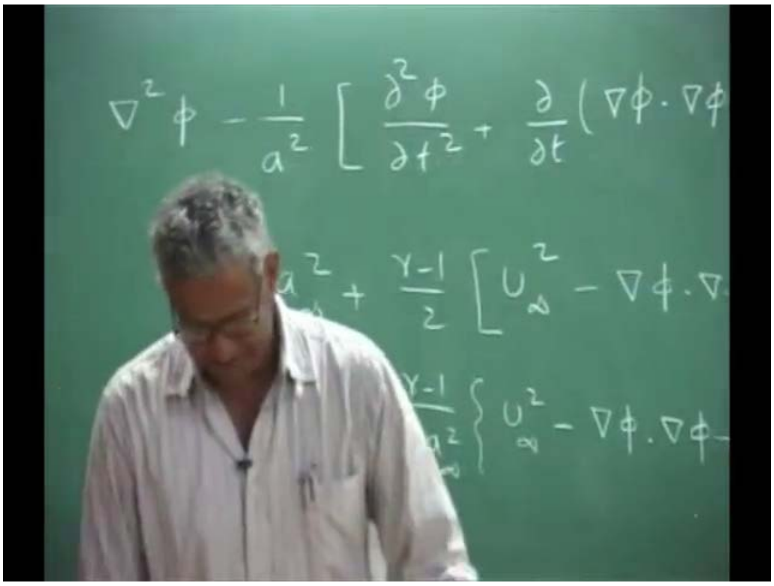
**Aero Elasticity**
**Prof. C. Venkatesan**
**Department of Aerospace Engineering**
**Indian Institute of Technology, Kanpur**

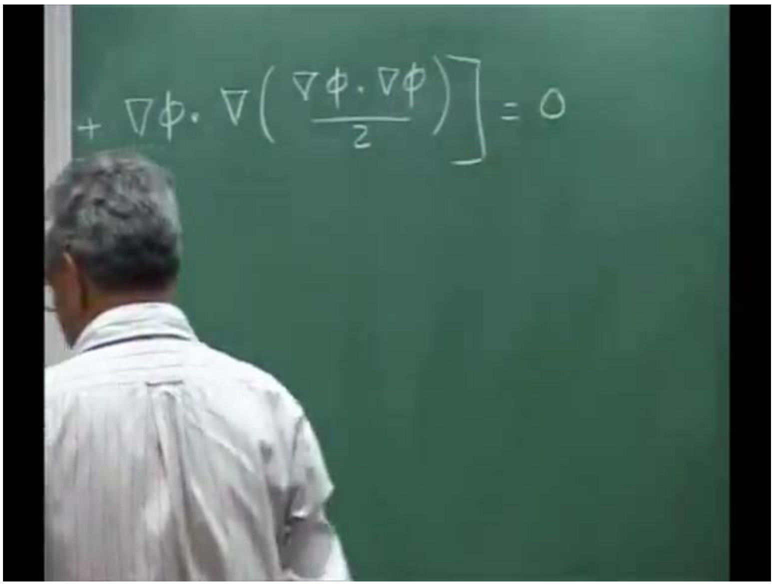
**Lecture – 17**

(Refer Slide Time: 00:15)

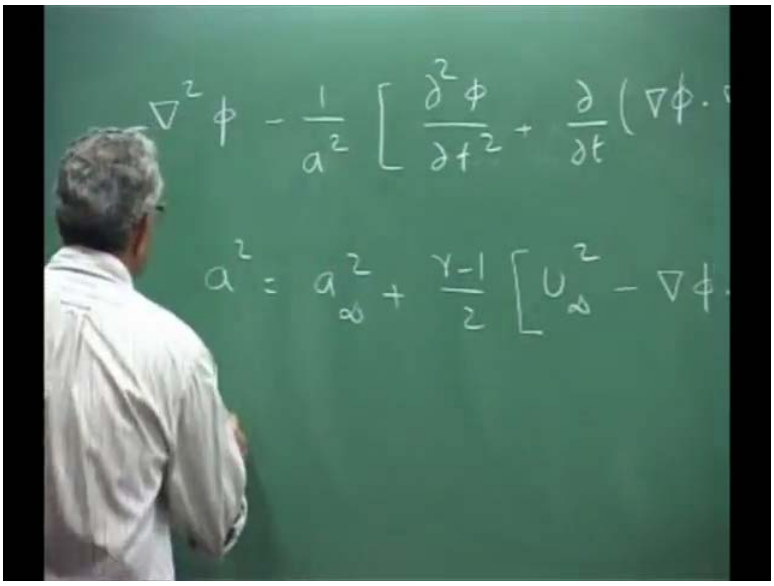


Del square phi over delta plus del phi dot del of del phi dot del phi over 2 is 0.

(Refer Slide Time: 00:39)

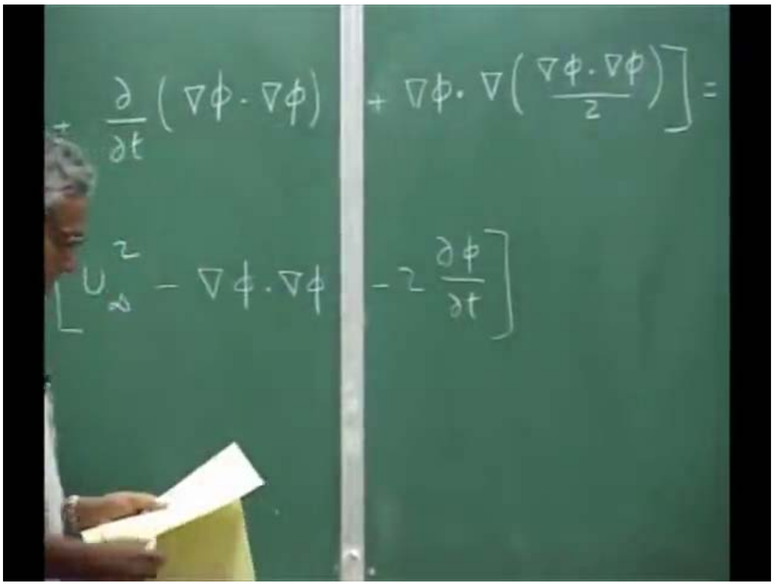


(Refer Slide Time: 00:55)



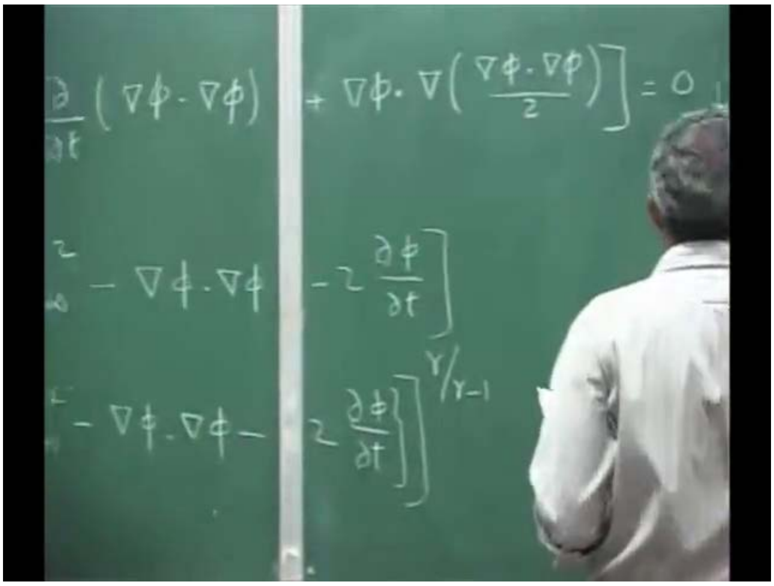
Then a square is equal to a infinity square plus gamma minus 1 over 2 u infinity square minus del phi dot del phi minus 2.

(Refer Slide Time: 01:29)



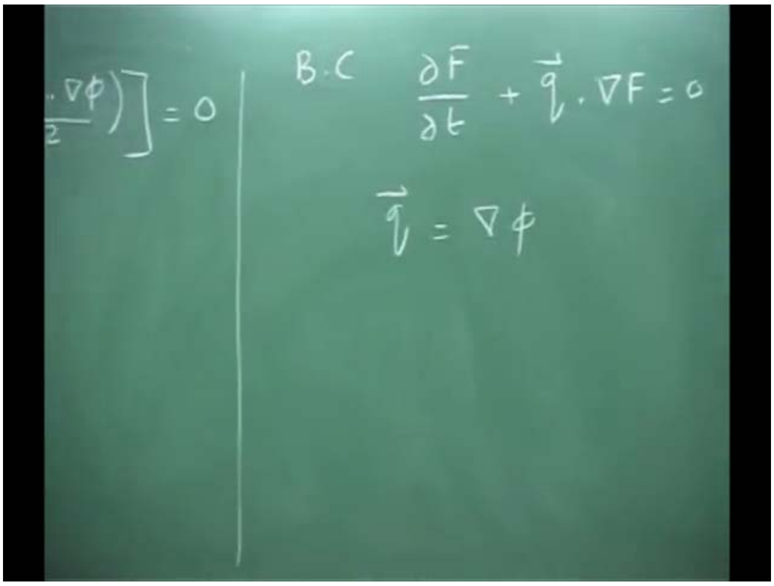
And then the pressure expression p equals p infinity into 1 plus gamma minus 1 over 2, a infinity square, u infinity square minus del phi dot del phi minus 2 delta phi over delta t power gamma over gamma minus 1.

(Refer Slide Time: 02:03)



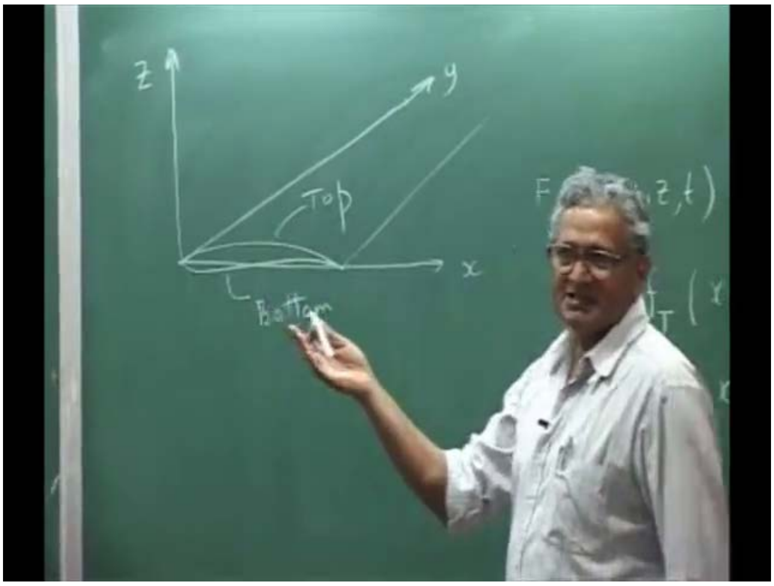
And then you have the boundary condition, which is the boundary condition is delta f over delta t plus q dot del f 0, where q is velocity which is del phi.

(Refer Slide Time: 02:14)



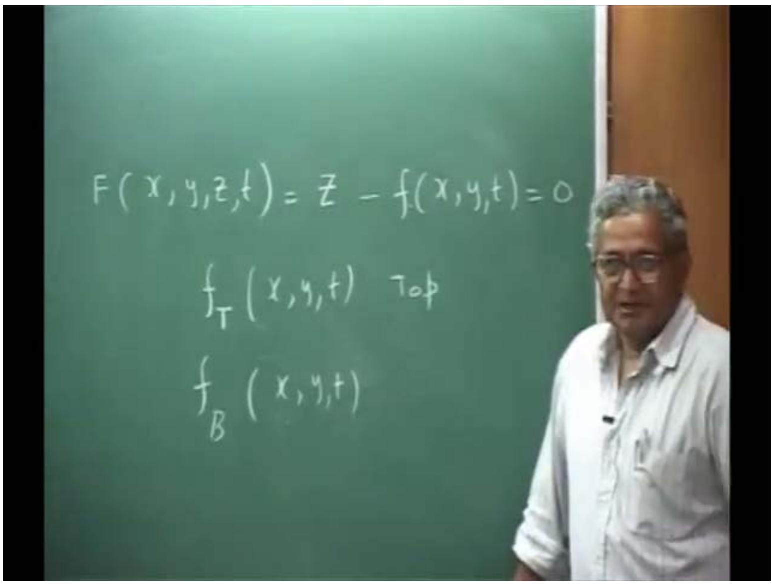

Now, this is our exact non-linear potential equation. The kind of problems which we have to deal in aero elasticity, that makes simplification to this entire set of equations and because this is highly non-linear. So, you have to go for the small disturbance potential equation, which is the linearize equation which will get. So, please note that is the small disturbance. Why is a small disturbance, because we are going to treat with wings limiting surface.

(Refer Slide Time: 03:39)



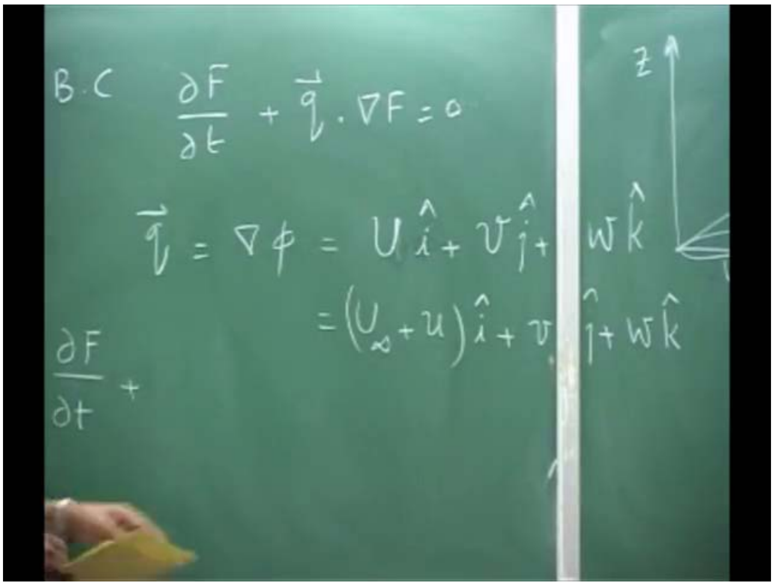

Limited surfaces we can see aerofoil surfaces, this is along x, this is you and you can assume this is the wing, so you can go and this is the wing. Now, the f is the equation to the surface of the body and which basically create the disturbance in the flow. Now, this approximation in representing f is actually responsible for reducing our equations to that small perturbation because how we applied. Since, it is a relatively thin surface, we will write this f which is a function of x comma, y comma, g comma t, as z is separate minus f of because equation of the surface.

(Refer Slide Time: 05:02)



Now, you can have a upper surface or the top surface and this is the bottom surface. Now, if you have a top surface that f t is for top surface, which will be a function of x comma, y comma t, this is top and the bottom is you can write f b, which is the x comma. Now, this is how we are saying that I am explaining somehow the result is coming out and I am writing the set, which is the location on the surface is basically a function of x and y. Now if I have this type of representation f t and f b, I will go back to the boundary condition.

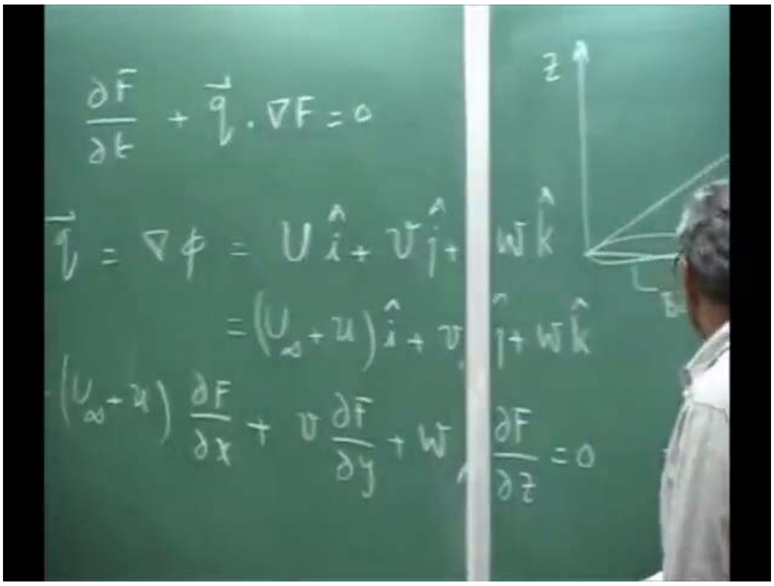
(Refer Slide Time: 06:25)




Now, what is delta f by delta t and this expression if I write, this is delta f over delta t plus q is del phi, which you can write it as U i plus V j plus W k. I am going to use the u a little bigger U because again I will split that into 2 because one is the free stream velocity, which is along the x axis. This I am going to write as U infinity plus lower case U i plus V j plus W k, that means my potential gives me the total velocity gradient of potential and my u infinity is the velocity per feet.

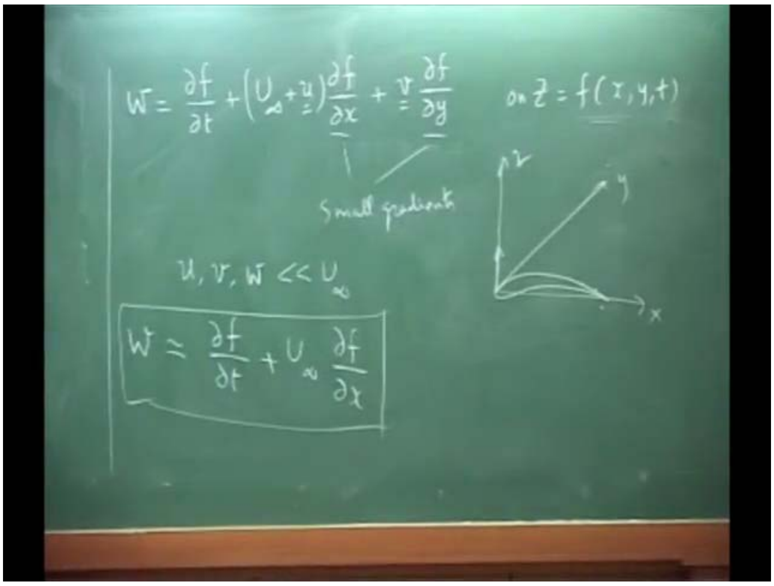
This lower case U V W is essentially bottom perturbations due to the wing are there aerofoil. Due to this submerge structure which is present in the flow. Now, if I write this because u infinity is the velocity of this whole thing, your speed of the aircraft or anything the wing and these are bottom perturbation in the flow due to the presence of the wing.

(Refer Slide Time: 08:24)



Now, I substitute this back here, you will get u infinity plus lower case u del of phi delta f by delta x because these are dot product then, I will have v delta f over delta y plus w delta f over delta g, this is 0. Now, I am going to substitute for f. This expression if I substitute this expression because f is the equation of the surface at any time. If I substitute and then delta f by delta g because this delta g by delta g that is 1 g, so that is 0. You will have w which is actually the flow velocity.

(Refer Slide Time: 09:34)



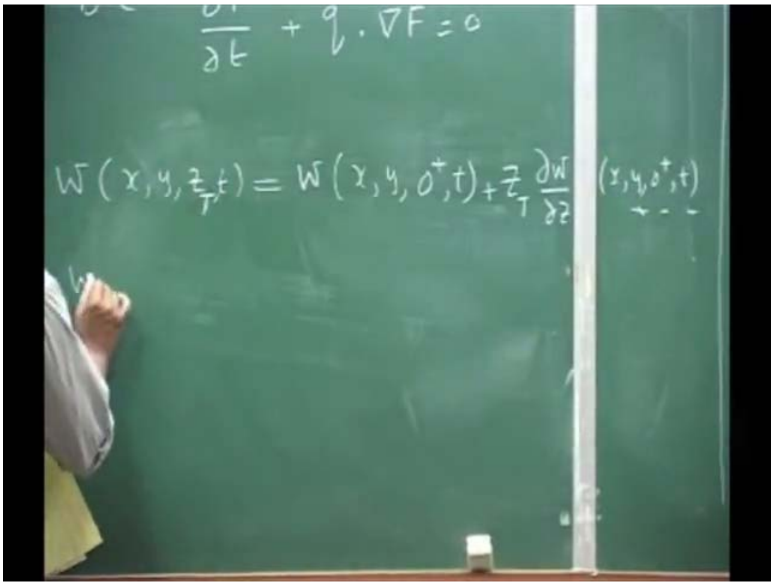
It will become like this w, which is the flow velocity is going to become this you will take it us delta f by delta g. Here all are lower case f because f is g f minus x g is the independent variable. Therefore, that is not a function of time. Similarly, here 0 not come here also 0 not come they will have a minus sign. So, you transfer the all term right hand side you will get delta f by delta t plus U infinity plus lower case u x plus v delta f by delta y.

Now, please understand on g plus f of x comma y comma t. This is the surface; this condition is on this surface. W is the flow velocity right and you know that f is this function. Now, you are going to make some assumptions here that are if the slop of the surface with respect to x because we have drawn this is g y and the x. Our aerofoil this slop with respect to x delta f by delta x are delta f by delta y. They are small because it is not a large slop.

So, these slops are small gradients. And then you say this is again you are making an assumption that my u and v are also relatively small in comparison to U infinity. So you say my U, V and w this is an assumption I making. Now, if I neglect terms which are products of small quantities that means this is the higher order term. Similarly, this is also higher order term. I will get here simply neglecting w approximately becomes plus U infinity delta f by delta x.

That is I am eliminating higher orders terms. This is my approximation later I will say this is equal to that is all. Now, I know the surface equation f of x y, this function I will substitute u infinity I know, I can know this. Only thing is that my flow velocity must be equal to this flow velocity normal whatever we said W, W is along the g direction. Here again you make another approximation that means W must be what? W must be among every surface point, but you are going to make an assumption.

(Refer Slide Time: 14:09)



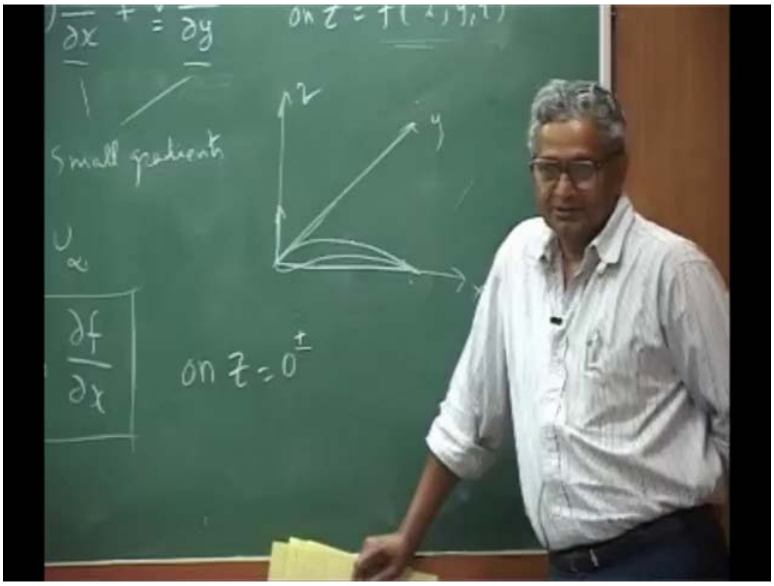

You are going to expanded that w, which is the function of f x comma y comma g, whether it is a top surface or a bottom surface that will depend on g, if you put it top because you want to calculate the flow velocity on the surface.

(Refer Slide Time: 14:16)


So, this can be if I put a top g t that means this is the top surface and you are going to expanded like a Mac Lorene series because this will write it W, which is x comma y comma 0 plus. This is an expansion about this g axis plus you will write g top delta W by delta z hat x comma y comma 0 plus gamma t plus higher order terms again. Similarly, for the bottom surface you will put this is d. You say that these are thickness is not very large and the slops delta w by delta g, that is the variation of the flow velocity with respect to g is not very large.

There is no sharp gradient in any of districts, this is an assumption. Then you say I neglect this term, I neglect y term. Then I say that w at x comma y comma g top surface t is 0 plus and I going to write similarly, for the bottom w x comma y comma 0 minus comma t. I am simply satisfying the condition on this line. Please understand I am just explain only on x y plane. I am satisfying the condition I am not really going on then satisfying the condition at the top surface. This is the aerofoil are the entire wing plane, I will satisfy on this that is why here I will write it on g equal 0, this I am satisfying the condition.

(Refer Slide Time: 17:10)




g is equal to 0 when I put plus that is the top surface then, I put minus that is the bottom surface. So, as far as I am concern my w is only on this plane, I am considering. Then you may ask what happens at the thickness of this. That is why now you say you have to do one approximation later we will go to that. The assumption we made is this surfaces are thin and we can make on the slops whatever the aerofoil shape, which you make the slops are also very small.

Therefore, we are neglected the higher order term that is the non-linear terms because another assumption that velocity U V W, which is are bottom perturbation quantities are small in comparison to the slop field which is U infinity. So, this is the basic assumption. Now, we say all my quantities that is the disturbance due to the presence of my aerofoil, they are small.

So, this is all the small disturbance potential flow. For that I am going to now modify the entire set of non-linear equations into corresponding linear equations. They will become a very simplified linear equation and that is what we will use for our solution of the flow supersonic, flow subsonic flow everything.

So, please understand that the assumptions we make everything is purely based on the boundary condition because you say my surface is thin. It is not going to disturb the flow you are not going to put a building over there and then try to solve this problem. In the sense you have so much of disturbance. We are saying our surfaces are thin surfaces lifting because that is where aero elasticity problems are important, that is why this assumption is made on.

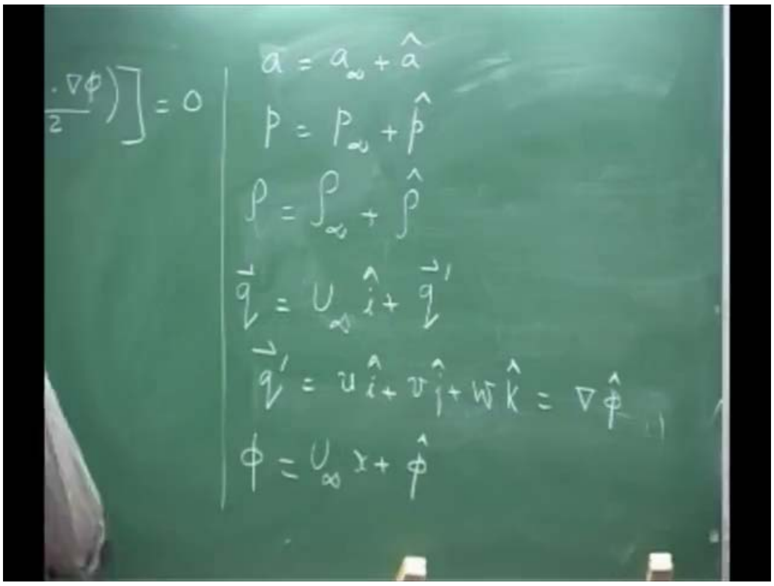
Thin aerofoil that is could done and the finally, the proof is if you can get a solution which is good, which is matches with an experiment that is where the theory gets validated. Now, let us get the formulation for a small disturbance potential theory that means the reduction of this equations as well as all the. So, the boundary conditions please understand this has been modified to this.

This is my boundary condition, now w is the flow velocity please understand, flow velocity. And we are looking w is flow velocity everywhere, but we are matching the flow velocity through the surface are g is equal to 0 plus minus only along the up to cart another wings fan that is all. If it is 3 d problem if it is wings fan, but it is a 2-d problem, its only cart. Now, there will be another what happens to the off body boundary condition.

There is something like that we do when what happens to the thickness. Does it do anything, that we will do off body boundary condition later because right now we will say this is my boundary condition. We will reduce the set of equations and then if possible we can do right away itself the tough body also because that will also give me we can do both.

Let me finish the linearization first and then I will take because that is a very interesting condition. Please understand this is the key we have converted this to this. Now, what is w? W is delta phi by delta g that is all delta phi by delta g at g equal to 0 plus or g equal to 0 minus. But please understand on the aerofoil g top surface bottom surface they are different do not mix this two. That is why this is very essential that when you put the top surface bottom surface, the s is correspondingly going to change f for top surface it will be top surface and f d it will be bottom surface. Now, let us take the reduction of the equation because I will keep this. This I leave as it is because this is important your boundary condition.

(Refer Slide Time: 23:04)



We are going to now use the condition that a speed of sound a infinity plus a hat. Some small these are small disturbance that is the small perturbation quantity and the pressure s p infinity plus p small and density rho, rho infinity plus rho hat. And your q that is the velocity, velocity is u infinity i plus where q is U i plus V j plus W k. Your free is U infinity x plus p hat. And you know that q prime this is nothing but del phi hat. Please note that del phi hat is this like what we wrote earlier.

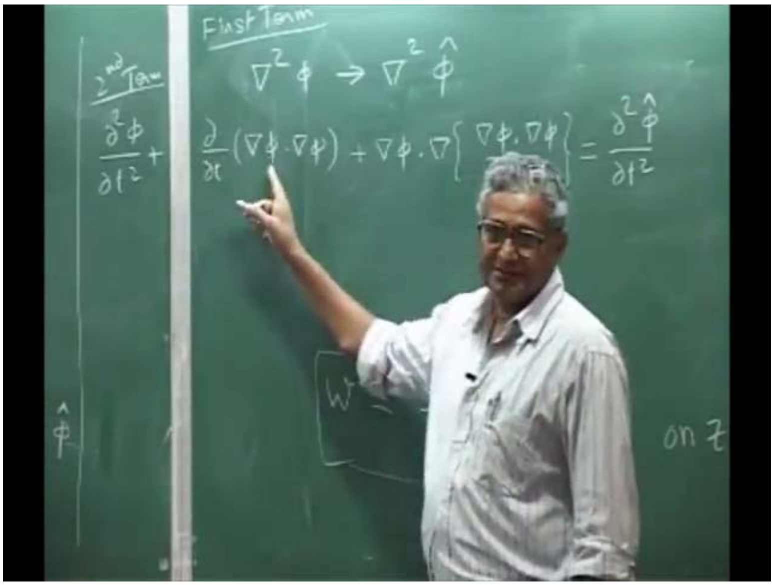
So, from the boundary condition only, I am bringing it here. Now why I am saying from the boundary condition you may ask the question. See, I am saying that the disturbance because this boundary condition is only near the surface of the wing. If I can make an assumption that my U V W, that is the perturbation velocities, they are small near the surface the boundary. Then I can make an assumption that far away from that they can be small.

Please understand this is an assumption because I say that disturbance to the flow is created by the presence of right, near the aerofoil the disturbances are small. Therefore, I say that far away from that the disturbance will be still smaller. Therefore, I am making all these assumptions that everywhere U V W are small in comparison with u infinity, but see this is a very interesting question, you know which comes. The disturbance is small near the boundary, which is the disturbance casing surface, but how can say that far away it will be less.

This is an assumption you make because this is there a counter example where it is not because this is very interesting. Suppose you take the tsunami, the disturbance wherever it is called this small, but then it built up far away from there and then you have floods only somewhere several miles away from the point where, the earth quake anything has happened us you follow. Now, the question is there how can you say that the disturbance is small there, but then it built there catastrophe in some several hundred miles away from that. It is a very difficult thing to answer.

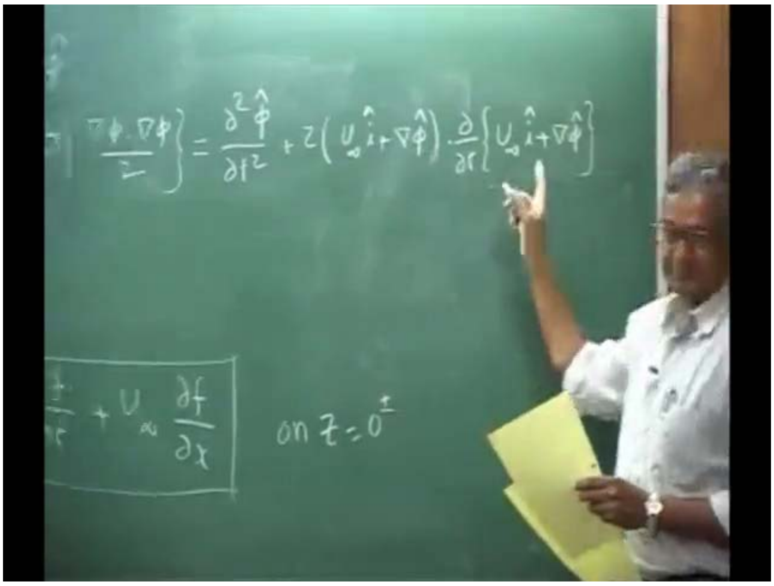
See, that is the surface where, but here we are making that assumption that whatever near the aerofoil disturbance small it is not going too far away. Now, with this assumption what we are going to do is, we are going to substitute these quantities because phi you know this I am going to put this back there del square phi. Now, when I put del square phi term by term if we go then, you will find everything will be ok. The first term del square because I am going to substitute this. When I substitute this that will become del square phi is going to become del square phi hat there is no problem.

(Refer Slide Time: 27:51)



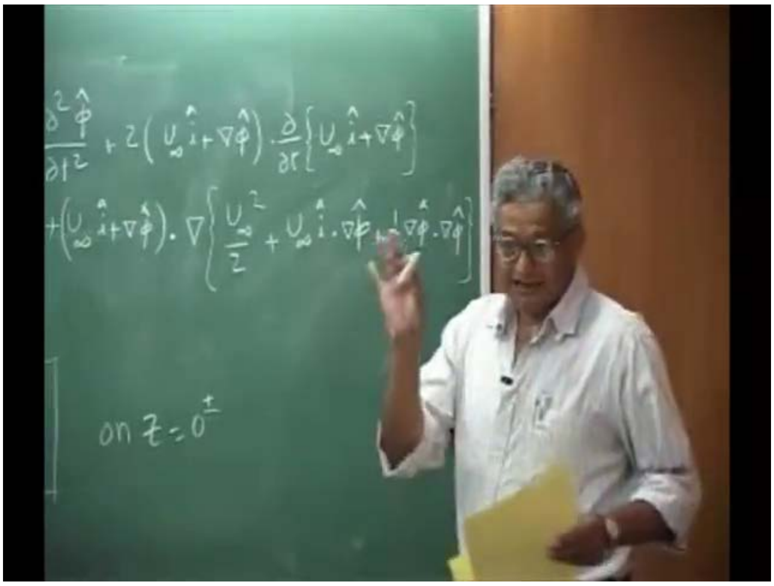

That is the first term because this is little bit tough algebra and the second term, which is del square I will write it here del square phi over delta t square plus delta by delta t of del phi del phi. This is a second term plus del phi dot del of del phi dot del phi over rho. This is because when I substitute this, this is independent of time because U infinity x is not there. So, this will become del square phi hat over delta t square, this is nothing but q dot q. So, q dot q is q square. So, you will get when you differentiate that will be 2 q. So, you can write it as plus 2 q dot delta by delta t of q. Q is given here, so I can write is as 2 u infinity i plus, this q prime is what? Q prime is del phi.

(Refer Slide Time: 29:43)



So, I am going to put plus del phi hat dot delta by delta t of U infinity i plus again del phi hat, that is this term same. Then you have to write last term.

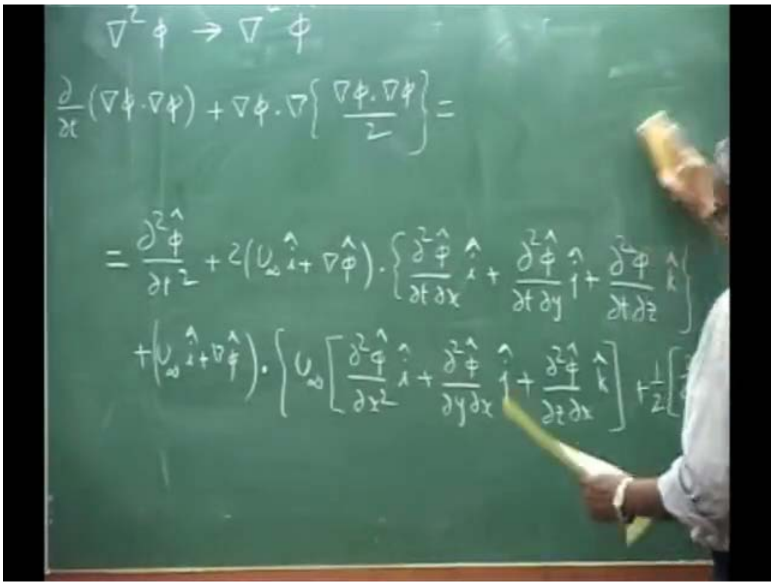
(Refer Slide Time: 20:06)



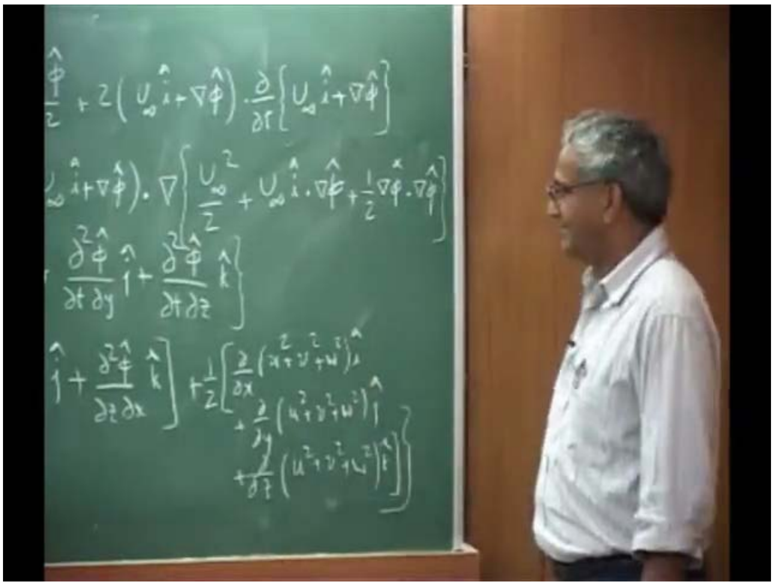
Last term is plus del phi is nothing but U infinity i plus del phi hat dot del of this is del phi dot del phi that is nothing but this dot this. This dot this is nothing but U infinity square over 2 plus U infinity i dot del phi hat plus half del phi hat dot del phi hat. And this is what you do is, you neglect all the non-linear terms. This will essentially reduce, this will again become this term will remind as it is, you will have what you do is?

You substitute everywhere delta by delta t this is gone because there will go to 0 there is no place to sent, but this del phi is nothing but delta phi by delta x I delta phi by delta x j etcetera, etcetera. Then this is also del phi, this del phi, this del phi they will become higher order terms. So, you take only U infinity i and the i of this term that will be a linear term.

(Refer Slide Time: 32:20)



(Refer Slide Time: 32:29)



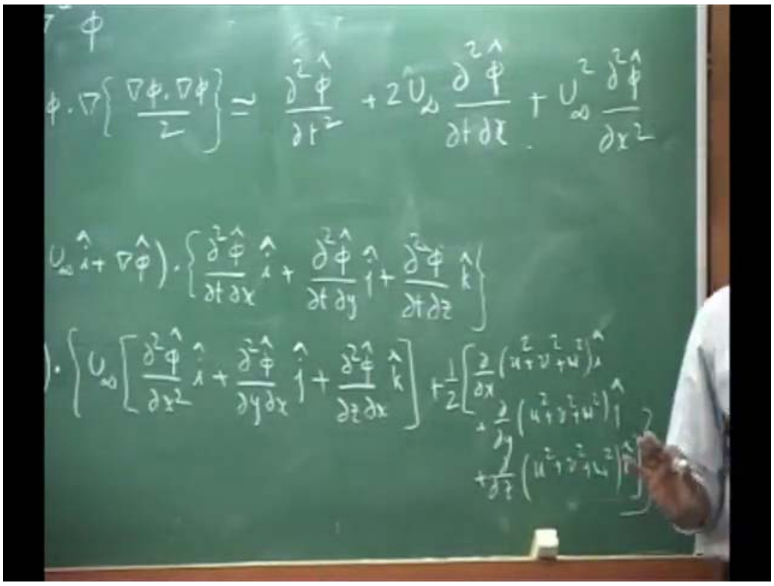
So, you neglect all the non-linear terms from here retain only the linear terms. So, if you want I will write the whole thing. Then, we will del square phi hat over delta t square plus 2 U infinity i plus del phi hat dot. You will have del square phi hat by delta t delta x i plus del square phi hat delta t delta y j plus del square phi hat by delta t delta g k plus you will have this entire term.

That entire term again you can write it as plus U infinity i plus del phi hat dot. See this is u infinity; this is del of u infinity square because u infinity is a constant. So, this will just drop up it will not exist, it has this gradient of the constant so, that will go up. So, you will have dot u infinity del square phi hat by delta x square i plus del square phi hat by delta y delta x j plus del square phi hat by delta g delta x k plus half this whole thing. Because I am taking dot product of what this into this because this term as it is remaining here, please understand.

I am taking this del i, I will have only i term because this is U infinity i. So, I will get only delta phi by delta x phi hat, that is why I have delta phi hat by delta x. This delta by delta x I, this is delta phi hat delta x then delta by delta y j delta x delta by delta g k and U infinity is just U infinity there. Because this term only i term will survivor or the sub term will be 0 because i dot j is 0, i dot k is 0. So, this will simply means u infinity delta phi by delta x delta phi and that you take a gradient a. And this is half the entire term that you can take it as half, you allow here i j k.

So, you will write it as delta by delta x of u square plus v square plus w square i plus delta by delta y of u square plus v square plus w square j plus delta by delta g of u square plus v square plus w square k. So, you see this is very complicated expression. You simply throw away all the non-linear terms that is what is done. Please understand otherwise if you expand this is going to be very big. So, you neglect the non-linear expression and write it as I will write the final result here.

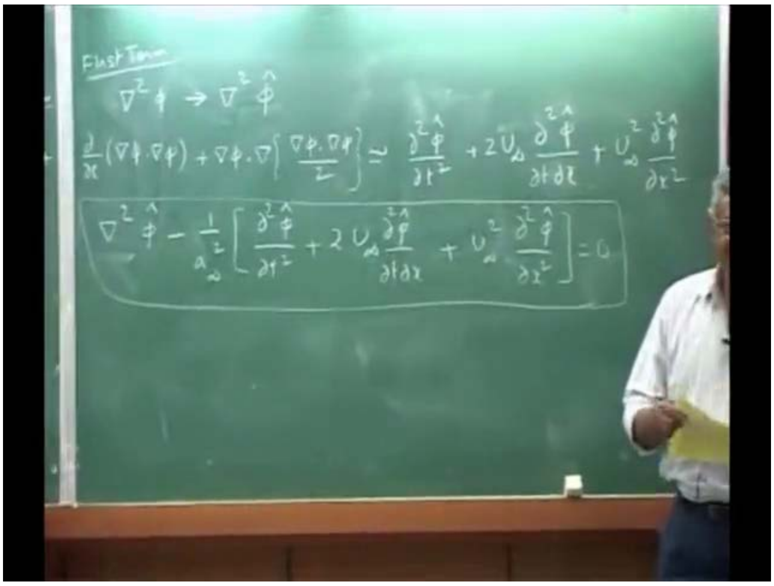
(Refer Slide Time: 36:37)



The final expression will be approximately del square p hat delta t square plus 2 U infinity that is this term del square phi hat by delta t delta x plus. You see this term U infinity i and u infinity square delta phi by delta x plus u infinity square del square phi hat away that is all, rest of the terms I am neglecting it. So, you see when I neglect rest of the terms, this becomes very simple. This is linear operator, so erase is this part and I will write my equations here.

Now, you will also have to see what I am going to do with this term because here it is a square, a square is again a function of phi, this is not an independent. Now, I say the perturbation and other things I am going to assume a square is a infinity square because the moment I make that it is not. Then it is again an non-linear equation because I am going to get phi here and this will be denominator.

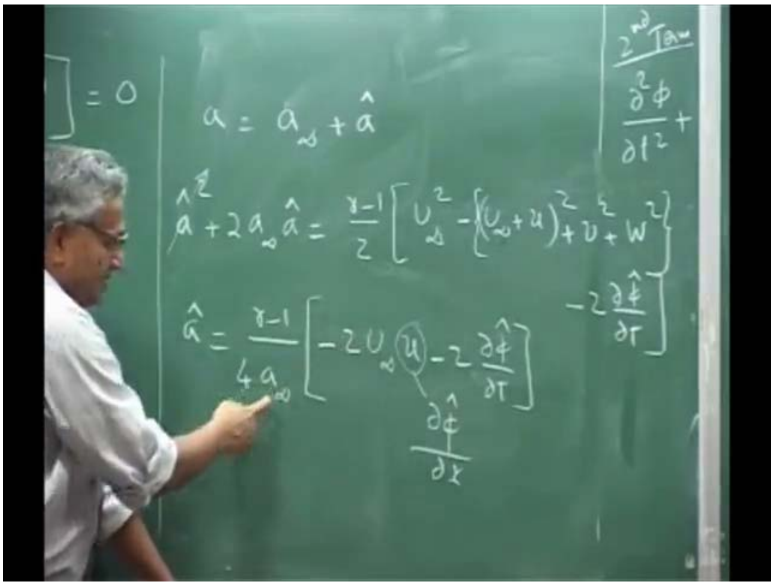
(Refer Slide Time: 38:38)



So, it will become again on non-linear that is why I will write this term on this and a square as simply a infinity. So, my small perturbation it will be del square phi 1 over a infinity square del square phi hat by delta t square plus 2 U infinity delta phi del square phi by delta t delta x plus U infinity square del square phi hat by delta x square. Please understand, this is my unsteady small disturbance linearize potential flow equation because this is the linear.

And this equation please understand this is linear completely absolutely no problem. You have a linear equation. Now, what we are going to do with the pressure expression. So, let us erase this part and we will write the pressure expression as well as the disturbance because even though I used here infinity square, but I can get an expression for local speed of sound. I can get an expression for a local speed of sound.

(Refer Slide Time: 40:19)



I am going to write that as a as some a infinity plus a. So, I substitute that here what happen to a infinity square we will cancel it out, so you will have a hat square plus 2 a infinity a hat equals gamma minus 1 over 2. you will have u infinity square minus, this is a del phi dot del phi which is nothing but again U infinity plus U whole square plus V square plus W square minus 2 delta phi over delta t.

Now, what I can do is I can again make a approximation that this a hat is small. Therefore, I throw this term out and I am just using only this term, which is like my hat will become gamma minus 1 over 4 a infinity. This is what U infinity square will go away leaving behind you will have minus 2 U infinity U. So, I am going to use only that term minus 2 U infinity U and rest of the V square W square I am saying that they are small I neglect that minus 2 delta phi over delta t.

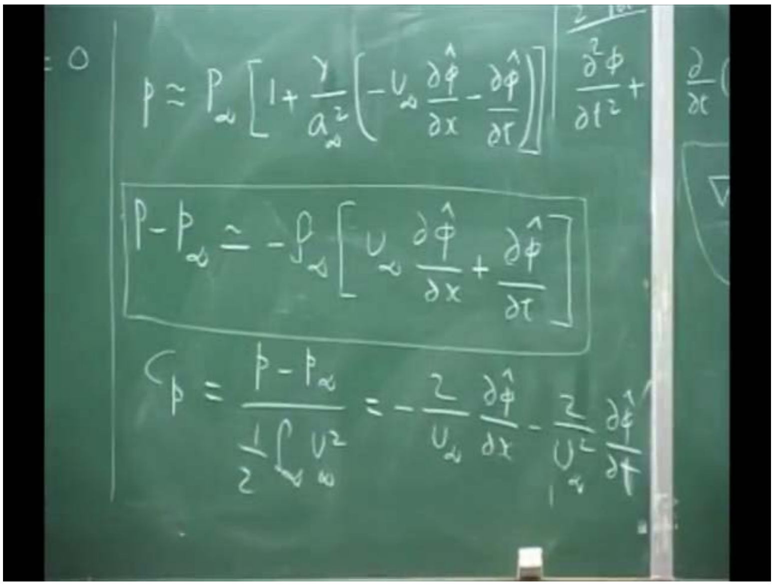
So, this is what I called it as change in the local speed of sound. Now, if you see u is delta phi by delta x, this term that is del phi hat by delta x that is what and this is anyway phi becomes this phi hat because there is no given prentices independent of time. Now, you see U infinity by a infinity, this is the Mac number. Schrodinger is the Mac number is not very large.

This speed of sound variation is not greatly affected, that is why Mac number, but very large what is very large is another question, that is why we will use. Please understand we are going to use this same equation for supersonic flow. We, are not going to use some other, can you Schrodinger the Mac number is not very large below Mac number 3 because there are another problems heating lawful heating everything will come way of isentropic flow.

We made of isentropic flow as an assumption. So, based on that assumption which we have to less than Mac 3 less than 3, this equation are parallel equal. And that is why these are used potential flow small disturbance. Now, let us go to the pressure expression because this is the changing a local speed of sound this is just for exercise. Now, let us look at I erase this part. Let me write the pressure expression.

The pressure is this I am again making assumption that all these are small quantities. So, gamma minus 1, I am going to expand by binomial. When I expand by binomial this will here inside and I written an only 1 term. I will not write more than any term because this 1 plus something power x something power. I said that this quantity is a small quantity. Therefore, I will get my pressure is p infinity times 1 plus because if I take this, this will become gamma over you can take it actually after simplifying. You will get a infinity square because U infinity U infinity will go up square and the factor 2 will cancel with the factor 2 with another factor 2 here, like what we did earlier.

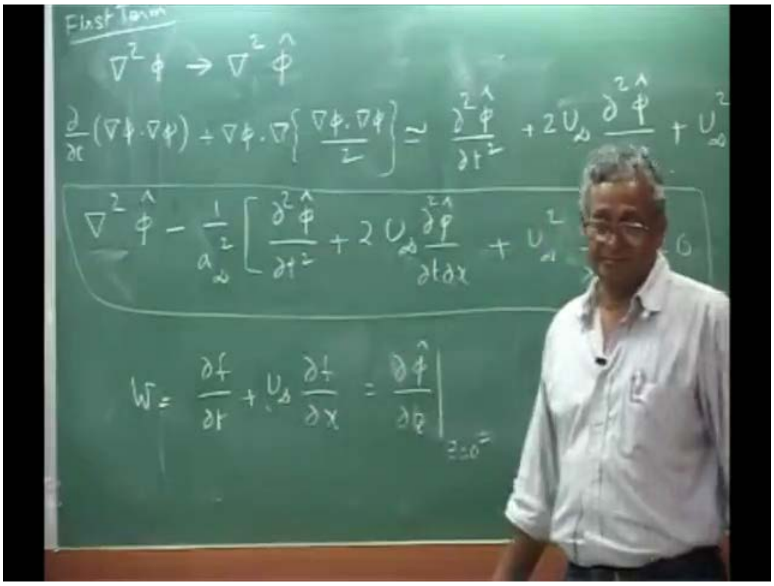
(Refer Slide Time: 45:18)



You will get basically minus u infinity into lower case u, which is delta phi hat over delta x minus delta p hat over delta t. This I can write it as p minus p infinity what is this? This is a infinity square is gamma, so you will get t infinity gamma over a infinity square is nothing but rho infinity. So, I will write this is rho infinity times u infinity delta phi hat over delta x plus delta phi hat over delta t, this is basically the change in pressure.

Change in pressure at any location in the flow this is my I can get the pressure quite efficient if you want c p. C p is nothing but p minus p infinity over half rho infinity U infinity square. You divide by that, so you will get minus 2 over U infinity delta phi hat over delta x minus 2 over U infinity square delta phi hat over delta t. Now, you see this is my equation and my boundary conditions are W s what?

(Refer Slide Time: 47:45)




Delta f by delta t plus u infinity delta f over delta x, but this w is delta phi hat over delta z at z is equal to 0. You can have plus or minus because f top f bottom that is all. Now, this completes your entire problem. Now, is that all is a question you follow. Now, you have seen I will not using the hat from now on because it is not necessary I have to because when I use phi, you assume that it is the perturbation only.

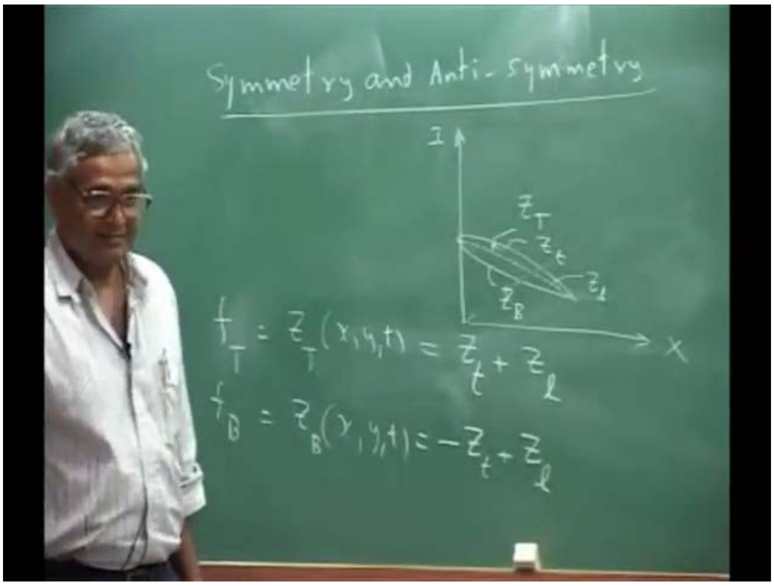
Just for again everywhere I have to put hat, hat, hat that is just for convenience, I am eliminating the hat from the all these equations. I will use only this equation, this equation and this equation that is all. My local speed of sound small disturbance you can get always what is the local speed of sound that is if you want to get it.

So, now this is your you can say your linearize about this is the small perturbation on I would say because these are all U infinity. You say it large in comparison to your perturbation. When we solve our equations whether it is a supersonic 2 d or subsonic, we are going to use only this, but then if you see in ((Refer Time: 49:41)) you go that means you are going to say a infinity is very large.

So, this term goes to 0. I am left with just a plus equation del square phi equal to 0 whether, it is a steady flow or unsteady flow. Please understand I have my equation is same. Now, the steady or unsteady depends on only the boundary condition. If my boundary condition f is independent of t that is all my boundary condition w is U infinity delta f by delta x that is all. And my pressure this will go up, this term will go up rho infinity u infinity delta phi by delta x.

So, I need to get only this change in pressure. Now, you will get pressure on the top surface pressure on the bottom surface and then you subtract that and you will get your left and you integrate over the cart of the airfoil. So you find finally, all over entire unsteady aerodynamic equations is reducing to solving this. Only thing is thickness till we are not protecting because we say this w the flow velocity on the surface, but that f we have differentiate between comma top surface and the bottom surface. Now, what we will do is, we with split that into the 2 parts the boundary condition itself taking into account the thickness of the aerofoil and then argument, we will say that the thickness is not this is ready to solve.

(Refer Slide Time: 51:45)

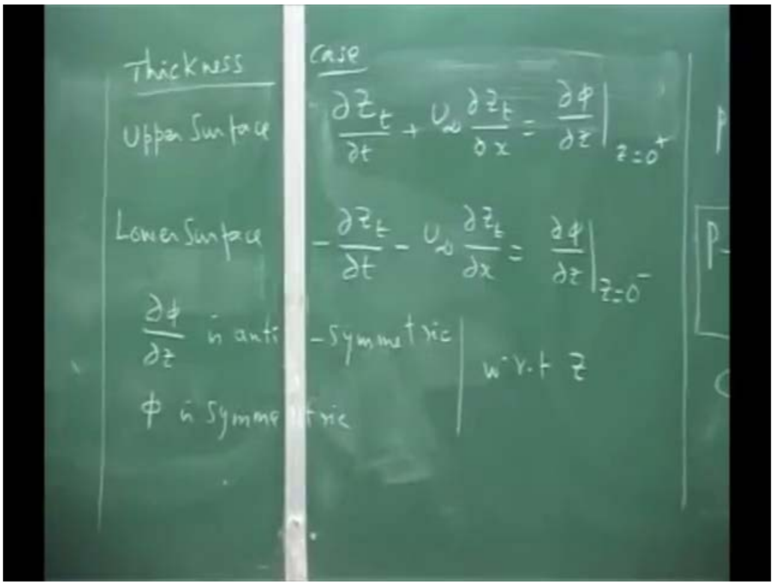


So, at symmetry and anti symmetry see we say this is our x, this is our y and this is a g and if we say this is my aerofoil, let me correct out of let me not bother about the way. This is the top surface, this is the botto1m surface, but I can write this as some good point I take it. So, I am going to write the upper surface f top surface, which is our is f is z top surface which is the function of x comma, y comma t because this is the function of x. I am going to write it of 2 parts, one is g t.

You take it as t is thickness, this is g t you may say g t by 2, but the 2 is the absurd in g itself plus g lifting, this is the that is although I can spilt it into 2 parts. One is the thickness another one is the lifting component that means I am writing the surface. This surface although this is this surface plus the small thickness. And similarly, f bottom this is g bottom, which I will write this minus because the thickness is negative. So, this is the mid plane plus some thickness mid plane minus thickness. Thickness is actually g t by t, but it does not matter.

Now, if I am going to use this condition that, I can spill my top surface and bottom surface into 2 parts rather although I can add this 2. Then I am going to substitute one is the lifting case, another one is the thickness case because I am writing in a linear combination and all my equations are linear please understand. This is a linear, this is linear, this is also linear, there is no non-linearity is the problem. Therefore, I am saying I am splitting my problem into thickness case separately, lifting case separately. Now, if I look at the boundary condition this is what that delta phi by delta g. If I write thickness case thickness case, what is the boundary condition?

(Refer Slide Time: 55:41)




Boundary condition will be upper surface and then lower surface we will have 2 conditions. In the upper surface I will have what I have to substitute here in the f upper surface top surface, top surface is Z t. So, I will put delta Z t over delta t plus U infinity delta Z t over delta x equals delta p over delta g, g equal to 0 plus. And on the lower surface which is the bottom surface I am going to put, what is the bottom surface for thickness is a minus Z t.

So, I will have minus that minus delta Z t minus U infinity delta Z t over delta x equal delta p over delta phi over delta g at phi equal 0 minus. Now you look at this. This is delta phi by delta g is anti symmetric because 0 plus, 0 minus. Zero plus is positive 0 minus is negative that means delta phi by delta g is for thickness case is anti symmetric, if this is anti symmetric phi is symmetric.

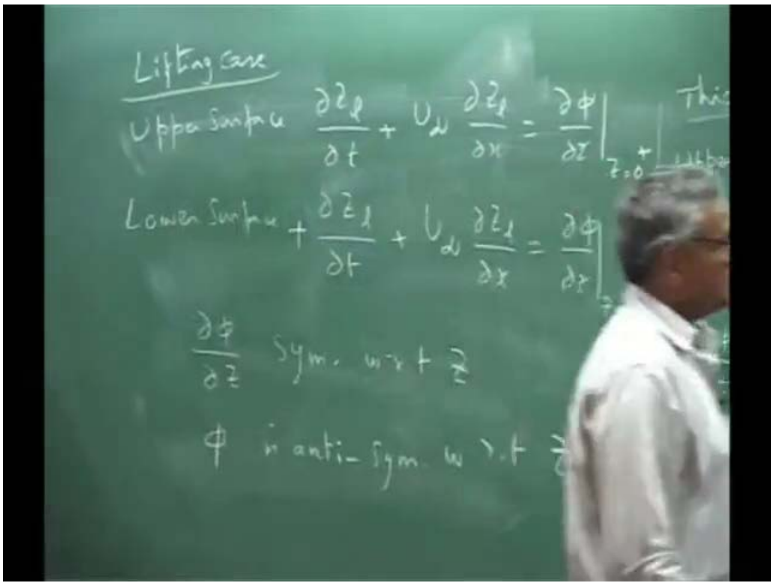
Phi is symmetric with respect to what, only respect to Z t symmetric, anti symmetric with respect to Z. If phi is symmetric that means the top surface phi bottom surface phi should have the same value. Now, if I go to the pressure p minus p infinity this is if I take in p top surface it will be p top surface minus p, p bottom surface it will be p bottom surface minus p. But my phi is symmetric with respect to g, but delta phi by delta x is same that means as 0 plus on 0 minus will have a same value. Therefore, it will not give me any lift. I cannot get any lift because of the thickness.

So, the thickness effect has no meaning. Only for all this linear problems I am telling you because it will not give. Phi is symmetric therefore, the pressure is also phi is symmetric means what pressure is also symmetric because pressure is delta phi by delta x only. With respect to Z, it is a symmetric that plus that minus it will have a same value, symmetric value. Therefore, this will not contribute anything to the pressure.

Now, if you go to the lifting case you will get delta phi by delta g, it will be symmetric. Therefore, if delta phi by delta g is symmetric that means phi is anti symmetric. If phi is anti symmetric then this is anti symmetric top surface bottom surface will have different value. Then you can get a pressure difference and that is what you will get itself the upper force.

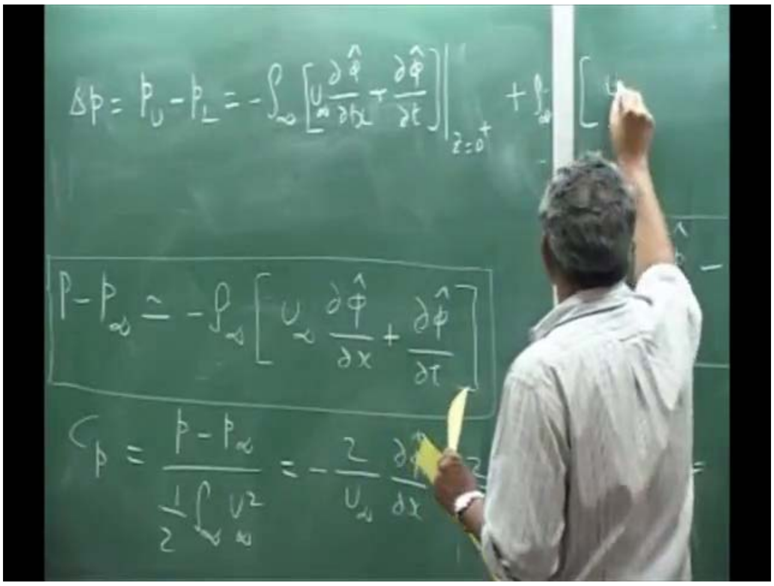
So, we will write that part in the lifting case now. You got it now, this is please understand this is the place where, your wing is there your surface because we are applying this boundary condition only on that not on free flow this is on the body. Thickness case and then let me write the other case I erase this part after that we will go to this is in the lifting case you will have upper surface as usual.

(Refer Slide Time: 1:00: 59)



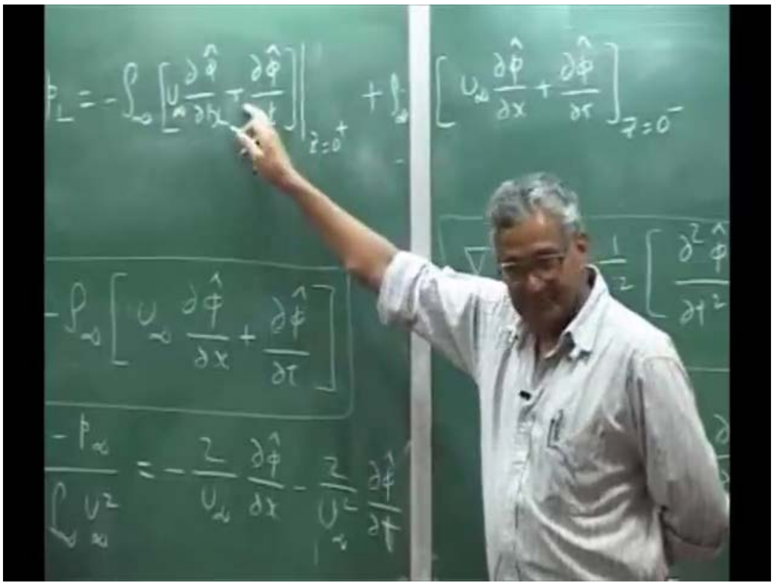

So, we will have delta g del over delta t plus U infinity g l over delta x, which is 0 plus and lower minus that is a minus sign sorry 0 plus sign plus U infinity 0 minus. Therefore, not this out you will have delta phi over delta g symmetric with respect to g. So, phi is anti symmetric with respect to g. If phi is anti symmetric with respect to g, I can write my pressure because this is the upper square lower square I can take. So, upper surface means p u minus p infinity, I will get one value, lower surface again p l minus p infinity is this. So, if I subtract upper minus lower which is the delta p change.

(Refer Slide Time: 1:02:58)



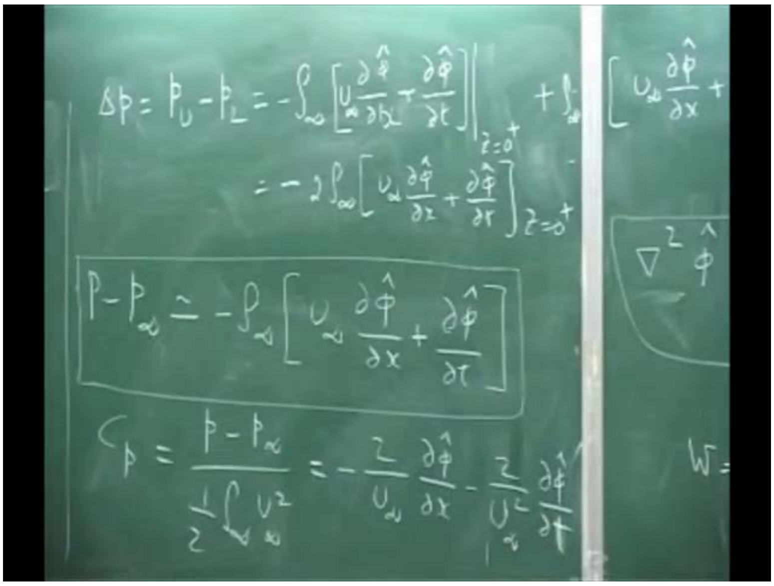
In the pressure differential pressure p upper, this is essentially I write this term because p infinity is anyway come and that will go off. You will have minus rho infinity delta phi over delta t plus delta x this is g equal 0 plus minus of minus.

(Refer Slide Time: 1:03:42)



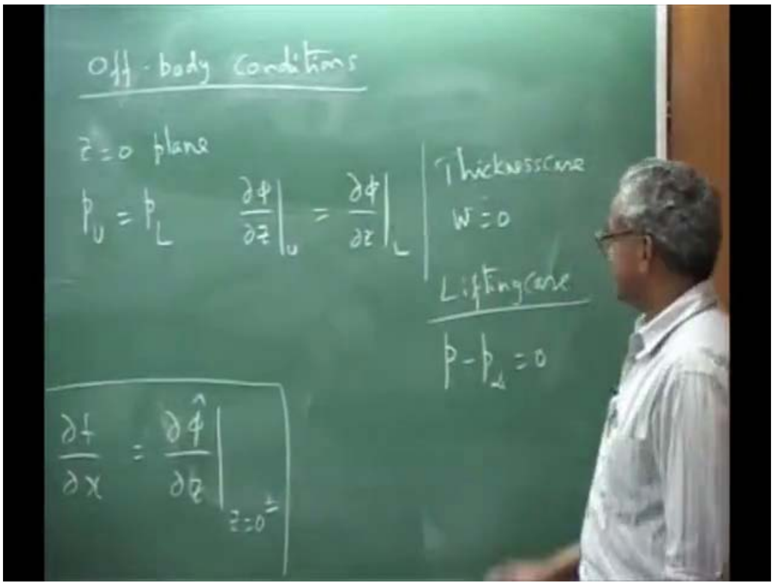
So, I will get plus rho infinity u infinity delta phi over delta x plus at g equals 0 so on. If phi is anti symmetric means then what will happens? This term will become minus on the top surface p s this will become minus.

(Refer Slide Time: 1:04:16)



So, your net term will become minus 2 rho infinity u infinity delta phi hat over delta x delta t g equal to 0 plus. So, you have a pressure difference that is because of your lifting thickness will not contribute. Now, the question is what of that of half body boundary condition. This is on the body. Now, off body boundary conditions you have to again write it for the lifting case and thickness case because this is essential if you want to solve rueful problems. So, I will write that part I will erase this here, it not necessary. I will write here off body boundary conditions. Off body conditions you can say.

(Refer Slide Time: 1:05:23)




See off the body means your velocity must be continuous and the pressure must be continuous that means let us look at it is only the g equal to 0 plane is important. So, g equal to 0 planes. You will have pressure upper is equal to pressure lower, it must be there and velocity must be continuous. So, you must get delta phi over delta g upper surface must be equal because particular this is very important for the q dot n because velocity is continuous null fall velocity have to.

Now, if you take condition if you look at here. From here thickness case, thickness case what does it say delta phi by delta g is anti symmetric that means, but I want delta phi by delta g to be same, which means thickness case what W 0 because there it says anti symmetric delta phi by delta g. Here it says that means I want w 0 that means there should be no vertical velocity in the backside.

Now, if you go to the lifting case, lifting case this phi is anti symmetric that means phi is anti symmetric means what? Your pressure upper and lower are opposite whereas, here it says p u must be equal to p n, which means the pressure should be 0. Pressure should be 0 means that pressure should be p infinity that is all. So, there is no pressure change. So, for lifting case pressure p minus p infinity must be 0, this is off body boundary condition. This is on g equal to 0, this is off body. Thickness case says that w must be 0 and then lifting case says that p should be actually p infinity. Now, this is what is used in getting solutions later off body boundary conditions.