Aero Elasticity

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Lecture – 18

Today we will see the steady flow or a thin aero foil in steady supersonic flow.

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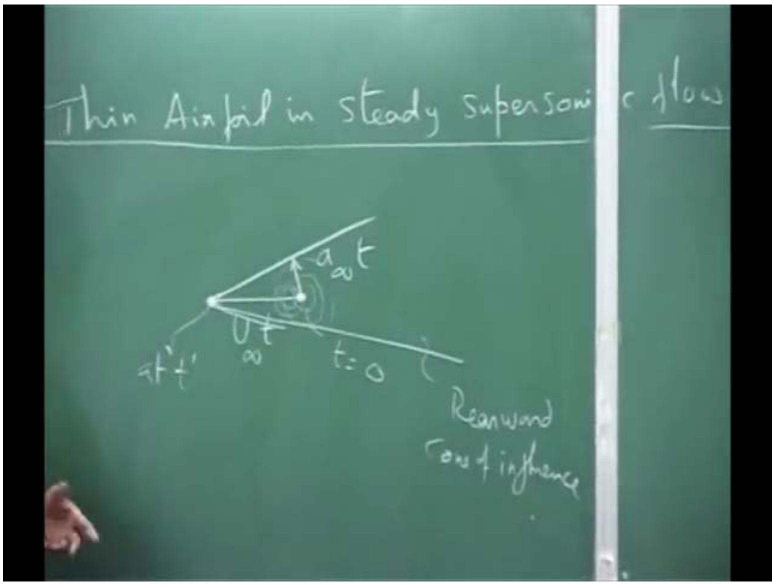

After that we will study the un study supersonic flow because this is the convenient one of the easiest problem, in solving the potential flow equation supersonic flow is the easiest. The reason is because flow is supersonic any disturbance in the flow it can go only at the speed of sound or we make the approximation that the local speed of sound is very small, the change therefore, it goes with the a infinity only. As a result the reason of influence is slightly restricted. We will see that particular part later you will understand oh that is why this is much easier to get the solution for a supersonic flow.

Suppose, you say the body this is moving at a supersonic speed that means in a time t should have gone through a distance of U infinity t because this is body, U infinity is the speed of the body is going in air. Now, but because it is in this position at t equal to 0, it will occupy this position at t because it was at this instant, it would have given that disturbance.

The disturbance will start travelling a speed which is only a infinity t because a infinity is speed of sound, but the disturbance travels at a infinity. Whereas, the body travels at U infinity and U infinity is greater than a infinity. Now, if you see at every point it will create a disturbance. So, here it is just at t it will create. So, you will find that there will be a cone of influence. In the sense when the body is travelling.

In these medians there is absolutely no disturbance to the flow and only the air within this cone is called actually mark cone or something like that. Only the disturbance will be restricted to the region behind. So, this is called the real world cone of influence. Suppose, if you say what will influence this body because I the problem in a different way saying that instead of body moving forward, I say that the body is stationary.

The flow is coming here, in which case this is the body, flow is U infinity what will happen the disturbance at time t equal to 0. Now, I am changing it, it would have given a disturbance that disturbance would have travelled by a distance if U infinity t, but the effect of that the speed of sound will go only at a infinity. Whereas, that disturbance will be washed backwards at the speed which is U infinity t.

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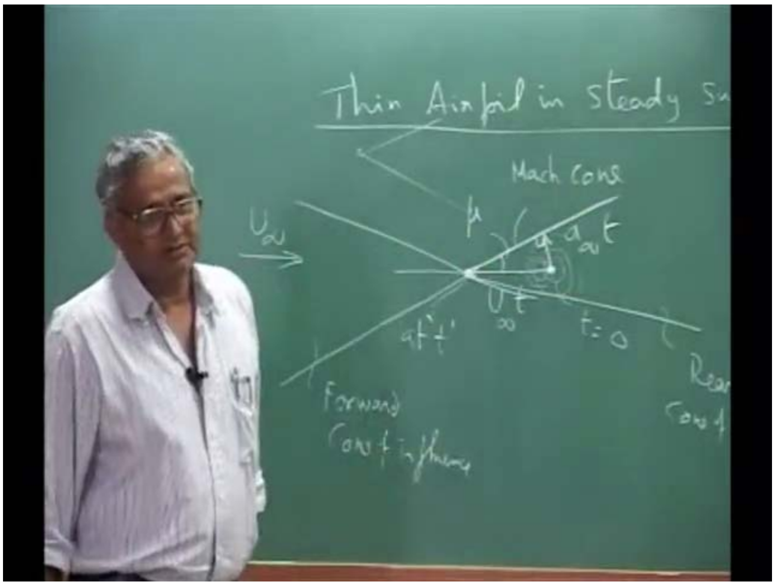

Now, if you see that if this is stationary which point will influence, means any point this is the forward cone of influence, that is if I have a point here, that will be disturbed by this, but it will be within this cone. Suppose, if I the point is here it will not be disturbed. Suppose, I have a point here the cone of influence will be like this, that means this point is outside that. So, it cannot influence this point. That means all the point which are in the forward cone of influence, they will affect the point of interest. So, that is why in the supersonic flow you always enforce this.

This is called the radiation condition or anything like that, what is the reason of influence, your flow will be affecting. This actually makes the problem much simpler whereas, there in a subsonic flow, the body moves slower whereas, the disturbances move faster. So, it will go and affect the flow everywhere. That will affect this also, that is the reason subsonic flow is much more complicated because the entire region will be affected. Whereas, in the supersonic flow only this picture region will be affected. This is called the mach cone.

Now, you can find out a this angle is mu and this is always 90 degree right because a infinity. That is the speed with which the disturbance is propagating speed of sound. So, a infinity. So, sin mu will be wait, that is this is the mach cone angle you can say. Now, using this basic simple thing you will go on then start solving the problem of a steady supersonic flow 2d problem. As I will not go into the lot of details in this and essentially the what is our equation, potential flow equation in steady supersonic flow that is.

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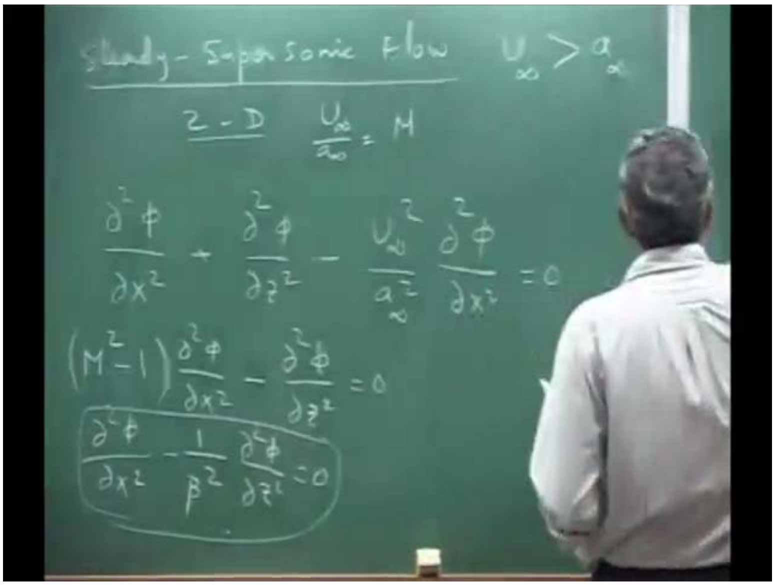

2 d that means U infinity is greater than a infinity. In 2 d your equation because small disturbance equation I am going to drop the hat which we have used. Your potential equation is this minus because this you can reduce it from your equation, which we

developed last class. This is my differential equation, small disturbance potential flow. I only remove the hat because I thought I do not want to carry the hat everywhere. So, this is just one. Now, this equation you can combine this term and write it as, this is U infinity over a infinity. So, call it mach M mach number.

So, you can call it M infinity or m. I am writing M square minus 1 del square phi over delta x square plus, oh not plus minus, minus del square phi over delta z square equal 0. This term I am going to call it as beta square because please note that this is greater than 1, this is greater than 1. Now, I can write this equation as del square phi over delta x square minus 1 over beta square equal 0. This is my steady 2 d, equation this is the wave equation, Now, for this the solution if you write.

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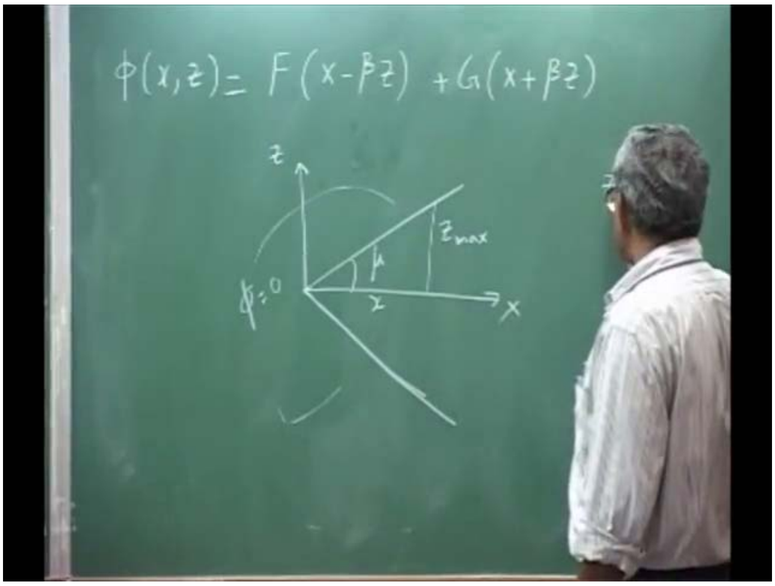

Solution, any function please understand of the form this, where F and G are functions. Any function of this form will satisfy this equation because what you do is you differentiate del square phi by delta x square. That means del square F by delta x square and the del square phi over delta G square means you will get beta minus meta one more differentiation you will get minus beta. So, beta square that beta square will cancel out with this paving behind it is only is. Similarly, weather you have this also, x plus beta G is some kind of variable. It will always come to this is the wave equation.

It is how wave equation, you find time and space, in use. This is actually the of wave equation, but we do not have time here. It is only in x and t. Now, you see this function any function F and G will satisfy this. The only condition which we have to use is, the bond a condition are the aero foil. Then this ration condition, this is we you will use it in identifying which solution, that is you have F is 1 set of function G is another set of function which is valid or what mentioned that is all.

So, let us take the radiation condition and this is our, this is our G this is our x axis. Please note this is my mark this is the leading edge you can take in the leading edge or any point or only take because you take the leading edge, later I will say. This is my mu everywhere here, phi is 0 because there is no disturbance because the flow is nothing, it is not disturbed at all because of the presence of the body here.

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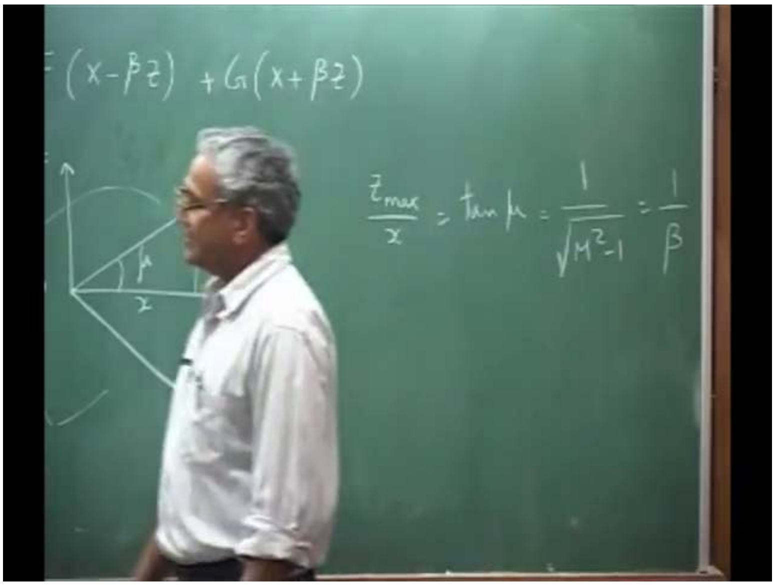

Now, if you write the equation for this cone. You will have, this is G max this is x.

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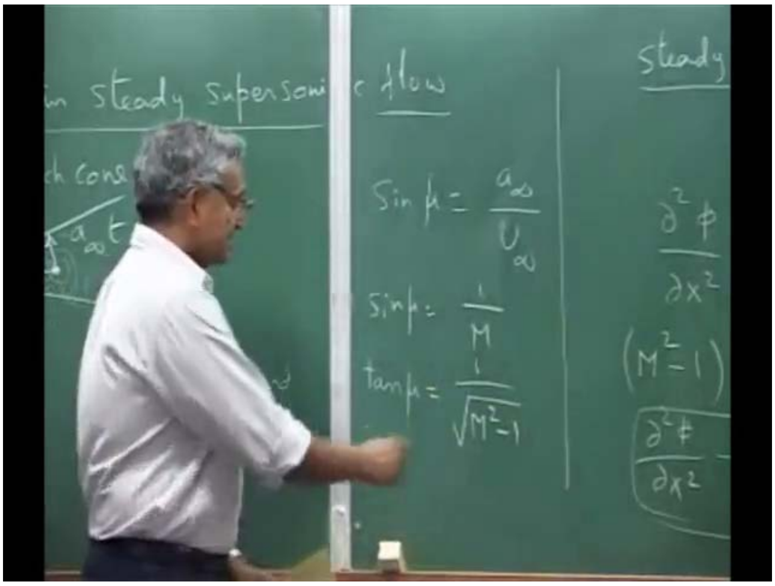

What is the maximum value of G, G max is over x is tan mu, but tan mu this is actually sine mu is 1 over M. So, tan of mu is what opposite side by hypotenuse 1 over what 1 over root of M square minus 1 this is tan mu. So, I am going to write is here tan mu 1 over root of M square minus 1 which is nothing but 1 over beta because we have defined beta as M square minus 1. Now, you see for any G because this is for the positive, please understand this is I am writing for 1 G max G is G is positive. So, the cone of influence, if I am looking at it the region of the positive z axis, positive z the value has to be G is less than almost equal to G max.

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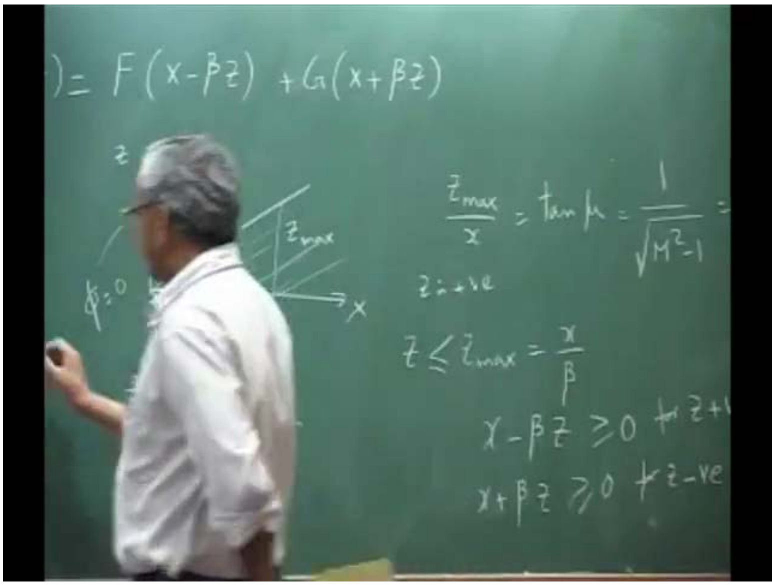

So, you can any point here, but G max is what x over beta or I can write x minus beta G because I multiply I bring it this side. This is for G positive, this condition should be satisfied x minus beta G must be positive ok. Now, when you take the negative side, then what will happen G max is minus sign. So, you will get for the other side, you will get x plus G, x plus beta G. This will be greater than 0, this will be greater than 0 for G negative.

That means essentially what it means is you can draw many lines like this, lot of line if you fix x minus beta G. Once if you fix a particular value it is positive value. If it is 0 that is this line with the slope temporarily mu because that is what is there. You will get it because x minus beta G is equation for a straight line and G is equal to you can put it G is that x over beta. It is a straight line with a positive slope of 1 over b that is all. This is another line, but the, only thing is greater than 0 any value you can fix that means if whether it is 1 2 3 4, whatever value. That means you will have a series of line, that is it. It will be the influence that is any point if you say my aero foil is only here.

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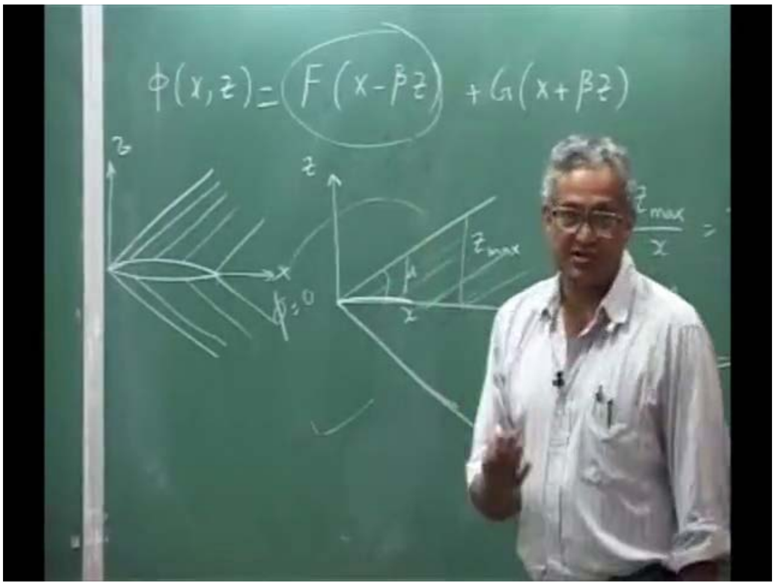

If I draw the aero foil like this may be I will draw a picture. My leading edge is here and this is my x this my z and it will have line like this, line like this right. These lines are constant values of x minus beta z you fix 1 value. This is some other value may be any other value you fix positive value you will get this line. This is x plus beta z constant line that means x minus z greater than 0 is for z, positive which means I. this particular function because x minus beta z or taking positive values because once z is negative it is always positive, please understand. When G is negative this is always positive, that is why that will be not it will restrict by this line. That is the reason x minus beta z is chosen for the solution when z is positive.

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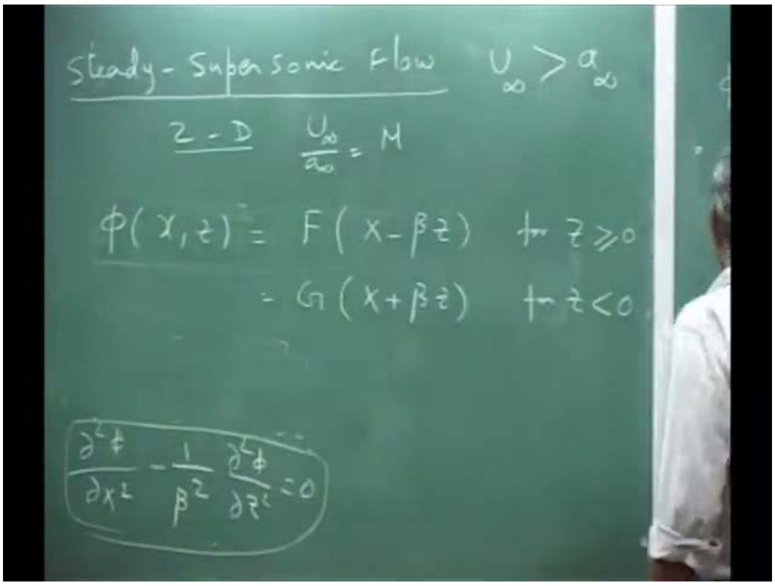

So, I will write the solution which is like this, my phi which is the function of x and z is equal to F which is x minus beta z for z. Then the other is same thing equal this is G x plus beta z, for z you can take less than 0, equal to 0 is almost that equality sign is standing everywhere. Now, you see I have two sheets of solution. Now, using this I can go back and then get my physically pressure etcetera everything. Let us apply the boundary condition what is the boundary condition.

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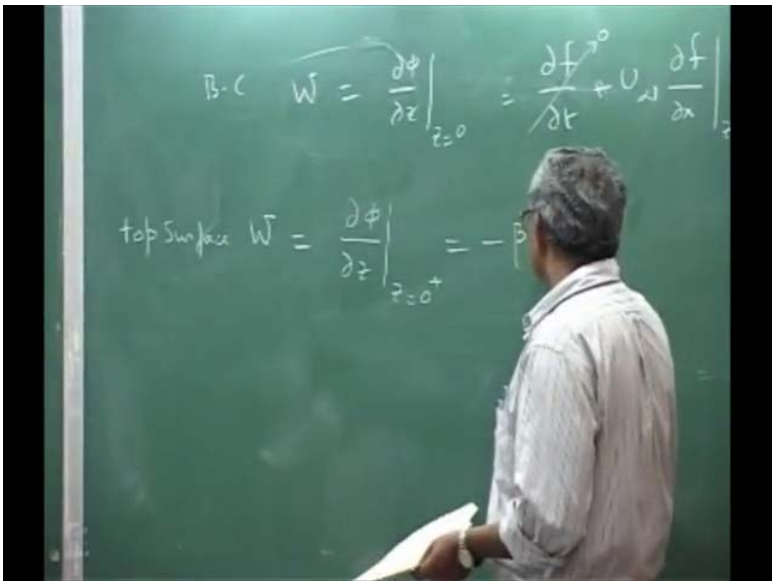

Boundary condition is w, which is the air flow in the normal direction, on the aero foil that must be equal to. This is basically delta phi by delta z at 0. This is for a, we wrote like this, U infinity delta x at plus or minus depending on top surface or bottom surface, but since it is a steady flow this is not 0. So, you left with simply w is U infinity delta F by delta x. Now, if you take surface w under top surface which is delta phi delta z.

At z equals 0 plus top surface top, top surface this is equal to. You know phi, delta phi by delta z is what nothing but minus beta delta F by, you can call this function as a and this as b. I am just calling them, just for because x minus beta z I am just calling it by the variable a and x plus beta z I am calling it as a variable b just for case of differentiation.

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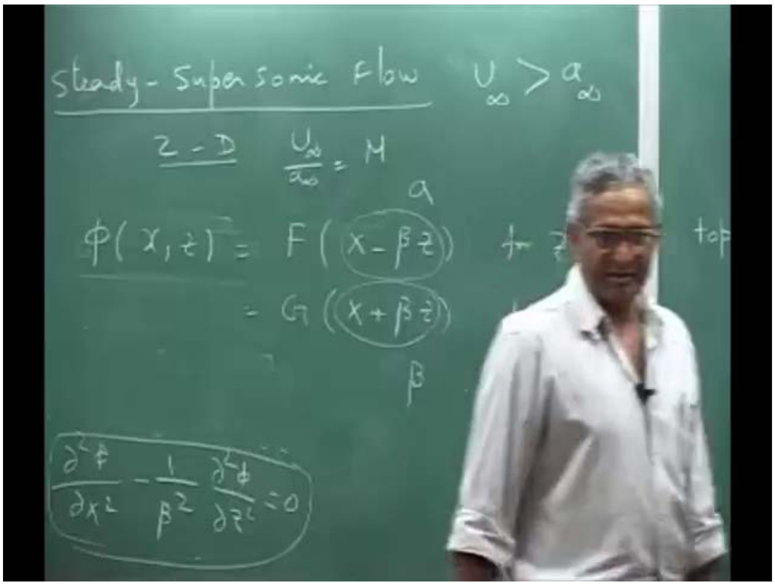

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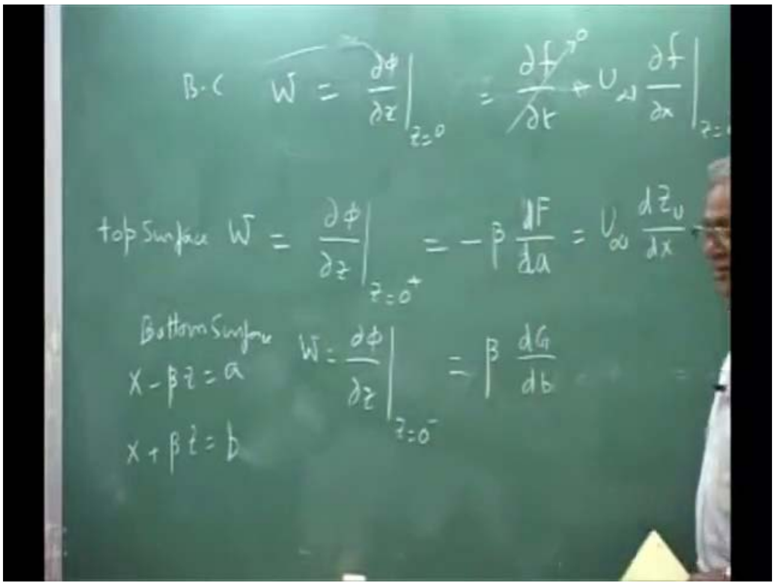

Since, it is only a function I have only totaled the ratio right. That is what you will get. Now, this is equal to U infinity top surface is d z top that is upper surface over d x, upper surface over x. When you go to the bottom surface, you w which is delta phi over delta z at z equal 0 minus because you know z less if this is the solution.

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So, you will get beta d z over d b, which is again assumed for U infinity d z lower surface by d x. Now, we need to what is delta F by delta x delta phi by delta x because the our pressure expression. May be I, I erase this part what is our pressure phi infinity phi minus phi infinity that is minus rho infinity U infinity because it is only a function of x.

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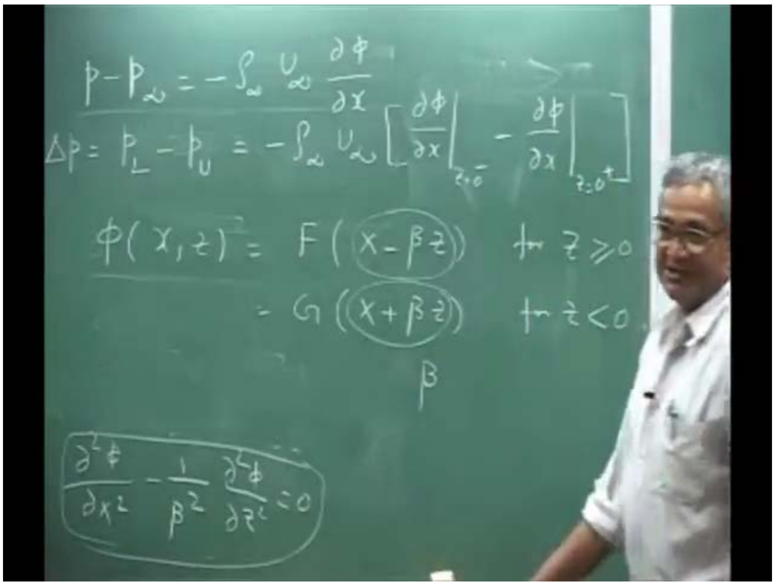

So, will have delta phi over delta x, this is the pressure at any point. Now, you say top surface you will fit upper and lower is phi lower, but delta phi. Now, let us take this idea may be I will write it. The pressure difference here phi l minus phi upper because I am taking differential as lower surface minus upper surface, phi lower it will be minus rho infinity U infinity. Please understand phi infinity is common that will cancel out. You will have delta phi over delta x.

Lower surface z equal to 0 minus, that is the lower surface minus because you are going to minus up the same thing for the upper surface, that will be delta phi over delta x at z equal to 0 plus. Now, you know the 0 minus is this function, delta phi by delta x is nothing but delta G by delta b, delta G by delta b inverse. So, you will get delta phi over delta x at z equal to 0 minus is nothing but d G by d b which is equal to U infinity over beta b.

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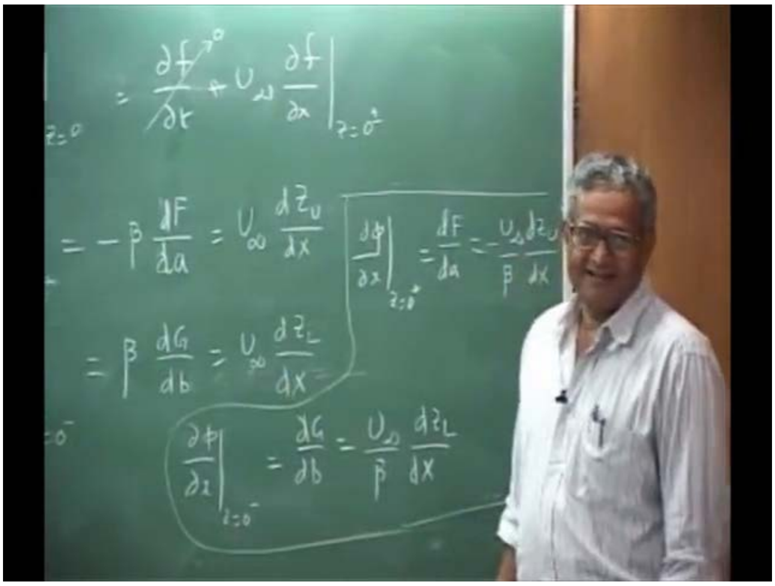

When you look at this delta phi by delta x z plus you have to put delta x at z equal to 0 plus is nothing but delta F by delta a because this is a same function. So, delta F by delta a that is d F over d a which is minus U infinity over beta b z U over d x. So, you now take this two and substitute here.

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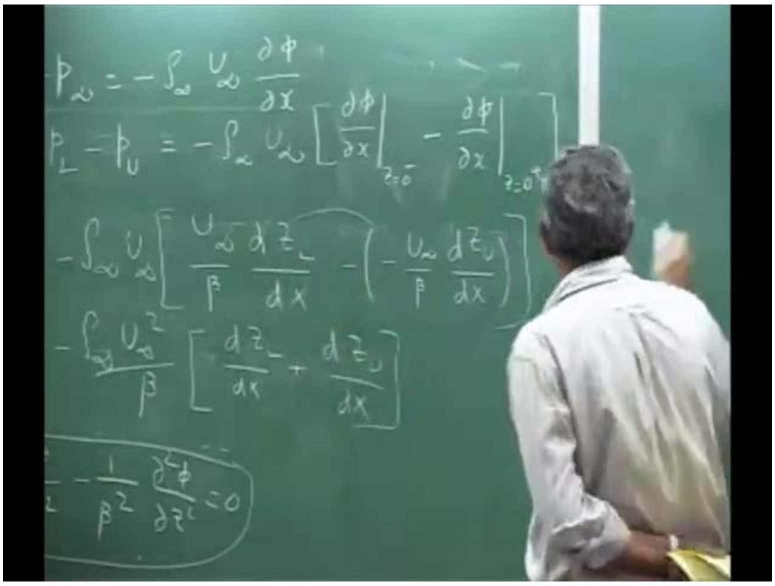

So, if you substitute you will get your, this is minus rho infinity U infinity phi by x 0 minus, that is this you will have U infinity over beta, d z lower by d x, that is a lower surface, minus of that other term minus. That will be minus U infinity over beta d z upper by d x. So, you combine all, you will get minus rho infinity U infinity square over beta d z l over d x plus d z U over d x, this is the pressure difference here. This is the local pressure difference top and bottom surface on the aero foil. Now, if you know please understand this is what it is. If you know that top surface equation, suppose if your aero foil is like this.

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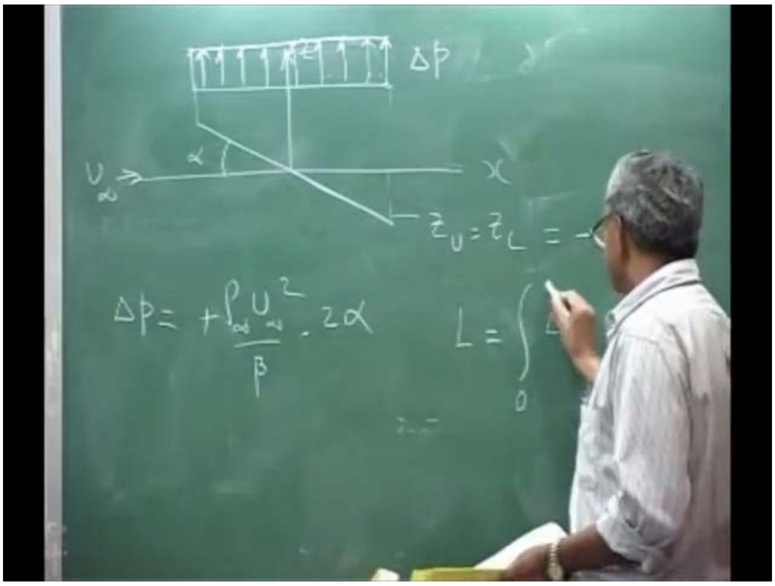

Why it is supersonic flow, supersonic air foils are this type of straight line as because they are just related on the slope. The local slope on the surface determines what is the pressure on the surface. Suppose, if I take a line this is my aero foil. I am having this is alpha U infinity, this is x I am taking here z, please understand I am taking z from the midpoint. Now, what is in if it is the air, this is the thin aero foil flat plate which is you can have the that is that because thickness does not contribute, you can take the in line.

Now, what happens if you take, this is z upper is both are same. What is a equation for the upper or lower surface this is nothing but minus alpha x right. The equation for this is z alpha x. Now, if take and substitute there I will get what the pressure delta phi will be. So, as a result d z l by d x is nothing but minus alpha this is also minus alpha. So, you will get your delta phi, the pressure difference between lower and upper surface, here for an angle of attack alpha that will be minus rho infinity, U infinity square over beta times two alpha plus.

So, if it is a positive angle of attack, this is the flow which is coming U infinity. So, you will get this is the pressure difference. Now, the pressure difference is at constant because this is a flat plate. So, your pressure difference will look like exactly it will be like this, maybe I will draw the pressure is this is a constant. This is my delta phi positive, that means I can get my lift, lift is just the integral of this. So, lift becomes integral delta phi over the whole chord 0 to c I can have d x.

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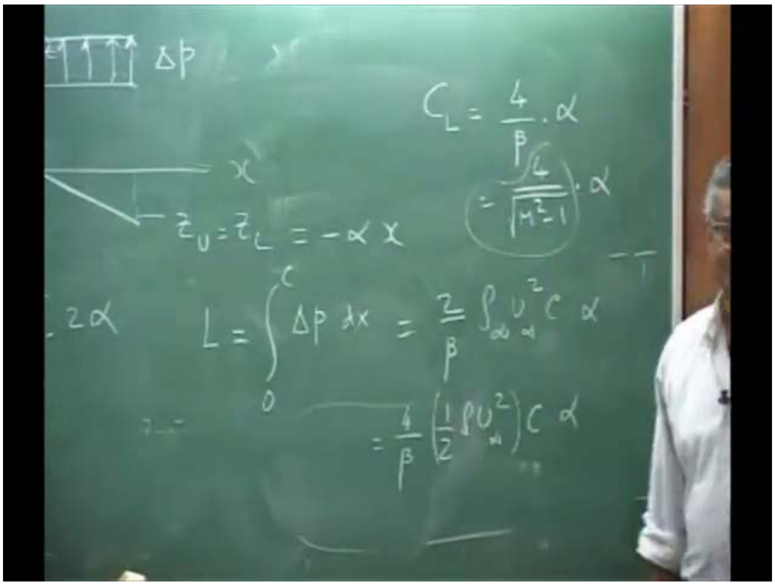

Since, this is a constant you will have this is nothing but two over beta rho infinity U infinity square chord into alpha, which you can call it as dynamic pressure, which you can call it four over beta dynamic pressure is half rho U infinity square, area is c and you will have c l which is. So, your C l for supersonic, C l for supersonic for the aero foil is four over beta which is beta is you know what that what is that, root of M square minus 1.

So, you will have four over square root of M square minus 1 times alpha. So, this is the quantity which is C l alpha, that is lift, is this. See subsonic you always say two pi where as in the supersonic this is four over root, but subsonic compressibility. When you take it you write it as two by root of M square minus 1, that is 1 minus M square you will write it, here it is M square minus 1 and this is the C l alpha.

Now, you know what is the pressure and where is the aero dynamic center, aero dynamic center definition is the point about which the moment is not changing. That means the moment does not change about the bit point bit point moment is 0, moment is independent of angle of attack. So, the aero dynamics center for supersonic flow is at mid chord aero dynamics center for supersonic flow is at mid chord, at mid chord only subsonic it is a quarter chord point because based on the depression you always have this moment is independent of angle of attack ok.

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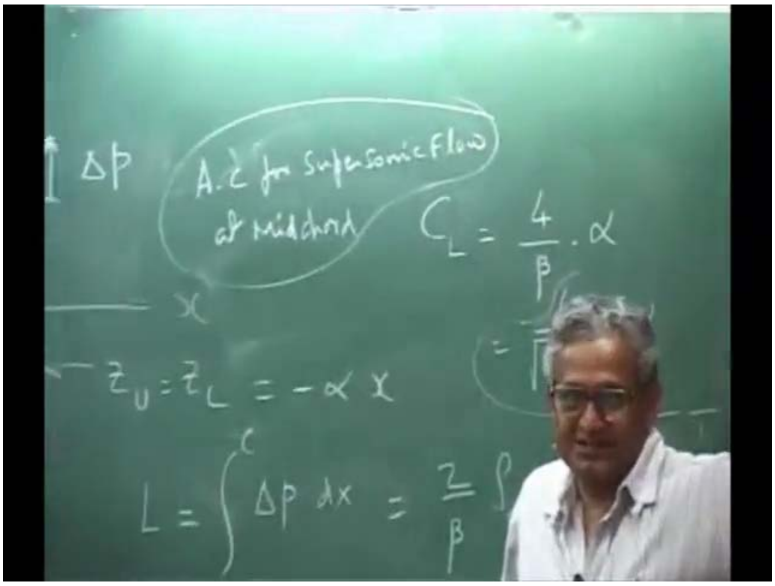

Now, you see you have an expression for lift, lift is purely four over beta into dynamic pressure chord alpha. Suppose, if it an unsteady aero foil, the same approximation which we made earlier when we were doing the simplistic analysis of flatter. You can use it, that is you see effective angle of attack.

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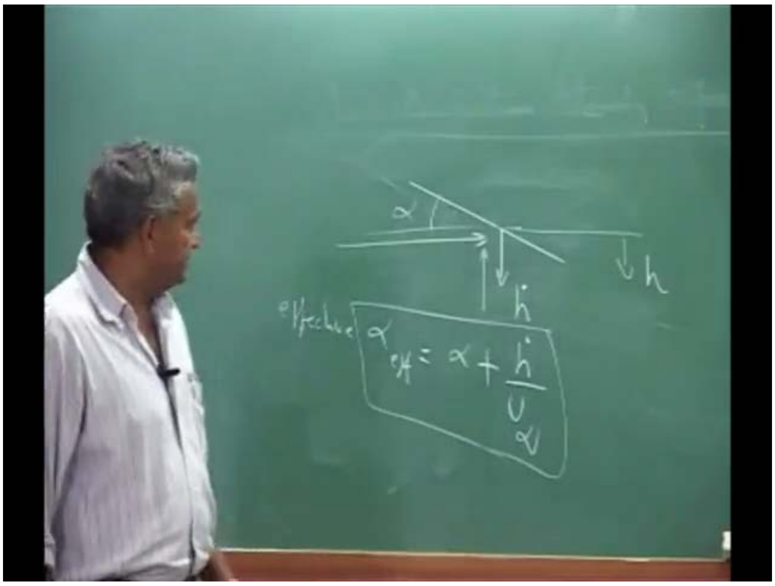

Suppose, your aero foil is also coming down with a velocity h dot because you know that this is how we define our h and alpha for aero foil which we define like this. Now, you simply say what is that effective angle of attack. Effective alpha effective is this will come alpha minus because this is body is going down. So, that is equivalent to the flow is coming up. So, it will be h dot over U infinity.

That means this expression directly you put it, that is why I know I am going ahead and solving an unsteady aero foil in a supersonic flow. This is this is very crude approximation, you follow. I think now we will have this is the steady flow, but you have done this steady flow you, you know it right, this part. These are all clear now we will go to unsteady supersonic flow, but again two dimensional alright.

See this problem of small oscillation of pings in supersonic flow, this was done in 1942 by complicated approach later in 1946, Garrick he solved this problem using some sources pulses, but now I will use totally different approach which is purely surplus transform method. Which will give you mathematically much easier, you will be able to get the result.

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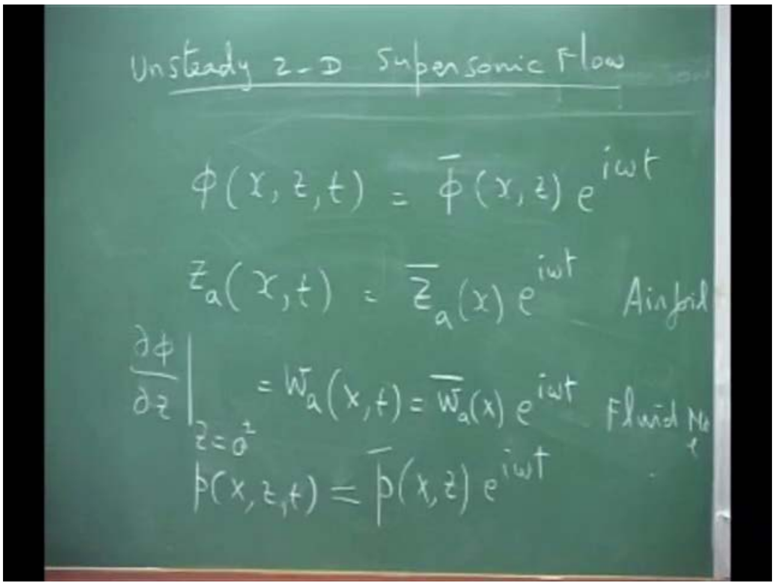

So, unsteady 2 d supersonic flow. Now, before I go on then start deriving for the arbitrary motion supersonic flow, what I normally do is, that my aero foil is executing a harmonic motion, please understand. All the unsteady aero dynamic theories, developed with the assumption that, the aero foil is executing a harmonic motion. Now, of some frequency that is treated as a omega because any signal, you can do for a transform because we are solving linear problem, linear small disturbance potential flow problems.

Therefore, for a transform we can use, that is the reason all the theories assume first that, my aero foil is executing a harmonic motion and develop the unsteady lift and moment on the aero foil. If this is my steady motion that means that is the starting point, even then you do subsonic. It is not that I take arbitrary unsteady means what, what is unsteady what kind of a motion the aero foil is doing. See in the steady it is clear, it is not doing anything except that the flow is steady.

Now, here what type of motion you are having. The flow is steady, but my aero foil has to do something. So, what is that motion I would not give suddenly some step input or change the angle of attack and reduce it those thing I can solve later. Once I know the solution for steady harmonic motion.

So, this the first assumption that harmonic motion of the aero foil is assumed, when you make that assumption you immediately say my phi x. So, automation potential I am going to write it like this, phi bar. Then z which is the aero foil just to indicate for my own convenience, we can instead here calling upper and lower, we will use a symbol a to indicate this is the aero foil because you have a upper surface, you have a lower surface z positive z plus negative something.

So, instead of calling both I am going to write the upper and lower surfaces by 1 symbol, later we can substitute wherever z a is there upper for upper lower for lower. This is which is a, function of x and t in the aero foil motion. This I can write it as again z bar a which is the function of x e power I omega t, this is air foil motion, please understand. So, technically what I am saying is, if my aero foil is steadily oscillating, my bottom pressure everywhere is also of the oscillatory type, with the same because it is the linear problem please understand.

If it is a non-linear I must add having higher frequencies various things whereas, in the linear problem if something has excitation frequency is omega. So, everywhere the flow is also of this same frequency. Now, your delta phi over delta z which is actually the velocity or flow at 0 z equal to 0, you can say 0 plus 0 minus, you can have anything you want I am going to use another symbol. This is w, which is the at 0 plus, but I am going to use again 1 symbol just to indicate I am looking at the w z equal to 0 plane.

My main interest is what happens at the aero foil. So, I will put 1 please do not confuse this I will put a subscript a, this is on z equal to 0 plane. This is again a function of x and t. This z a is aero foil motion, w a is fluid motion on 0 plus. This again I am writing is as w bar a x e power e omega t, this is the fluid motion fluid motion at z equal to 0. You can take plus minus wherever it is, if it at the aero foil, you will write it at the aero foil, if it is behind the aero foil it will z plus minus that is all because you should have continuity of the flow.

Now, my pressure, also varies may be I will write it here. My pressure at every point is also varying as phi bar x comma z e power I omega t, pressure is also. This is at any point. Now, using this I am going to write my unsteady supersonic flow equation in 2 d. Then the boundary condition, boundary condition is what this is equal to delta F by delta t plus U infinity delta F by delta x and I am saying F is aero foil this is z a.

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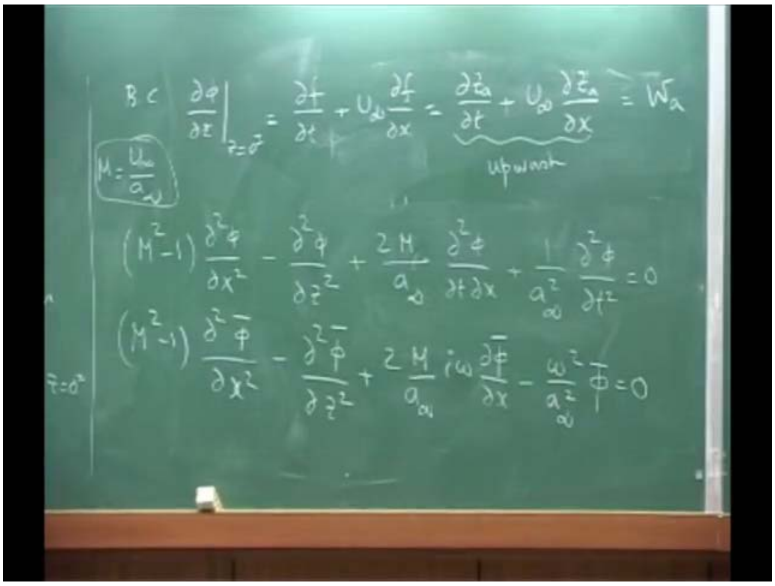

So, I will write it as my B C is delta phi over delta z at 0 plus minus, you can have both top and bottom which is nothing but this is what we normally use. U infinity delta F by delta x, but since I am writing F first z a, I will write it as delta t plus U infinity delta z a over delta x. This is on the aero foil it is nothing but if I restrict to the aero foil, that is nothing but w a. If I look at only the aero foil section this is nothing but delta phi by delta z which is w a.

Only in the aero foil zone, now there is a term which is this particular quantity is called the up wash. This term, this is purely defined based on please understand air foil motion because z a is aero foil. So, delta z a, if you know the motion of the aero foil you can write delta z a by delta t time derivative you can take and this the space and multiply U infinity. This is the up wash at every point that up wash is basically equal to the fluid velocity at that point that is all.

Now, this is all our basic, I would say the starting point. Now, that has gone right the velocity potential equation which is M square minus 1 del square phi over delta x square. This is the supersonic flow minus del square phi over delta z square plus two M infinity sorry M over a infinity del square phi over delta t delta x plus 1 over a infinity square del square phi over delta t square equal 0. This is my 2 d unsteady supersonic flow, where mach number as usual U infinity U infinity over a infinity.

So, M some people may put M infinity because just to denote that mach number, but we are made assumption that local speed of sound is basically a infinity, that is the reason we are not distinguishing between M infinity and M. Now, you say my phi be a harmonic function right. That means I can substitute this in this place and e power I omega t will completely cancel out from here. Let us substitute that when you substitute you will have M square minus 1 del square phi bar over delta x square minus del square phi bar over delta z square plus two.

This is delta phi by delta t you will have I omega. So, M over a infinity I omega delta phi bar by delta x minus omega square over a infinity square phi bar this is my equation. Now, though it is a complex please understand I am getting a I omega here. It is a partial deferential equation what I will apply is because you know from theory of Laplace transform because 1 of the important thing is, if we pick over x axis system on the say this is my aero foil this is my x axis.

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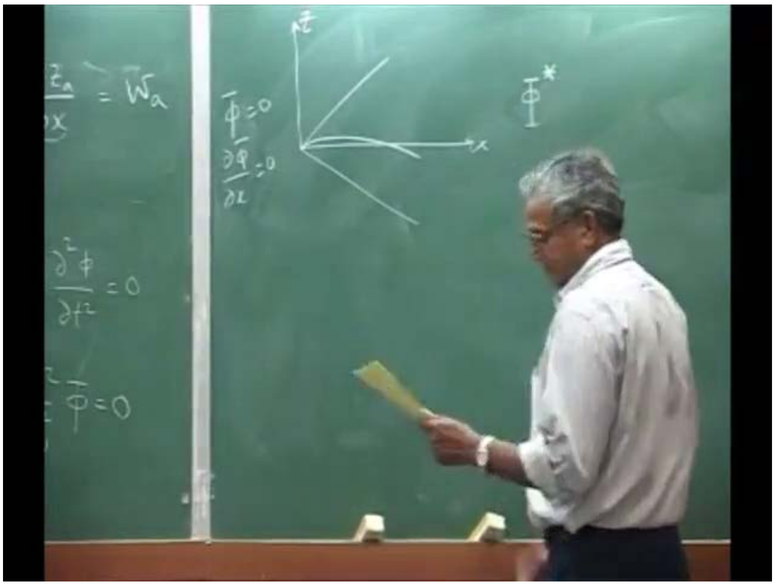

This is my z axis, you know that you will get a mark cone. That means all these places when x is negative 0 and then derivative is also 0. There is no disturbance to the flow you can say delta phi bar over delta x 0 delta phi bar by delta z is 0 everything is 0 for x less than 0 right even beyond some z they are 0, but that is ok.

Now, this is phi d I can apply Laplace transform, by applying Laplace transform the partial Laplace transform in what, to x please understand. I will do a Laplace transform to x then this phi d will be a ordinary deferential equation in z. That is what is done in getting the quick solution, but this is where mathematically it becomes a little messier, but the approach is, you apply Laplace transform to this equation for x, the variable is x then you will get a word e. Then word e you can write the solution.

Then you apply the radiation condition after that you will get a solution. Then you have to inverse Laplace transform to get my phi bar, phi bar which is the function of x. Once I have inverse Laplace transform phi bar, I can go and get the pressure, pressure is expression minus rho infinity U infinity delta phi by delta t that term will come. So, you have the pressure expression. Once you have pressure expression, you can get the lift you can get the moment.

In words I have described this, but how do we do this part. I will go up to some step, say that this is what is being done. After that we will simplify for, low frequency approximation and very high frequency approximation. That means low frequency high frequency they write main this omega because then you can get a closed form solution. Otherwise it will be left with some integral, this integrals have been evaluated in some publications which may have used.

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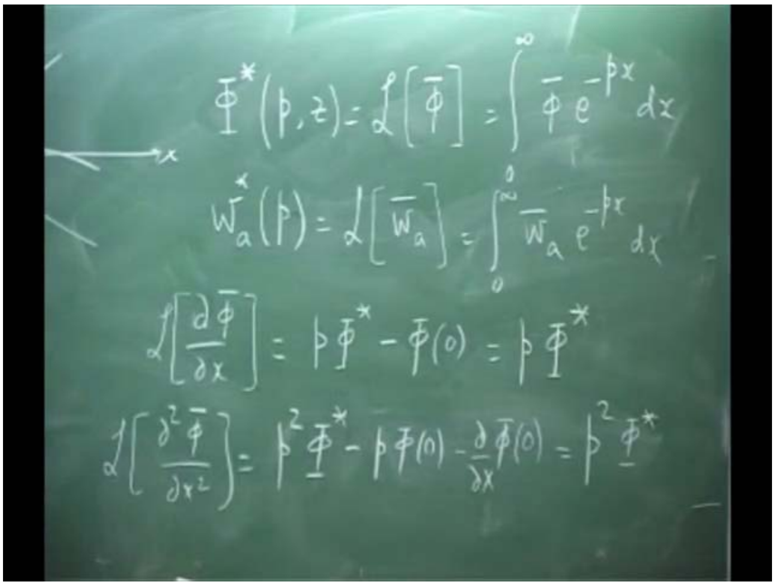

Now, let us apply Laplace transform in x. So, that is because of this p 0 this is 0, let us write Laplace transform I am going to use the symbol p star, p is my Laplace variable p, G is Laplace of phi bar which is defined as 0 to infinity phi bar e power minus p x d z this is the definition. Now similarly, for the w the fluid motion that also has to be changed to Laplace. So, w star a Laplace of w bar a because this is the function of. So, I can get Laplace, this is integral 0 to infinity w bar a e power minus phi x d z because this is the fluid velocity and you apply to boundary condition also everything you have to apply, that Laplace transform.

Then you know that Laplace transform of d phi by sorry I put delta phi by delta x, phi bar. This is p, p star minus initial condition 0. So, you say I am going to have my p initially 0. So, this becomes p, p star. Similarly, Laplace of second derivative, it will have del square phi bar over delta x square this is p square, p star minus p, p bar 0 minus delta by delta x phi bar 0 that is gradient.

So, you just assume again all my initial conditions are 0. Therefore, this is phi square phi star. Now, what you do is you have the Laplace definition here, convert this equation apply Laplace transform to this. So, I am erasing this part completely and write the Laplace transform. What will happen is, maybe I will go here I will start here. So, what will happen is your equation will become if you apply please understand Laplace transform to that equation this will be your.

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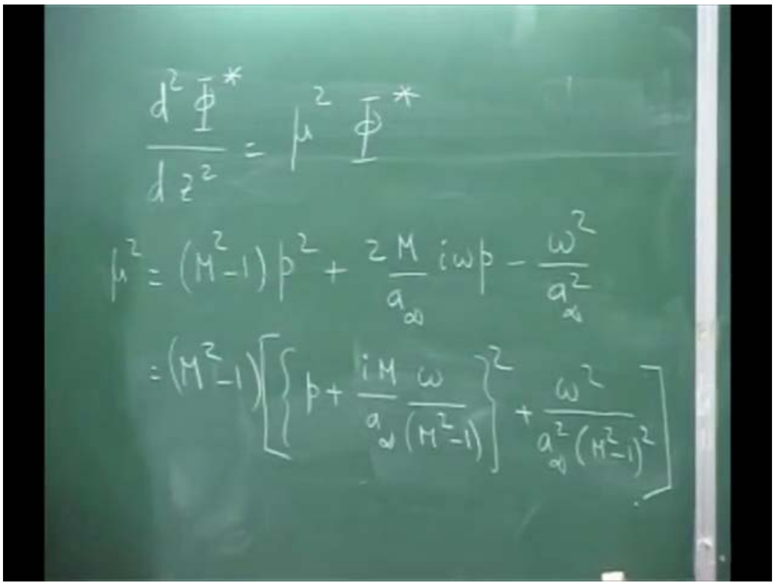

Where mu square is M square minus 1 into p square that is what I am saying. This is phi bar surplus of this will become p square p star, it will stay as it is. Here this will become phi and the last 1 will be as it is. Now, you are essentially writing the a infinity I omega p minus omega square over a. This is the mu square and this term will become partial derivative will become the total derivative. This will got the right hand side I have taken it.

Now, you see I can write this, this is a quadratic expression p square p something. So, I am going to put in a form which is sum of two squares p plus, i M over a infinity, omega over M square minus 1, whole square. Then plus omega square over a infinity square into M square minus 1 whole square. This mu is not the macron under, yeah you have brought its this is some variable name. Since, you brought is this is not the macron angle I am using the same symbol this has nothing to do.

This is the just some variable mu, if you want to ballet comma, if you want to ballet anything else you can do it. So, yeah thanks for finding it out this is. Now you see this is if you take it M square minus 1 you are taking common. So, that will be in the denominator. So, you will have two times this value, that is this and then the square of that and then you add this you will get your basically this term.

Now, you have represented your mu square in the quadratic form, I can first write the solution of the equation for this because the solution of the equation and the boundary condition we can first let us write the solution. The solution of this equation is p star z is a e to the power mu z plus b.

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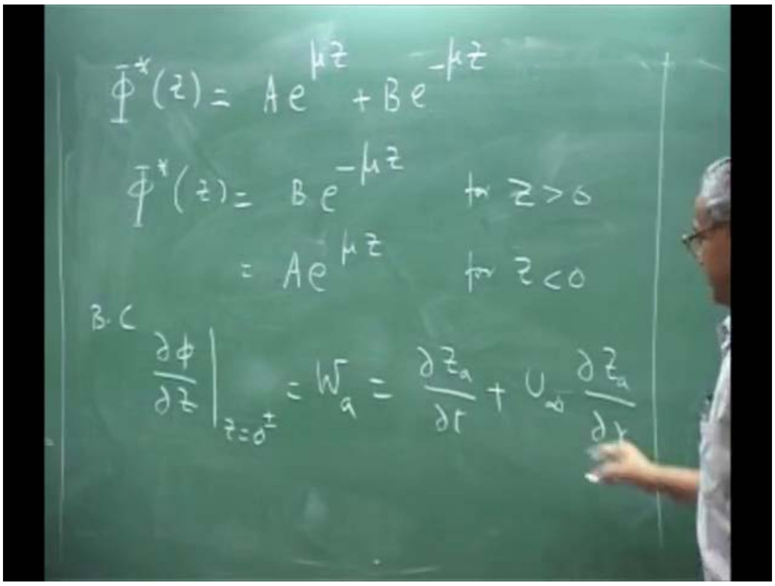

Now, you say if my z is positive this is where the radiation condition, you will apply if z I positive as z increases what will happen. This will keep on blowing up, therefore, you say my solution should be, I will write like this phi star z is equal to phi into e to the power minus mu z for z greater than 0. Then this is equal to a e to the power mu z for z less than 0. This is precisely similar to what that x minus beta z x plus beta z t previous example which we did.

Now, I have the solution, but I have to apply the boundary condition. Now, what is the boundary condition I am having, boundary condition is delta phi by delta z at z equal 0 plus minus is which is w a. I am putting at the near the aero foil, this is going to be delta z a over delta t plus U infinity delta z a over delta x. Now, if I substitute all the harmonic motion in the boundary condition I will end up with w bar a.

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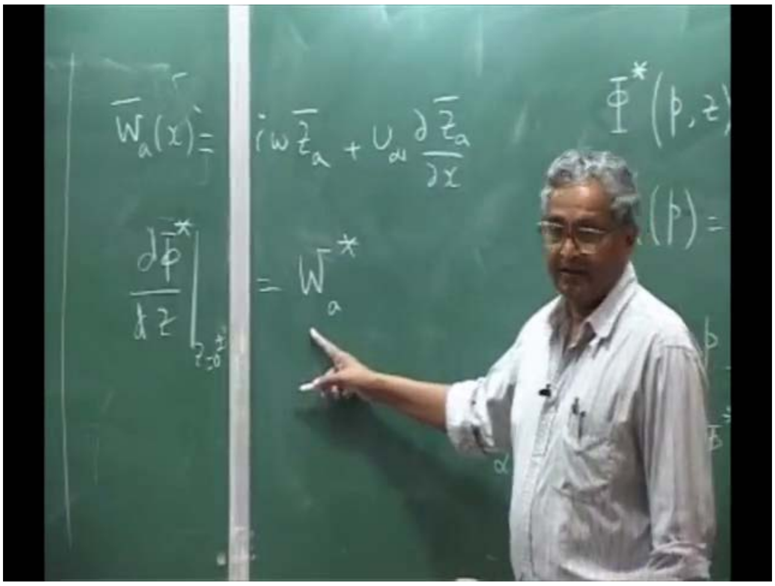

Which is the function of x equals, this is I omega z bar a. This is U infinity right, this is what I will have, but Laplace transform half delta phi by delta z which is w a x. This Laplace transform of this quantity I have to I am going to call it as that is d phi star by d z at z equal to 0 plus minus. I am calling this as w a which is here because d phi by d x sorry d phi by d z is this. So, basically I am saying Laplace transform of this is d phi by d z star. Now, you know that this is my boundary condition. I have the solution here I will substitute the boundary condition there, what is my boundary condition delta phi by delta z is from the solution that is minus, delta d phi star by d z is minus b mu e power minus mu z at z equal to 0.

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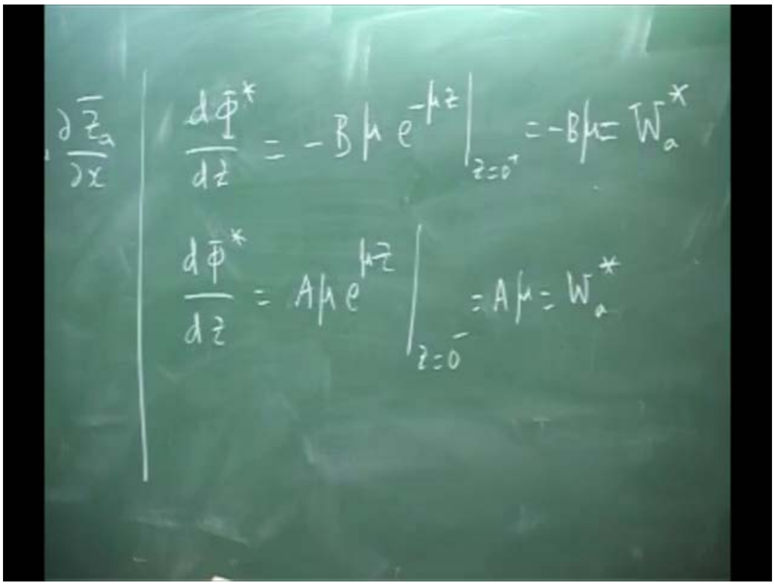

You have to the s power minus mu z at z 0 plus is nothing but minus b mu, which is essentially w a star. When you do the same thing for the lower surface d phi star over d z minus sorry not minus, this is plus a mu e to the power mu z at 0 minus. This will give me a mu which is again w a star. So, you can get b is what a is what phi equals w a star over mu with a minus sign this is w a star over mu with a plus sign. So, I can write my solution like this. Let me erase this let it be there may be I do not erase this part. So, I can write my, but please understand my mu contains the Laplace variable phi therefore, I am writing it as just for curative.

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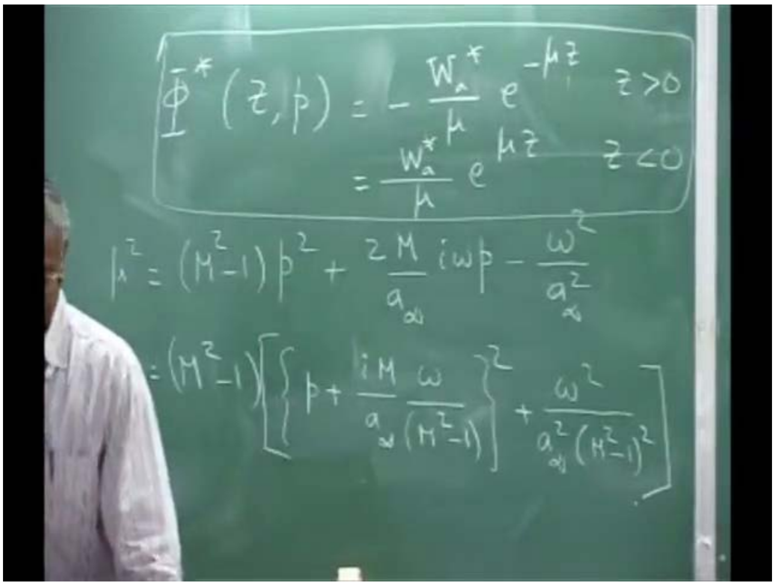

This is the Laplace and this is nothing but minus w a star over mu e to the power what z greater than 0 and equal to mu e to the power mu z for z less than 0. So, this is my. Now, you know that this is a Laplace of w a the function which we define mu is a Laplace variable and mu is sitting. Here what you need to do is you have to do Laplace inverse. Now, I will not go and do Laplace inverse for the entire ground because I am not interested in getting the solution everywhere. I am interested in getting the solution where z equal to 0 plane because that is where my aero foil is sitting. Otherwise you have to get this Laplace inverse of this entire term because knowing the w a that is why this becomes messy stuff.

What is done is see I am not going to do this full term Laplace inverse. I am going to assume the since I am interested only on the z equal to 0 plane whereby aero foil is also sitting I will put z 0. The moment I put z 0, this gone, this is gone. Then I am having only z plus z minus the phi, but you see the anti symmetry. That is why you get a lift phi is anti symmetric and delta phi by delta x is a it will become x delta phi by delta z will be symmetric, but phi is anti symmetric.

Therefore, you are getting a lift, like last time class we told you about the 1 was the thickness problem another was the lifting problem this is the lifting problem. Now, this is the Laplace inverse since this is a Laplace. This is the Laplace term phi. now this is the product of two functions which are on the Laplace domain that is with z 0. Now, if I want the Laplace inverse, you use convolution theorem.

That is why I said that inverse the product of two functions in Laplace domain, if you want to convert them back into the original domain. Then it is actually a convolution integral of these two functions. The proof I am not going give, I am going to simply write that step. Then you will have an understanding this is what is done everywhere because this is the product of two Laplace transforms. So, let us say I erase here, I will just derive that expression, that is if you have Laplace of please understand.

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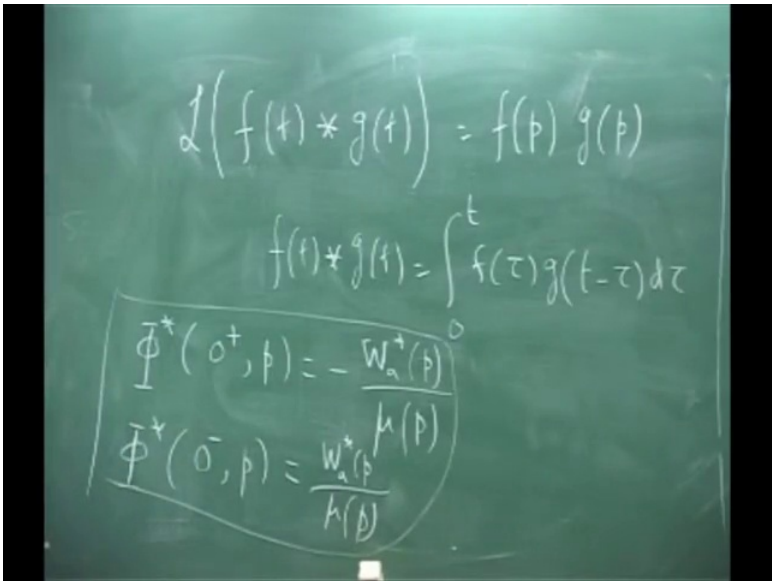

I am going to use a symbol f of t star g t, this is f of phi g f phi t is the Laplace t is in original domain what is the meaning of this star is this is called convolution integral f of tau g t minus tau d tau. This is the convolution integral, mathematically this can be shown easily this is not very difficult. If have two functions f and g in this form. You are integrating from 0 to two, this is called convolution because if you have studied vibration you will know that part what is a convolution integral.

If take a Laplace of this you will get product of the Laplace transform of the individual functions. So, what we are having here is the same thing. When I said z 0 that is z equal to 0 plane this is 1 Laplace function mu is another Laplace function. So, I can always do the inverse Laplace transform. So, that is what is done in the solution, what we will do is, we will take phi star, please understand phi star z plus comma phi is nothing but minus w a star which is the Laplace mu, which is the function of phi.

Similarly, phi star 0 minus phi is w a star phi over mu phi and I having these two not 0 plus 0 minus. I will just go and apply the inverse Laplace transform. So, I will just write that little bit then we can see how to get the solution. So, if I want phi 0 plus comma x please understand because phi is getting converted into original space x.

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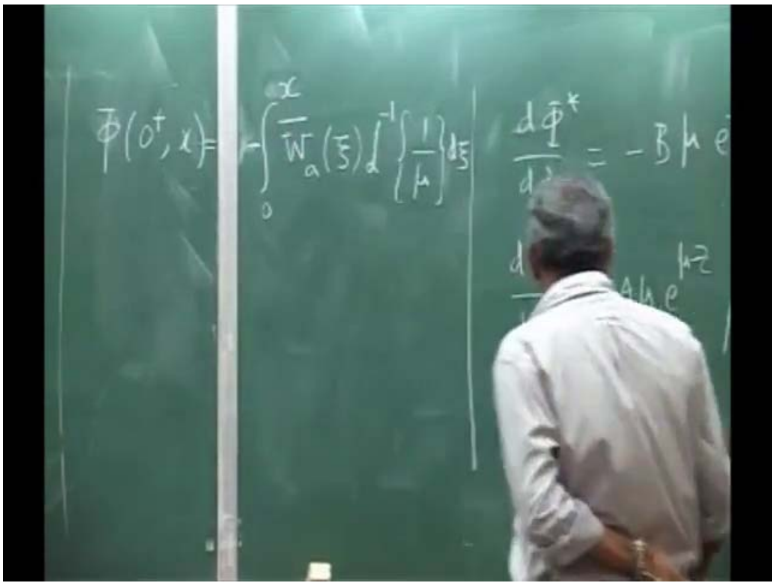

This is integral with the minus sign 0 to infinity. Now, w bar a I use a symbol because that integration symbol I am getting. Then Laplace inverse of with the convoluted, sorry this not infinity this is at any x, sorry any x yes this is z not infinity. Now, you see I need to and get what is this because this is I know is not assuming that, you know this function somehow because on the aero foil you know the motion of the aero foil. Behind aero foil you do not know, but you are interested in the on the aero foil.

This particular Laplace inverse of that mu which is given here, that is given in terms of threshold function, with a because you know that phi is sifted because you know that Laplace s plus a, if you have something like that what you have learned. It is a shifting theorem. So, you have to use the shifting theorem and this particular term if you do the Laplace inverse it will lead to the cell function.