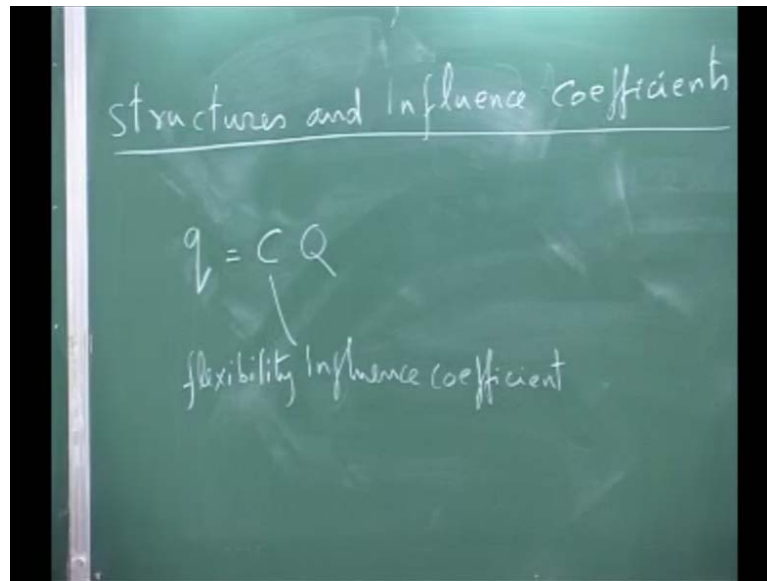


Aero Elasticity
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Lecture – 2

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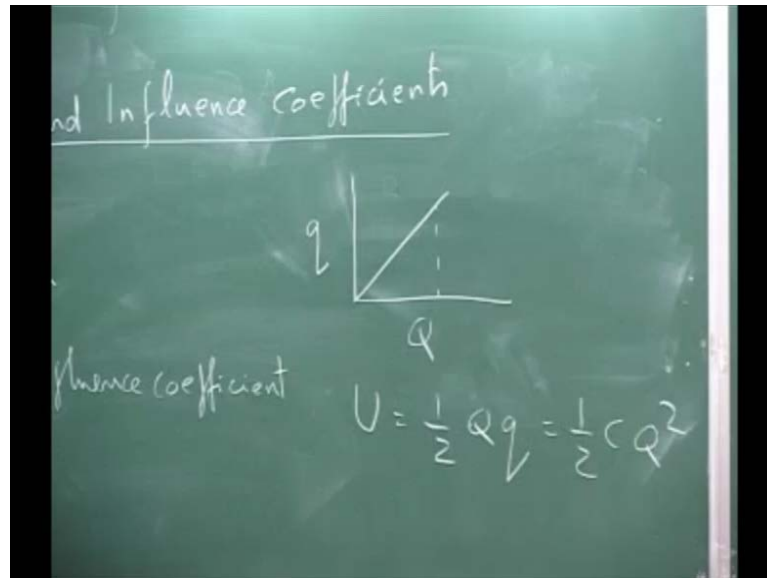
Today, what we will do is the analyses of this structural model. Essentially we will do the deformation of structures and influence coefficients, because we have said that aero elastic problem consist of structural modeling, aero dynamic modeling, inertia modeling. So, the first thing is we will start with the structural modeling and just for the sake of understating you say candle liver beam and this is a highly approximate idolization of a wing you can take it. If you put a load somewhere, that is going to deflect.

Now, assuming linearity we will say load verse deflection, Q is the load and you say that this lower case q is the deflection at the point of application of the load. The load can be a force it can be a moment, if you write linear relation you will have deflection is some constant times q load; you call this as the influence coefficient, because this sometime also refer us flexibility influence coefficient.

Because, everything is within the elastic limit I apply the load it will forms, I remove the loads, it go back to the original stay. And now if you want to know what is the work

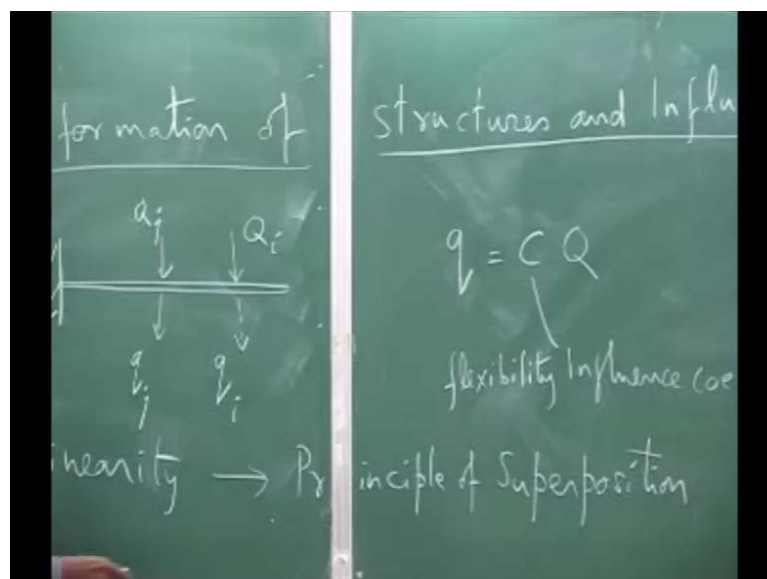
done, the work done by the load essentially goes and stays as the energy stored in the structure.

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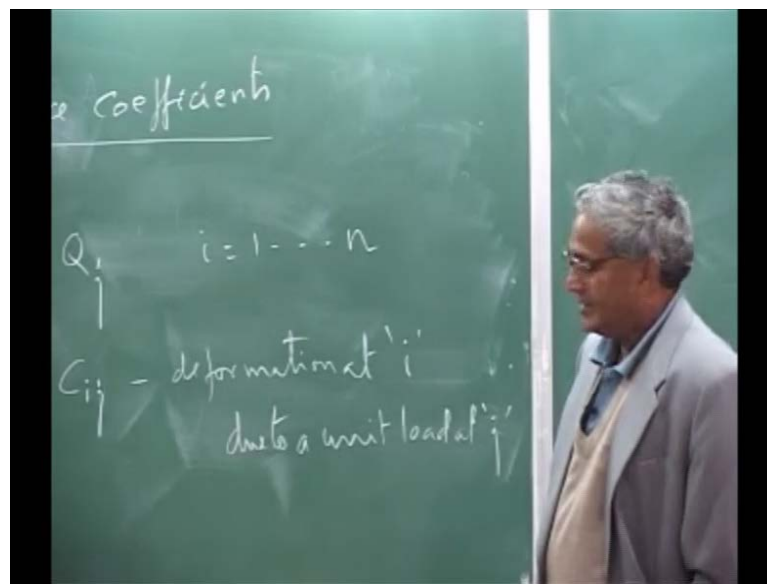
So, that you will write it as because you are writing this like this, so you will have deformation verses Q , it will be like a straight line. Now, the energy which you call it energy stored in the structure you will write it as half $Q q$ that is all any way this is the strain energy you call it. You can substitute for this and you will write it as half c square this is one way of writing the whole thing. Now this is for simple singular very basic thing.

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Now, if you have more number of loads, it is not just only one you may put q_i , you may have another one like q_j like that, they have several load. You assume because it is the very first thing is you assume it is linear problem, linearity is assumed the loading is within the elastic limit. Therefore, and it is an linear strain, type of load deflection curve, then you can directly go ahead and write principle of super position is applicable, so you say linearity means, I apply principle of super position it is applicable.

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Then you can write your deflection in a submission form q_i is the deflection at point i due to q_j , j running from 1 to n you may have n loads. Now i also run from 1 to n locations that means, c_{ij} now becomes, you call it influence coefficient matrix because this is like deformation at i due to a unit load at j and you are just summing it out.

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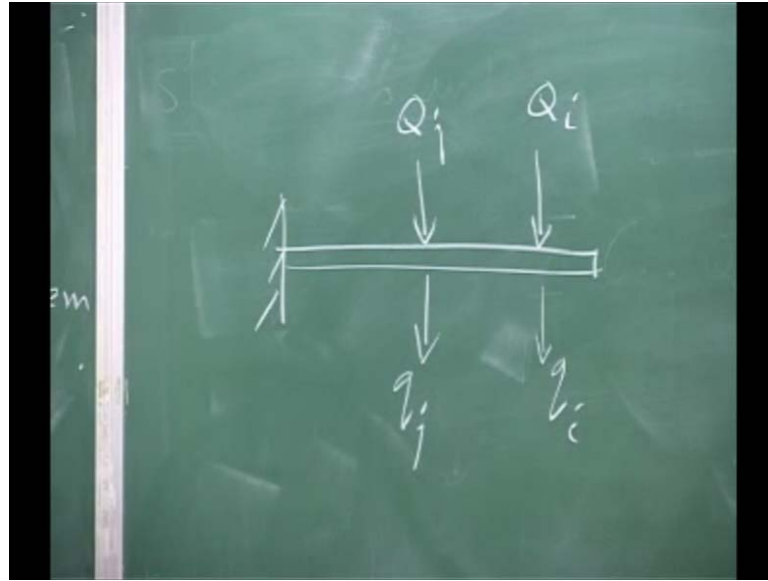
The image shows a chalkboard with handwritten mathematical equations. At the top, the text 'flexibility influence coefficient' is written. Below it, the equation $\{q\} = [c]\{Q\}$ is written. In the middle, the equation $\{Q\} = [k]\{q\}$ is written, with a circled $[k][c]$ term to its right. Below this, the text 'stiffness influence coefficient' is written. A small 'I' is written above the circled term.

Now, this you can just put at in matrix form which will come as q equal to c is Q . Similarly, you can write the load in terms of deflection like spring you write it f is equal to $k x$, where k is the stiffness please understand here c is flexibility, where you write f equals $k x$, k is stiffness. Now, you can also write using the stiffness type of thing load is given in terms of, this is stiffness influence coefficient matrix are you can leave it, as it is.

This is flexibility influence coefficient now, what is the relation between this can substitute this q here then, you will get $k c q$, that mean this must be identify. So, stiffness matrix is inverse of flexibility influence coefficient, either or this is inverse of this that all. Now, apart from this is written in matrix form, the influence coefficient matrix has another important property which is it is symmetric.

Now, the symmetry has to be proved first we will do in this matrices notation, later we will take a continuous loading. How do you get this symmetry, because this is an important property of the influence coefficient metrics. This is based on this is the reciprocal theorem they call it Maxwell and Betti reciprocal theorem.

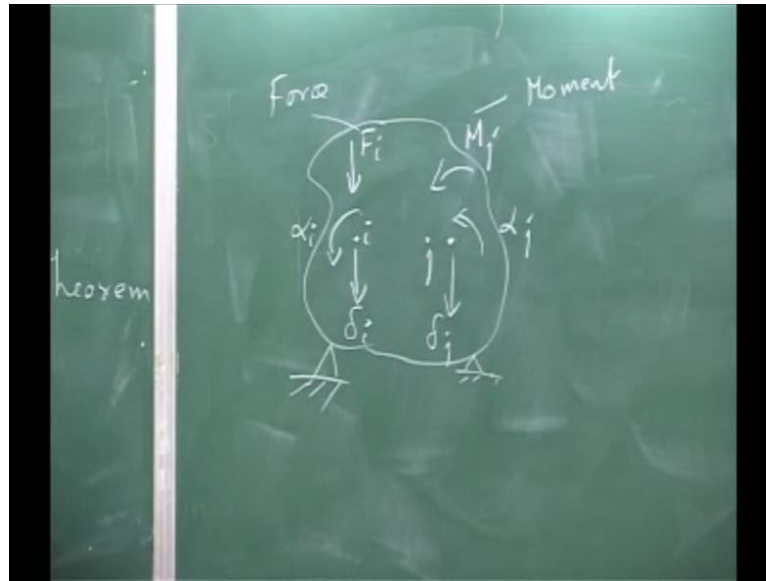
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This all goes down to the strain energy, the energy store in the structure during the deformation. It is like this suppose, we have taken this case we apply two loads they applied actually two loads like one here and another one here this we call it Q_j . Now which sequence there are apply, whether you apply this first and then this or you apply this first and then this ((Refer Time: 11:06)). Now in terms of final deformation of the structure you say it is independent of the sequence of loading.

And that means, the energy stored in the structure, because of the application of this load it really does not matter which one is applied first which one is applied later. Because, this is linearity is again very, very important, so we will just a prove that because that is the very important thing.

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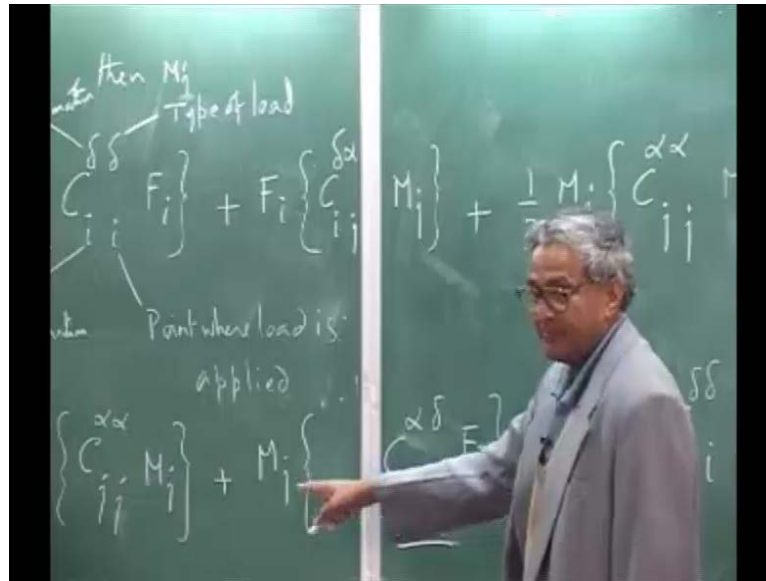


Suppose, you take a structure and fix it here have some fix it here and there are some notation which will use it. Because, this we say delta i and this is alpha I, delta i is this is the location i, delta i is the linear deflection. This is angular deflection at the point I at that location, you can take it. Because, point does not have rotation you can always say that zone now, you can take another which is j here, this is delta j and this is alpha j 2 point.

Because, we are taking both translation as well as rotation in that because that structure can deform. It can also have a rotational deformation, angular deformation that is why it is taken generally. Now, if I have a load F_i applied I am only assuming that two loads are applied I can apply four and that location j, I am putting M_j moment. So, this is the force this is the moment m, now using this let us try to find out, what is in strain energy of the structure due to this loading.

If you say the strain energy U we first start with sequence is I first apply f_i and then m_j that means, first F_i then m_j . Now you need to write the same energy what is the first when I apply F_i it is going to have some deflection. That deflection at point of application because you do not other point also will deflect, but you have to look at what is the deformation at the point where the loading apply.

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So, you will write half F_i deflection due to load F_i . So, I am going to use that influence coefficient form $C_{ij}^{\alpha\delta}$, δ please note that this is the notation this is point of deformation, point or point at which you are measuring the deformation you can call it. And this is point where the load is applied, now this is type of deformation and this is type of load.

So, you see I am using actually four, two subscript, two superscript to indicate, one where is the location, my load is applied and what type of load is applied, whether it is a force or a moment then similarly, the point where I am measuring the deformation, whether it is a translational or rotational deformation. So, one is the point of load another one point at which you measure the deformation. Now I have to add the other this is first the energy due to only applying load F_i .

Because this is nothing but q that point that deflection at that point, now I said first F_i apply. So, this is this strain energy it store, then I am applying M_j that means, I have add the energy due to the load in this. Now, this load F_i is fixed load and that will move through from some distance right, what is that, that would be F_i into you will have $C_{ij}^{\alpha\delta}$.

Because M_j is the moment which I am applying here because of this moment what is the deflection here, so that is why j is the location my load is apply and what type of load it is a moment, that is why α , the rotational load. Where is the deformation at location i

what type it is a translation or linear deformation and load is F into distance that the energy work done by this load. Now work done by the load M j will write that will be half M j C j j alpha alpha M j is this clear.

So, that why the notion is very important where you measure the deformation, where the load is apply and what type of deformation, what type of load. Now if you change the sequence change the sequence means, I am applying M j first and then F i. I will have u is half M j C j j alpha alpha M j moment angular rotation, this is the first one. Next I am applying the load F i that mean, this moments which is already applied here it will rotate some other angle.

So, that means I will have M j put C j alpha and I delta f this is the rotation at point j, due to your load at F i plus half F i then you will have C i delta i delta. Now you say that it is independent of the sequence of loading once, you specify you will find that you have to equate both of them both these expression. When you equate you find that M i F i M j F i here C i delta j alpha, here it is C j alpha i delta and if the strain energy of the structure is independent of the sequence of loading, that means that must be same equal.

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$$+ \frac{1}{2} M_j \left\{ C_{ij}^{\alpha\alpha} M_j \right\}$$

$$F_i + \frac{1}{2} F_i \left\{ C_{ii}^{\delta\delta} F_i \right\}$$

$$C_{ij}^{\delta\alpha} = C_{ji}^{\alpha\delta}$$

So, you will have C i j delta alpha is equal to C j alpha i delta this is the now, we can write very general expression, which I will write it, you can write the strain energy also in a different form, so I will erase this.

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$$\begin{Bmatrix} \delta_i \\ \alpha_i \\ \delta_j \\ \alpha_j \end{Bmatrix} = \begin{bmatrix} C_{ii} & \alpha_i & C_{ij} & \alpha_j \\ \alpha_i & C_{ii} & \alpha_j & C_{ij} \\ C_{ji} & \alpha_j & C_{ji} & \alpha_j \\ C_{ji} & \alpha_j & C_{ji} & \alpha_j \end{bmatrix} \begin{Bmatrix} F_i \\ M_i \\ F_j \\ M_j \end{Bmatrix}$$

So, put it in a suppose this is delta i alpha i delta j alpha j you can fill of the full 4 by 4 matrix, that will be C i i delta delta C i i delta alpha C i j delta delta C i j delta alpha, this is multiplied by F i M i F j M j. So, I will fill the whole thing you will have C i alpha i delta i i alpha alpha i j alpha delta C i j alpha alpha, C j i delta delta j i delta alpha C j j delta delta C j j delta alpha and then C j i alpha delta j i alpha alpha C j j alpha delta and C j j alpha alpha.

This is the complete for the two loads please understand this how it would like, now if you have more number of loads. The size matrix I is going to become bigger and bigger now, later if you have distributor loading we are going put everything instead of matrix rotation, we will put it in the integral form. Now we can write the strain energy this part also in a different form using this. Now, for the same case what will I will erase this part because this is maybe I will erase these parts.

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$$M_i + C_{ij} F_j = \frac{1}{2} F_i \delta_i + \frac{1}{2} M_j \alpha_j$$

$$+ \frac{1}{2} M_j \begin{Bmatrix} \delta \\ \alpha \end{Bmatrix} \begin{Bmatrix} C_{ij} \\ M_j \end{Bmatrix}$$

$$\begin{Bmatrix} \delta \\ \alpha \end{Bmatrix} \begin{Bmatrix} C_{ij} \\ M_j \end{Bmatrix} + \frac{1}{2} \begin{Bmatrix} F_i \\ C_{ij} \end{Bmatrix} \begin{Bmatrix} \delta \\ \alpha \end{Bmatrix} \begin{Bmatrix} M_j \end{Bmatrix}$$

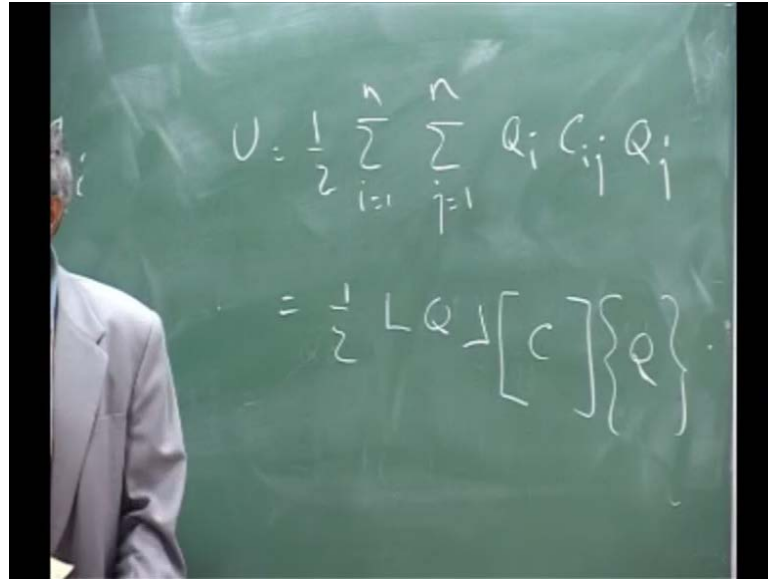
$$\begin{Bmatrix} \delta \\ \alpha \end{Bmatrix} \begin{Bmatrix} C_{ij} \\ M_j \end{Bmatrix} = C$$

Now, using this if you only have these two loads your delta I is $c_{ij} \delta_j$ plus $c_{ij} \delta_j$ sorry $c_{ij} \delta_j$ plus $c_{ij} \delta_j$. Because, this is the total deflection at point I due to both the loads because here you have $f_i m_j$ these two are 0. So, you have $I \delta_j$ delta delta this term. Now similarly, you can have α_j that is the total rotational deformation at point j due to both the loads, you will have $c_{ji} \alpha_j$ plus $c_{jj} \alpha_j$. Now what you do is, this expression you split it into two parts you write it as half.

So, maybe I will put it you can write this particular expression right half $f_i c_{ij} \delta_j$ plus $c_{ij} \delta_j$. So, I can write it as $c_{ij} \delta_j$ plus $c_{ij} \delta_j$, this is the first another half I am writing it as half again you can write it f_i but I will change it to because you know that this is same as this. So, I can put half of it that means, $c_{ji} \alpha_j$ but I remove this I will put it like. Now what I do is I take this term added to this part, I take this term added to this part then, my expression will be $u = \frac{1}{2} f_i \delta_i$ this term will remain as $c_{ij} \delta_j$ plus, I will have $c_{ij} \delta_j$ plus half I am taking this term $m_j c_{jj} \alpha_j$ plus I am writing this term.

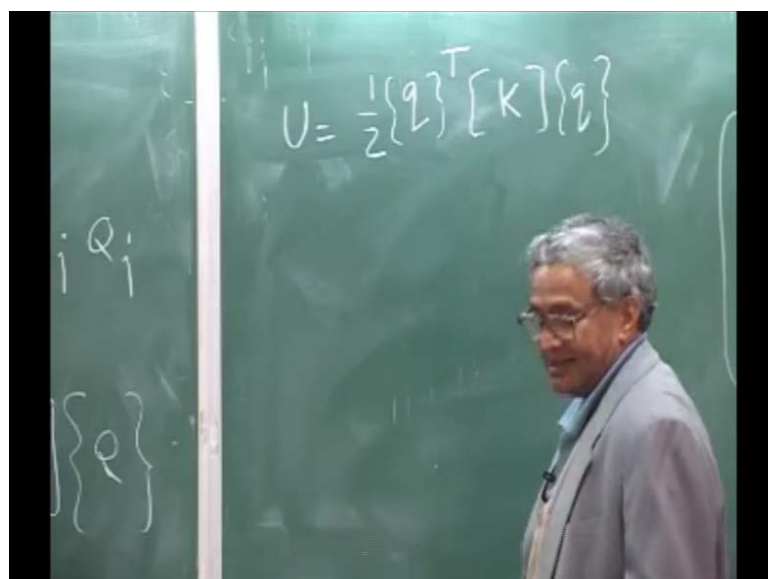
Because, this is m_j I am taking out, so $c_{ji} \alpha_j$, so $c_{ji} \alpha_j$ now you see this term is nothing but δ_i and this is nothing but α_j . So, you write your this is half $f_i \delta_i$ plus half $m_j \alpha_j$, now you see in that form of expression it is the load and the total deflection at that point of application of the load and half. So, you can write your very general energy expression as may be I will erase all this part.

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$$U = \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n Q_i C_{ij} Q_j$$
$$= \frac{1}{2} [Q]^T [C] \{Q\}$$

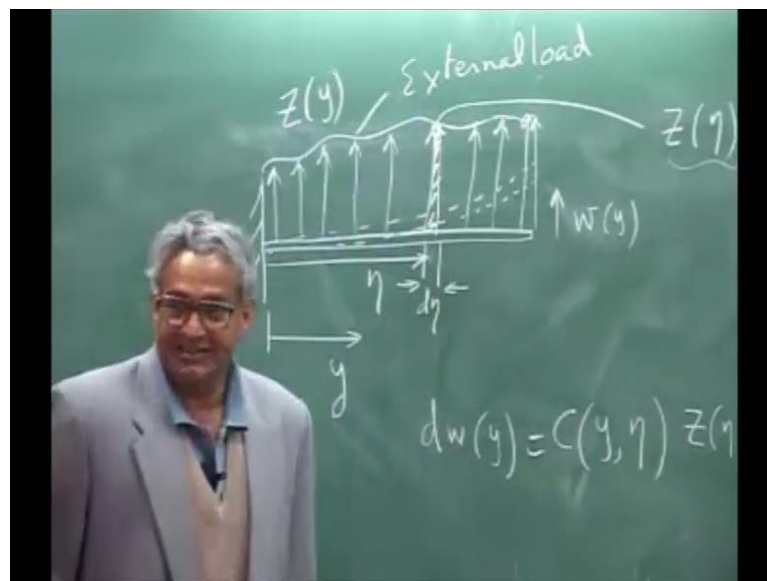
Now, not necessary u is half summation q I q I that is all, now if there are n loads not two load n loads you will simply put I running from 1 to n you can replace. But you know that how you represented the q I, q I you have represented as summation c I j q j this j running from 1 to n right. I can go back and substitute here and write my strain energy expression, I can write my strain energy expression as u half summation. I running from 1 to n summation j running from 1 to n, I will just put q I c I q j and this itself can be written in matrix form as half q c q or you can say q transpose c q and you can put it in stiffness form also either way. So, this is the energy expression.

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$$U = \frac{1}{2} \{Q\}^T [K] \{Q\}$$

If you want to write in stiffness expression this will be what u is $k I j q j$, so again you will be able to write it as $\frac{1}{2} q^T k q$. This is the stiffness form, this is in flexibility form, this is how you define the strain energy in the structure and important property is it is symmetric. I think I can see have we have till now we consider only discrete load, suppose you take a continuous load that means, loading is not specific because this is the matrix form regard let us take for example.

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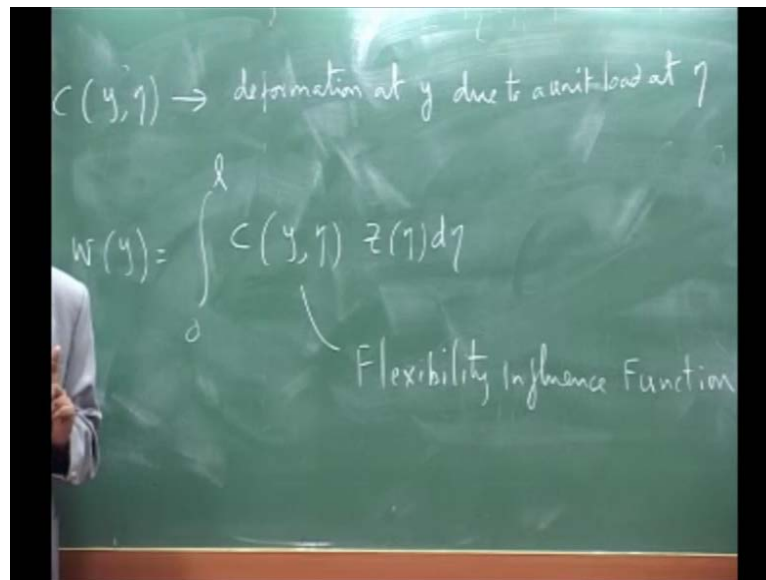
Same the wing is represent as a canten liver beam, this is acted on by a aero dynamic loads or external load i will put it. This is acting, this is the external load, this will distributor, all right and this is going to deformed, this will have some this will have a deformation. Now for this we need to get, because this is what realistic case is you have a distributor load you want to know how the wing deforms, under the action of distributed loads.

Whether it is bending or transfer here we take a bending thing and let us define axis system. First let us say this is I think I use the symbol y and I also use another notation, which I call it η and this is $d\eta$ and external load and I am going to call it as ψy because this is the function of y . Now, the load that acts at this location is essentially $d\eta$ over a small elemental link, the load acting is this.

Now, the deflection at any other point due to only this load is you will write this as d I am taking deformation this is w which is the function of y . W is the function of y that is

bending deformation, now this is the elemental load I will have $d\eta$ which is the function of y is y is running very variable. Please understand where as this η is the fixed at this point, now you know that the loading is $z(\eta) d\eta$ there was some influence function, that influence function we will write it as $c(y, \eta)$.

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That means this factor $c(y, \eta)$ represents at deformation at y due to a unit load at η , now if you want total deflection, what you have to do is you have to integrate. So, total deflection means deflection due to out the loads now that integration it is over $d\eta$. So, here that will becomes, so now you can write it as w at any y will be integral 0 to l . l is the length of the if you say this is the total length is l this will be $c(y, \eta) z(\eta) d\eta$.

Now, this is called again flexibility influence function actually if you really see this is easy to make a measurement in experiment because what you do is you put units load at several location simply go on measure the deflection at different every point, on the other hand. The manipulate form is stiffness influence function that difficult to formulate because it is not easy because stiffness influence you will be writing in this fashion actually, what do you want you want deflection that is force at any point in terms of deflection at all the points.

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$$= \int_0^l \underbrace{k(y, \eta)}_{\text{Stiffness influence function}} w(\eta) d\eta$$

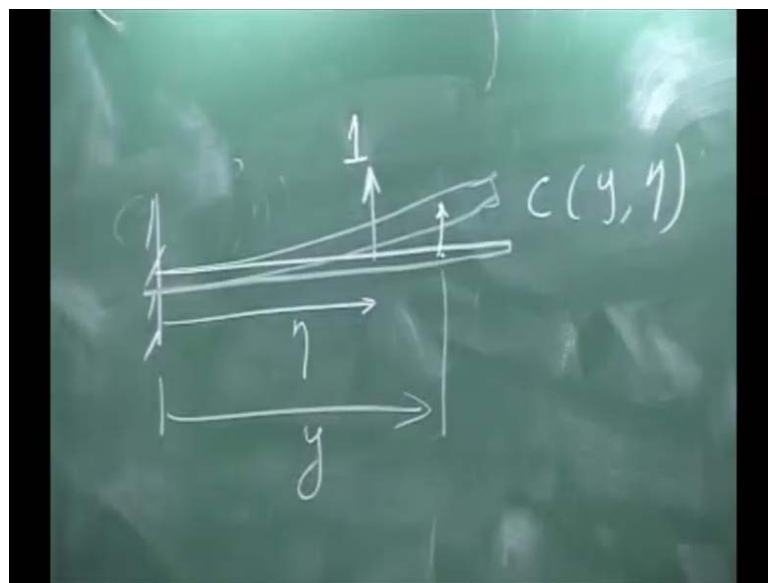
That means you will be writing some k this will be w eta, this is difficult to this is a stiffness influence function this is difficult to get. A discrete system it is easy but experimental measurement always you will find, this flexibility influence coefficient is easy you can go take a wing put it on a string put suitable boundary condition and then apply the load and then, keep making measurement at a way point you can form matrix because you have to make a grid and then you can get this then you can have interpolation.

Now, you see this is how the structural deformation is suppose you know pressure on the aero foil various point. Because, this is what the c f d calculation or any the calculations you do not know to pressure into some area there is a load. Find out the deflection you know what is the total deflection on the entire wing but now the question is how do you get this.

We have given everything in terms of just mathematical form, now you need to know how do I get this sometime this also call influence function. I write people use sometimes as green function but we will use as a flexibility influence function, influence coefficient function if you want to put coefficient also that is fine. This is where the structural analysis comes, how do I get this function for a given structure this is what you have learned in structures one course.

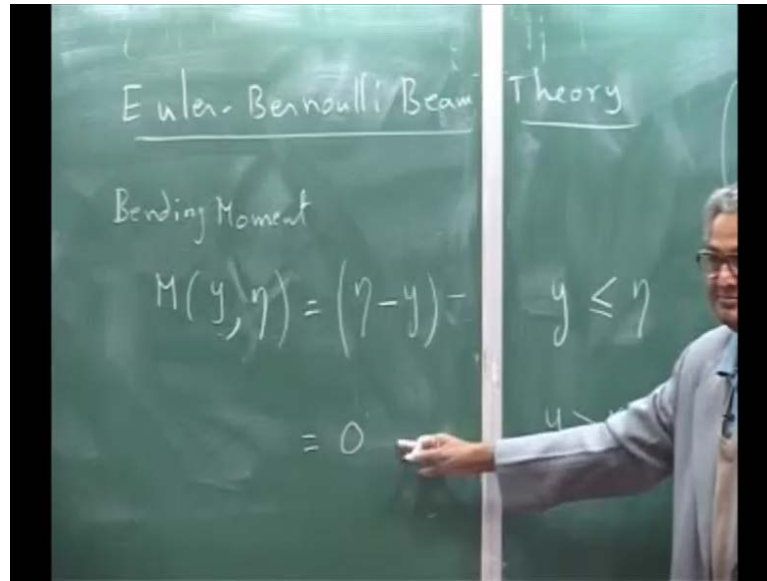
If it is the beam, under bending or plate model or beam model passion torsions passion all those theory are applied here to get that flexibility influence function. So, what we will do is we will take the same problem and please note this is symmetric in the sense $c(y, \eta)$ is equal to $c(\eta, y)$. So, in a functional form also this is symmetric, so $c(y, \eta)$ is equal to $c(\eta, y)$. This symmetric function, symmetric means you change the position of this change the wherever y there is you put η wherever, η is there you y function symmetry sense. Now, we will take a same example of the beam problem and we will try to get, what is the $c(y, \eta)$ and then write this general expression.

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Now, let us take same problem let me first we need to our idea is to evaluate this, this is the same canten liver. Let us assume a unit load is applied at location η and you want to find out, the deformation this is $c(y, \eta)$ and this is y . Now, for this case we assume that Euler Bernoulli beam theory.

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Now, you see what Euler Bernoulli beam theory, we use it you all know what is the Euler Bernoulli beam theory is just simple case plain section remains, plain and perpendicular to the reference line before and after deformation. Because that there is no shear deformation on the beam, now what is the bending movement at various locations. The bending movement, bending movement m at any y please note that my load is at a fix location η , bending movement at any location y that depends on I will put, if you want more specifically m η .

η is the load application point y is the location where you are getting the bending movements, this will be η minus y when y is and this bending movement is 0. Where y greater then actually y equal to η its take exactly 0 that means, you are dividing the split the whole beam into two part. One part you have a bending movement which is η minus y that means, when y is in this zone and y behind this point.

Now, you solve the equation of just bending deformation you know the bending movement and Euler Bernoulli beam theory, says that bending movement is equal to what $e I$. This is the load this is $E I d^2 w / dy^2 = w$, this is the standard second derivative of the deformation. Now what is w because we know that w no other load is there except only one load.

So, this $d w$ is w itself that is nothing but $d y \eta$ only So, this becomes essentially $E I d^2 w / dy^2 = c$ of y comma η . So, this is my equation you have to solve you

know the moment is given here substitute the moment here, solve for c y η but in two part of the beam. So, with the boundary condition that is the key, now let us take the first case.

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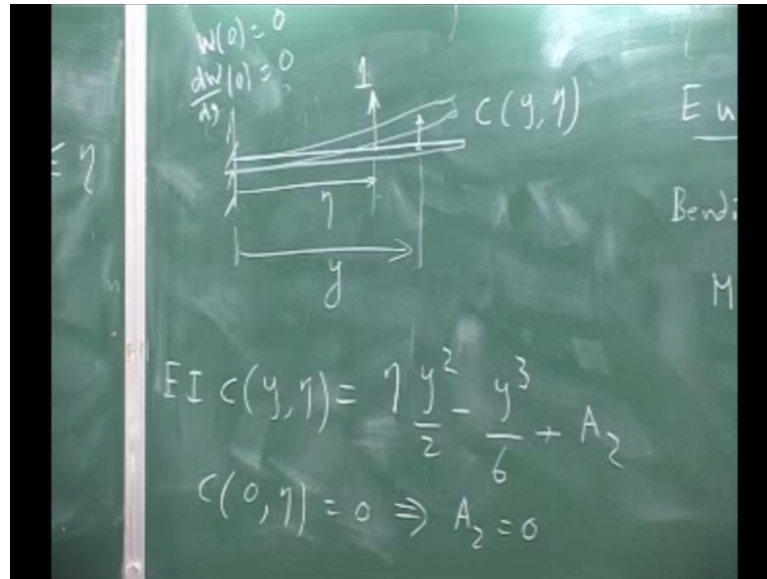
$$EI \frac{d^2 c(y, \eta)}{dy^2} = \eta - y \quad y \leq \eta$$

$$EI \frac{dc(y, \eta)}{dy} = \eta y - \frac{y^2}{2} + A_1$$

$$\frac{dc(0, \eta)}{dy} = 0 \Rightarrow A_1 = 0$$

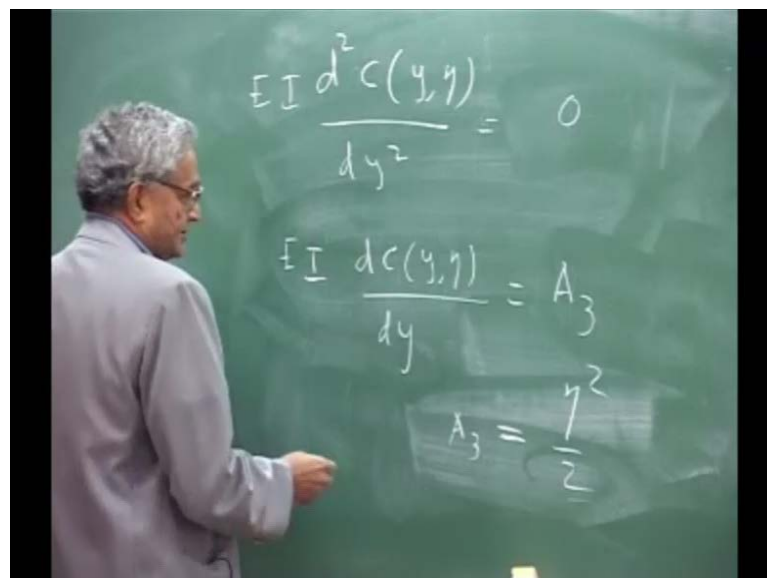
The first case is $E I d^2 c d y^2$ equal η minus y for y less than or equal to η . So, we solve this part now you can get $E I d c$ equal ηy minus y square over 2 plus some constant A_1 . I am integrating now you know that boundary condition when y is 0 load is applied at η unit load then y is 0. The slope of the beam because it is canteen liver beam that your assumption starts coming now what is my boundary condition, boundary condition I say w at 0 is 0 $d w$ by $d y$ at 0 is 0 $d w$ y becomes basically because w is c . So, the prime therefore, you will have the boundary condition $d c 0$ comma η over $d y$ this is 0.

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Now, when I substitute $y = 0$ I will get this condition leads to $A_1 = 0$ then, what I do I do one more integration and when I do one more integration. I will have $EI c'(y, \eta) = \eta y - \frac{y^2}{2} + A_1$. Now I am integrating this will be $\eta y^2 / 2 - y^3 / 6 + A_2$, I know that deflection at the root is 0. So, I will have $c(0, \eta) = 0$. So, this gives me $A_2 = 0$ all right, now you have got the function $c(y, \eta)$ in the zone y is less than η . So, we will write it may be I write here possibly.

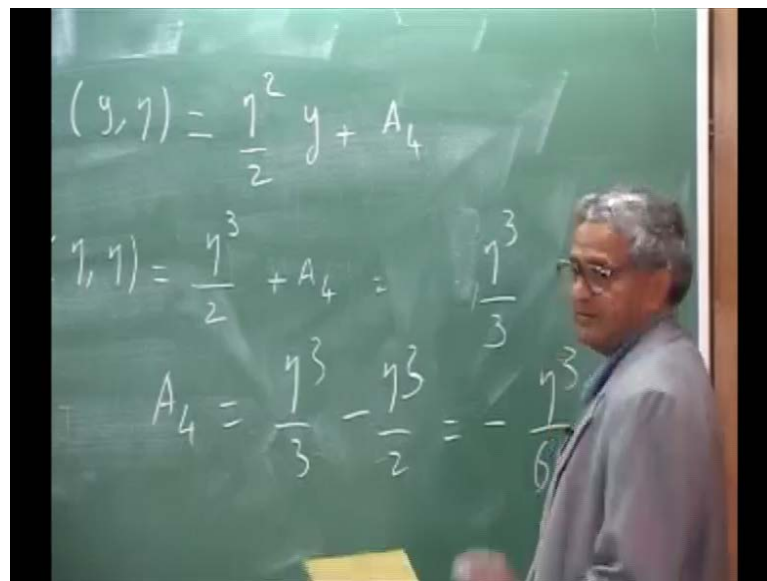
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So, I will put it here $c y, \eta$ equals ηy^2 over 2 minus y^3 over 6, 1 over $E I$, this is for y all right. Now let us go do the other part, where the second half where y is greater than η , the bending moment is 0. So, I will have now what I will have a constant this I call it as A_3 , now what is the value of A_3 , you have to determine from the earlier case at y equals η what is the value, that is if you write it may be I should write it here, where do I write I will write here that is the best.

Because, we got what is that $c y, \eta$ this was actually 1 over $E I \eta y$ minus y^2 over 2 and when y equals η . This will become $c y, \eta$ sorry $c y, \eta$ becomes, what is the value y equal to η this will be η^2 over 2. Now that is what you will be having here it here of course $E I$ is brought in here, so you will have your A_3 will be $E I$ no η a three is it correct one minute I will get A_3 no one minute what is that A_3 is become, I have taken 1 over $E I$, that is y otherwise if you want to write it this, entire thing is A_3 then you will be having it this is what you will have I will maybe I will put it this term will go here right this what we had write. So, you will have $E I$, so η^2 over 2, so I can write this as A_3 as η^2 over 2. Now once you substitute you can again differentiate it and then get now where do I write problem. Because, it is very little place here I erase this part again come here this is the also.

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Let us say, I will have $E I c y$ comma η it will become, A^3 is η^2 over 2, so you will have η^2 over 2 y plus you will have some A^4 . Now again you have to put y equal to η when you put y equal to η , that is this expression you will have that is $E I c$ η η becomes, this is η^3 by 2 plus A^4 from this expression. When you put y equal to η this will be η^3 by 2 takes it will be what is that 1 by 3 right 1 by 3 or 1 by 6. This is if I take 6 this is 3 no 2 by 3 what not 2 by 3, 1 this is 1 by 3.

So, you will have this will be $E I \eta^3$ over 3 this one, now A^4 becomes, no is there something I missed here no there not be any no, there is no $E I$. So, A^4 becomes η^3 over 3 minus η^2 over 2 which will be 6. A^4 becomes η^3 over 6 now you can write your function may be I again go here and erase the whole thing.

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$$E I c(y, \eta) = \frac{\eta^2}{2} y - \frac{\eta^3}{6} \quad y > \eta$$

$$c(y, \eta) = \frac{1}{E I} \left(\frac{\eta^2}{2} y - \frac{\eta^3}{6} \right) \quad \nearrow$$

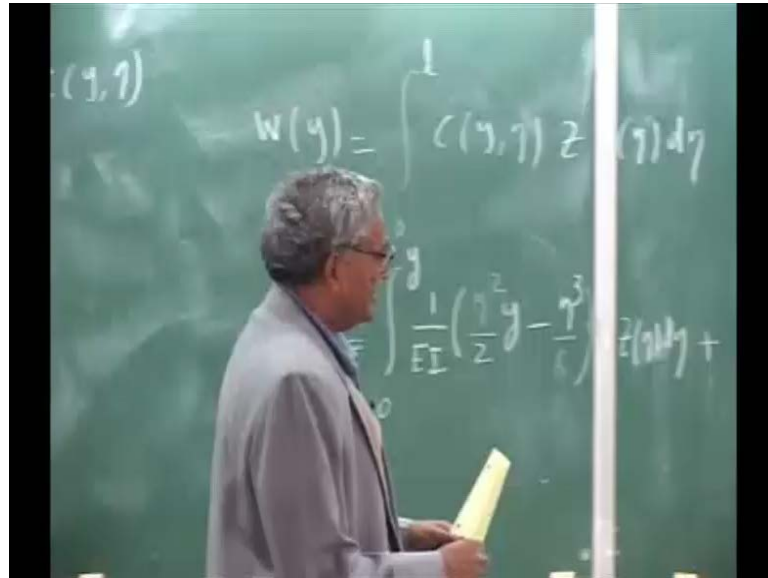
$$c(y, \eta) = \frac{1}{E I} \left(\frac{y^2}{2} - \frac{y^3}{6} \right) \Rightarrow y \leq \eta$$

You will have $E I c y$ comma η is η^2 over 2 y , A^4 is minus η^3 by 6 now otherwise, I can write my $c y$ comma η is 1 over $E I$ η^2 over 2 y minus η^3 over 6. This is for y greater than η and you see y less than η , that is this expression $c y$ comma η is 1 over $E I$. I am writing it repeating it the same thing that is ηy^2 over 2 minus y^3 over 6 for y less than or equal to η , this is for this and this for this.

Now, you see the symmetry here I can change y 2 η if I change y 2 η , this will become y is η , η is y and. So, the flexibility influence coefficient function is also symmetric now using this function, you have to write the total deformation of the

structure now how do we write that deformation. So, first we will take it because you know that this is the function we want I erase this part.

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Now, you can have w at y this is how we define 0 to l , c y comma η this is how we have define, we need to substitute this function c . So, what you do is we will split this function into two parts this integral, we will write it as integral 0 to y that means, you are considering. The load that acts on the beam till the point y that means, η less than y η less than y is sorry η less than y is this function you follow, what you are doing is you are splitting the integral into two parts please understand I am putting it this is w y .

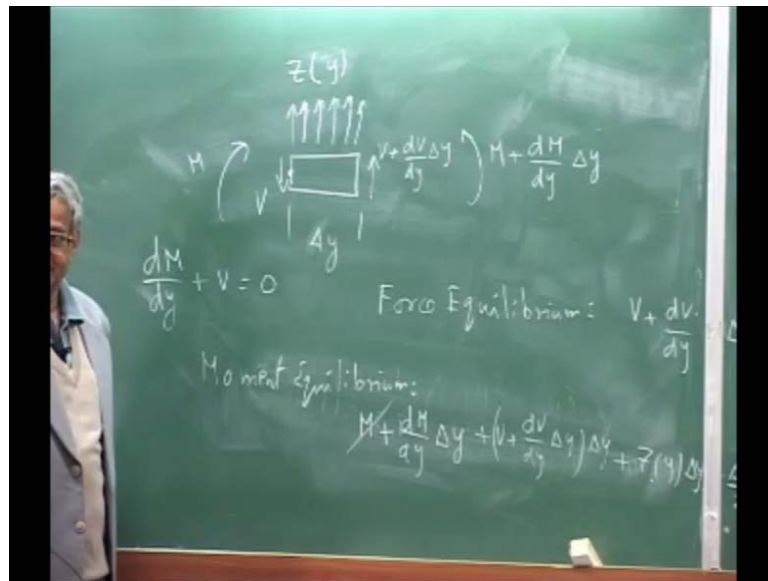
I am splitting the loading that means, 0 to y here I will have what loading is z η d η , η I am going from zero to y but my y is outboard of η . So, y is outboard means this is the one, so I will be putting 1 over $E I$ η square over 2 y minus η cube over 6 z η d η . But, here y to l again I will have z η d η , I am measuring the deflection at y the loading is beyond that point that means, η is beyond y that means, this function.

So, I will put this function here which is 1 over $E I$ η y square over 2 minus y cube over 6 is it clear, because this is the little confusing always if is it see this is you always confusion rises here, the confusion is this integral is what you are measuring the deflection at point y right. The loading goes from where to where 0 to y that means, where is the loading, loading is in board of the deflection point. Loading is in board of the deflection point is where this is loading y is greater y is value measure.

So, loading is in board of the deflection point. So, that is y the first one and in the second case, the loading is y to l that means, loading is outboard of where you measure the deflection, z is outboard of where you measure the deflection this why this function should go there is it clear. Because, I know that this is always a confusing thing for many students I even some time we get confuse please, understand this is the where you apply the load where you measure.

The deflection whether the load is out board of deflection sign are load is in board of deflection point that all. Now you have seen here very interesting relationship but getting c is not that easy because you have to solve like this it is a very simple problem which we have solved. Now you can take a plate vibration, you can take a torsion beam, you can do anything you want torsion of beam you can solve this. But how it is mathematically represent is you know equation of a motion for sorry not motion equation for beam bending very simple, because this you most have I will Just write mathematically figures how will you write the beam equation, now I erase this part.

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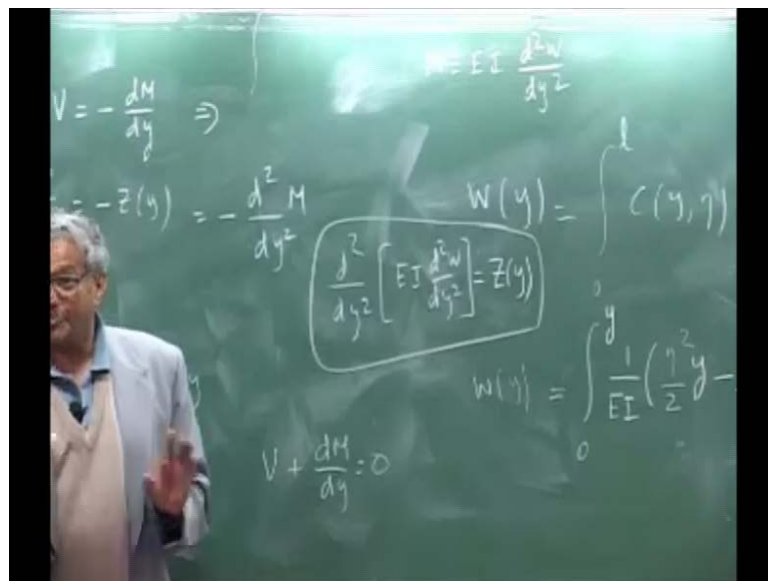
Because, this is just a mathematical see you will have you take small element of the beam and this is act on by a load. This is $d y$ what will you say let us say this is shear force, at this is downward shear force v this is the v plus $d v$ by $d y$ into Δy , you can call this as Δy and then, you can have a moment here you will put a moment m bending moment, m plus right this is the standard beam.

A small element of a beam you have taken and you are put on, a right face what over the shear force and the bending moment, left face the shear force and the bending moment. Now you write the equilibrium of this element force equilibrium that means, all the forces, this is just force equilibrium this is just a you will write what that y into Δy that means, this v and minus v .

So, you will v plus $d v$ over $d y$ del y minus v plus $z y$ del y and this will give you $d v$ over $d y$ is minus z y right then, you take a moment equilibrium. This is the force equilibrium then, you can write moment equilibrium moment you can take moment about the midpoint. The moment about you can take, you can take moment about this point, you will have m plus $d m$ by $d y$ del y and then this into this distance.

So, you will have v plus $d v$ by $d y$ del y into Δy moment because all are counter close positive and then you take it as though. The resultant is acting in the middle you may take plus $z y$ Δy into Δy over 2 and then minus this moment 0, you will write it this is the moment equilibrium then, what will happen this moment will go up moment will go up then, you stay in the limit you divide by Δy grow up and then, set Δy goes to 0 then, what will happen. If you divide by Δy you will get m by d capital M by $d y$ plus v into Δy right plus v equals 0, that what you will have right.

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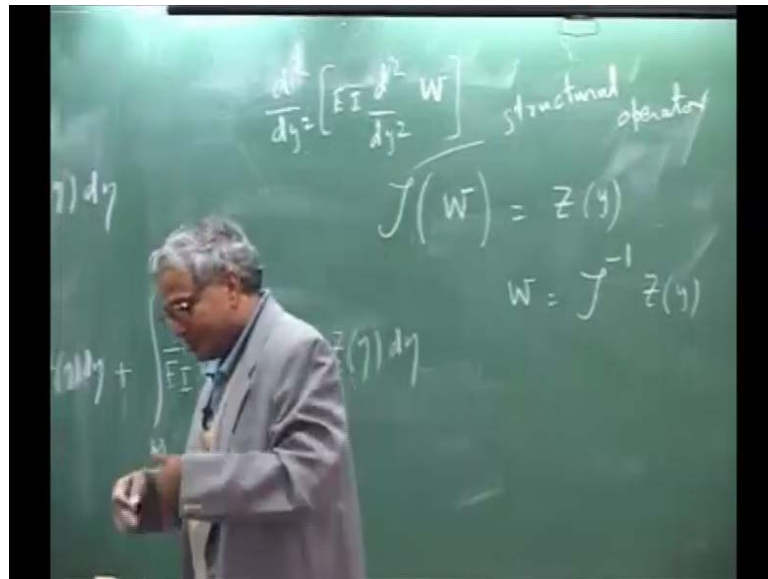
Now, you come here you had one equilibrium here this is the force equation moment equation says, the v plus $d m$ over $d y$ is 0 all right v equals minus $d m$ by $d y$. So, you

say therefore, is minus but you know $d^2 w$ by $d^2 y$ is you differentiate this one that will become, $d^2 w$ by $d^2 y$ becomes d^4 .

So, you have minus d^4 over $d^2 y$ into m but you know moment based on elementary, this is moment is $E I d^2 w$ over $d^2 y$ square put it here, what will you have sorry d^4 over $d^2 y$ square of $E I d^2 w$ by $d^2 y$ square equals this is the beam equation right, this is the beam equation equal z y is the distributed load and this is the $E I$ you know I is the transitional area moment.

Now, if you look at this I have a deflection equation please, understand this is the fourth order deflection equation I need four boundary condition. I need to solve properly here I have a solution, which is w in terms of delta loading. Now this is what people called operator this is the differential operator.

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The differential operator we can write it as d^4 by $d^2 y$ square $E I$ this is actually operating on w y . So, you may call it this is structural operator, structural operator operating on w gives the distributed load right. So, I am calling this is the structural operator but what I have written here w in terms of z that means, I am writing inverse the operator is inverter. The inversion operator is basically this is what about this is symbolically written in this part.

Now, there are several issues associated with this because this is a fourth order differential equation, we need to get the solution to that and you have to put approximate, if here you can get a close form exactly this that is the advantage of this. whereas form b issue in the inversion. So, all over aero elastic problem some time people write it as in operator form w is the deformation looking one more.

I have an structural operator acting on w , I have an inertia operator acting on w that is external load which is the aero dynamic load which is again dependent on w , so structural operator, inertia operator, aero dynamic operator. They just mathematically write represent that form here in is this here, this is the differential equation form this is the interval this is you do not say this is the solution.

Because, we still we do not know what is that because that is the aero dynamic load that load depend on w itself please, understand. It is not that is we saying there is the load this course you will start studying now how do we get the structure. This is purely structural part of the problem starting from influence coefficient next, we will study how energy methods are applied for structural deformation, how do you get the approximate solution to the problem.

Because, $E I$ uniform you are getting all this ((Refer Time: 1:18:01)) synobrilio suppose $E I$, that is bending stiffness is a not constant. Because, the air craft it is not a constant it is varying now how you do, this is that is why please remember I put $E I$ inside. If it is constant I can take it out again, I can write the solution I can allow e a to d a function of y itself, that is the solution is not.

Because, this equation solution is off signed with uniform $E I$ property please, understand I this is not if I have a variable $e I$ completely. I can write this place this is the example problem which is started with a constant $e I$ because that equation we use. So, you have to clearly know what constant we have made at one point in getting this to this, this is very general thing but again it is Euler Bernoulli theory separate this whole thing this equal to that y . So, next class we will study how energy method is applied for getting the deformations.