Aero Elasticity

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Lecture - 22

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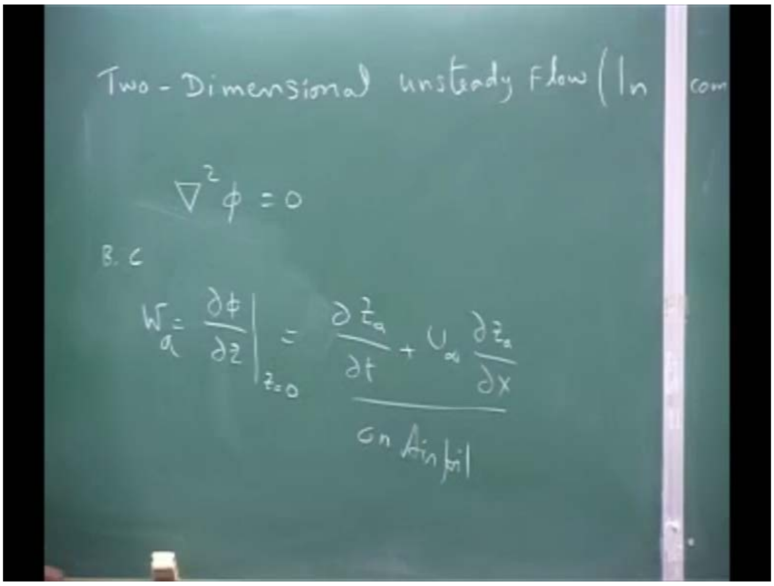

We will start with the two dimensional unsteady flow incompressible, again this is governed by the equation del square phi is 0, and the boundary condition w, which is delta phi over delta z. We call it at z equal to 0, we call it w a, let us stop, this is nothing but on the air foil delta z a over delta t plus u infinity delta x, this is on aero foil. Now, if you look at it the only different between the steady case, unsteady case is in the boundary conditions and in the dimension, and otherwise my equation is same that is why phi is same only thing is this is extra term that is used.

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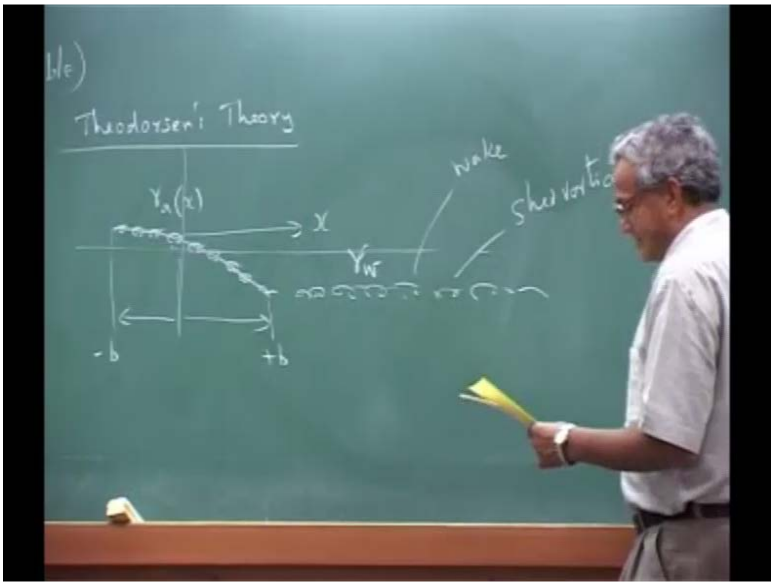

Now, how this was solved in 1934 Theodorsene's, this is a actually Theodorsene's theory in 1934, but he used sources and ((Refer Time: 02:14)). And then he calculate, but what we follow is basically Theodorsene's theory, but we will do the approach of inversion integral, that conjoins inversion integral. But, how the unsteady flow because of the air foil motion, what really happens to the entire problem if you look at which of the aero foil.

We put it like this, so this is plus b, this is gamma a, this is on the aerofoil I have ((Refer Time: 03:17)) the whole diagram, this is on the aero foil. Now, in the bake you will have some structure, this is we call it gamma w, we did again a function of x, but our x always start from the midpoint this is our x. Now, you see this is the vortices, which are called shed vortices, now you assume in this problem that the aero foil is continuously oscillating; that means, you will have a wake.

This is actually wake, with because the lift is changing my circulation on that is changing, the changing in circulation is every time shedding something the vortex, this vertex will flow with the oncoming velocity. Now, what was assumed that this vortex structure, this extend from the laying edge to infinity, and it is assumed that it is on a line, x is equal to it is a long line x axis that equal to 0 line.

It is not that this is distracted; that means, this is called the prescribe wake structure, that I prescribe the structure of the wake, which in this simply, because simplest, because I use the word prescribed and pre, these are all much later started. My wake structure is laying on line, and this moves with the velocity infinity, because anything addition is going there, this is all the basic structure. Now, using the method of sources and sinks get off ((Refer Time: 05:56)), but what we will use that chain in other approach.

Now, what is the boundary condition we apply, because we said that this is the circulation distribution on the air foil, this is on the wake. One is that is a relation between this and this, because of the change here only this is happening in the wake, therefore there is a relationship that relation also should also identify first. What is that gamma w, how it is related to gamma a, gamma a is over ((Refer Time: 06:38)) some integral value, and the second one is this is going to have a influence on the velocity here, because any w delta phi by delta z, this is the potential.

So, all these wake will have a potential, so it is the integration, so the wake will have a affect on the flow on the aero foil, that is the key you cannot neglect this wake. If you neglect this wake completely, it is relate to the study case, because you have not taken any gamma that what we did last class. Now, with this have the bases, we start how we will proceed with the solution, see finally you want the pressure on the upper surface, lower surface, and then integrated it, you get the lift, and take movement about any point you get the, this is one the entire problem is.

Now, following the case which we did for study we follow single approach, but only condition what we will apply is, because this is my equation, this is the boundary condition on the aero foil. Of the aero foil means of this for the lifting case we said that disturbance pressure is 0, so in the wake the disturbance pressure is 0, but you will have a velocity u, which is different because of the wake. Because, you remember we relate that differential gamma a, we related to the difference in u disturbance velocity, so you will have again the here, because e is the symmetric for the lifting case.

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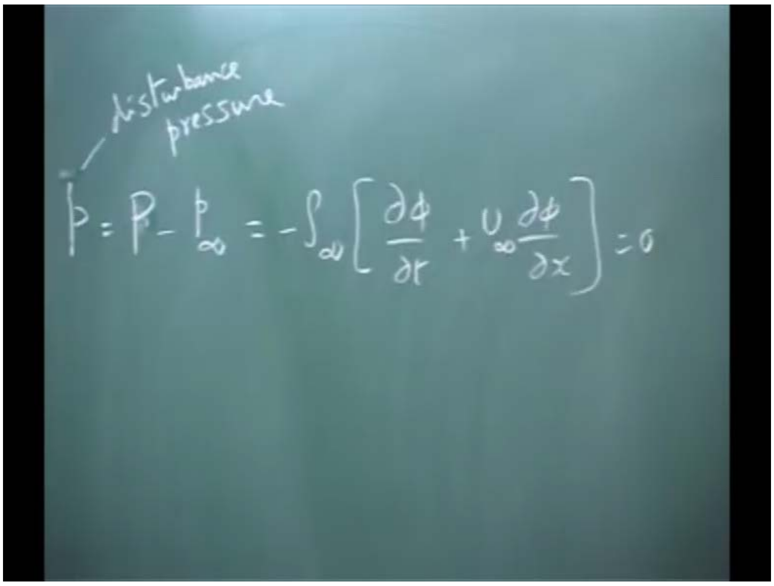

So, what the condition that is applied is the pressure of the aero foil, so that is basically you say p, which is we had the p, p is enough this is lower case p, this is p minus p infinity. This is the disturbance pressure, which is given by you know that 2 minus rho infinity delta p over delta t plus u infinity, this is my disturbance of pressure. This is 0 for x greater than d and x less than d on z equal to 0, because z equal to 0 is our key, because we do not consider that this is the small motion only.

We go back in your formulation we always say my velocity, I calculate that z equal to 0 plane form here, that is why all this motions are small motions small mistakes. Now, that condition I will use that is another system you can say got it upon boundary condition, and of course on the aero foil I have to use this condition, because on the aero foil you will have a differencing pressure. This is x is greater than b x less than b, but on the air foil the pressure will decrease, now using this let us start with our formulation of we directly use straight away, because we know the wake structure.

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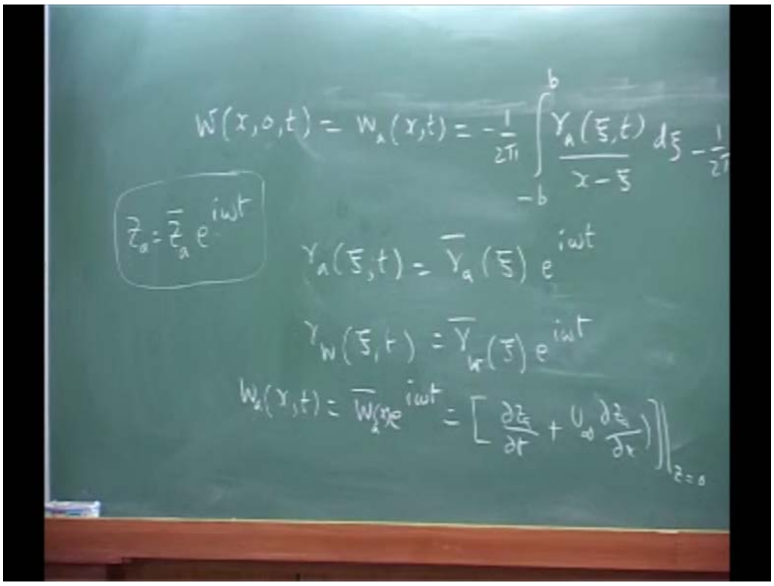

What is the velocity induce at z equal to 0 plane, you use the same this is w a x comma t, this is minus 1 over 2 pi minus b plus b gamma a over x minus. This is due to the wake, this is vortices on the aero foil minus again one over 2 pi, you will have b to infinity, this is due to the vortex strength on the aero foil, this is due to the vortex strength in the ((Refer Time: 12:33)).

Now, I have this relationship, and you know w a, w a is related to delta z by delta d plus u is equal to delta z that is the aero foil motion, if I get the relation for gamma w, in term of gamma a some integrated value of gamma of a I will get it. And then I will just substitute here, now I will contribute like some form, I will use w a, and this I will bring to left hand side, then I use which is identical to the formulation which I had felt. So, I will use inversion integral conjures inversion formula, I will directly get gamma a in terms of w and some integrated value.

So, I just probably mention the procedure the procedure is identical to what we did, because earlier in the study case what we did was this is not there. So, simply we inverted this came here, and we got the gamma a in terms of w, then you integrated rho u gamma is the lift same you will do here. You will get the pressure difference, substitute the pressure difference, integrate the pressure difference over this minus b to plus b. You get the unstudied flow and then movement of n point you get the delta unstudied point, so this is the basic line.

Now, how do we get the relations between this and this, that is the first point, see what we do is we are going to assume that my gamma a bar. This is the running variable along the x axis e power i omega t, I am always assuming that my aero foil is executing a harmonic motion, we start some harmonic motion, so the theory assumes that and my gamma w, again it is gamma bar w e power i omega t.

Now, your e power i omega, which is given as because you know on the aero foil delta z a over delta t, that I call it plus u infinity or z is equal to 0, but if I write my z a is my aero foil is executing a harmonic motion. Then I can substitute here, I can get out e i omega t outside, this part we will use it later, at the final step we will use this to get the results final result I have applied. So, now, let us just formulate with this assumptions, so my formulation assumes that my air foil is executing a motion of this, but z valuate I will write in terms of pitching and plunging.

Z value is nothing but the what is that displacement, that is any point on the air foil, that will be written in terms of plunging and pitching motion, so you will have this is the 2 degree of freedom motion. That relation we will get it at end right now, we will keep it as it is, now substitute this not here if you substitute e i omega t will cancel out everywhere.

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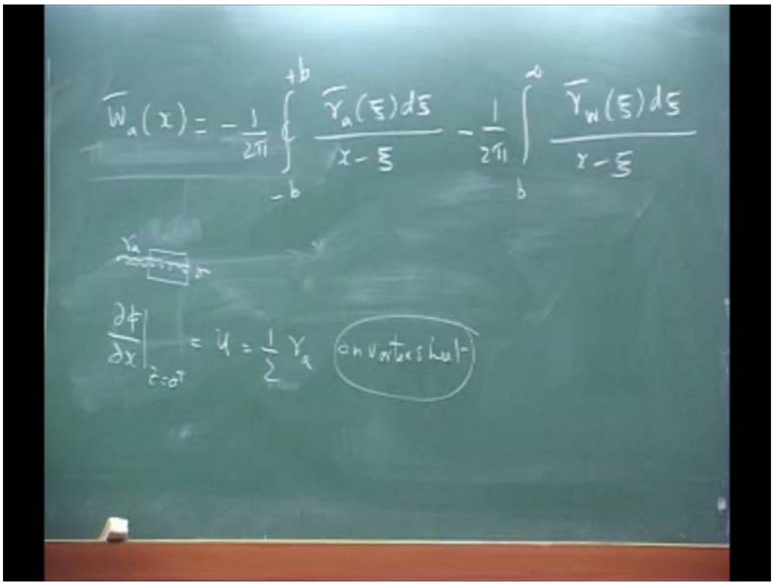

Now, I go back then write the expression, which is w bar a 1 over 2 pi minus b, because this is plus b gamma bar a psi d psi over x minus 1 by 2 pi d to infinity, but please note that my axis I always start with the midpoint of the mid card. This is gamma w bar psi e; that means, this is not z equal to 0 plane only x is the location, I get the velocity, this is the integration.

Now, what we will do is let us evaluate what is my phi, because earlier last class we mentioned because when I distribute my vertex like this, if I take a circulation integration. We showed that delta p by delta x is equal to 0 plus is on the aero foil last time this is gamma a, we said u this is u, which is half gamma a, this is on vortex sheet in the since just above and below, u integrate is disturbance velocity.

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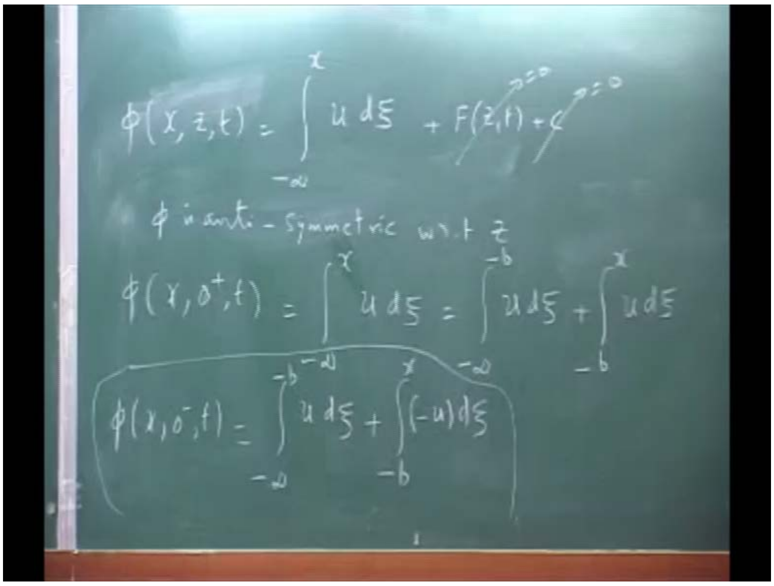

Now, if I want z I simply have to integrate this, if I integrate for phi I will get it I can write phi x comma, but I am not interested at phi far away, I will make it. This is minus infinity to any x u plus some function this function, finally I will set it to 0 what this thing am setting 0, because phi is anti symmetric with respect to. Now, I am interested to only phi, because of anti symmetric these are all 0, I do not need to have, any phi at x 0 plus this is minus infinity x u d, but if I want 0 minus.

I will have, because this is anti symmetric, so I will put the minus, but now I am going to do this integration into 2 parts, minus infinity to minus d u d plus minus b to x. Now, if I write for 0 minus comma t, this integral is same, but you know that minus infinity to minus b, you do not have any vortex in front. So, u has to be continuous, u is not 0, so you will find this will be minus infinity to minus b this is same, but when I go to minus b to x, I will have, this is in front of the air foil.

There is no other vertex, whereas c l will be the once, you come on to the vortex there is the difference in the velocity, now you have phi expression for to this go back and calculate the pressure difference, I erase this part.

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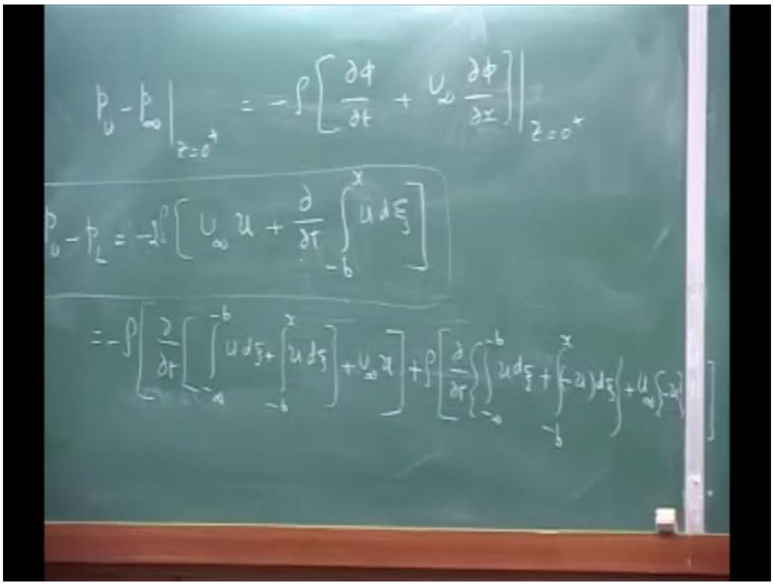

The disturbance pressure that is p upper surface minus p infinity z is equal to plus, this is you will write it minus rho delta p over delta t 0 plus and p lower surface, you will have same thing. Upper surface, lower surface only thing is t l minus t infinity you will have similar expression, now you subtract upper minus lower; that means, p u minus p l, which is upper surface is this plus z is equal to 0 minus of minus.

So, I am putting minus minus rho delta x that is z equal 0 minus, now delta p by delta t, I know phi here, lower surface, upper surface I have the phi and delta phi by delta x is nothing but u at any point. So, delta phi delta x is delta p by delta t I am going to write now minus rho, upper surface that will be delta over delta t of you write that delta p over minus infinity to minus b u d psi minus b to x u t psi plus u infinity into u.

This is on that equal to 0 here you will have minus and minus that will become plus, so am putting plus rho delta p by delta t 0 minus, I will have delta by delta t of 0 minus I will have minus infinity minus b u d psi. That is plus minus d x, I have minus u d psi I and then delta p is equal to delta x is u, because delta p by delta x is just a u expression with the lower surface, you will have let me what is that, this bracket is closed plus u infinity minus u.

Now, if you look at all the terms, you will find that delta by delta t u infinity minus infinity to minus b, they will have the opposite sign, so they will cancel out when you consider delta b by delta t minus d x u, you will get minus u. So, they will have double, similarly this is minus u, so that is also double, so your pressure I erase this here and write that expression. Pressure is given by minus 2 rho u infinity u plus delta by delta t of minus v x, this is my pressure difference z is equal to 0 plus 0 minus.

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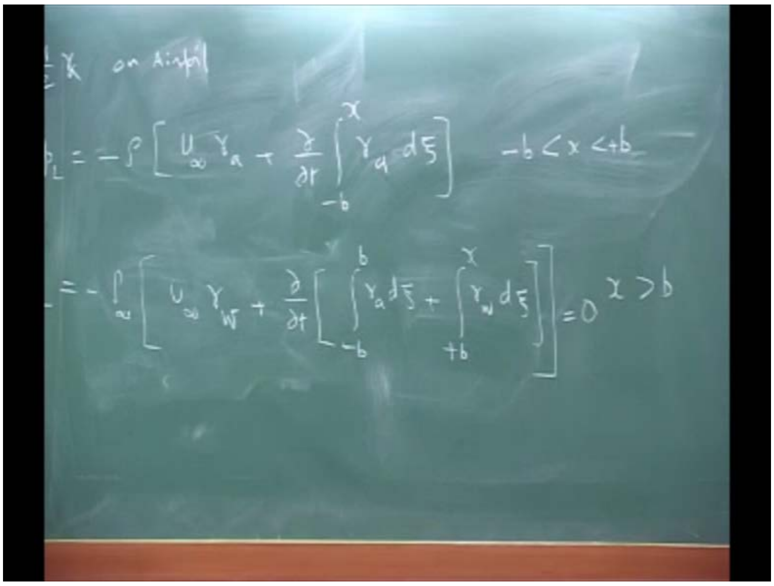

Now, I go back and let me write my what is my boundary condition, you know that u is equal to half gamma a on aero foil, now let us look at p u minus p l minus rho u infinity and I can put a 2 factor this is gamma a. So, I will have 2 u is gamma a and then plus delta by delta t minus b to x gamma a, this is for please note, this is on the aero foil, when I go outside the aero foil p u minus p l this again same expression, but now I will have what, u outside the aero foil beyond plus b to u become gamma.

So, you will have u infinity gamma w plus delta by delta t of, you will have minus b to plus b gamma a d psi plus minus b plus b to x gamma w. Disturbance pressure for x greater than b is this, but you know that in the wake disturbance pressure, because this will not lifting. Now, this gives me the relationships between gamma w and gamma a, how that is you write this relation.

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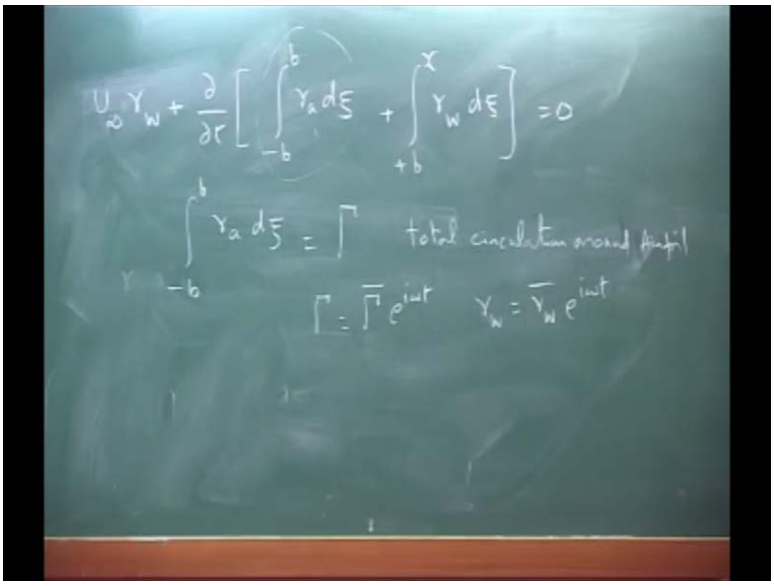

This is u infinity gamma w plus delta by delta t of minus b to plus b gamma a b psi plus b to x is 0, now what we do is you assume, because gamma a is the function of i omega t gamma w also e power i omega t. Now, you will see this is nothing but we call it the total circulation minus b to plus b gamma a d plus we call it total circulation around the air foil, now we assume that this is the harmonic motion. So, we always, we can assume this is gamma bar t i omega t and gamma w, we assume that this is gamma w bar e i omega t, you can substitute these and you will get the equation.

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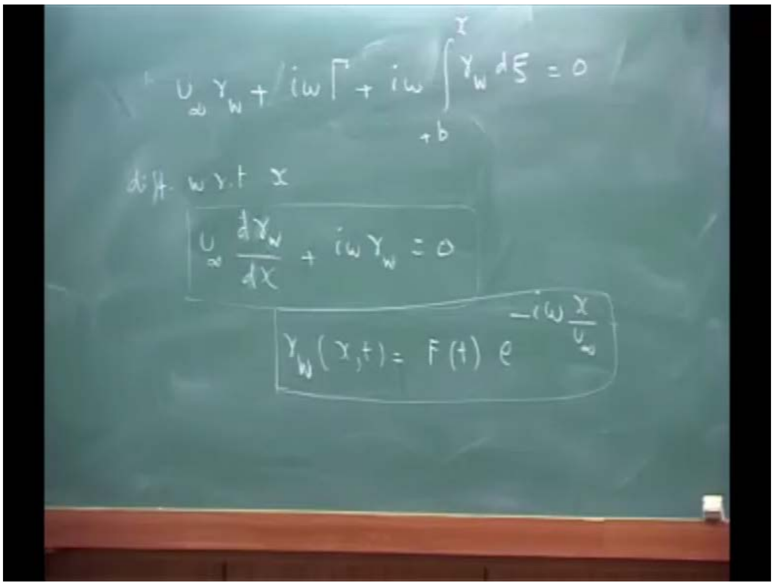

You will have u infinity gamma w plus delta by delta t that will be capital gamma, that is i omega capital gamma, because delta by delta t that one, and then gamma w is again that will be plus i omega this is 0. Now, what you do is you differentiate this with respect to x, because this is in the wake, when you differentiate with respect to x, you will have this is u infinity gamma w by delta x. Gamma w is only a function of x either it can be total you can put it, nothing because it is also function of time, we are putting it like this is 0, because gamma w is independent of x.

And then this will be just i omega, this is equal to 0, now this is my equation for gamma w, so the solution of this, this is the first order equation. So, your gamma w you can write it has which is the function of x and t, you can write it has f of t e to the power minus i omega x over u, because the solution the time is gamma w is the function of time, that is why I put the f of t function this is like i omega over u infinity x. Now, what I do is I take this, I substitute here, I will get a equation like this, all this exercise is to get gamma w in terms of gamma a.

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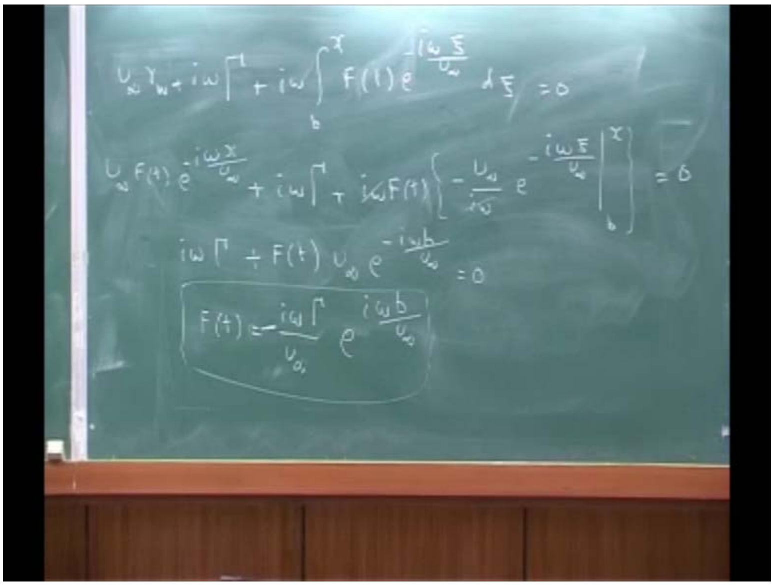

So, take this substitute, you will have u infinity gamma w plus i omega gamma plus i omega b to x, gamma w is this you will have f of t e power i omega, the minus sign over u infinity here. You can integrate this, your integral because you say that far phi, when you integrate you will have, because gamma w is given here, so u infinity f of t e to the power i minus i omega x over u infinity plus i omega gamma plus i omega f of t, because you can integrate this is nothing but e to the power something.

So, you will have minus u infinity over i omega e to the power minus i omega psi over u infinity, limit b to x this is 0. Now, what you will see that i omega cancel that. In the lower limit and in the upper limit if you send the upper limit, this will be x that term will canceled out with first term, leaving the lower limit term as this. So, you will get from here gamma, because i omega is common, you will get the gamma will be, let us write this is i omega gamma this is cancel out.

So, you will have minus f of t when you set the lower limit that is going to become plus sign, so you will get plus u infinity e to the power minus i omega v over u infinity. That means, you can get f of t, f of t it becomes you take it that side, this will be i omega gamma over u infinity e to the power i omega b over u, put the minus sign, now I can substitute this here I will get my gamma w.

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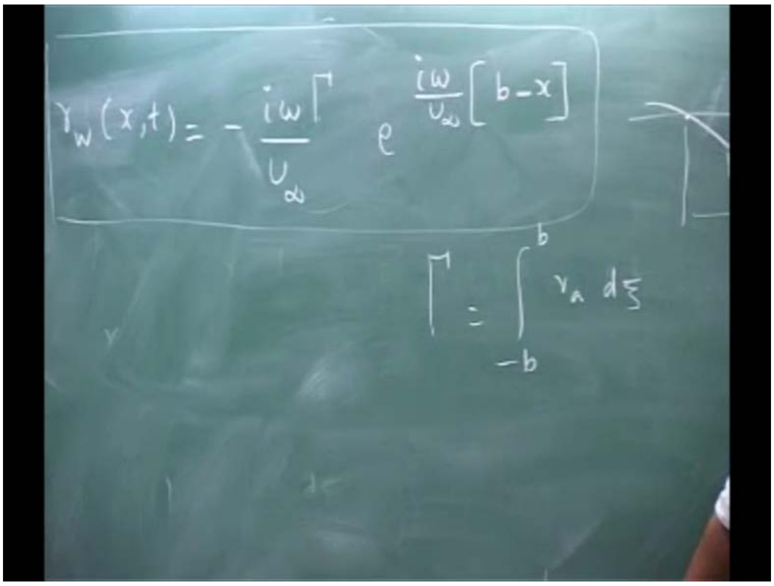

So, we will write that gamma w, gamma w x comma t is nothing but you take this term put it there, you will get minus i omega gamma over u infinity e to the power, because this is plus that is minus e to the power i omega over u infinity b. This is the relationship and please do not forget my gamma is gamma a d psi; that means, I have a relationship between wake and the circulation on the air foil, but the interesting, what it implies look at it.

The movement a vertex shed that is going to go far away, because with the velocity of u infinity it moves, if you take x minus b, because you air foil is here, you took this as b and my x is here. X minus b put the minus sign x minus b gives you the distance travelled by the vortex u infinity, divided by the u infinity is what x minus t over u infinity is the time taken just taken to travel. That means, at time t if you look at it the vortex, which is at x that must have been shed earlier, because this is moving with the velocity with u infinity, u infinity is carrying.

If you shed a vortex now, after some time this vortex will go somewhere else, basically this term tells you the time, it has taken; that means, the vortex which is here at the particular time was shed. Earlier, by the distance it has travelled over your infinity that much time earlier, the vortex was shed earlier by the distance, it have travel by u infinity, that much time earlier the vortex was shed. It is not that whatever it shed immediately, it shedding it is close to that, so we have a continuous motion of the aero foil vortex keep on shedding, but they keep moving.

So, basically you find the relation between vortex at this location, any location x is related to the circulation on the aero foil at some other previous time and that is this. Now, you have a gamma w in terms of this immediately go back, but you can write the interesting part here is I want to non dimensionlized certain things, then write it two terms, because that is where we bring in certain quantities that is you can write.

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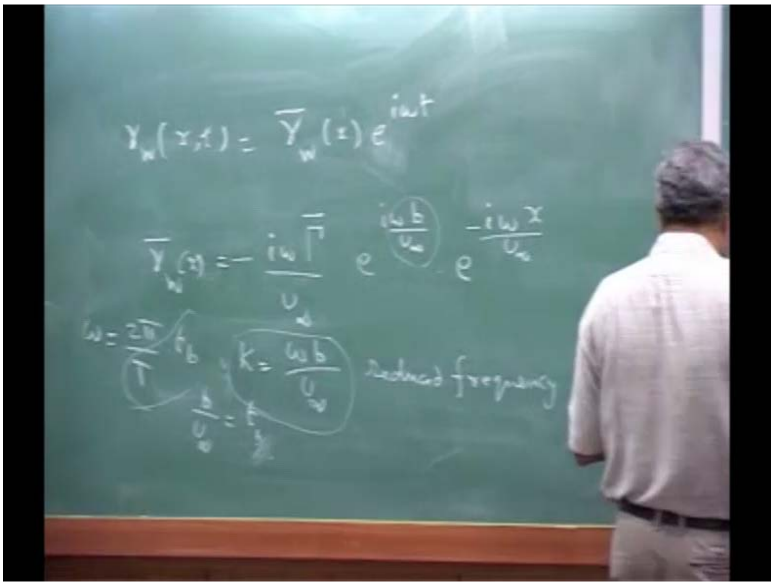

Now, we assume gamma w, which is the function of x of t t z gamma w bar t power i omega t, you can put that way also we have gamma w. Now, our gamma bar w you can write it as minus i omega, because this is gamma bar u infinity, because that circulation gamma a is the function of time. That is why that become gamma bar into e to the power you will have i omega b over infinity into e to the power minus i omega x over u.

Suppose, this is x it will change the symbol to some other running symbol only this symbol will change, now this one omega d by u infinity that is called the reduced frequency, which indirectly it means b over u infinity is what. The time it take for the vortex to travel a distance p, now that is time omega is the oscillation, so omega t in the b over u, how many card length, card means this is semi card, how many semi card length the vortex move in one osculation of this.

Whether, it moves much faster omega is very small very slow, this moves very slowly, omega is very fast, because omega t because this is nothing but the distance traveled by the vortex every air craft. So, this kind of represent that how it move behind the aero foil, technically you can take it otherwise you can combine both of them to see how much distance the wake as to go. How long, because the u infinity is very fast, because these are all sub sonic case these are subsonic incomplete, so I cannot put supersonic here.

That is why this kind of reduce frequency also represents, how much the wake has or the shed vertex has moved the time taken for it to move, behind that thing aero foil. And this is the non dimensional basically this is the number, they call it reduce frequency parameter, and this can vary depending on what is the different osculation of the aero foil, and oncoming flow. D fixed d the semi card of the aero foil, because you take the omega as 2 pi by capital T, which is nothing but what the time taken for one osculation.

Now, b over u infinity is time for one some time to travel b by 2, you put both of them you will see that this, product essentially gives you t times it go it becomes 2 pi 2 pi in one hour osculation, how much it has moved. Basically, because both of them are same that is over u infinity is nothing but capital T you will find that moved distance d, what is that 2 pi like that I am taking omega into t, because this you can relate into for time. It is a what is it omega t u pi, basically can you write in terms of distance itself, because this is the distance, this is the velocity and so this is the time.

The time taken for wake to move to the distance of this is not b, by this is the osculation in other words can be represent the k as a parameter representing, if I take this ratio t b by t, what does it really imply. The time takes to travel a distance d t divided by the osculation time of the aero foil, suppose if the osculation is very slow this will be, the t is very large, k will be very small; that means, the reduce frequencies is very small value.

We can make approximation later, when k is very small, it is almost that is where it call it ((Refer Time: 50:53)) study aero dynamic and k is very slow. I make the approximation suppose if I say my k is not small, then the values will be big, I need to take that because the wake is moving, when you see this is very large capital T. This is very large means the wake has not gone further, it is still closer to the aero foil, am I right, because this takes much larger d b is the time, it has taken for it go through some card, that is much smaller than this osculation.

So, the reduce frequency, when it is almost tending towards 0, 0 means this is infinity, this is whatever it may have; that means, this wake affects and really not bothered about that why you will find, slowly you will find the wake affect will go away. When I say k is, but even though, do not say my aero foil is not a osculating it is still osculating, but I make my approximation based on the fact my reduce frequency is very small. And later when we plant, because this is factor which comes on this, now you have the expression here let us go back and write the, I will write the final w that is now, am going to write this.

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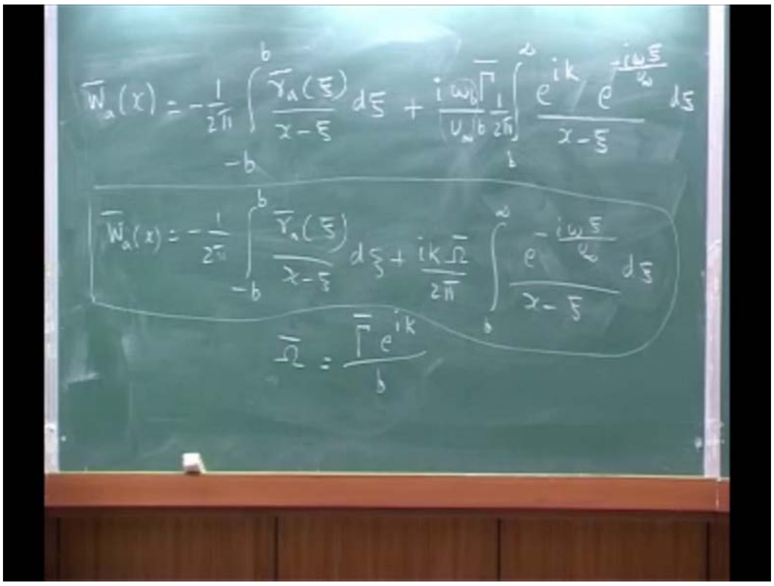

I am still in this, 1 over 2 pi minus b to plus b gamma a bar psi over x minus, this is gamma w, I am substituting this expression, now if I substitute gamma w that minus and minus that will become plus. So, i omega what I will do is I will multiply by a phi divide by a phi then what will happen, i omega gamma bar b infinity e to the power I this is k, but that is independent of it will come outside. And e to the power minus i omega psi over u infinity d over f minus, now acutely I am writing by another term I made a small approximation here, because let me simplify if I multiplied by a b and divide by a b that is going to become comma bar a x minus b sin this is omega d is i k.

So, I will write it has i k gamma bar over d, I am going to call it omega bar, there is there 2 pi is this, because 1 over 2 pi is there please take that over 2 pi what I will do is gamma bar e power i k over d is my omega bar. Now, this will be d 2 infinity e to the power minus i omega psi over u infinity d psi, now you see this is my equation, now what is done is I will just do one simplification here. Finally, I will go the results part, what is done is you keep this term on one side this, you keep and then this you bring this into another side.

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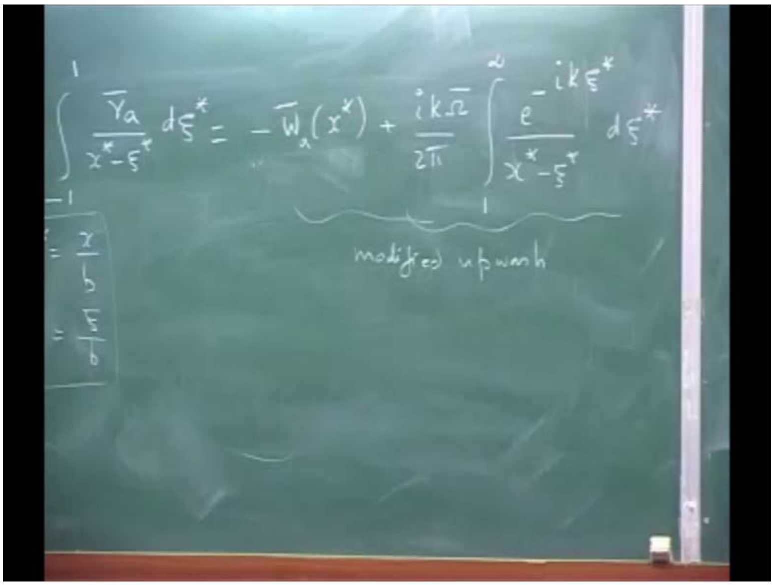

Now, your equation will be like this you just look back your notes, and then you will see, and I am non-dimensionalizing it with respect to x star is x over d psi star is psi. 1 over 2 pi this is minus 1 to plus 1 gamma bar a over x star minus x I star d, this is equal to because I am taking this term left side, and I am taking that side. This will be minus w bar a x star plus i k omega bar over 2 pi 1 to infinity, e power minus i, because I am going to use multiply by p divide by b, so this is will become i k psi star over x star minus psi star.

Now, if you look back your notes, what you will find is because of this motion, steady case if you consider steady case only these two term will be there in the steady case, in the unsteady case this is the additional term. And this is we call it as the, this is the original if it is the study case this is the up wash, then you have unsteady case this is the modified up wash.

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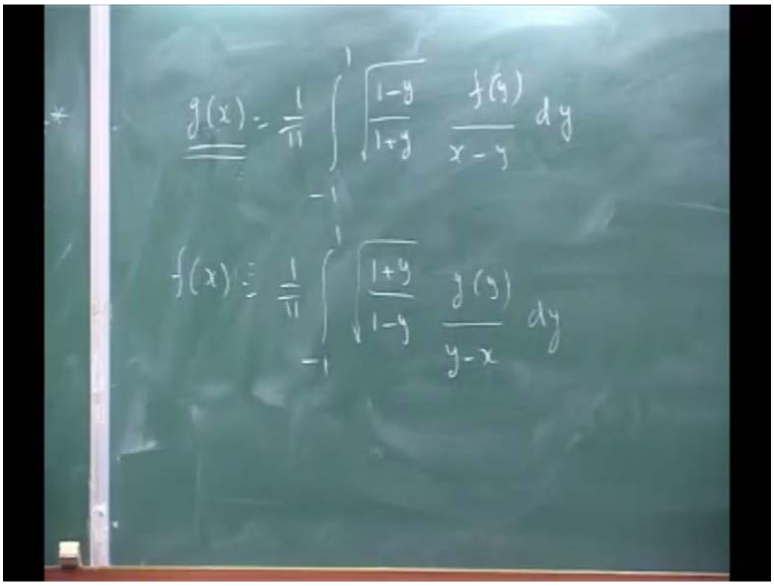

Now, this is the I told you last time we had this, now I erase all of them here, we had the transform inversion that is g of x is 1 over pi minus 1 plus 1, 1 minus y over 1 plus y f of y x minus y d y. This is written as f of x is 1 over pi minus 1 plus 1 1 minus pi g f y over y minus x, simply this inversion is used, this is the you take f of y, this is my g of x is this right hand side.

And my roots 1 minus y overall 1 plus y f y is gamma a, now what I will do is I will simply inverse, I will get gamma a which is this term in terms of this term. I will not get into the, I will do that path, I will write it you will get this expressions now onwards it is going to be really messy long expression, I will just write that expression and finally I will go to the happen.

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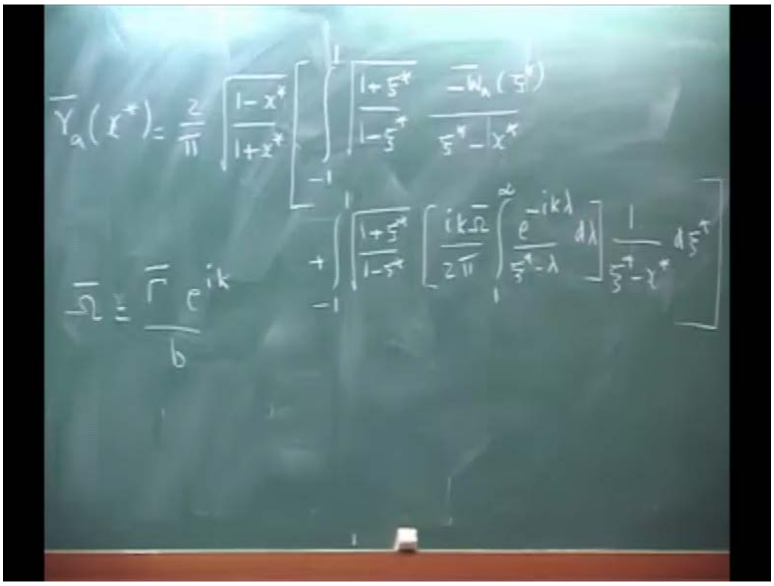

it is going to be very long expression gamma bar k x star 2 over phi 1 minus x star minus 1 over plus x star, open the bracket integral minus 1 to plus 1 1 plus psi star minus 1 minus star minus w a bar over x star. This is one term then you have another term, plus integral minus 1 to plus 1 and the root 1 plus psi star minus 1 minus psi star, and here you will have one more integral i k omega bar over 2 pi 1 to infinity e to the power minus i k lambda over psi star minus lambda d lambda 1 over psi star minus x star d x star.

And here you close the brackets, this is the full expression, see this term what is here is basically the second term 1 to infinity that is this term, this is w a, minus w a term, so rest of the term remain identical. Now, you need to get still gamma a omega bar is related to, again, because omega bar we wrote it as what gamma e power i k over some k b, so you have that relationship from here, finally you get the gamma a star. I will not go to the integration of this, because this integration takes lot of calculations several steps and I simply go to your final possibly I think I will go to your.

Student: What is lambda?

Lambda is just the change of the variable, because I have just to change, because it is one to infinity I just change the variable to lambda, thus ultimately you need to get the, because this is the lot of procedure is involved in between. I got my gamma a, now w a is known and you can use your pressure expression, because p u minus p l again, and what I will do is my aero foil expression I will write.

And finally, go back and write the different movement expression, otherwise there is again, because please remember there is lot of the inter mediate step involved, inter mediate steps you know what is the omega bar. Omega bar is gamma bar i k over b, this gamma bar is integral what gamma bar a d this is you can have one d will be there something, because you know that gamma is this expression.

So, this is related to gamma a; that means, gamma bar is gamma a related integrated value this is the individual gamma bar, so what you do is you take an integral of this equation again. And then you will finally, you get an expression for this capital gamma, and from there you go back capital this omega bar expression will become that will be again substituted here, that will be the in terms of the Henkel function, which are again in terms of lesser functions.

So, you find that there is the lot of intermediate steps, which will run through several pages, so what I thought was I will not get into the details of all this every step, because this is pure integration. I will directly go and give you the expression the for lift and the movement on the aero foil, which is given in terms of I will write the list of movement expression.

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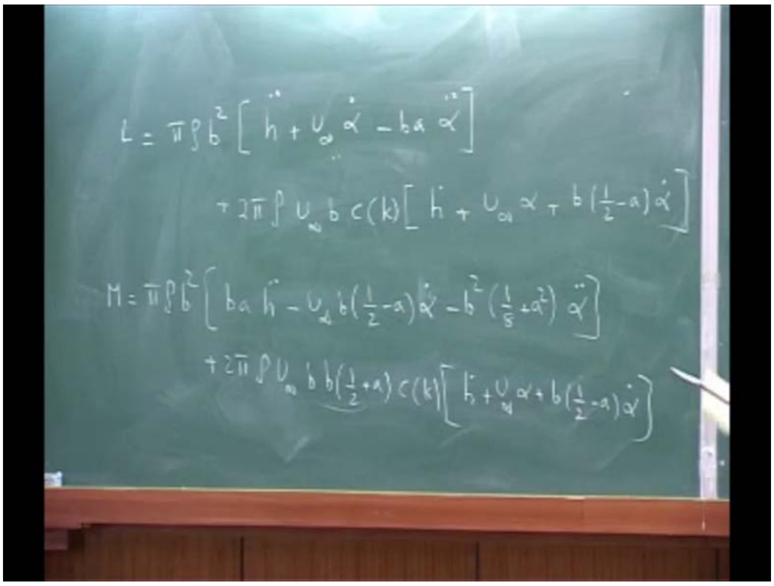

Finally, an osculating aero foil lift, which is at the quarter card point, I am writing the final expression h double dot u infinity alpha dot minus b a alpha double dot, plus 2 pi rho u infinity b c of k. I will mention what is c of k h star plus u infinity alpha plus b into half a alpha dot, you take this and this is the nodes of movement pi rho b square b a h double dot u infinity d half minus a alpha dot minus b square 1 over 8 plus a square alpha double dot plus 2 pi rho infinity d into b half plus a. That is b into b that is b square you say c of k times, h dot plus u infinity alpha plus b half minus a alpha dot.

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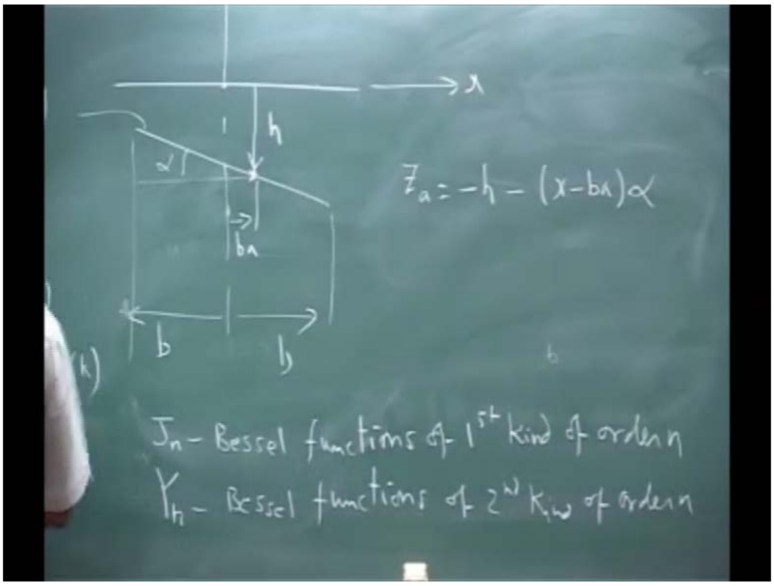

Now, I will give a triangle to this air foil which is like this, because this is my reference this is my z axis, this is my x axis my aero foil comes like this, this is b a and this is h, this is d. So, z a is minus h and this is alpha, x is measured from the mid card minus h minus x minus b a times alpha, so this is the air foil, this is the convention I am using, this is nose up movement pitching. This is the lifting, it tells please because the reason is this is given, there is a c of k and thus write that expression, and today will close it and again we will continue in the next class, where right here c of k is h 1 2 k over h 1 2 k plus i h 0 2 k, what is that h I have to explain to you what that h is I will give you, H n 2 is j n minus i y n. Now, j n are Bessel function of first kind of order n, and then y n are Bessel functions of second kind of order n, and h is also called Hankel functions of second kind order n that is this, this is c of k complex number, these are all purely based on integrals.