**Aero Elasticity**
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**Lecture – 19**

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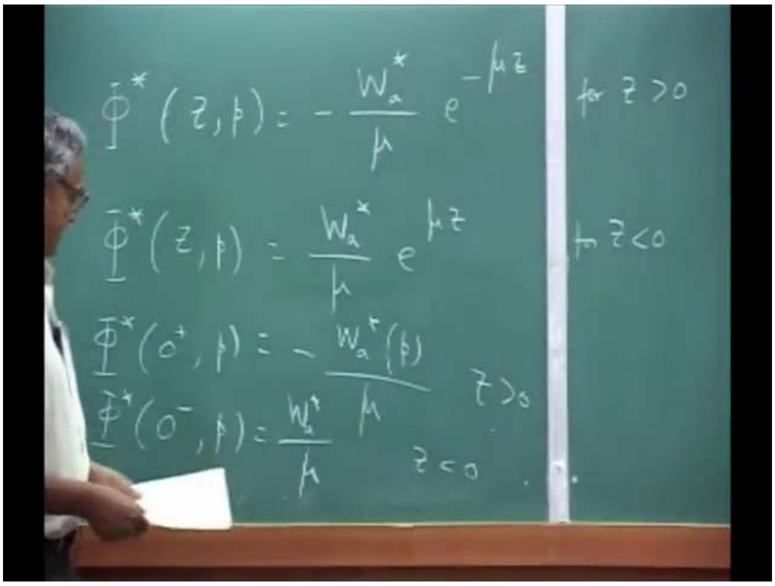

Have you wrote the solution of the potential that phi star minus this is W on the aerofoil minus mu z for greater than 0. Then at this for less than 0. So, you see that anti symmetry, but this is the phi star at any location z, but if you want to get the phi how to do the inverse Laplace transform, but we are interested in the solution where z is equal to 0 because we are not interested in getting the potential at every point. Only on the aerofoil we get the pressure then we can write the solution.

Therefore we said equal to 0. Then we will do the 0 plus comma p which is minus W a star which is, this is the velocity at the aerofoil. Similarly, phi star theta minus p you will have W a star over mu. This z is greater than 0 this less than 0. Now, this is a product of two Laplace transform because mu is a function of phi. So, when you have products of two Laplace transform.

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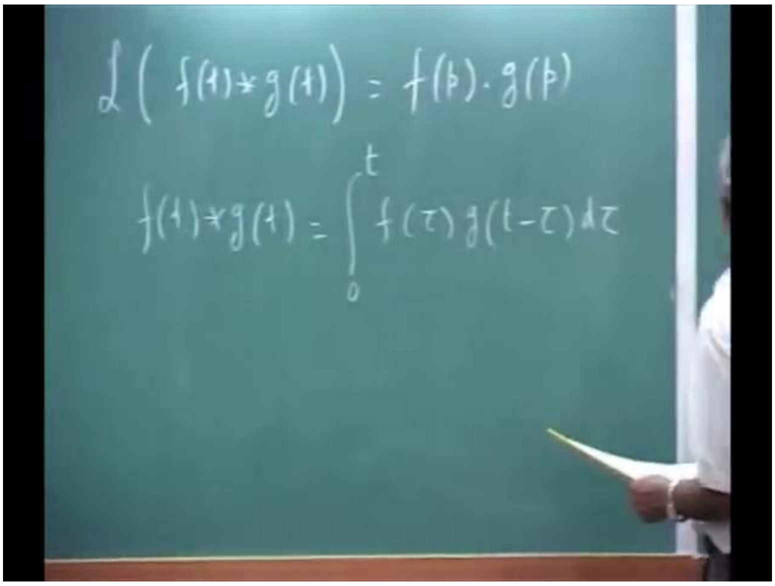

Then actually this is where the Laplace of, if you have two functions. This is basically f of p g of p, that means products of two Laplace transform, is basically the convolution of the two functions. Convolution is we will write it like this, g of t which is integral 0 to t f of tau g t minus tau d tau. Now, we simply apply this solution technique. When we apply we will get phi bar 0 plus comma which is a function of p becomes x because phi is the Laplace transform for x.

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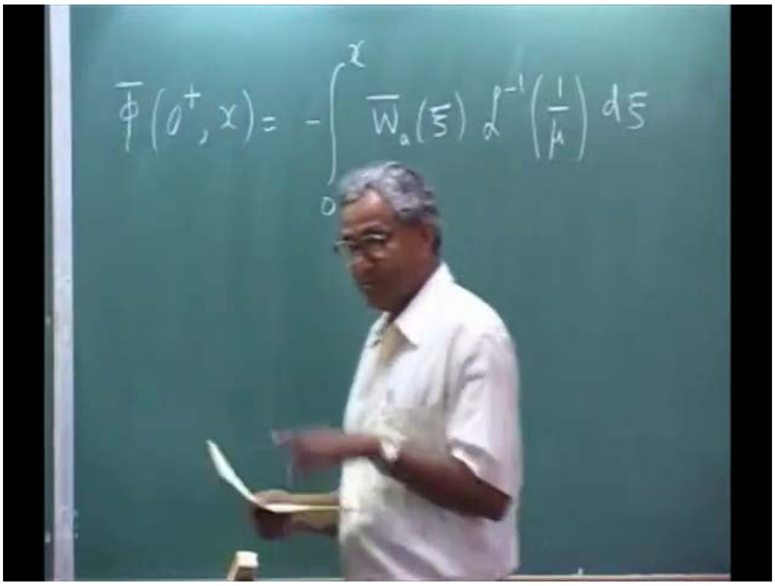

This will be minus integral, same thing 0 to x because this minus is because of this minus. W a star that will becomes W bar a, which is a function of some phi because this x is going to be there. Then Laplace of that inverse of 1 over mu this into d psi. Now, Laplace inverse of 1 over mu because if you look at it with, please understand this I have to have like t minus tau. So, I will have x minus psi or this when I write the Laplace, but mu you go back to your notes. Then look at what is the form of the mu which we wrote if you see that we wrote mu square.

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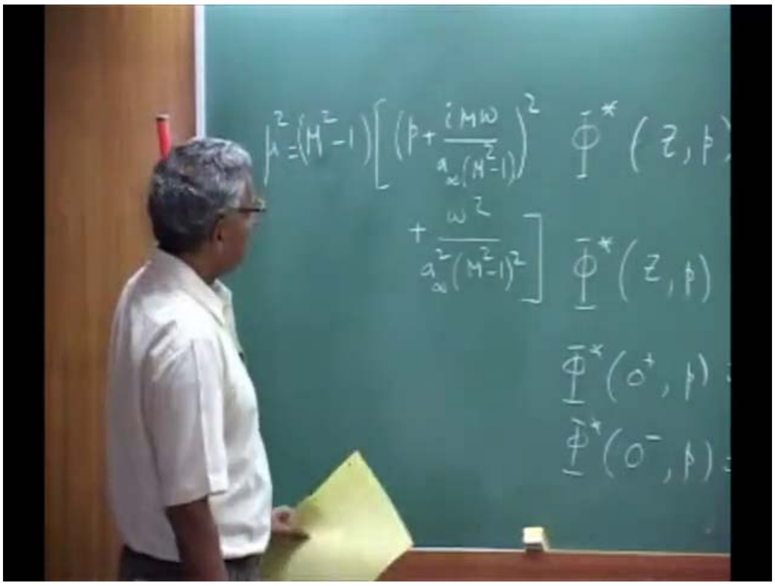

I will just write it here. So, that it is clear mu square, we wrote M we use M in infinity or m. So, M square minus 1 p plus i M omega over a infinity M square minus 1 whole square plus omega square over a infinity square M square minus 1 whole square.

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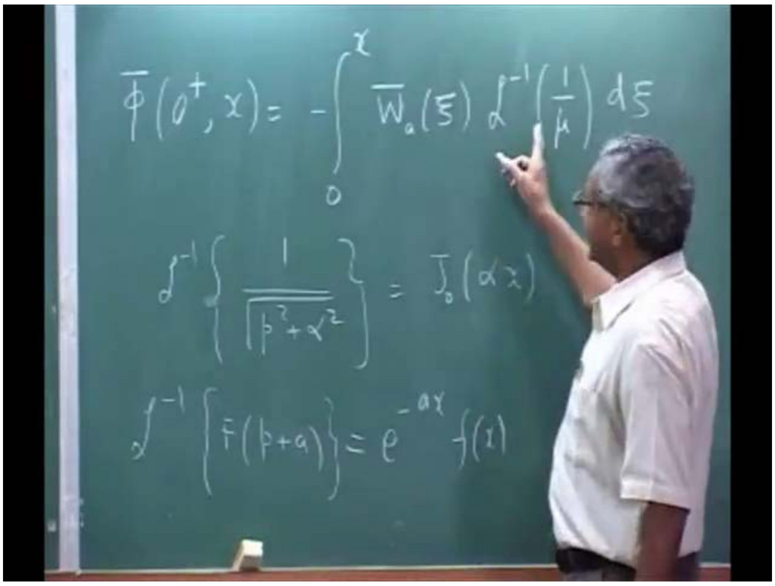

This is like 1 square quantity plus another square quantity. Now, if we have that is the mu Laplace inverse of 1 over, sorry square root of p square plus alpha square. This is actually a basal function. Then if p is added with because p plus some quantity that means I am doing a shifting theorem. So, if you have a shifting because Laplace inverse of you know f of p plus a is nothing but e power minus a x f of x. So, you use these two to write the solution for this.

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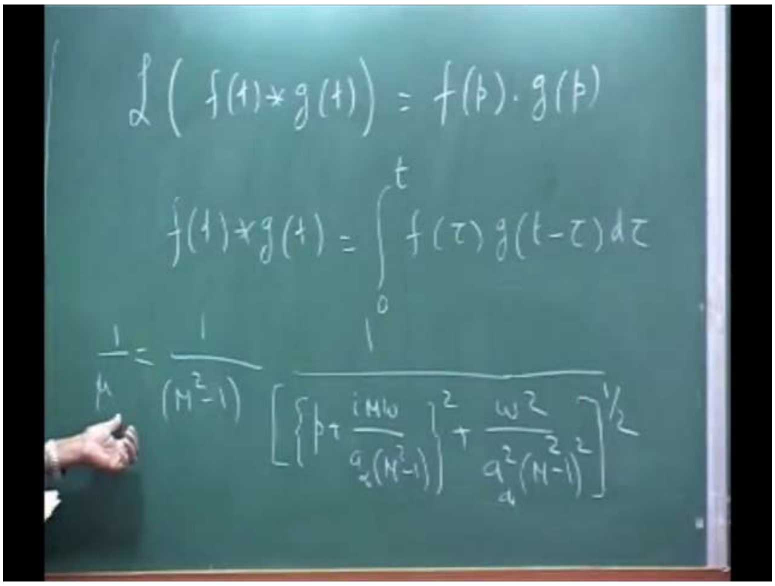

Now, the solution will be because you know that mu I will write the expression for 1 over mu 1 over mu is 1 over M square minus 1, 1 divided by bracket open p plus i M omega over a infinity M square minus 1 whole square plus omega square over a infinity square M square minus 1 whole square over half. So, this is my mu. We have the Laplace inverse for this. So, I will write my final solution may be area is here because this is going to be a long expression this will be.

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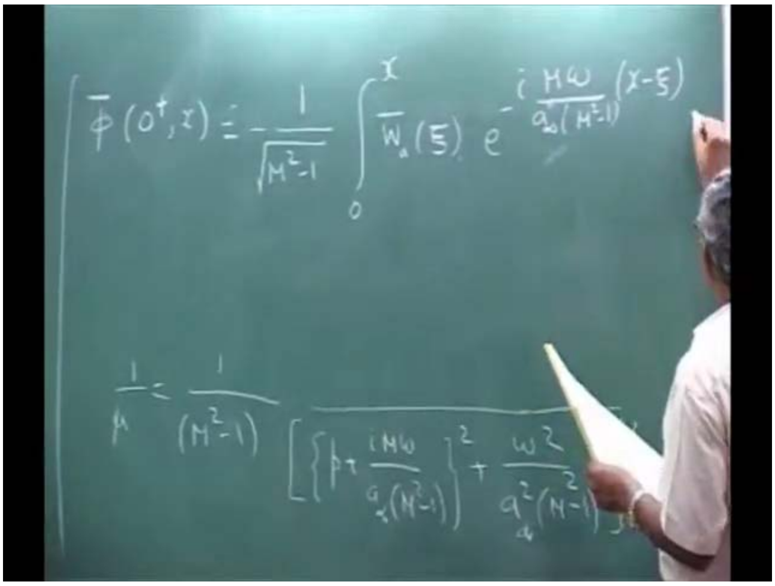

Now, the solution will be because you know that mu I will write the expression for 1 over mu 1 over mu is 1 over M square p bar 0 plus comma x is nothing but minus 1 over M square minus 1 0 to x W bar a psi e to the power minus i, M omega over a infinity square, sorry a infinity into M square minus 1. That is this term that is the shifting and shift we have to because this is convolution. We have to the convolution for that convolution is t minus tau. So, this x will become x minus psi into j naught may be i erase this part because this entire term is exponential into j 0 of omega because alpha x alpha is this term.

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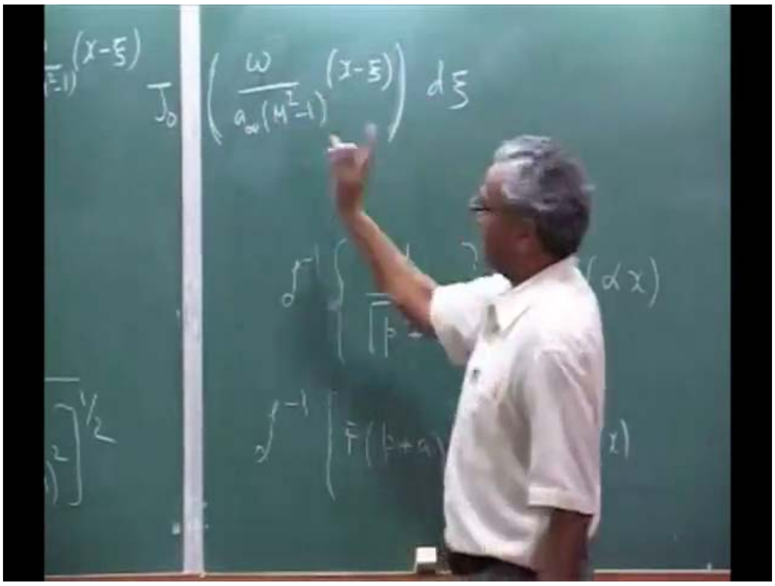

This is alpha square half of that omega over a infinity M square minus 1 into x minus into d psi please understand this is within the bracket. I will put it like this oh sorry this is a square root I am sorry this is square root. Now, this is my phi you can actually calculate this because we will we are going to do make approximations later. What we are doing is we will non dimensionalise this. Write it in a compact form, but there is a paper in 19. Actually it is a NACA report they have applying this. Then it gives a table I will write that reference to you it gives the complete value of this integration because this is the basal function. This is an exponential term and we will non dimensionalise it by writing.

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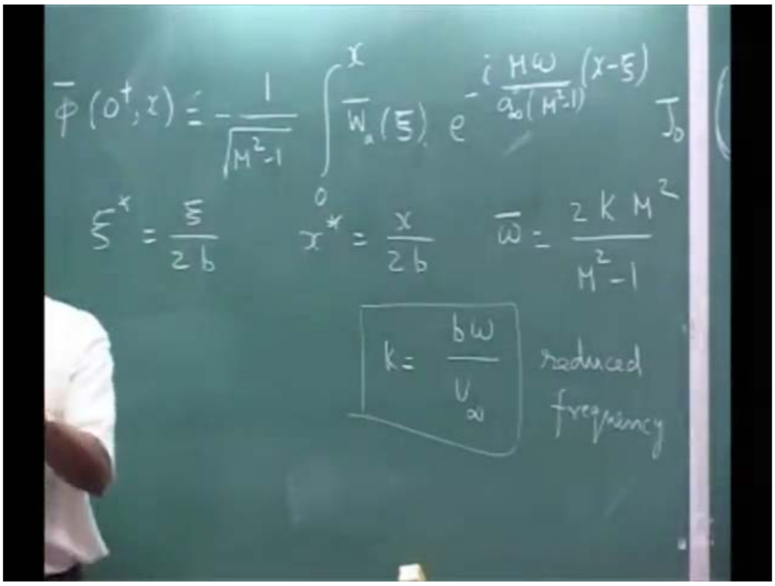


The length we non dimensionalise with respect to card that is 2 b. We will put a star x star is x over 2 b. So, you see if I want phi only on the because phi at any x 0 plus my integration starts from 0 to x. If I want only over the aerofoil I will just integrate this only up to the aerofoil. This is nothing but the velocity of the fluid of the aerofoil. Basically the W velocity you say the W velocity is nothing but the velocity of the aerofoil itself.

So, you know this motion, then you can always get this phi. Once you get phi you get the pressure expression. So, that is how it is done. Now, let us write this. Then you define 1 more which is omega bar as two k M square over M square minus 1. Now, what is this k, k is b omega over u infinity. This is non dimensional parameter which is called reduced frequency.

This is called reduced frequency this is nothing but omega because we have assumed the that the aerofoil is oscillating because if you look back it is a harmonic motion. If it is having a omega b over u infinity, u infinity is the far field velocity. This is the reduced frequency, later we will do approximations based on only this, very high omega, very low omega because in between you have to use this expression only. Now, your expression in non dimensional form it will become.

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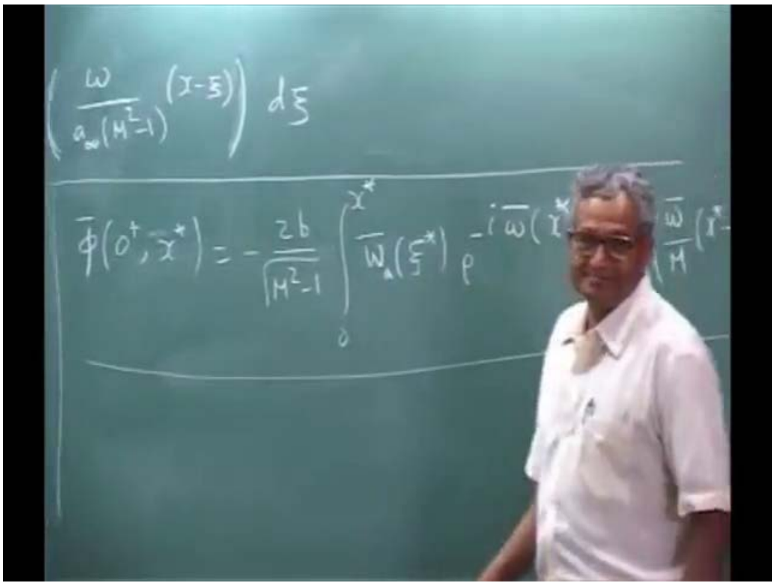

I am writing it here phi back of 0 plus comma x star this is minus 2 b over root of M square minus 1 integral zero to x star W a psi star e power minus i omega bar x star minus psi star j naught omega bar over M x star minus x y d psi, this is my solution.

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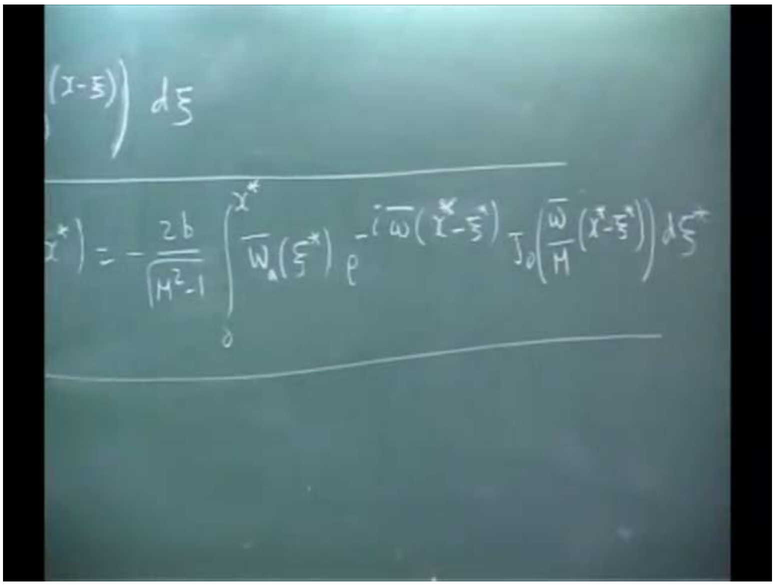

Now, you go back you have to use their unsteady pressure expression. The pressure difference, I will write the final solution, the pressure on the upper surface.

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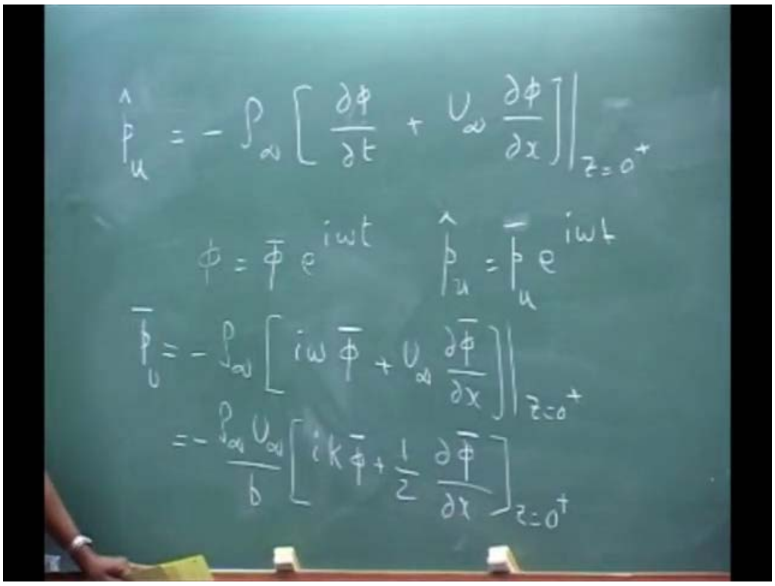

This is p minus p infinity that what I am putting it as this is rho infinity delta phi by delta t at this is our pressure expression p minus p infinity. Now, if my phi because what we assume our phi is phi bar e power i omega t is what we have assumed. Now, I am going to substitute here, if I substitute this will become p bar because p also we said pressure is also fluctuating p. This is p hat may be sorry may be I put it like this i omega t I may have put a u because when this oscillating pressure is oscillating everything is steady oscillation. This will be minus rho infinity i omega phi bar because e power i omega t.

I am cancelling out plus u infinity delta phi bar over delta x at equal 0 plus p upper I am sorry p upper. Now, this itself can be non dimensionalised by writing it minus rho infinity. You take out u infinity outside and you divide by d this will become i k phi bar plus half right. Now, you can also have lower surface lower surface. You will have just change l because please do not this I should change it to x star I am sorry because star stopped. Please understand the factor do comes because it is x over two b that is why is that star.

Now, we you see this is the pressure on the upper surface. Similarly, you will have pressure on the lower surface with the same expression one thing is minus. One you have to subtract the other one. Then you subtract this minus of the same quantity you will get that will become plus, but you know that phi top surface and the bottom surface they are anti symmetric therefore, your p u minus p l upper surface minus lower surface I will write it here.

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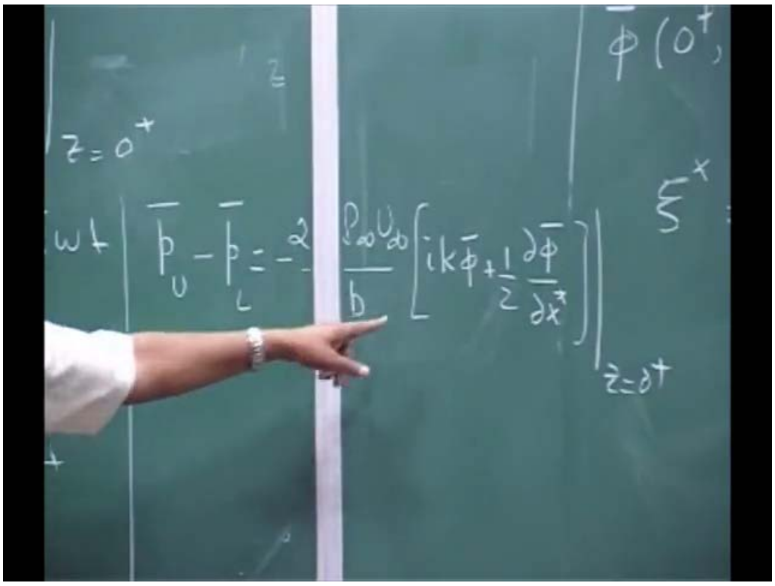


May be there is a little space, it will become p bar u minus p bar lower surface. This minus 2 rho infinity u infinity over d i k phi bar plus half delta phi bar over delta x star at equal to 0 plus. So, this is my differential pressure. Now, I know the pressure the phi bar I know it here that means only thing is, if I know this integral please understand this integral is a function of this omega, that means for every omega bar you will have a value you understand. That is how the integrals are defined later. You will learn how to apply if I have this type how do I solve my flutter problem that we will learn later. First we will get only the unsteady aerodynamic load. Now, let us take this is the pressure that means if I know the pressure I can get the lift, I can get the moment about any point. So, we will write the expression for that and maybe I erase this entire part now.

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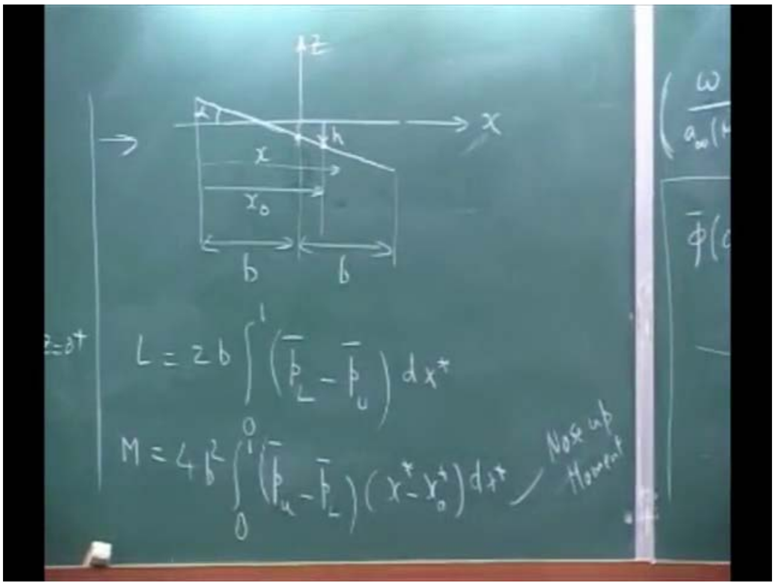


So, my lift expression because if I have my aerofoil, please take this. This is my thing what you will have you will come down because you say this may be your b and this may be your location where it is coming down. This is my h and this angle is alpha this is the centre of the aerofoil this is where my springs are attached. If you go back to you old thing reference. So, this h and your flow is coming this way and you define your distance this is x 0, I put this is semi card this is also semi card b and x is along this direction.

So, you measure x from the leading edge, please understand our integral you measure from the leading edge because the phi ahead of the. Therefore, you always have the integration starts from the leading edge. Then you proceed to any x. So your leading edge always becomes the that is why I put x naught. So, you define this is at any x this is any x and upward, this is your write your lift as two b integral 0 to 1 p bar lower minus p bar upper into d x star lower minus upper means, I just put a minus sign minus and minus that will become plus.

So, this is my left and moment about the elastic axis are the point about which I am mentioning the rotation. So, my moment on the aerofoil becomes 4 b square again 0 to 1. Now, please note I use p bar upper minus p bar lower into x star minus x naught star d x star because this is my nose of moment is taken as positive. So, please note that this is nose up moment because alpha is taken this way. So, p upper minus p lower is the force that into x minus x naught is this. That is what I am saying and that moment is in the nose up direction.

Now, you find out see this expression you have obtained first phi then this phi you have to substitute here. Then you lifted. Now, for the sake of this I will write that this is NACA 846 this is in 1946. Please understand Garrick and Rubinow, how they have represented is they wrote my left as four rho infinity u infinity rho infinity u infinity is that that b k square, please understand h naught over b.

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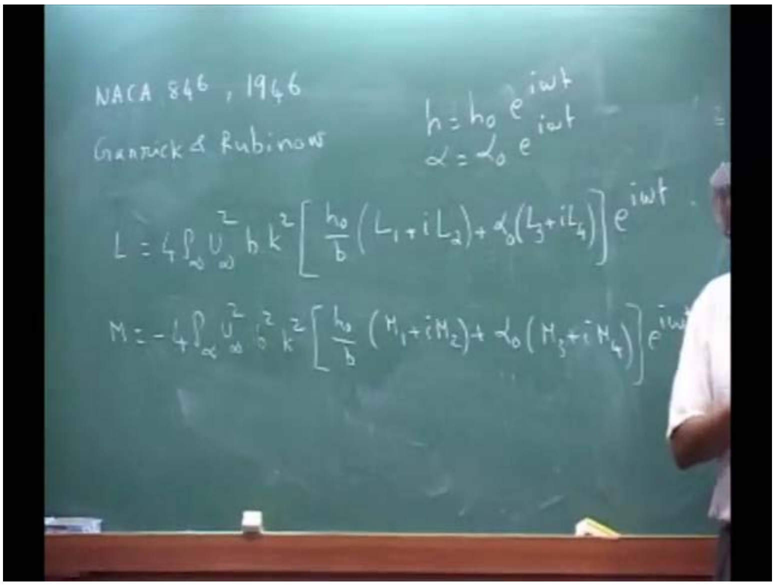


l 1 plus i l 2 plus alpha where h equals h naught e i omega t alpha equals alpha naught e i omega t that means in my motion. I represent them as harmonic and I am writing my lift expression, but please understand it is a complex number l 1 i l 2 l 3 i l 4. Similarly, my moment expression is given as minus 4 rho infinity u infinity square. This is also square I am sorry about that 4 rho infinity u infinity square b square k square h naught over b plus alpha naught into M 3 plus i M 4 e power i omega t.

Now, that reference gives you all these integrals all these l 1 l two M 1 M complete this is an unsteady supersonic flow and they do a flutter analysis also, but flutter analysis part we will do later I am just giving this reference gives the complete details of this. Now, you know in supersonics you can get the lift and moment in this, but of course, you need to know that integral that is the key. Now, can we make approximation basically the pressure expression because you know that this integral you need to know of course, this is available that is what I am saying this integral is available.

It is given in this form please understand even in subsonic the similar form even subsonic flow, but this taper has taken even a trailing edge surface. So, this is not just an aerofoil alone it has taken a trailing edge also the trailing edge can have its own motion. Then they have added 1 more, l 5 l 6 M 5 M 6 and this is the complete unsteady aerodynamic load. The next part is how do you do the flutter that I thought we will first finish all the theory. Then learn how to do the flutter analysis even in the calculation. That you how do you get the lift expression, but the key points are it is a complex number, number 1.

The complex number essentially represents complex number mean what it has a magnitude it has a face because you can write the entire thing as some constant, some magnitude into e power some i phi. Some other r i some face angel motion is at omega t you will have a lift is having a face difference between the motion that is why lift will actually be a little slower depending on how the face comes out to be.

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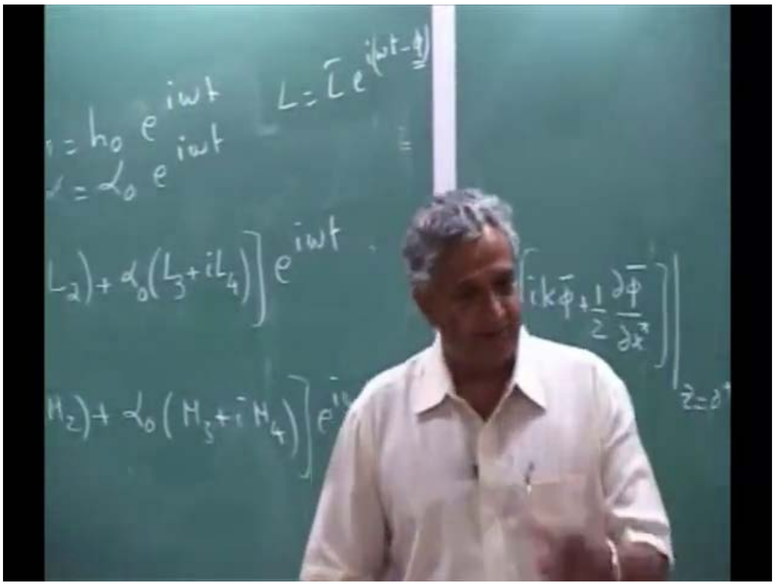

It is like you can write alpha and edge, you say my lift is some l bar e power i omega t. Similarly, my moment this depends on what is it and this face difference is the one which causes actually it will lead to the flutter and when it is adding because you know in resonance in any of the, if you have studied the single degree of freedom system. When you have resonance, resonance you call it what when you say your input you always study the output. Whenever the damping, actually your input value balances the damping force, you will find the resonance will come.

That is what you will have that ninety degree face shift will always be shown when you draw the magnitude on the face. You will find the face curve will go like this at resonance the face is 90 degree. This is similar to that because is a delay in the motion of the fluid which is going around the aerofoil. If the aerofoil is executing harmonic motion the lift will not be at the same face, it will have a different face depending on your omega.

That is what you basically capture because if you assume quasi immediately like what we said every instantaneous angle of attack. You take it you find out the lift then there is no face difference between motion and lift, but as if you include the unsteady aerodynamic theory then you have a face difference between motion and lift. That is like input output like a control system because control theory into this. To see how we have modelled the unsteady aerodynamic in the subsonic case input is motion of the aerofoil output is lift.

Now, this like a control system and you can always find a face difference and that face gives the essentially responsible for all river flutter various types of problems, that is why the flutter analysis itself is done using this theory. You can do flutter analysis like what I did earlier that is you assume that it is a, no face difference very simplistic formulation. I have assumed lift is every instant whatever is the angle of attack that is proportional. If I use that you will get still you will do, but that is a very conservation analysis, this is more sophisticated approach to solving the unsteady aerodynamic problem.

Now, let us see this is without making any assumption on the omega. Suppose, if we make low frequency approximation, that is one end other end is I am making high frequency approximation. These are the two approximations I will make and i will get a very simple closed form expression for pressure very simple.

Then which I directly go ahead then start for my unsteady aerodynamic, I will not get into this, but of course, this there in this reference that is I wrote down because you can download it from the internet. Now, let us take the two simplistic cases. I am erasing this part what was our, let us write the expression for the mu square and from there we will do the low frequency high frequency approximation.

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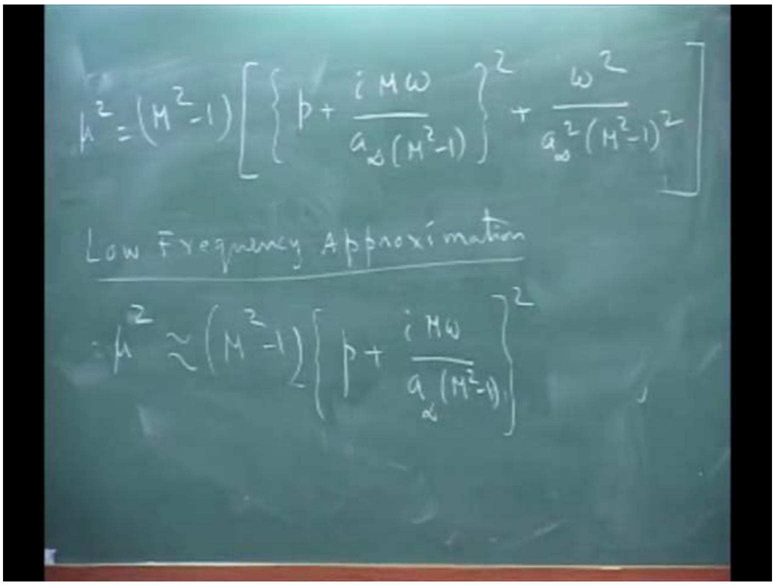


We wrote mu square is M square minus 1 p plus i M omega over a infinity M square minus 1 whole square plus. Let us say I want to do low frequency approximation that means my aerofoil omega is very small. Now, when I make very small I am going to throw this, this term then my mu square I am approximating M square minus 1 into b plus i M omega over a infinity M square minus 1. That is all I am writing only this because I am not neglecting this because if you throw everything out, then you will not get an answer I am only throwing this term out.

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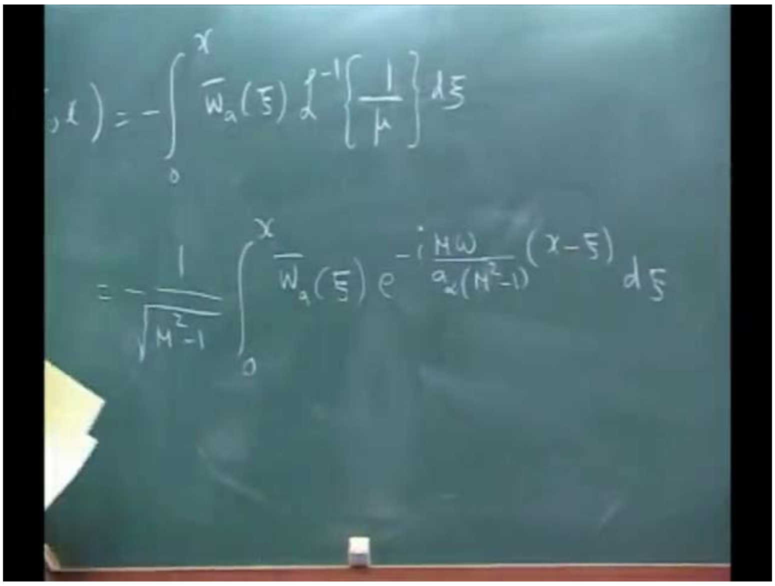


Now, if I write my Laplace inverse because I have simplified this mu square. Therefore, my this is minus as usual, this is what we wrote W bar a Laplace inverse of 1 over mu. Now, Laplace inverse of 1 over mu is very simple because this is nothing but p plus a whole square. So, it is just a exponential term.

So, you will write you answer as I am directly writing the answer, 1 by root of M square minus 1 0 to x W bar a psi e power minus i M omega over a in infinity M square minus 1 into x minus psi d psi, that is all.  This is my 0 plus comma x, you see the difference between this integral and this. Only this is I have a that naught which is a basal function here I do not have that and this is very easy to integrate. What we will do is let us find out the solution. Now, pressure.

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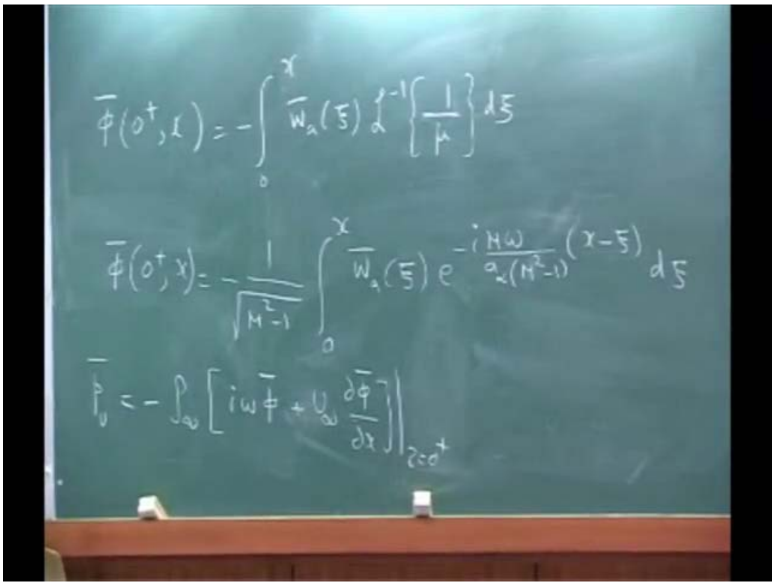


P bar upper is nothing but minus rho infinity i omega p bar plus u infinity delta p bar by delta x at equal 0 plus. Then will subtract this we will multiply by 2 you will get a upper minus lower pressure if I substitute this expression here delta p by delta x. You will find that quite a few terms will cancel out. This is be going to be a very long exercise because it is not may be it will take the entire board I am just for you to that is I am substituting p bar here I will have may be I will write here.

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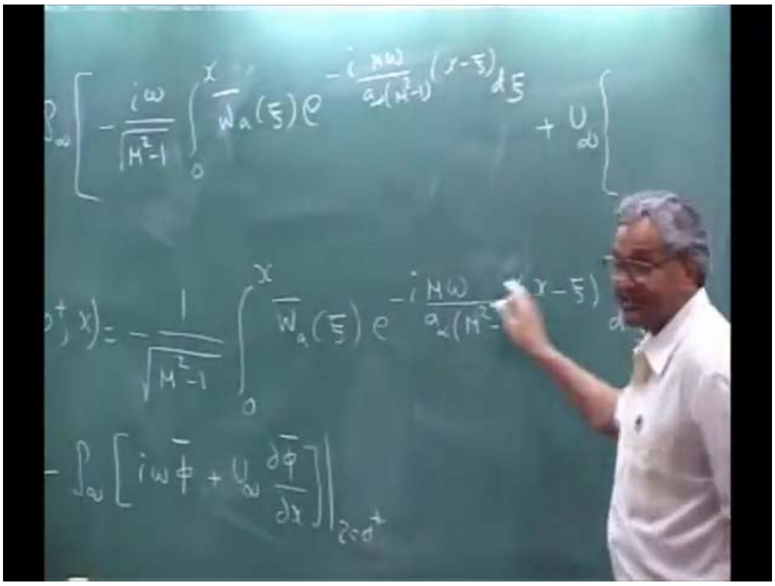

So, that I can go full then we will erase that part that is p bar minus rho infinity i omega phi bar phi bar is here. So, you substitute minus i omega over square room of M square minus 1 integral, this whole expression 0 to x e power i sorry that W bar a psi e power minus omega over a infinity M square minus 1 x minus psi. This is the first term plus you infinity into delta phi by delta x.

You have 1 x term e power minus i M omega a infinity x, that is independent of this integration. So, you can take it outside you can differentiate it another 1 is 0 to x, that x is upper limit again that differentiation. So, 0 to x you will get the integrant itself. So, you will have two terms 1 with the integration another 1 just the integrant. So, you will, I will write the full expression.

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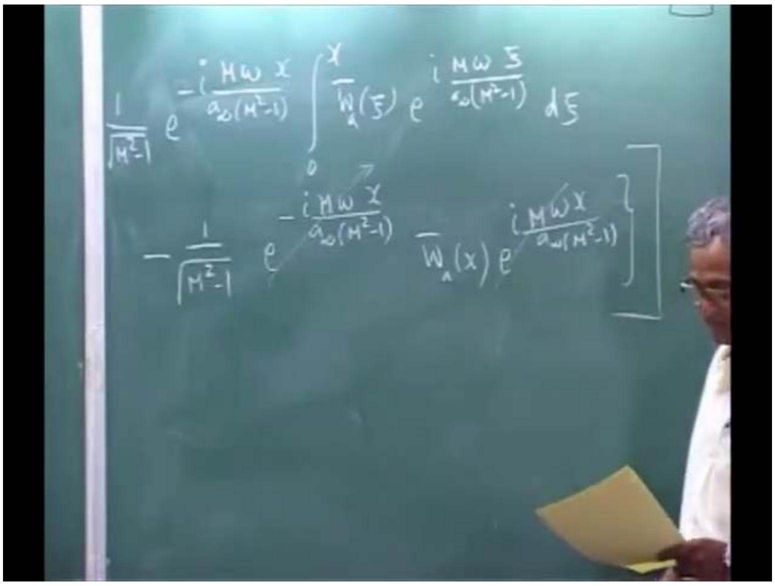

i M omega over a infinity M square minus 1, 1 over root of M square minus 1 e power minus i because I have first differential with respect to x i M omega x over a infinity M square minus 1 integral 0 to x into e power i M omega psi over a infinity M square minus 1 d psi this is I have differentiated only this term. Then in the next 1 I keep it as it and that will be the same integral. So, I will get minus 1 by square root of M square minus 1 e to the power minus i M omega x over a infinity M square minus 1 that is this term, this kept outside what is left inside is, this term which is an integrant 0 to x integral.

Then differential with. So, you will be left with basically W bar a x e to the power i M omega x over a infinity M square minus 1. Now, you see this term you will find this will cancel with this you allow W bar a x over root of M infinity square minus 1 into u infinity. These two terms this can go inside you will get similar, you can have omega and omega sitting there. Now, you what we made was it is a low frequency approximation.

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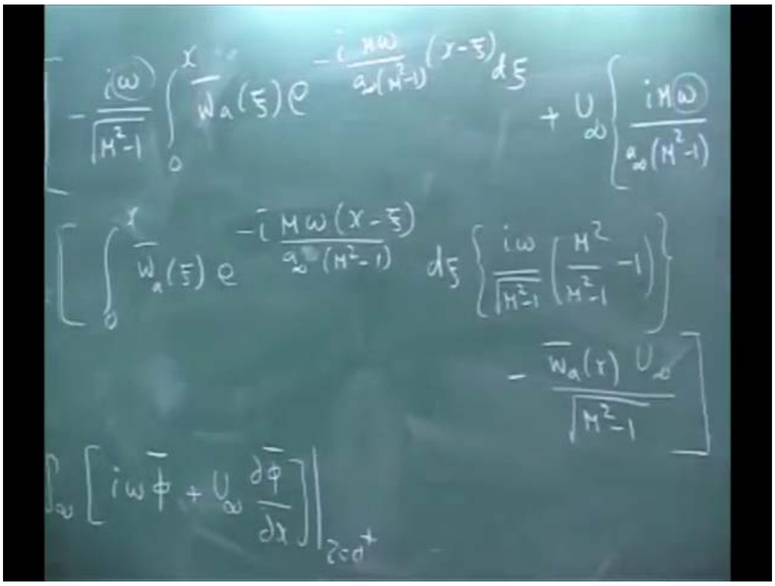

Let me write that final expression, then you will see how it will become highly simplified, that is p bar u is minus rho infinity integral 0 to x W bar a psi e power minus i M omega into x minus psi over a infinity square, a infinity M square minus 1 d psi i omega over root of M square minus 1, into M square over M square minus 1, minus 1 this is the first two terms and the next term will be plus sorry minus, W bar a x u infinity over root of M square minus 1 that is all.

Now, here only you make the that is why this is a messy algebra, but final expression is going to be highly simplified expression because. Now, I make low frequency approximation, that means my omega here I am setting it to 0. I said it is very small in comparison with this term expression like this, I erase this part fully and write a very simplistic answer. This term will be highly simplified p bar upper is because I have substituted everything here.

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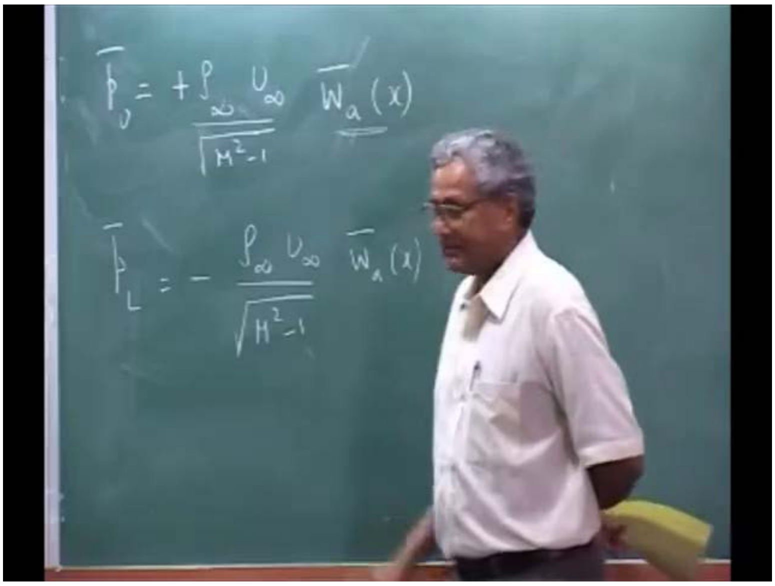


I have got p bar upper is minus rho infinity and this minus rho infinity minus, minus and minus it going to be plus rho infinity u infinity over M square minus 1 W bar a x that is all. This is pressure upper mean, this is a pressure on the upper surface plus pressure lower you will have a minus sign minus rho infinity u infinity over root of M square minus 1 W bar a this is plus this minus. Now, p upper minus p lower it will be two times this. Now, W bar a is nothing but the velocity of the fluid on the aerofoil where it is kept. So, you can write that. Now, I will write that expression because this is highly simplified form. Let us write p lower minus p upper you will have e power i omega t.

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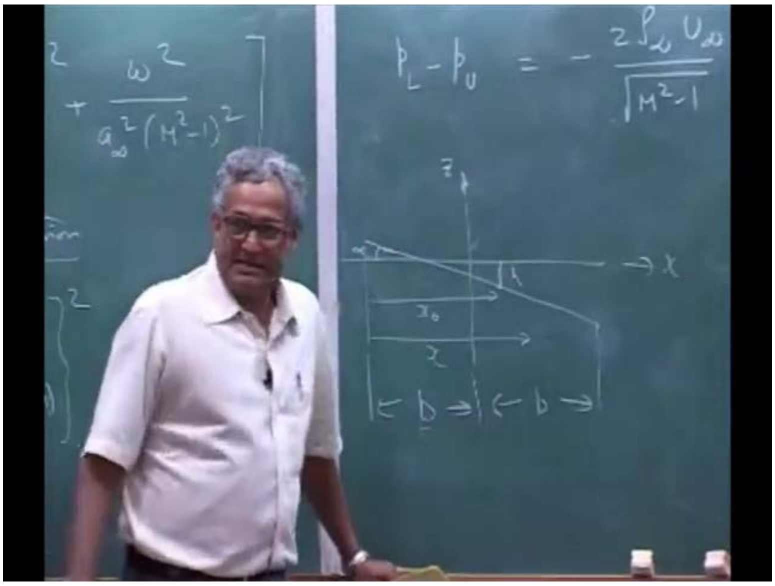

It will come that will be actually p lower minus upper means this minus that. So, you will have two minus two rho infinity u infinity over square root of M square minus 1 W a e i omega t ,that is sorry W a we wrote it as W bar a e i omega t right. So, I have to put e i omega t multiply then I will get pressure this will be W a, velocity of the fluid on the aerofoil. Now, we have to take as usual our reference, that is this point is coming down with h. This angle is alpha and this is the leading edge, you have x naught and this is any x this is b and this is b because this is my and this is my x.

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Now, what is my W a because if you look at your motion W a is written. this is given if you look at the aerofoil thing go back to your notes, you will see W a is W bar a e i omega t which is delta a. This is the aerofoil the displacement of the aerofoil, this is what you are you look back your boundary condition. Now, what is z bar a, a sorry z a is the displacement of the surface because this is a thin line. Now, I am taking a line straight line.

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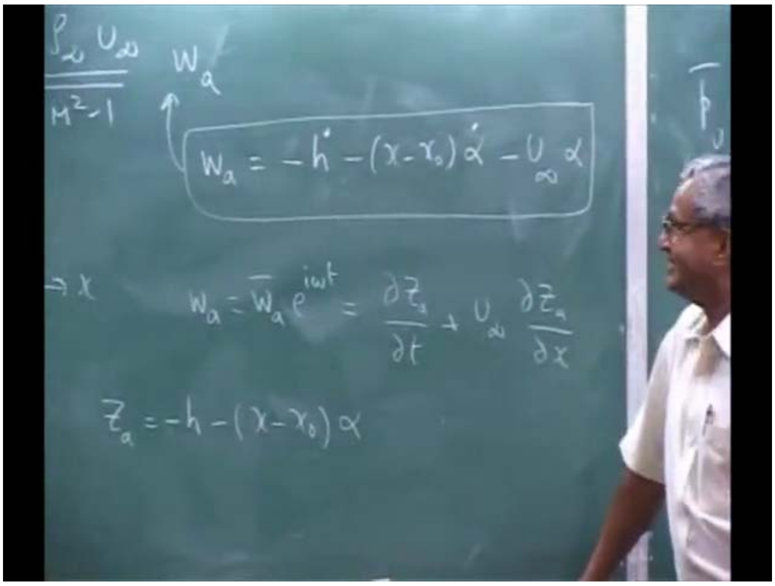


So, my a, because whether it is upper surface lower surface it is a same thing because this a line. This will be minus h minus x minus x naught alpha this is my z a right. Now, what is W a. Now, I have to substitute this back here if I have to substitute this, this is delta a over delta t means my W a becomes may be I write it here my W a becomes delta a by delta t this is minus h dot minus x minus x naught alpha dot. Then u infinity delta a by delta x delta a by delta x means this is nothing but alpha.

So, I will have. Now, pressure upper minus pressure lower is what you just substitute here. Now, I have my pressure difference I can get the lift. I can get the moment in closed form. This is the low frequency approximation, you will get two b this x anyway is a running variable x minus x naught because x naught is a constant. So, you can have the expression for the pressure difference, you can get lift as defined by earlier expression and the moment also by the earlier expression. If you want I can again write it for your convenience because that will become simpler for you. So, I let it be here I write it here.

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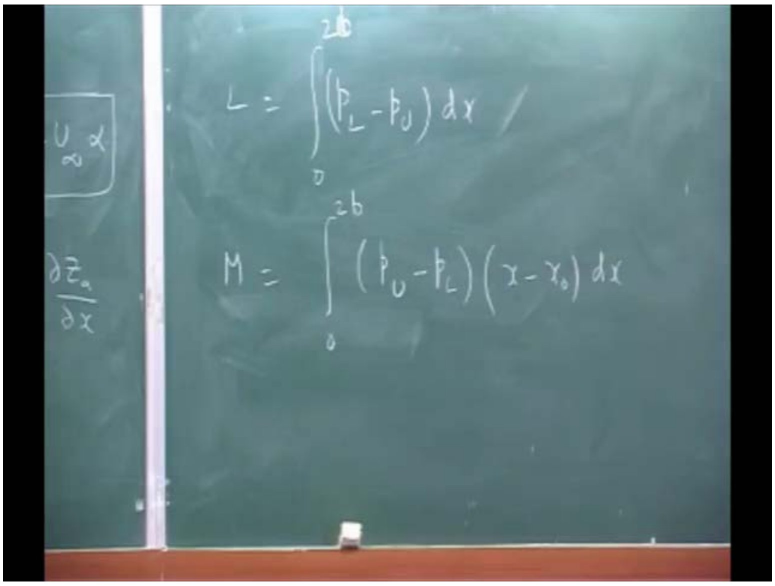

Your lift is integral p lower minus upper into 0 to sorry 2 b 0 to 2 b and the moment you can write it integral again 0 to 2 b we because no sub moment we are taking as positive. So, p upper minus p lower into x minus x naught into d x this is my moment expression what you have to do you just have to take this expression substitute that. Now, you see the pressure on the aerofoil is a highly simplified form highly simplified, just the rho infinity u infinity over M infinity under root. Now, let us then there is another approximation which is sorry this is called the low frequency approximation. Now, there is 1 more approximation which is called the high frequency approximation.

That is this clear because I will erase this part. That will also give you a closed form expression with a little difference you will see this expression. There it will be little different this expression will be different that is all. That high frequency what is done is in the high frequency expression, this term M square is actually what you do is you take it inside you write mu square this particular term, you expand the whole thing p square this will be p square plus two p into this plus this term right.

You multiply this inside this M square minus 1 will cancel with some other terms, but it will also have M square minus 1 p square. You say it is a high frequency because the large value whether it is M square minus 1 or M square, it does not make much difference. It is not a high mark number I am using p as the reference.

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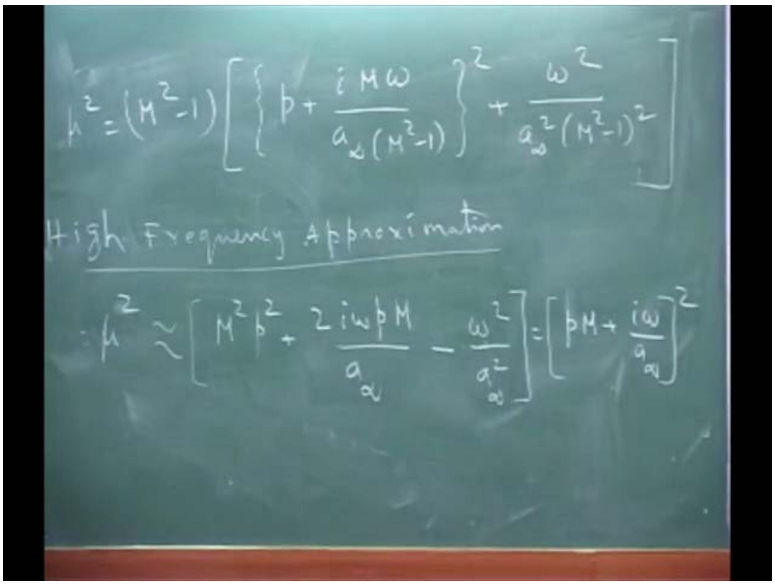


It is going to be a very high frequency. So, I am approximating it please understand M square p square plus M over this term because two times minus omega square over a infinity square which is written as I sorry p M plus i omega over a infinity square. This is the approximation I make you look back your original mu square expression in that make this. Now, this is very simple mu square, then mu is p M plus i omega over a infinity then again I erase this part.

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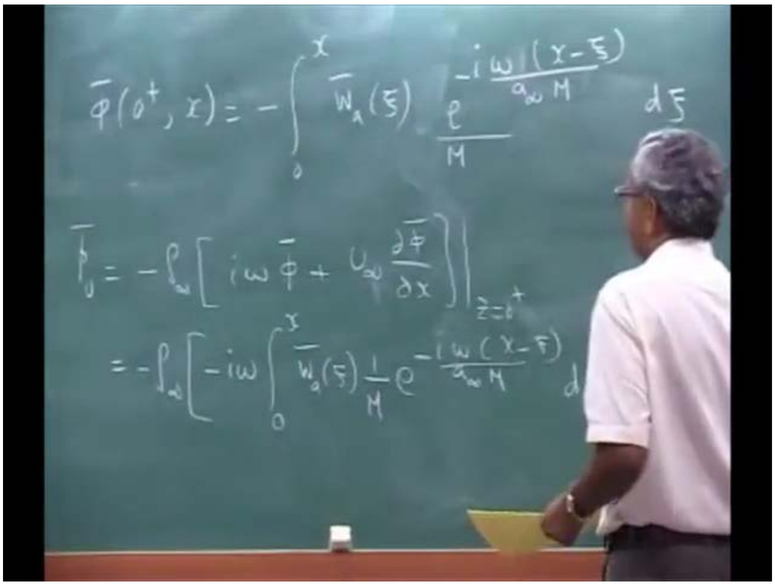

So, your p bar 0 plus comma x becomes W bar a right Laplace inverse of this because Laplace inverse of 1 over mu, that is the same thing you will have e power i omega x minus psi over a infinity because there is p m. Now, with sorry minus sign minus sign this is d psi this is again you substitute in the p bar upper which is minus rho infinity i omega p bar plus u infinity delta p over delta x this p bar. This is equal to 0 plus because this is p upper.

When you do that, this will become again much simplified expression I will just write that minus rho infinity i omega this, this will be minus i omega integral 0 to x W bar a psi 1 over M e power sorry there is a 1 over M divided by some M is there, sorry 1 over M is there Laplace inverse get 1 over, M denominator e power minus i omega x minus psi over a infinity M. This is the first term then u infinity delta p by delta x. Again this has two terms. So, you will get again you will have two expressions it will become with i omega.

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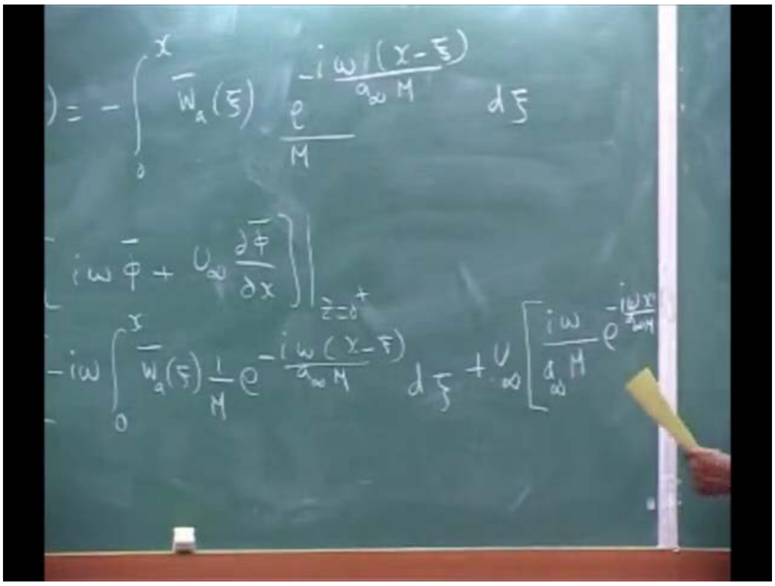

First you differentiate with respect to this. You will have a infinity M e power minus i omega x over a infinity M. Let me erase this part this is going to be a long integral 0 to x.

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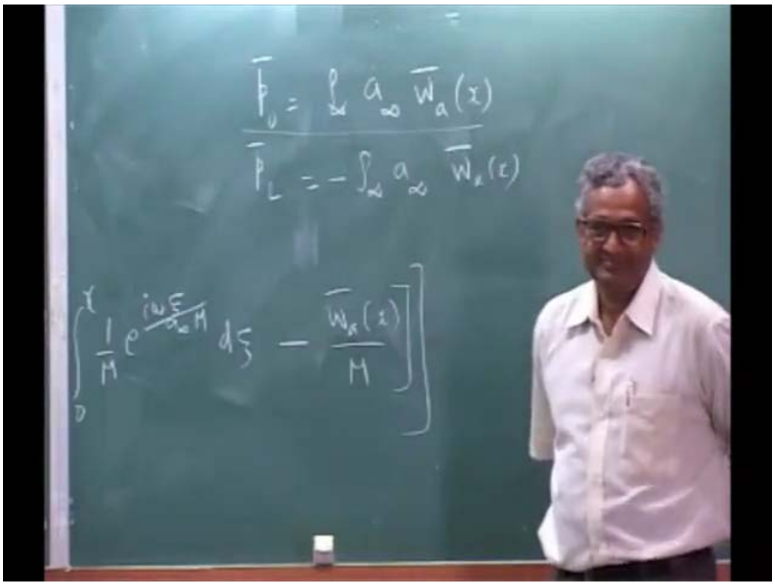

1 over M e power i omega psi over a infinity M d sorry minus W bar a x over m. Here you will find these two terms will cancel out and you will be left with very simple expression, where p u is rho infinity a infinity because u infinity over M is nothing but a infinity. So, this is your expression for the pressure. Similarly, the lower surface p bar l you will get minus rho infinity a infinity W bar a x.

So, you see high frequency low frequency only difference is the term corresponding to this because you will find that these two terms will cancel out because you will see what is no there is 1 W right here. I must put that should be a there is a double because this term is W bar a because you will have i omega because a infinity M is what this is nothing but u infinity, u infinity will cancel out. Then this term will go inside this term is nothing but this term.

So, this is same as this, this is with the minus sign this is with the plus sign. So, you will find these two identically get cancelled leaving behind only this term. This will have u infinity over M with a minus sign which is basically a infinity and that minus and minus with the rho it will become. So, this becomes as the high frequency approximation.

This is nothing but density at infinity speed of sound at infinity. This is nothing but the velocity at the point where you are having W a. This theory is called piston theory and this is what is used in supersonic panel flutter calculation everything. So, that is called the piston theory I erase this part. I will just briefly describe why it is called piston theory. So, you had two approximations one is the high frequency approximation. Another one is the low frequency approximation this is high low frequency, you had what are they take place.

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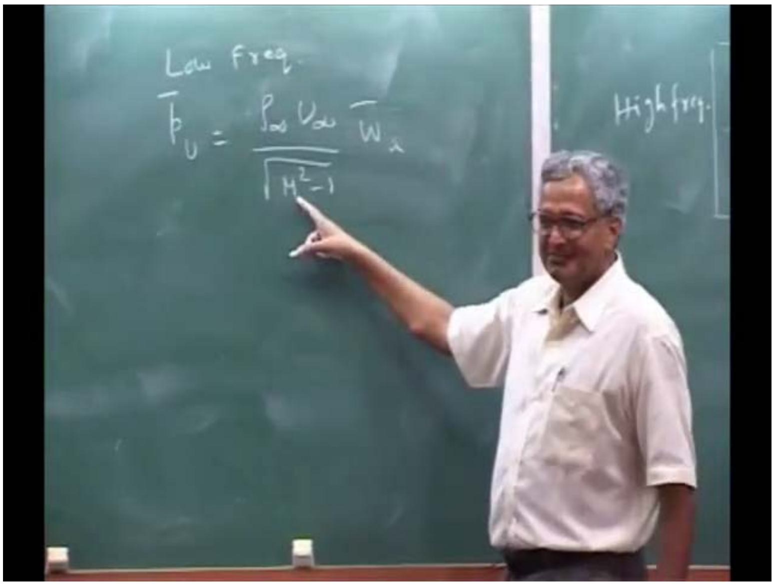


So, if you right the low frequency and high frequency. You will see this is the low frequency, you will have p bar u as this is the high frequency. So, you see the difference only thing is instead of u infinity over. Now, you may say if the mark number is very large this is nothing but what this is one. You will cancel out, you will get speed of sound, then that sorry u infinity over a, a infinity u infinity will cancel out. You will go to a infinity that is what piston theory is but, however theory is valid up to mark number two three.

So, that is why one is a low frequency approximation another one is called the high frequency approximation. So, we essentially have. Now, in supersonic theory a closed form expression for pressure on the aerofoil. Only thing is we have to define what is my W a, W a is you know that that is delta z over delta t plus u infinity delta z by delta x because W a is the velocity of the fluid on the aerofoil. Motion of the aerofoil is given by z a please understand, that is why you should not confuse with that here.

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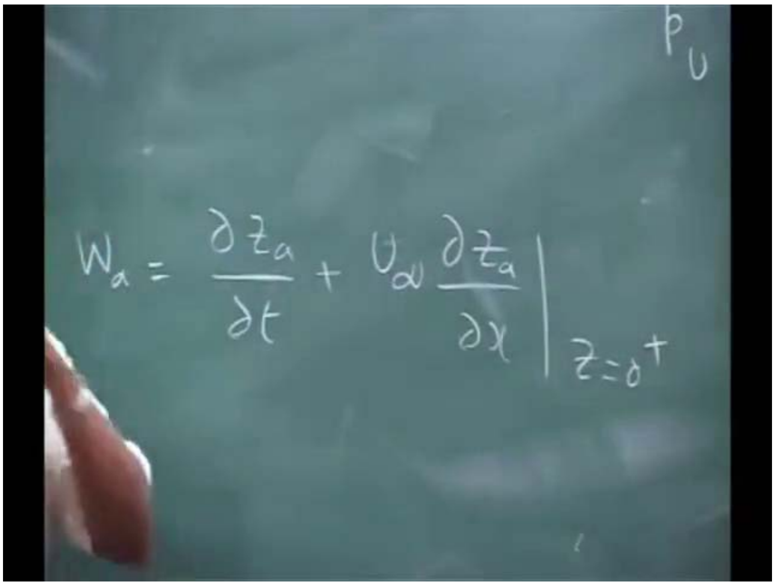


You remember we wrote W a is delta t u infinity delta x at equal to 0 plus minus. This is flow velocity this is the body velocity. Now, you know the body motion, flow velocity is equal. So, you simply substitute body motion into this. That means you have your pressure in terms of the body motion. Then you can solve your equation that means my load is known and I can use essentially what is the external force.

External force is now a function of the body motion please understand, but this is not a complex number here you know here what happens is this directly you are giving only a factor and the motion. You understand h data all product you will directly put it there use the exact nose approximation for omega then you will start having the complex. So, basically you will solve for flutter problem complex Eigen value theory. Actually complex Eigen value problem you will not solve, but that we will learn at the later part.

Now, I thought I briefly mentioned this is for two dimensional structure. You know, you can always write the equation of the surface based on our geometry what is the motion and you can plus 0 minus is only because you know thickness is not generating any lift. So, will have the mean line, only mean line and you can immediately write your load. Now, this theory is very simple, even if you have a panel like a supersonic flight the panels. The panel will vibrate and the piston theory essentially means each small area is considered as though it is a piston.

It is a long piston and you just move whatever is the pressure there, that is the pressure given by that expression. That is why it is called the piston theory very approximation because you can do I have that derivation. May be I will just briefly mention to you why it is called the piston theory. I will just briefly do very simplistic problem. Suppose, you have a very this is the highly simplified you have a very long piston. You say you assume and here.

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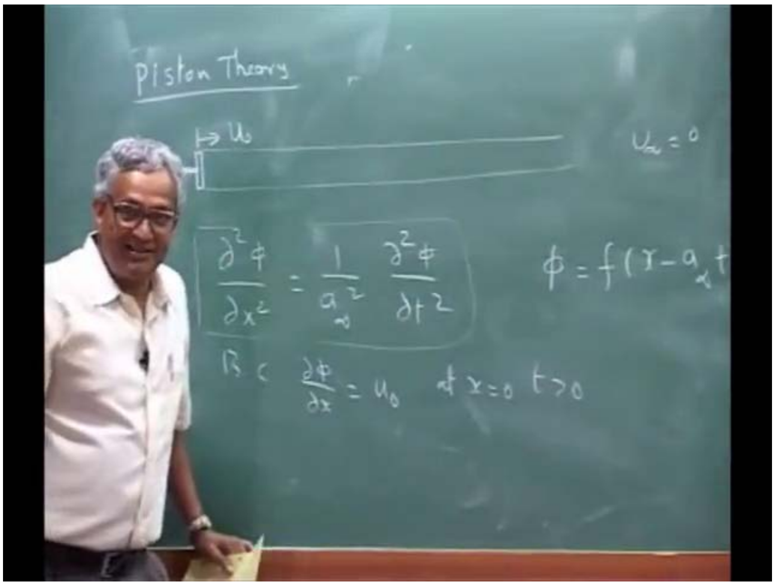


There is no flow this is a stationary you put a piston here the piston is given a velocity say u 0, what is the pressure of the decks on the piston. So, you see this is a long tube I move the piston with the velocity, what is the pressure that acts on this piston. That is what our problem, once I have the pressure then that is exactly equal to that then I will know. This is a 1 dimensional disturbance problem. So, potential flow 1 dimensional is, 1 dimensional problem with boundary condition delta p over delta x is u naught at x equal to 0 for t greater than 0 because x is 0 is this part.

Now, for this the solution, this is the wave equation, you can always write solution as phi as f of x minus a infinity t, but I will take only the a minus a infinity t because the wave which is propagating on the right side because you can have wave propagating on this side wave propagating. On this side this is the wave which is propagating on the right side.

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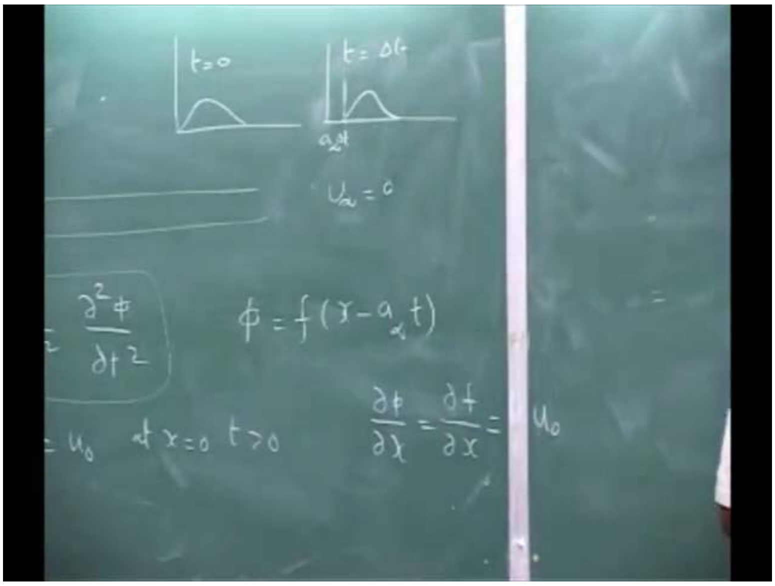


If you have a wave something at t equal to 0. This wave moves at t equal to some delta t it has moved here like this. That means this equation is what because you are only shifting my because the wave moves with the velocity a infinity delta t. So, my x axis gets simply shifted. This is the right travelling wave left travelling wave you put a plus sign. That is why you use this solution only for this problem with the piston moving because you do not take the other solution.

So, when you have this is the solution you can find out what is your pressure because you know that delta phi over delta x is nothing, but delta f over delta x which is u naught. The pressure on the piston what is that p minus p infinity this is given as minus rho infinity what was that delta phi over.

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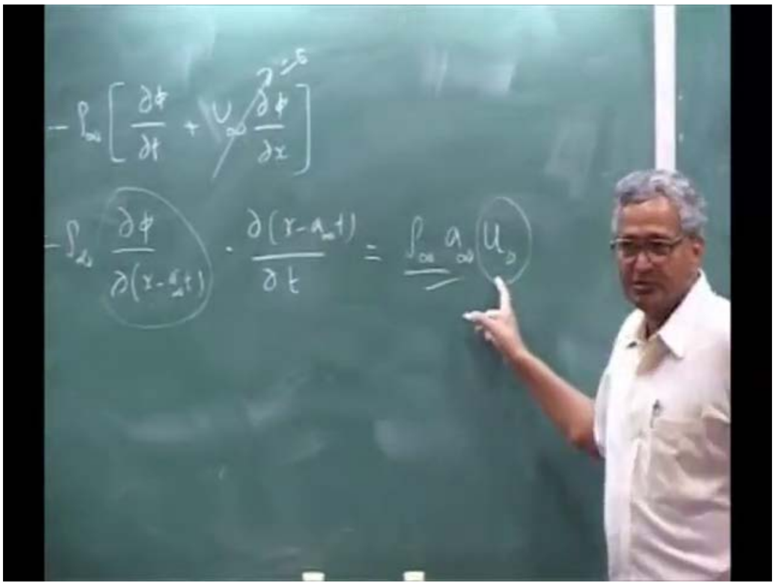

Right, but this is 0 because u infinity is 0 because it is a stationary fluid. So, you will this will be what minus rho infinity delta phi by delta t delta phi by delta t will be delta phi over. This term into you can write it as delta phi over delta x minus a infinity t because this is 1, 1 term into delta of x minus a over delta t which is nothing but this is what delta phi by delta x that u naught. This is u naught this is minus a infinity.

So, you will have plus. So, rho infinity a infinity u zero. Now, you see what is W naught, W naught is velocity of the piston at the left hand W a is what this is nothing but the fluid velocity which is basically, you have to write it as the piston velocity because W a is given by.

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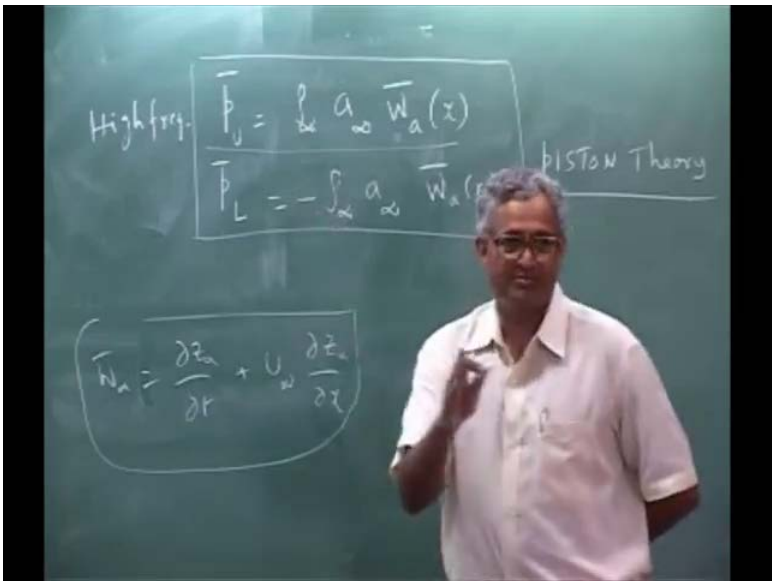


What you know that delta a over delta t plus u infinity delta, this is the body motion. So, essentially the velocity of the body at that point. So, this is the piston theory, that is why in supersonic flow on the surface, if you consider every small region act like a independent piston, with this 1 long cylindrical like, if this is kept up like this because in the aerofoil if you take it like this, this is like a long cylinder. You find out whatever is the velocity there you put it there. So, each is a cylindrical long infinite cylinder. So, this is the piston theory, but high frequency, low frequency is little different.