, please understand this is use extensively, in all the calculation in the result also people use. But, we in the helicopter field we have slight modification, and we use the same thing C of k 1, but of course, in our research currently we have model different, some stall effect etcetera. Then we are bringing in C of k that part I teach you finite state modeling the unsteady aerodynamics, how it is come from here.

Now, because if you have a spring mass system, the aerofoil mass system m x double dot m, because you are going to write the, because you are putting what there is a spring here, and there is the static spring here this is how aero elastic equation are there. And mass of the aerofoil will be there, that will come as h double that and i alpha, i into alpha double dot, but the mass of the aero foil is order to mass of the air which is surrounding that, but mass of the air surrounding that is very small.

So, you do not make a lot of error in neglecting these term, but then dot term is like a damping term, because first derivative is the dative. So, some people they say no, no I will neglect these two, but I make this term, because this is the damping term, because I will have a damping I have a damping term here. Now, you see different approximation, which you are really throwing out different effects, but that depends on the kind of problems. But, here what we will learn is, you will learn taking it full how you solve the flutter for a aerofoil, using this expression of unsteady lift and moment, including C of k.

If C of k is 1, you can solve the problems, because you have solve the problems with this assumption, you can also take C of k 1, solve the problem again you can do it, now you can solve the same problem with quasi steady aero dynamics assumption. So, either you have neglected damping, you have included damping, now I say little bit more sophisticated models now, that you can solve also. Next is you can take C of k itself, so you see the same problem you can solve different ways and then find out how I flutter frequency is varying with respect to the kind of assumption I make in my aerodynamics model, unsteady aero dynamic model basically.

Now, you now know you wanted that approximation what is the meaning of that quasi static, quasi steady, unsteady, so if somebody says quasi steady means he has neglected, he set C of k equal to 1, that is the wake effects, that forms. Now, just a deviation, same theory, because this was Theodorson's theory, later lowly for helicopter rooter, he use the different wake structure. Then there is a white heads theory that is for casket that one, another Green berg theory what he did was Green berg, in this my flow is coming at steady velocity.

And my aero foil is executing from one motion what he did was my flow is function that means, U infinity itself it is varying with time, and I have a constant angle of attack over which I have a osculation. Because, C is alpha is 0 angle of attack and it is occulting, he has use the constant angle of attack over which it is osculating, so an aerofoil an osculating aerofoil with a mean angle of attack in a pulsating on coming through that is the Green berg theory. So, you see Theodorson, Green berg then Lowey, then white head I think lowly is a lowe any way, these are the various theories.

Now, that is the different just the modification of the same theory different application, but we use Green berg theory in the helicopters, lowly theory is the little bit complex, so we do not incorporate that. But, white head theory we have used one of the masters even published paper on that, now you have the full model, now I am going to briefly give you a different some one small aspect I will describe. What is this particular terms ((Refer Time: 01:04:33)), h dot U infinity alpha plus b into half minus a alpha dot, what is the term represents, b by 2 is here, b by 2 minus b a into alpha dot is the velocity here.

Suppose, if I want my velocity at 3 quarter card that is nothing but what h dot plus alpha dot, suppose you take out the U infinity outside that means, h dot over U infinity alpha. Then this is the velocity h dot plus d half minus a alpha dot is the velocity at 3 quarter card point, downward velocity divided by U infinity is the instantaneous angle of attack, please understand. This is the instantaneous angle of attack I have taken at this point, suppose I take the instantaneous angle of attack at 3 quarter card point, that is that velocity, that is this expression.

So, this expression is called w 3 4 c, that is downwards at 3 4th card point, now you see circulatory lift is acting at quarter card point based on the angle of attack at 3 quarter card point. Because, angle of attack is what everywhere the velocity is changing that means, I am writing my velocity alpha instantaneous angle of attack in alpha plus velocity by U infinity. But, the velocity at or you can say this speed at the down ward velocity at 3 quarter card point, so the quarter card point and 3 quarter card point are very important in aero dynamic.

You take the down ward 3 quarter card point, but put the lift the quarter card, now you understand the vortex latex methods, various methods, they calculated the downwards three quarter card in a box, lift the quarter card that pressure is put at quarter card downward itself to the quarters card. Now, this are very important points, now this particular term if I assume h equal to h bar e i omega t, then I can put them in omega square all of them, alpha alpha bar. So, I briefly write that particular expression, because that is what is going to be used later, because this is given in a different format, but essentially it is the same thing.

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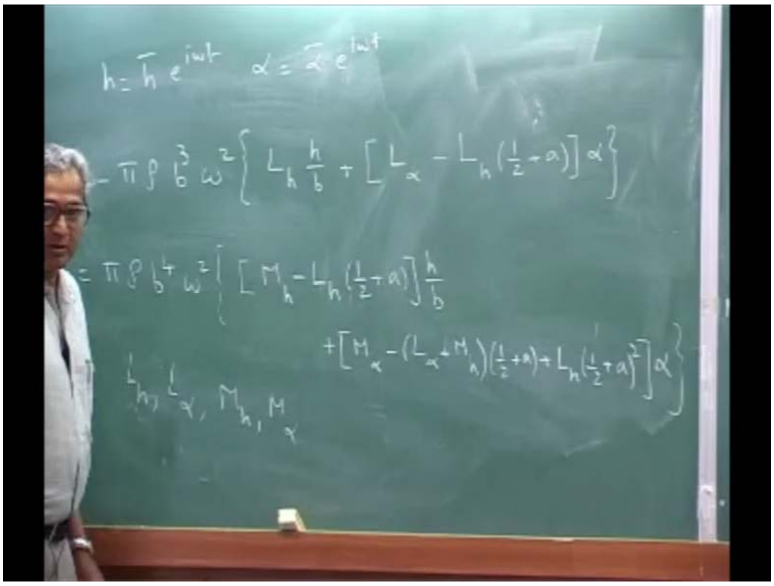

So, that is h equal to h bar e i omega t and alpha equal alpha bar e i omega t, I am going to write my lift as minus pi rho b cube omega square L h h over b plus L alpha minus L h half plus k alpha, please understand. I am writing, I go substitute this in those equations I collect the terms of h that is the plunging motion, I collect the terms of alpha you follow and then similarly I write my moment pi rho b to the power 4 omega square M h minus L h half plus a h over b, h over b is a non dimensional plus M sub alpha minus L alpha plus M h half plus a plus l h half plus a whole square alpha, please understand.

Now, I have L h L alpha M h M alpha these are my aero dynamics coefficients I call it, un steady aero dynamic coefficient, these are given in terms of reduced frequency, mark number. And this is the big tabular column, you can have now you understand whether it is the sub sonic, supersonic, any things you always write it in.

This form in a different in the sense h you separate, alpha you separate and then write the coefficients. Now, I give you what this 4 coefficient are, please understand these four you can get the you know I have given you the expression here, you substitute this in this two. And then start collect all the h term, collect all the alpha terms and then equate them and you will get this form, because this form is what we are going to use later, because otherwise every time writing this whole things you know it is rather a lengthy stuff. So, I would briefly write those coefficient and you can 1 minute I will these are given in tabulated form I will just directly, because I have I have here.

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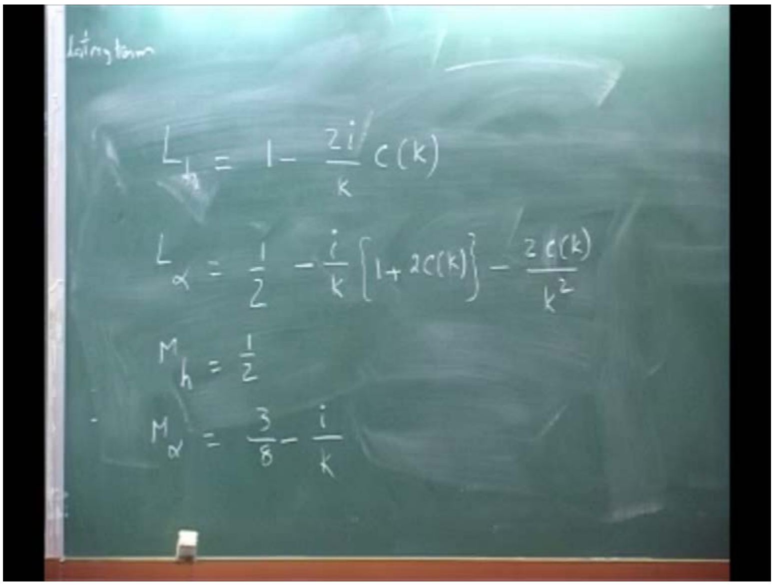

L h is 1 minus 2 i over k C of k this is L h L alpha half minus I over k 1 plus 2 C f k minus 2 C, 2 C of k over 8 square then M h is half, then M alpha k is 3 over 8 minus I over k. Please understand this you verify it, I want you to check, please do it like a home work or exercise or anything. So, L h is 1 minus 2 i over k C k, l alpha is half minus i over k 1 plus 2 C k minus 2 C f k over k square; then M h is half m half by g over 8 minus this is clear. Now, what we will do is the next class we will study how to use these things.

So, our final unsteady aerodynamics expression in the incompressible you can say subsonic flow is here, using this how do we flutter calculation, number 1 and there is if you have a arbitrary motion. Suppose, you have gust, how you will analysis what is the gust ((Refer Time: 01:15:31)) those things we will study and then I will also teach a little bit about finite stage model basically using this.