Aero Elasticity

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Lecture - 20

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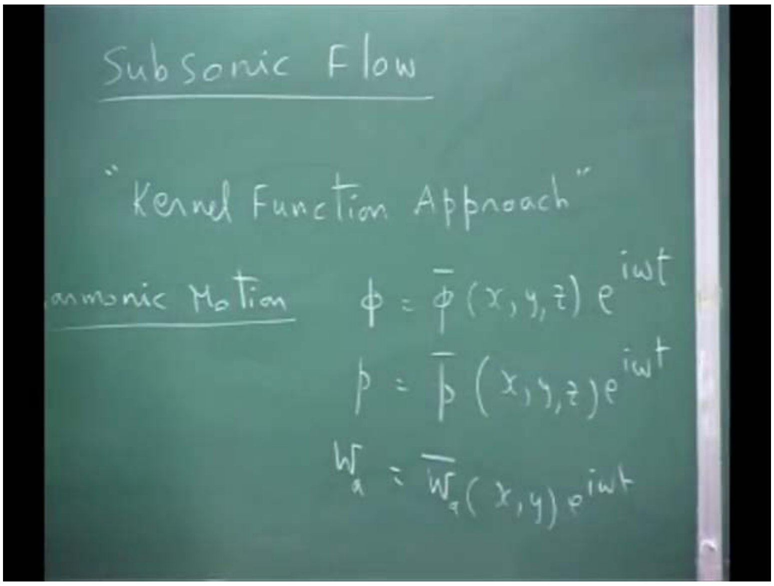

Today, we will take the subsonic flow and this is the first time, give me the derivation of the unsteady aerodynamics for a lifting surface in subsonic flow, but it will be purely mathematical. Later we will go for the incompressible 2 d theory, which is essential predation theory, because this formulation is a most general formulation and people were working on this since 1915 or even earlier. So, I will give some references which are 1955, 64, 66, various types of texts and you will know even today people publish articles on the approach, how to get the surface pressure on the oscillating lifting surface.

Now, let us take because in the subsonic flow, the key is the flow gets influenced by the motion of the wing, everywhere it is not like a supersonic flow where that disturbance does not propagate forward, here it propagates all over the place. So, what we will do is I will describe by a approach, which is basically by transform method which will lead to smothering called Kernel function approach. This is similar to what we did for supersonic flow identical procedures that is why you will find whether supersonic 3 d also can be solve by similar approach.

I will not get the full solution, I will get the final expression for the solution, but there are several integrals which will come in between which lead to the evaluator, but how they start this full process is we assume harmonic motion this is the key. Harmonic motion is assumed, so you have your velocity potential which is d bar x, y, g, e for i omega t. Similarly, the pressure is the disturbance pressure again it is e, x, y, g, e for i omega t and then you take induce or the delta phi by delta g at g equal to 0. This is the velocity which you will write it as will bar a x, y e for i omega t, because this is the j e d component of the velocity.

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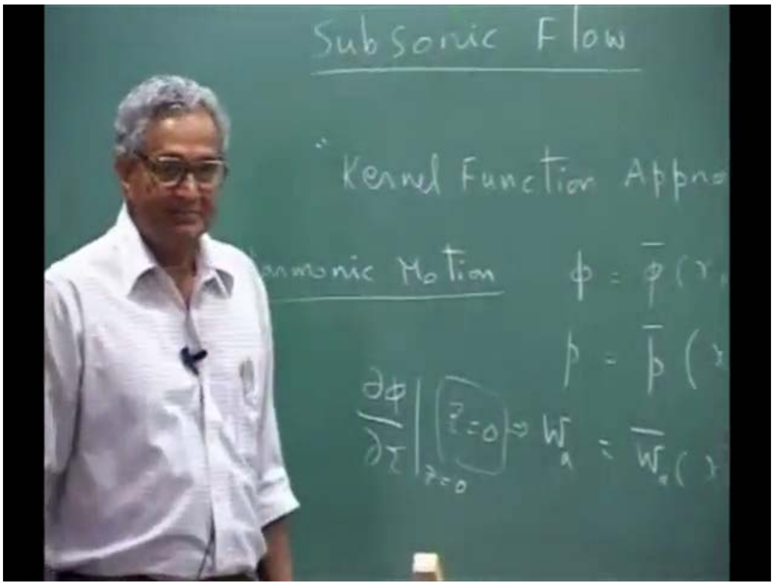

In the plane, z is equal to 0 plane delta phi by delta g, so if you want to write this, you can write it delta g at g is equal to 0.

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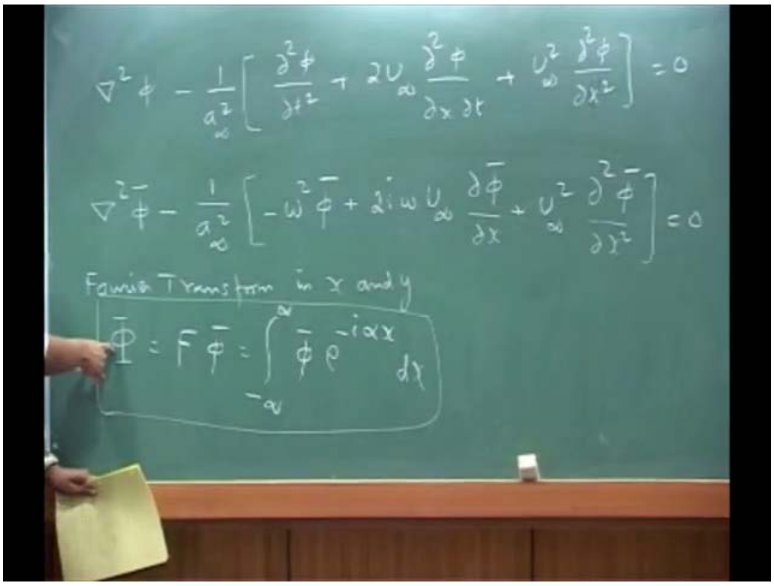

Now, using this you directly got your linearized potential equation, which is you look back your note, we are derive this part del square phi by delta t square plus 2 u infinity delta x plus delta. This is our velocity potential small disturbance linearized equation substitute the phi and then cancel out e power i omega t from all the terms. You will have del square phi minus sorry phi bar 1 over a infinity square minus omega square phi bar plus 2 i omega u infinity delta phi bar by delta x plus u infinity square del square phi bar by delta x.

That means time is taken out of this unsteady equation, it is written in omega, now what you do is you apply because the disturbance goes from flow in this is the partial differential equation. You do transform, but you apply fumier transform, whereas in the supersonic, you apply the plus transform along the x and if you are a 2 d problem in sense if you have a finite wing, you will also take another transform, which is the Fourier transform.

So, here since the flow gets affected everywhere, you do there Fourier transform and x and y. So, we will write that actually a first, if you do this is phi capital phi, this is a Fourier of phi bar minus infinity plus infinity phi bar e power minus alpha i alpha x t. This is the Fourier transform, but this will be actual y and z will be there and of course it will be alpha dependent, but then y and z will be there. Then, you do another transform with respect to y, and then you will call it as phi star, I will write this. Now, when you do this, but please note that integral minus infinity plus infinity delta phi bar over delta x e power minus i, this is the first that you wrote up the function.

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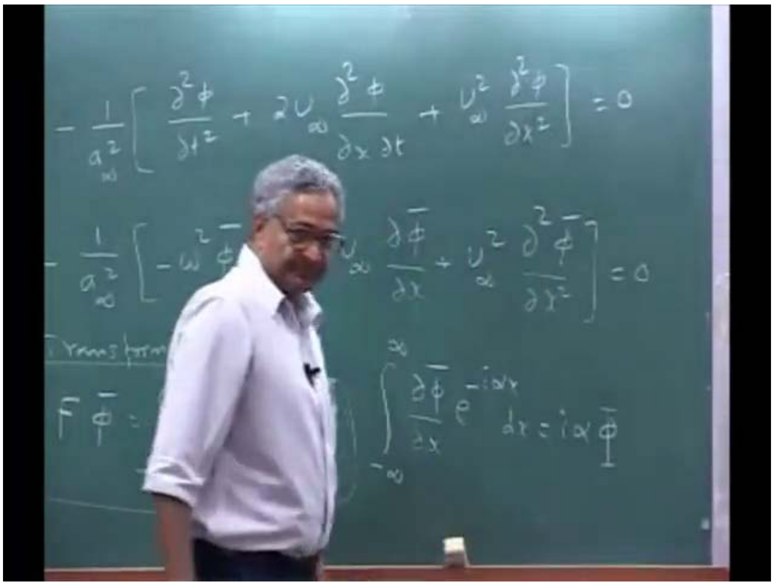

This is the function, this is the function, this will be i alpha phi.

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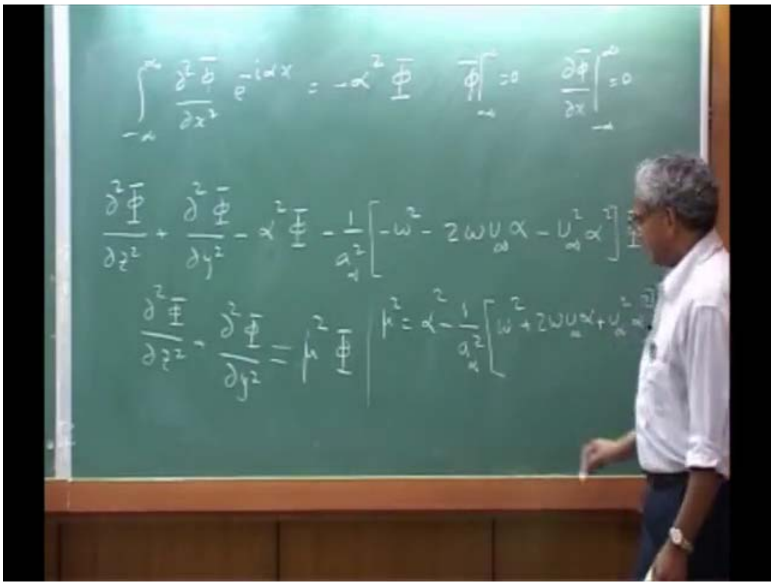

If you take the second that you wrote you that is equals minus alpha square and you with the condition phi bar 0 and delta phi bar by delta x at this is also 0 because what we saying is the disturbance dies out at infinity. The gradient of the disturbance also dies out that means basically the initial condition kind of thing, but now using this you change this equation. When you do this you will have del square phi bar, sorry not phi delta g square del square phi by delta y square minus alpha square phi.

This is the transform of the del square delta x square of phi bar minus 1 over a infinity square minus omega square minus 2 because this will become i alpha. So, I will become minus 1, so you will have 2 omega u infinity alpha minus u infinity square alpha square phi. Now, this entire term what you will do is you will come these two and you will call that as some mu square.

So, you will be writing like this del square phi over delta g square del square phi over delta y square equals some mu square phi mu is this entire expression this is function of part. That is mu square is essentially alpha square minus 1 over a infinity square omega square plus 2 omega u infinity alpha plus u infinity square alpha square, this is all. Now, what you do is you do one more transform here another Fourier transform with respect to y, then you write back instead of alpha you use gamma.

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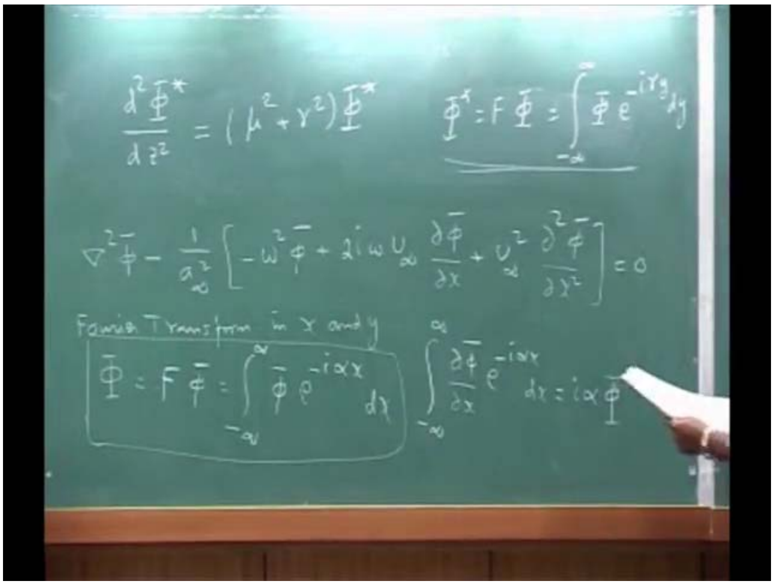

I write that part here and then you will be left with one more transform and that transform you are writing it as phi star, because when you do this is a second term. Similarly, i gamma y will become minus gamma square phi, so I will put it as mu square plus gamma square, where phi star is again a Fourier of this phi, which is e for minus i gamma y phi y, this is the now this is one Fourier that is another Fourier.

Now, if you write the general expression, because this is not necessary, now you will put is as this is also, because this is nothing but this part. So, I am putting phi bar, this is my velocity potential e to the power minus i alpha x minus i gamma y e x y. This is my

Fourier transform, what I apply two step is actually at one step, you can directly get this form.

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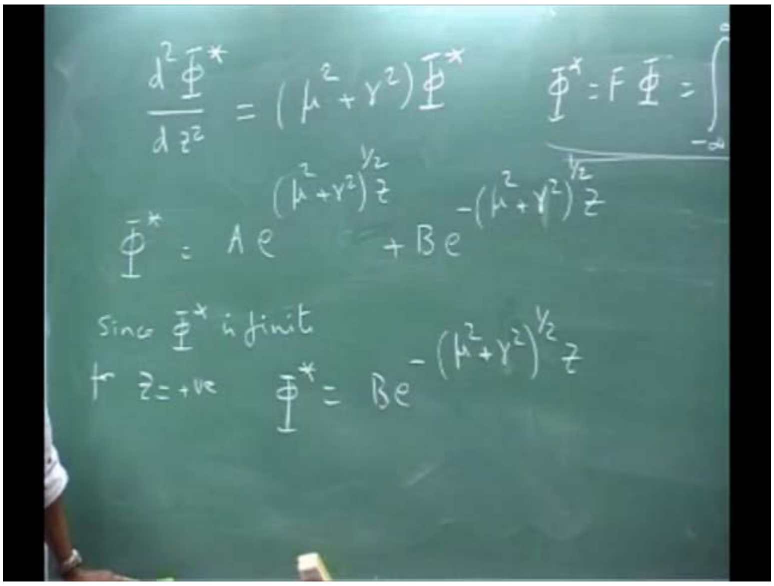

Now, you go and substitute your boundary condition after obtaining the solution, because this, now ordinary differential equation you can get phi star this will be a e to the power mu square plus gamma square g plus b e to the power minus power up. Now, you impose the condition that for positive g phi cannot increase exponentially that means this term is 0 for g plus for g minus that term is this. So, you get that basically phi is for positive, you write this solution for g negative, you write this solution because phi has to die out as you go far.

Otherwise, with those positive that means this will blow up, so you write since phi is finite so for g positive, you will have phi star is b into e to the power minus mu square plus not g square gamma square power half for z first. Now, you apply the boundary condition with the positive g plus.

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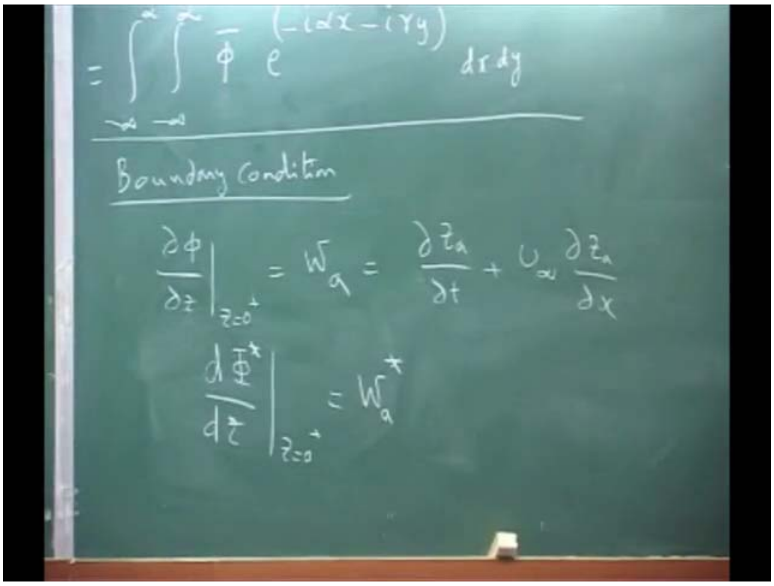

Now, let us look at the boundary condition, you will have a the boundary condition is on the aerofoil W a that is on the lifting surface not aerofoil on the lifting surface W a should be equal to what? This we have delta phi by delta g at g 0 plus minus whatever take it this is W a, which is delta g a over delta t plus u infinity. This is the motion of the surface g a can be a function of x and y please understand, but what we will do is we will not worry about this part.

We will only take this is my motion of the lifting surface, take it later, we can once you know the motion you can always substitute here that is not a problem. So, let us look at this you do the transform again because this is in original because p is a function of x y. So, you have to do the Fourier transform when you do the Fourier transform of this, you have to do the Fourier transform of this also, you will get your boundary condition d phi star t g at g 0 plus minus this is W a star.

Now, what you do is this is the transform the boundary condition zero plus substitute; here you can take a derivative of this. Then, put it back this will be with respect to g, you will get a minus mu square gamma square half into b and then W a k. So, you will get substituting this here, you will basically get what you will get because if I differentiate this, this will be phi star minus this entire term.

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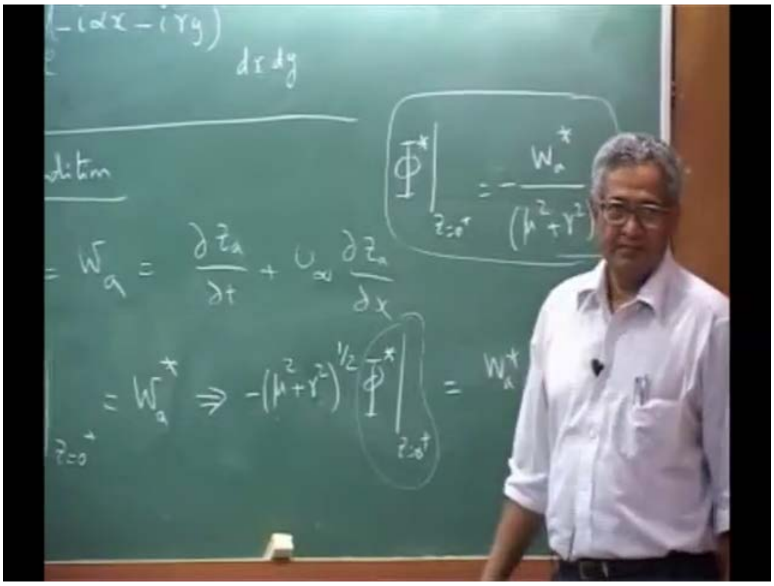

That is I will get minus of mu square plus g square over, sorry mu square plus gamma square gamma square half phi star hat this is equal to W a star. In another words, this is given by W a star over this with a minus sign, so I can write phi star at g equal to 0 plus this over 0 plus this is basically what is b. That is all you can get the b value, now the 4 pi is this problem, how do you resolve, you go and solve or this is a velocity potential phi transform velocity potential you directly go for pressure. Now, the disturbance pressure I will write the expression this side, I will erase it here.

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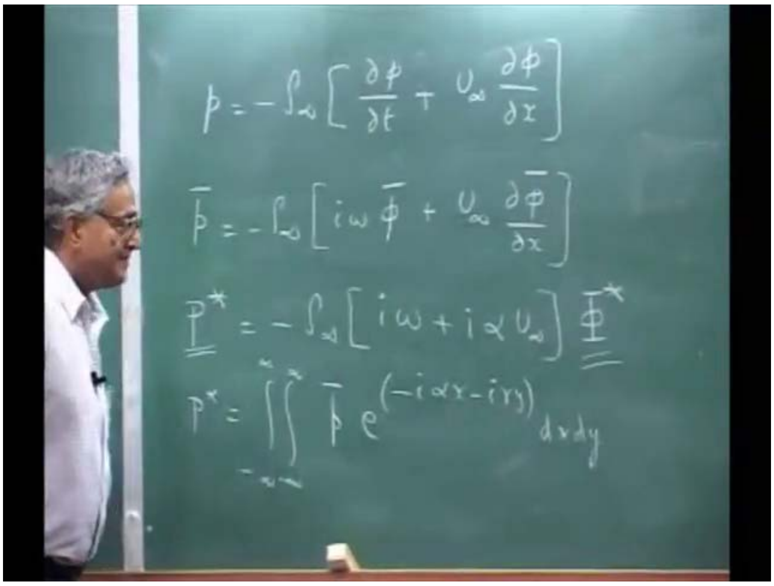

The disturbance pressure is given by minus rho infinity, if you look at your note back you will get this is your disturbance pressure. You do substitute harmonic, you will get phi bar minus rho infinity, this is i omega phi bar plus u infinity delta phi bar over delta x. This is a pressure disturbance pressure, now you again do a Fourier transform when you do the Fourier transform of phi bar that is going to become phi star p you write it as p star.

So, you do p star that is i am doing Fourier transform by minus rho infinity, I will have i omega plus i alpha and your p star is p bar e to the power minus i alpha x i gamma y d x d y. Now, you see I have a relation between pressure and p star this is what i will introduce. The keys we are interested in what is happen on the z equal to 0 plane that is where our lifting surface is going thicker, now z equal to 0 plane, you know this quantity, so I can directly go there z equal to 0 plane 0 plus I will get 0 minus I will get the minus that. Then, I will actually my lift is 2 times the value, I will get it essentially, we do is you substitute p z equal to 0 plus, then this will become z equal to 0 plus this is written in terms of W a which you called it as the up wash.

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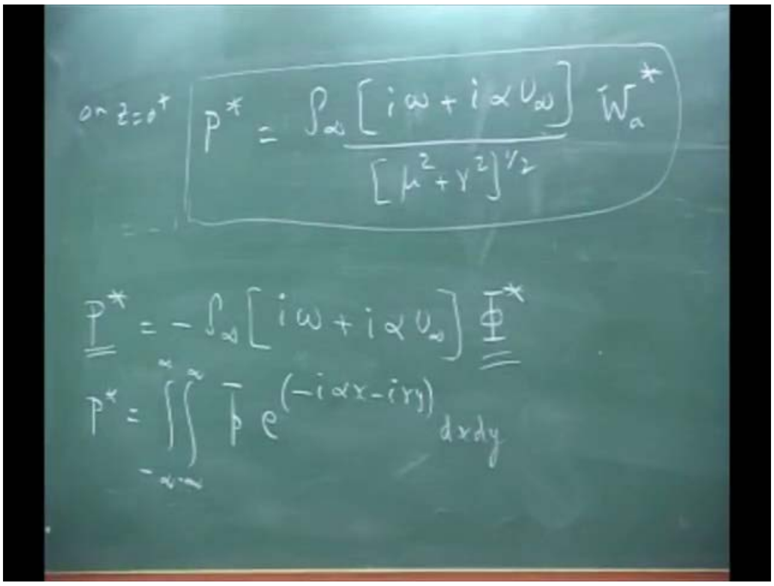

Now, let us write that part on z equal to 0 plus I erase this part, you will say on p star is this is minus rho infinity. You substitute this term you will get rho infinity i omega plus i alpha u infinity over mu square plus gamma square over half. I can actually do some small changes here and write it in a different from, what I will do is I will write W a in terms of pressure.

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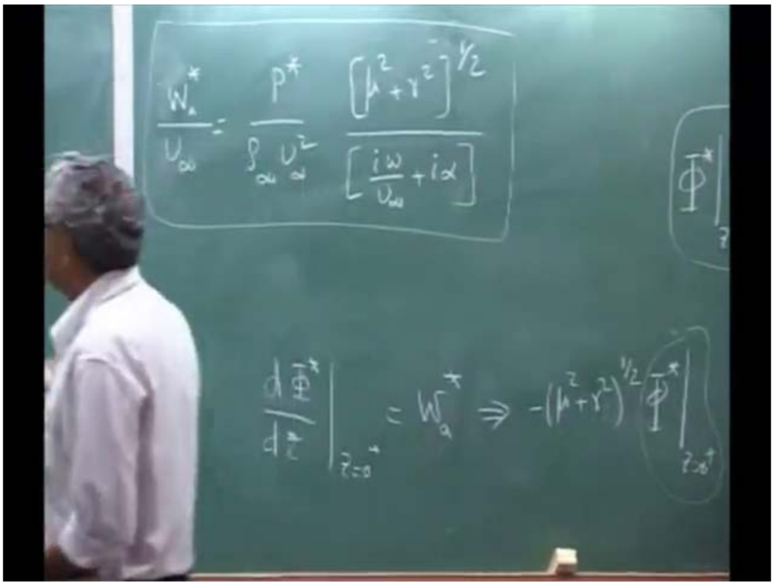

That means I am going to put p, sorry W a star over u infinity is p rho infinity u infinity square mu square plus gamma square over half over i omega over u infinity plus i up. Now, this is my in that transform doming, this is the relationship, I go back and do the inverse transform, inverse transform is what I will write here, now the inverse transform.

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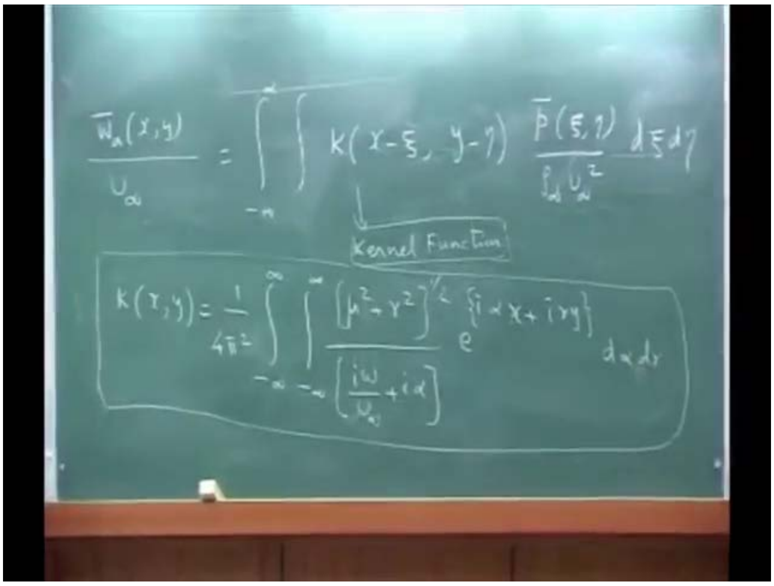

This is given by W bar a which is now a function of x and y because this is z equal to 0 plane that is inverse this is this also minus infinity, but the book they write it like this. You will have k x minus sorry y minus eta p bar over rho infinity u infinity square t p this is because I am doing inverse transform, I did two first in the sense Fourier transform in x and y. Now, do the inverse transform this will be, now this looks like this particular, this is called the Kernel.

This is the kernel function, what it does say is I write the expression for that please note this is the convolution, but integral is over the full domain minus infinity. That means if I have a pressure over this area only, this area p eta from the one location, what is the W that is the up wash at x y this k gives you that wash. This is like essentially it is like a if you have a unit pressure pulse, this is the up wash that function and actually this is given by k.

I am given only x and y this is the inverse this is think four pi square minus infinity plus infinity mu square plus gamma square over half over i omega over u infinity plus i alpha exponential e power i alpha x plus i gamma y d alpha d r this is my Kernel function. Now you see basically the inverse of this term, this is the kernel function once you get you have to put x minus psi y minus eta to get this integral. If you look at this we said earlier the boundary condition when we develop the disturbance pressure away from the lifting body is 0.

That means this minus infinity integral you do not do over the full doming what is require is this integral please understand this particular integral is evaluated only on the lifting surface because p bar disturbance pressure outside the lifting surface 0. Therefore, usually people do not put infinity, they will just put s that means this is only over the lifting surface.

Now, I am interested only in the lifting surface, I am not interested in other places and this W a is nothing, but what is the velocity of t lifting surface. That is basically the motion, but this integral at this integral and this integral evaluation has to be done. In addition there is also one more condition which you do not do in supersonic flow.

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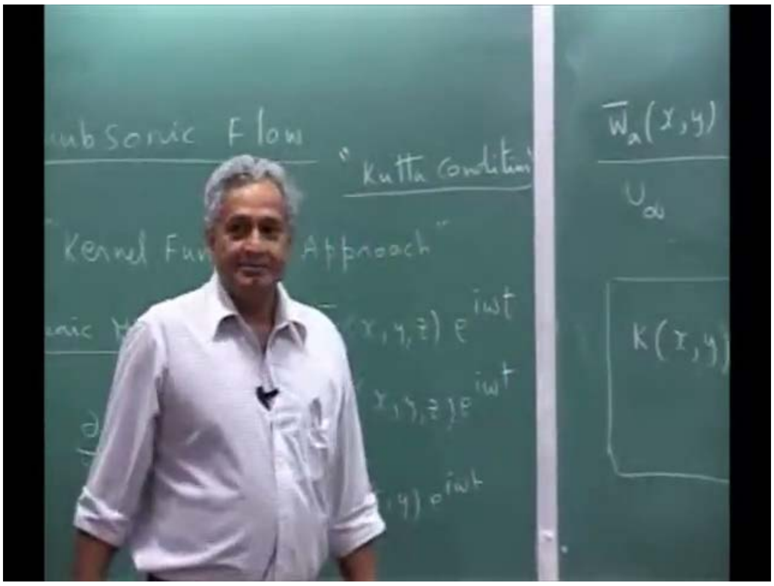

In the subsonic flow we introduce that condition that is the Kutta condition. The trailing each you say my disturbance pressure is 0, but you can say even leading hedge, but then that will not give me correct result. Usually, this experimental observation, so you said my thrilling hedge that means that pressure thrilling hedge is my x variable is x i, so wherever is trailing edge there the disturbance pressure is 0.

Now, how do I really solve this problem, this is where the whole thing is because you got this whole. Now, I will just modify a little bit here, this integral as you have see evaluation of this integral is one problem. There were several apportion on this efficient method of heading this kernel function that is why I said since I am not working this particular completely this formulation, but you have several research publications on the Kernel function approach.

Now, assuming that you know this because you have to do numerically because there is no postpone solution, but people got certain expression for this, I have give publication, I will write that reference. You have several even is there 2003, you have seen publication efficient method of calculating the kernel function approach for a lifting surface.

They have in their original thing the lifting surface also non planer that means the wing will bend like this. So, it is not a planer jumps so they took the variable jumps that non planer lifting surface also, but essentially these are the equation, but how they solve is this integral. You can write it as a summation over small elemental area you say that this pressure is constant over small elemental area. This is over the entire area of your lifting surface, so you put it as a W summation and then I will take that.

You can make some approximations, because we know that the pressure has to satisfy the Kutta condition, so I will choose certain type of functions which will try to satisfy my Kutta condition and then get the unknown co efficient. I will just briefly mention this part, because this is where the key thing is that is, suppose I am just using, I erase this part, now you got that all essence issue only thing is your supersonic instead of minus infinity plus infinity, it will become a convolution for the modification that is why, so that is why even in the supersonic flow this approach is called the Mach box.

It is supersonic 3 d planer, because it is just an integration only thing is the kernel function will be different one will be plus another one will be Fourier. So, this warm method, but things you in the very neat mathematical form, but only question is how do you get that. It is very allegiant, the approach is very allegiant, but the getting the solution is the integration, this is where lot of boat has gone up the past several year because each company has his phone code in the some of these aero elastic codes, which you may find. They will have that approach not just see I am not talking see of or not this is purely potential flow, there which is used extensively in the aircraft industry.

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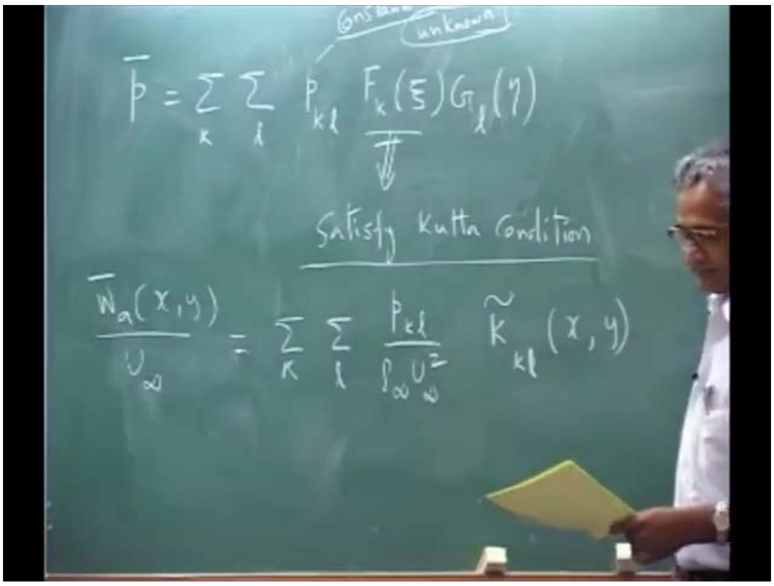

Now, let us write the pressure has because we know that just to satisfy the so i will put it as these are constant. I am writing my pressure as a function of two functions, this function is along the curd this function is along the sand, now what you do is since it has to satisfy the Kutta condition, I will choose those functions for which the trailing edge f k is 0. The function f k when is go trailing, its value is 0 that means automatically I am satisfying my Kutta condition. Now, what you do is you substitute because this function satisfies this satisfies all this f k's satisfy Kutta condition, but these are constant, but they are unknowns.

Now, this is only over the surface I am interested only on the surface, because I know this only on the surface of the wing. I do not know W a outside the surface, I know in only what is my boundary velocity W a on the surface, this is directly given in terms my body motion. So, what I will do is I substitute that term here and then write it in this passion here, this is W bar a x comma y over u infinity is summation k summation l. I have putting it here and then I am integrating the rest of the things, this will be p k l over rho infinity u infinity square some k x, now I will write this expressions here I put it here.

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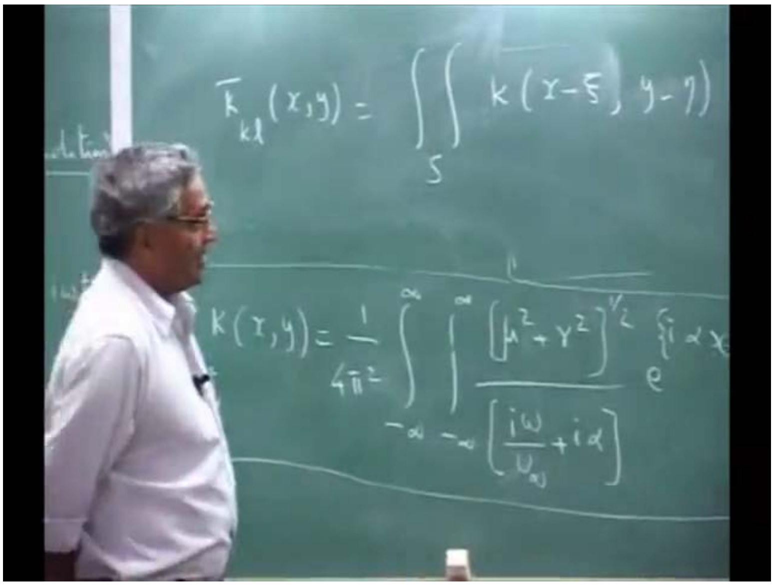

That is k bar k l x comma y is nothing but this is over the surface k x minus delta minus y minus eta f k psi and g l eta d psi t.

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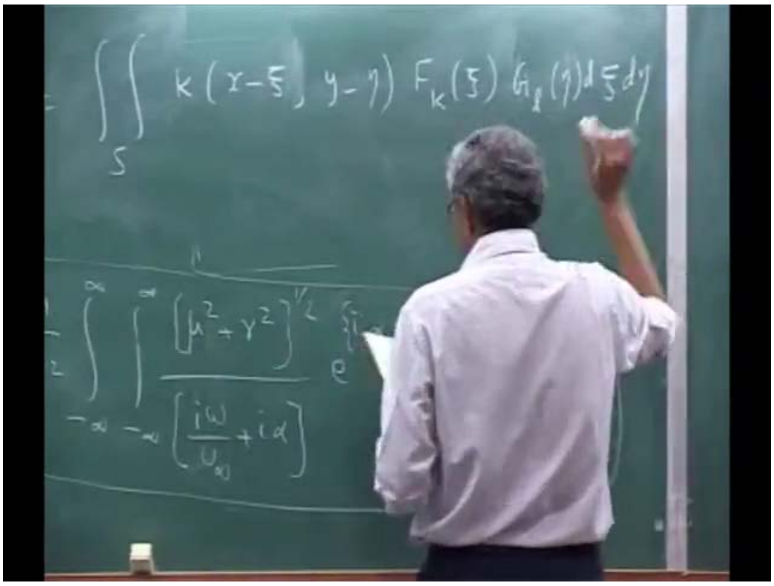

So, I just need to evaluate this integral that this surface integral over the wing surface. Now, you see what I do is I have here so many unknowns, what I will do is I will just pick as many unknowns, I have I will pick those points only that many points of W a, how many unknowns, I will pick only that many points. Then, I will have because x y is any point of on the lifting surface, so I will pick that many points on the surface for which I have the number of unknowns, so I will put in a forms this, I can write it as I erase this part.

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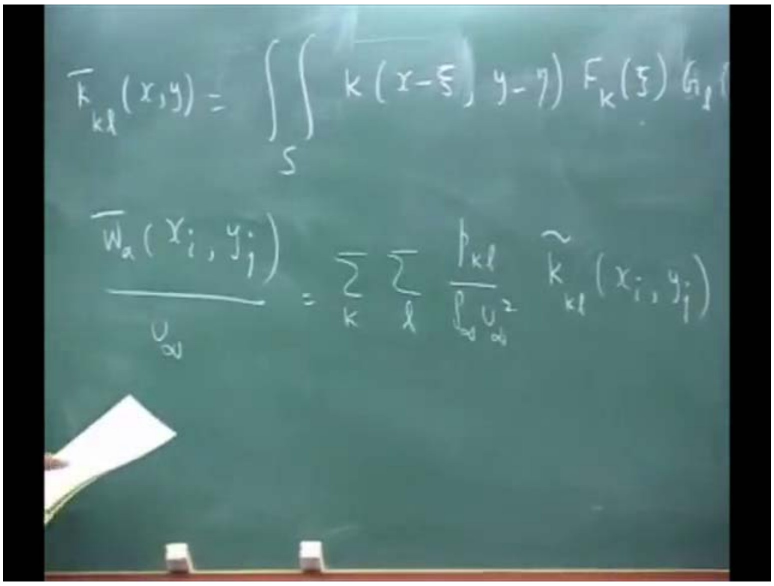

You will have W bar a x i comma y j, this is the specific location u infinity s summation k summation l p k l over rho infinity u infinity square this is x i comma y j.

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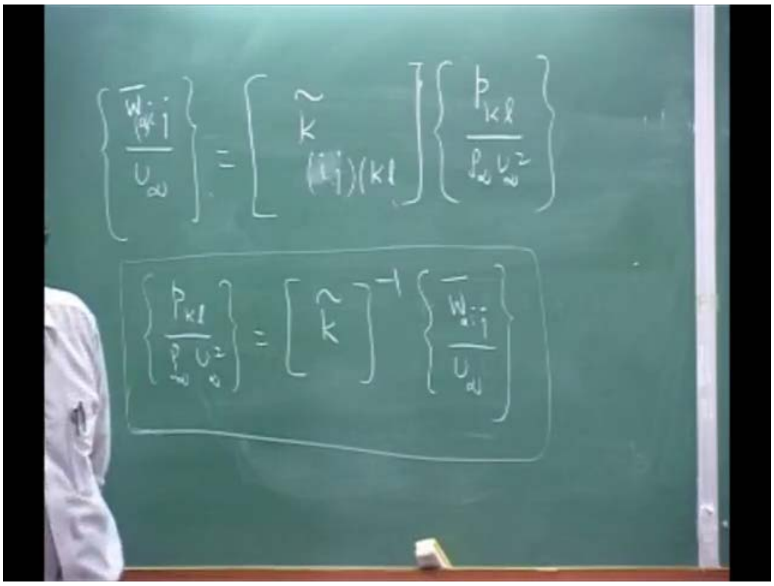

Now, I called this as put in the matrix form, I erase this part, so we have the put in the matrix form is equal to k l, sorry i j k l i can have i j and k l, because this is the k l function I am using at i j point multiplied by p k l. If you want to inverted, you can take directly pressure in terms of body motion that means you need to only this. You can say this kernel function that is all, so you inverted you can also write p at any point rho infinity u infinity square, this is k hat inverse and you can write W bar i j, sorry k i put a k i here, let me I am sorry about k over u.

Now, this is my and this approach, you called the kernel function approach, but there of lot of questions one can ask how many points that you should choose what is the function. I must have how the optimum is, how we like it this is why my whole several studies have gone and this is very similar to the supersonic mach box method, because supersonic mach box also is identical.

So, you see how in the actual wing problem lifting surface problem, we will see what is done you know the motion of the wing in every mode please understand every mode. That means first mode second mode third mode etcetera that means I know the shape. So, I know this and I can get what should be the pressure depending on what is the frequency it is oscillating, so that part come later because frequency per meter 6 year, because we put e for i omega t what is the two omega you follow.

So, the latter part of the calculation is different, please understand this is purely getting the pressure on the surface given the motion later comes, how we calculate because once we know the pressure able to top surface bottom surface. You have a difference with the pressure which is the same because of the lifting and you can get the lift, you can get the moment. So, you will say if my wing is oscillating with this particular frequency in this particular mode shape, I will have this integrate this pressure, then you integrated over the whole mean you get the generalized motion.

These are all complex numbers because there is an i u everything is sitting and that is where the entire research, I would say there are several specialist groups specialist group stay work on this. If they are working on subsonic flow, they keep on working on subsonic flow and the NASA there funded the lot of research on this and then people developed the codes. I will give some of the few references because which you will because these are old once see one, I write given reference some few four references, I will just write because there were plenty.

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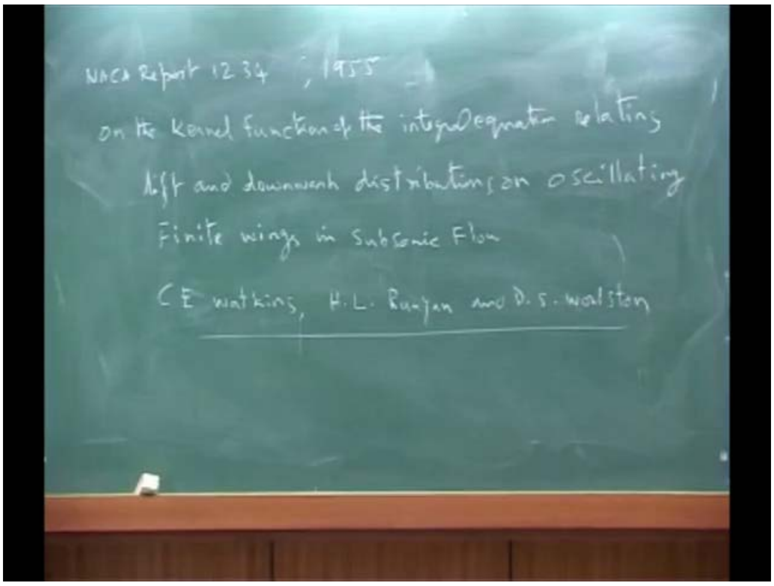

This is NASA report 1, 2, 3, 4, this is on the kernel function of the integral equation relating lift an down wash lift and downwash distributions on oscillating wings, because the oscillating finite wings, finite wings in subsonic flow. This is by c e watching and then H L Ruayan and that D S Woulston, this is one reference and this was actually I would to say one minute I will tell, this is 1955.

Then, what happen is another 1967, then 1966, I give 66, 67, 69 that sequence, so you will have an idea this is 55, then see that is why it is a pressure to downwash relation that is all kernel function of project. Suppose, you look at it one has to really solve this problem, develop their own codes and then come up with that, because people have gone through lot of efforts.

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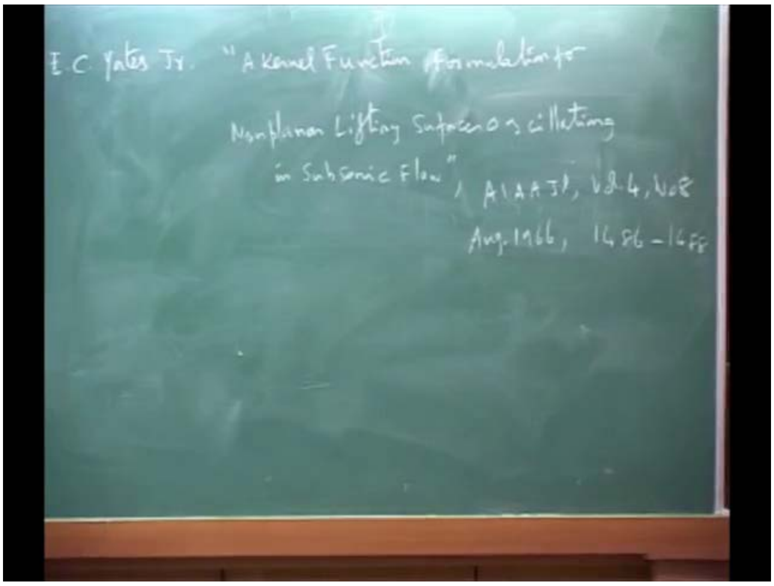

This is another one by kernel function; this is actually E C Yates unit gates, this is a kernel function approximation, sorry kernel function formulation for non planer lifting surfaces oscillating in subsonic flow. This is I think journal AIAA volume 4 number 8 19 august 1966 and page number is a 1486 to 1489, I think 1488, this is 1966.

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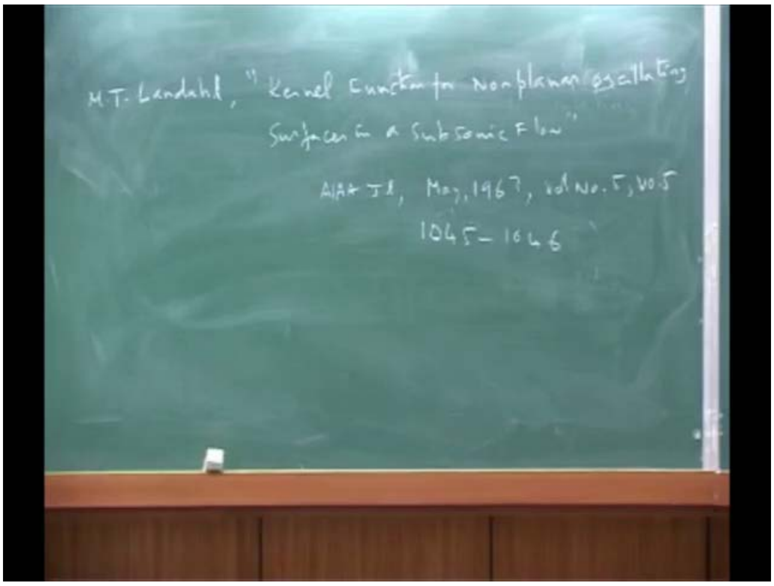

I will just give you few references, then finally, write 69, one paper came and this is another one sixty seven this is M T Landahal, this is again kernel function for non-planer oscillating surfaces in subsonic in a subsonic flow. This is again AIAA journal, I think may 1967 and number volume 5, number 5 page number is what is 1045 to 1046. Then came another paper, this is the one of the, because evolution of this was done using its called doublet lattice, because some similar those appear in all this things. What he did was he has a pressure and W, you have to get pressure and w, you do not do at the same location, you keep the pressure at quarter cart what three-fourth cart.

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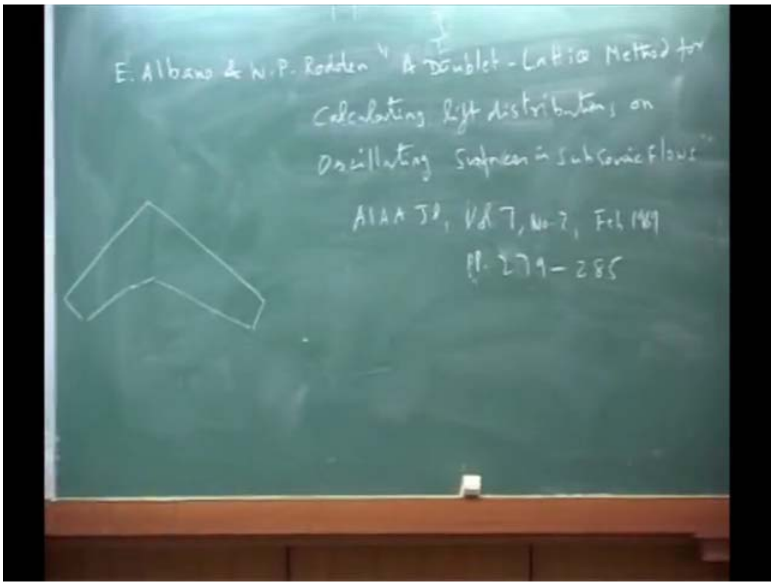

In the sense what he did was I will just write this, this is Albano and W P Rodden, this is called doublet lattice method for calculating lift distribution on oscillating surfaces in subsonic flows. This is also AIAA journal, which is volume 7, number 2, February 1969, this page number 279 to this is a 285.

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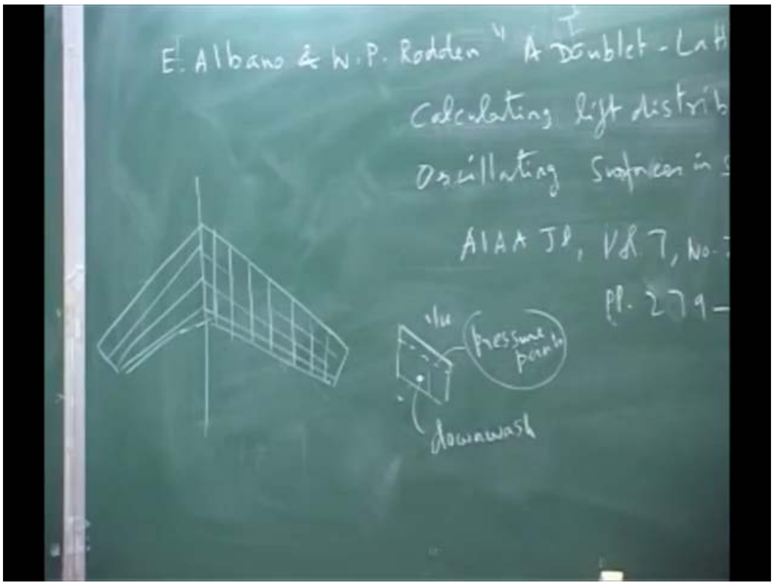

What is the doublet lattice, what he did was he took the wing, any wing this is divided into like that lot of box and in each box, this is the downwash this is the line of doublets is that the pressure points. This is the pressure points, you may call it dublet or because doublet is possible negative pressure, now this is the quarter cart of this box. So, you can say you placed the pressure point that the quarter span of this not spans quarter of this along the cart. You get the down water three force and using this he calculated for minor twinge correlate with experimental data everything and that is very good correlation and actually this technique is used that in the industry of course.

Today, of course because these are these are all plate and this is what he used even in a any industrial that is a all the commercial wing analysis, this is what it is. So, it is clear because this approach is used actually even today research is going on in improving the correlation in evaluating this. You know efficient approaches are modify method of evaluating this various integrals and that is the area of research because this is subsonic you can go transonic in any level.

Of course, it is only linearized potential flow that is why now transonic field they will say let me solve the non linear equation and then solve the linear, so up to compressible thing, this approach is very good. If you used the till transonic aerodynamic is a problem, but that is again another group of researchers are working out there. So, you find supersonic flow is a easiest flow subsonic is more complicated, and essentially this is the relationship which you get, because as a mathematical, the approach is very allegiant.

That is why I prefer doing it in the course at least to explosion, because it is not I have done research in this field to really say that this is what he done because each group, they work on a specific problem in the aero elastic. This is very allegiant approach and there are several publications if you are interested to take it, another problem, and then you should expose to that. This is the kind of work he has done in evaluating the pressure on a relating wings, because next class what we will do is we will take a 2 d problem in compulsively.

There also you will find all of this transformation, we will use a similar one and again it will be lot of integrals in definite integral everything, but we will get a flows form solution. So, from next lecture on what that is in Wednesday, what we will have subsonic, but in compulsive flow, so that second term your equation will be only del square phi equal to 0 or plus equation. So, that is the in compulsive flow for which we have a close form solution.