Aero Elasticity
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Lecture – 16

Today I am mentioning about the Kelvin's theorem.

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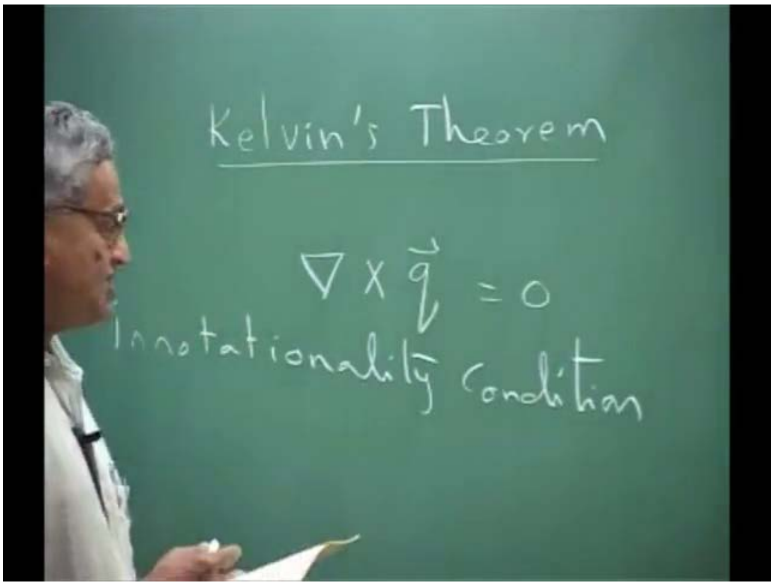

That is the fluid is initially irrotational, irrotational means is 0, this is irrotationality condition, if the fluid is initially irrotational, then it will be irrotational always.

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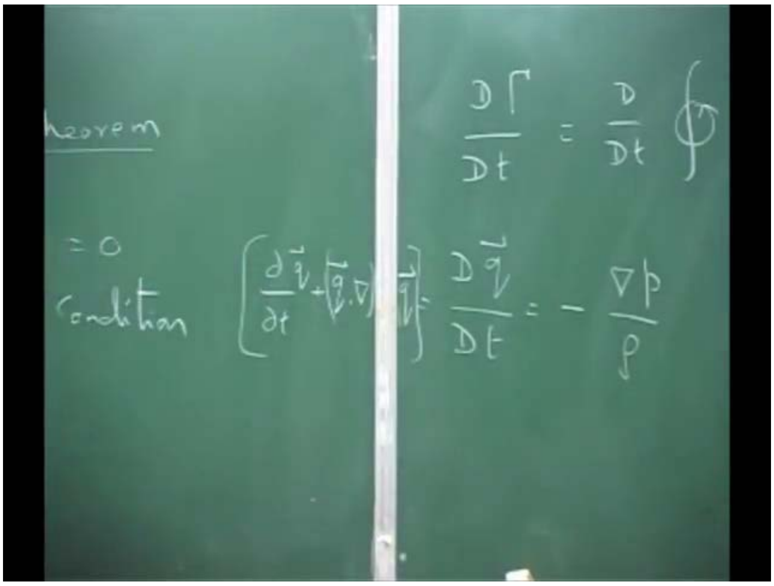

So, the proof for this is because this is over a region that is why you have taken the this is nth, but, D over D t up because circulation is q dot D r. Now, from over momentum equation, we know D q over D t, this is the momentum equation is minus because this is nth, but that delta by delta t up q plus q del q. This is what this is now what you do is you take a curl of this and then because this d by d t and curl, they are linear operator, so you can shift them wherever you want.

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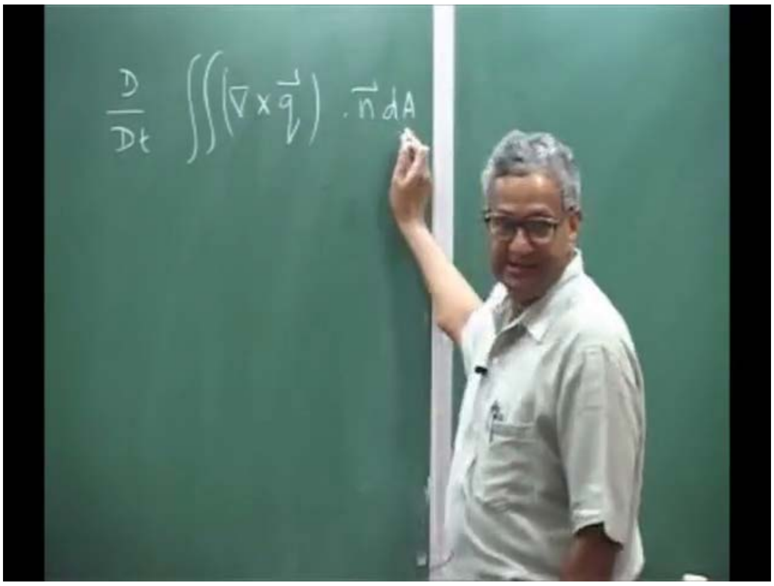

Then, you write like this D by D t, take a curl that is I am taking del cross this and then take an integral over a area. That means basically you are calculating the circulation and then n that D a.

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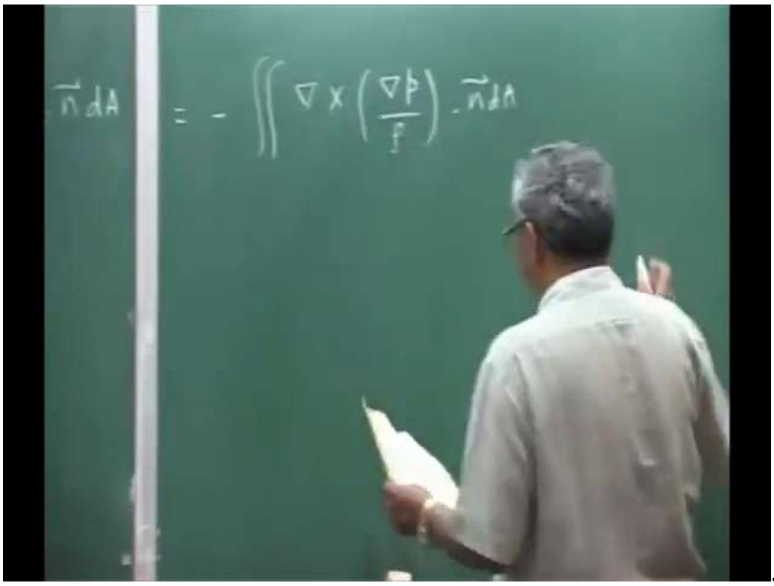

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This is again minus you will have because d q by d t, you are putting del cross, so del cross means that will come here, the same del cross an integral will be there. So, you will have del cross delta p over o, that n d A these is a surface, now you apply stokes thorem.

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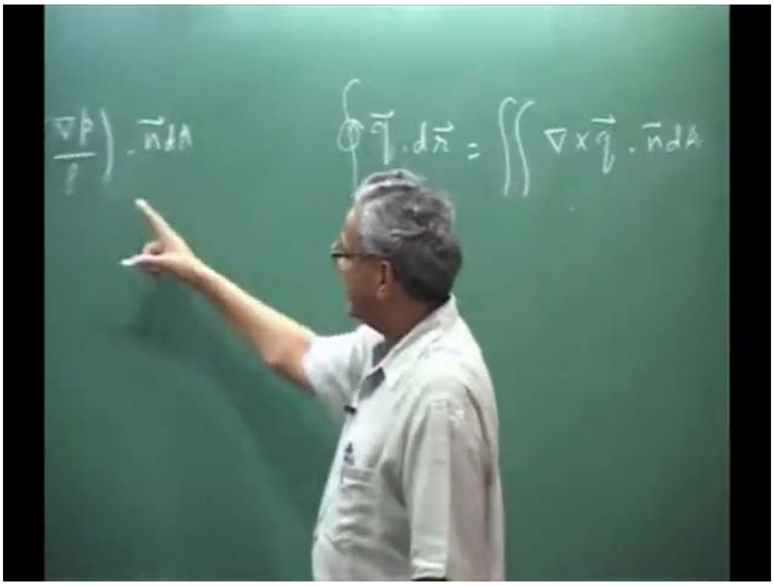

The Stoke’s theorem says that q dot d r is n d A, the surface integral converted into a line integral, now this is the surface integral del cross n.

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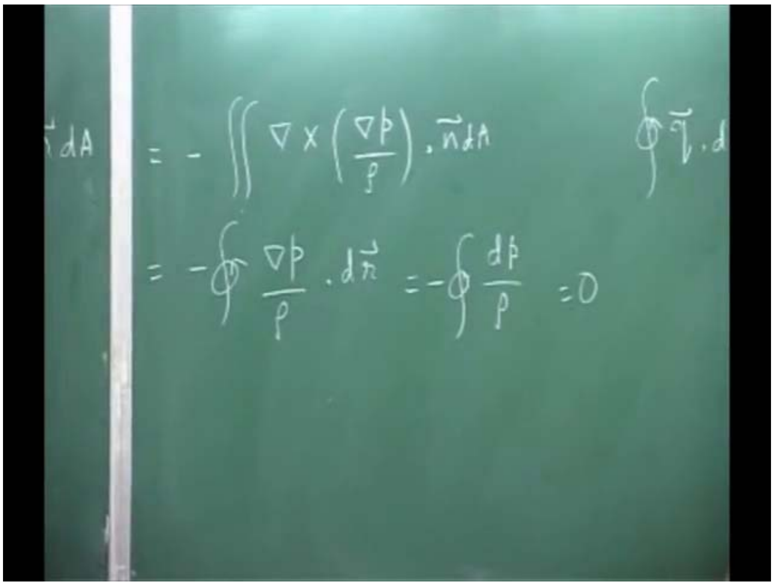

So, this will become minus this into del p over rho dot d r del dot is nothing but d p, because del p is delta p by delta by delta x i delta p by delta y is j delta p by delta k is a k. That product is nothing but d p, now you know that the pressure is only a function of density, therefore this close interval is 0, because we said that because of and on the isotropic flow. So, first we gave that proof isotropic flow in which it will give you that pressure is only a function of density and after that you evoke the Stoke’s theorem on the Kelvin’s theorem to show that if my circulation. If this is 0, initially it will always be 0, because it will always be 0, if this is initially 0, it will always be 0.

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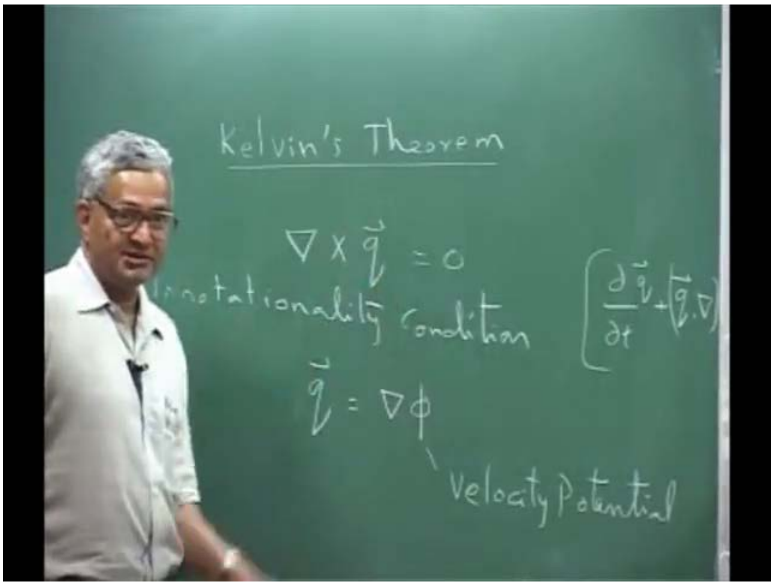

Now, what does this means, it means is if my del cross q is 0, this is irrotationality condition, then it will remind that is the Kelvin's theorem. If this is 0, initially it will continue to be 0, now if you how this condition call up a vector is 0 means that q can be written as a gradient upper potential. This potential we call it a velocity potential because this is 0 will be in this form, because it will substitute, it will be always identically 0, now that is how the potential flow as coming to picture, so you said first this isentropic than irrotation.

So, inviscid isentropic gives you barothropic that is pressure as a function of only density, our density is only a function of pressure as a function of that relation p over rope over gamma is the constraint pressure is the relationship. Then, Kelvin's theorem that is the Stoke's theorem you define what is circulation gamma q dot d r the Stoke's theorem come, because the line integral is that area integral.

Then, you invoke kelvin's theorem, once we are proved this much that my velocity and be written as a gradient of a potential. That is how the potential flow has come into picture you call it a potential flow because q can be written as a gradient component. Now, we will have a several derivations, but what is the meaning of del cross q 0 what is this?

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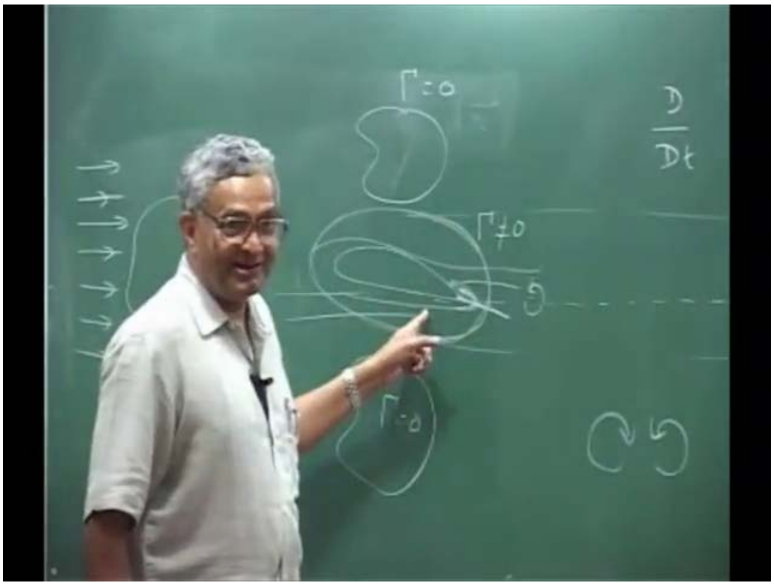

If you take an aerofoil point, just briefly just victor elate you because if the flow is coming from initially flow static, it start with the uniform velocity m u l, then circulate is 0, because q is same. So, you take a closed path, you will gain a close path gamma is q dot gamma is a, but we have a aerofoil point, but please understand that this path you squeeze it. It comes to your point, it does not cross any boundaries and you can take here here also it is zero gamma is 0, here you take a close path, here on the gamma is 0, but if you take a close path around this that means you are crossing the wake.

This is squeezing into a your boundary, so here you can how this gamma not 0 if you take this whole that is full wake, then what happens is usually when the flow starts, you will have one circulation or you can say one vortex will shed. This is called the shed vortex, how would it shed, why it is shed that by experiment, you what happens is the flow comes the stagnation point initially maybe on the because it is a term angle of stag stagnation point will be here.

That means stagnation point of a velocity is 0, the flow will come and then go here, the flow has to come and then make a turn. Now, making a turn the quarts enormous qwords, now these is one of the condition initially when it said it tries to g that way, but after because you have to have lot of qwords this point that is due to viscosity effect, how that is created.

That you have to have viscosity without viscosity, you cannot create all text, but then slowly this point will move here. That is why the stagnation point, you make it there triangle, but in real flow, it will be something like this, it will go like this, it will extend beyond that that is because of the viscosity under the effect, but here we are neglecting with viscosity. Then, this comes this is like a flow is going back that this comes here that means this word text, it will shed and then this point will become the flow.

This will become the stagnation point and if you include this because the circulation initially is 0 everywhere starting that means the shed one circulation I must have an equal opposite circulation to make it 0. The opposite circulation is this you take around that around the aerofoil that is like if I have one more text like this, another word text must be there, so that the total sum is 0, so this will be around the aerofoil.

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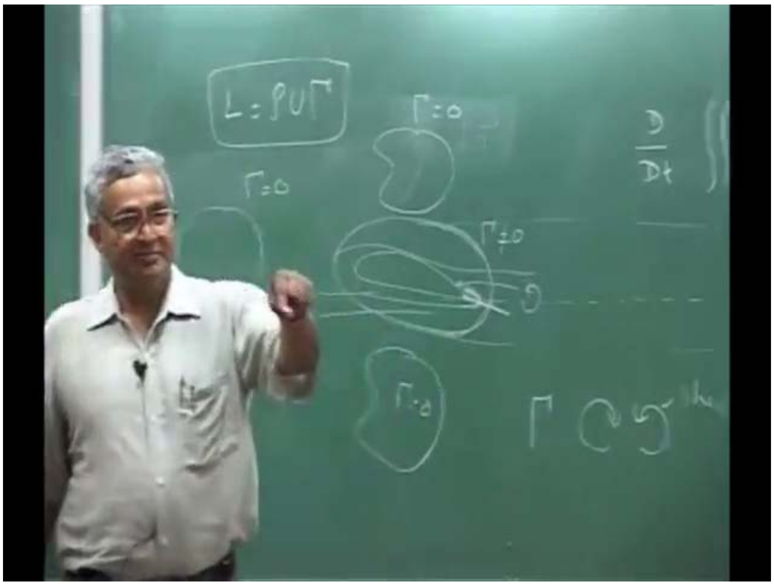

This is shed will simply go away because that is from the speed of it will be taken, but then once it goes far away, the effect of this on the aerofoil is negligible, because this flow once it is infinity, they will say this goes to very far away. Then, the only influence are around the aerofoil is this, this circulation, we defined that circulation you must have seen gamma. Now, you say my lift on the aerofoil is rho u gamma, this is what you use, but only thing is you think up gamma. You know density, you know velocity strength of gamma is dictated by the conditions that the triangle becomes a stagnation point this is what sub sonic case.

So, supersonic you do not how to that is how the Kutta condition is, Kutta conditions tells you the triangle does the stagnation point that the lift comes out with this. Now, in the case of this is when the aerofoil is steady that is not moving nothing but far field flow is always steady flow not even if starts moving up and down, the aerofoil are ugly thing. Then, what will happen is you will continue to shed, word test is you will have many this is what the trailing word text.

Now, you cannot neglect them, because you say everything from the triangle to infinity whatever word shed first, it will be there and it comes. So, it moves on going, now the effect of these word thesis have to be included if you are doing that and study aero dynamics. If you neglect that is what you are pass study quads aero dynamics once I study aero dynamic theory, you can say if you neglect that effect directly. So, this is where the complex complexity comes in what is number one, you do not know what is strict and the structure, how it will be.

This is where initially for all later we will develop this theory; the structure we will say that word text will always be in a straight line behind the aerofoil up to infinity. There is no interaction between this and this, because this and this may turn it this way, the structure can change. That is why we use those are all complex theories, we will not get into that we will use that there is no interaction between these vortices that is like a prescribed wake. This is what is called the prescribed wake structure I said it will be on the straight line behind the aerofoil, but if it is a free wake that means I allow the interaction between this.

That means they can interact they can change the orientation them computationally that will become more complex. That is why free wake analysis prescribed wake analysis all these things are in there potential flow only thing the strength of the word text. You do not know what you calculate later, this is what the whole potential theory is about, you assume a structure even in rotor blades. We have something out of a word text theory and then you prescribed are free wake analysis to get the effect of this. Let us now go back to the formulation I just want to derive you important things that is key conclusion of all this is velocity is a gradient up of potential.

Now, I have to develop an equation for the potential, because develop on equation means basically what I will do is we have continuity equation, we have the momentum equation and of course we got that pressure is a function of density. We use these five and simply reduce them into one equation and that equation will be in terms of free, but in that process you get some one more speed of sound do comes.

That means you should have another equation for the speed of song, so we will have two variable two equation that is all your full potential flow theory is represented by two equations, whether it is steady, whether it is un steady. We are deriving un steady equation, you will find that the these two equations are, now we will get that equation, so what is do is that there are some relationships which we first get.

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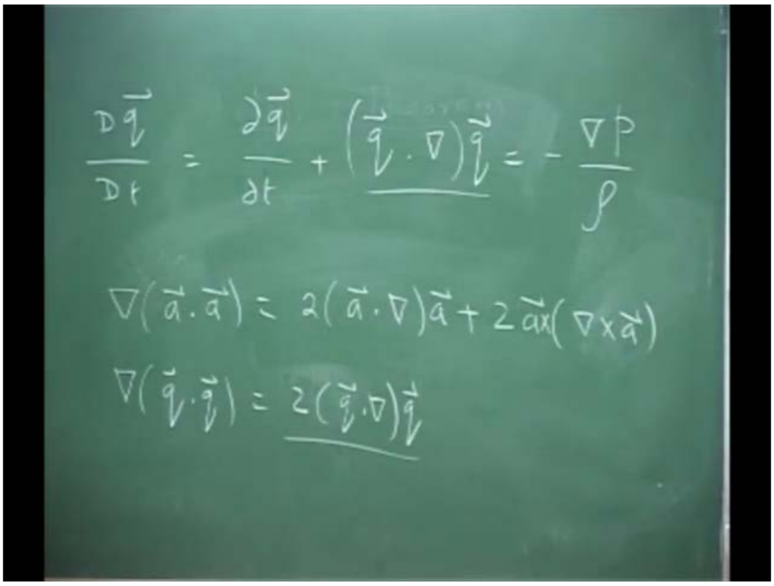

You know that this is your momentum equation, this is delta q over delta t plus q dot del q is minus delta p is over rho. If you have a vector identity, this is a vector identity two plus 2 a del cross, now if a is q that means del of q dot q if a is q. You will get del cross q but, del cross q that is 0, because it is a rotational fluid, therefore, if a is q if I write this as q dot q, this will become 2 q dot del. Why am I writing this is because this term is basically this term, now you can replace this and q is delta automatically del cross q.

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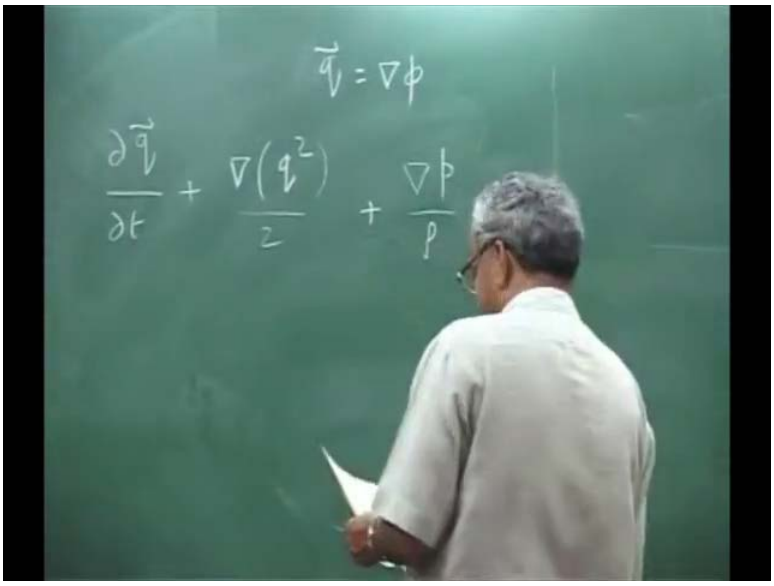

So, you will get the momentum equation becomes delta q over delta t plus del of q square, because q dot q is magnitude of q. That is why q square plus delta p over rho is 0, what you will do in this is you know the q is delta, you try to this expression also as a del of some term. Then, you can take del outside that mean that whatever term is inside, it is constant everywhere, because it is the independent of the position that is what we will do is particular term.

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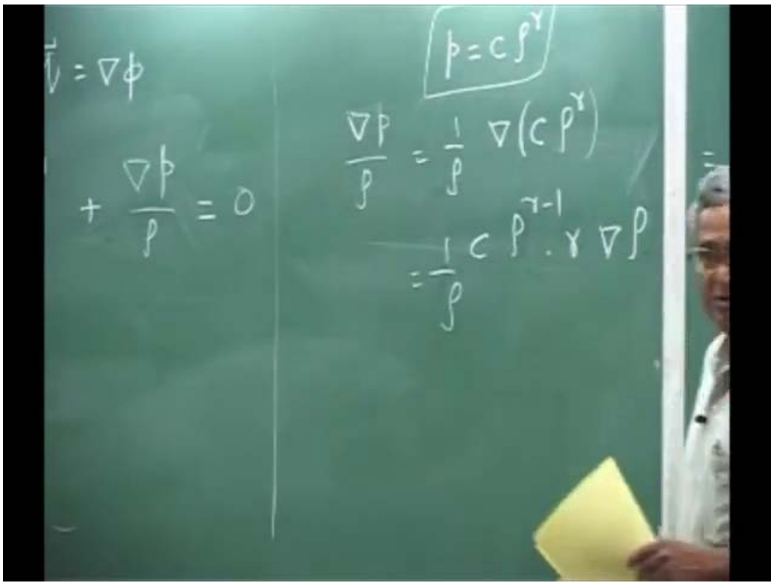

We will write it as delta p you know that it is p is a function of only density this is nothing but, one over rho delta you can write it as del c rho over gamma, because we said that pressure is some constant rope over gamma. This will become 1 by rho c rho gamma minus 1, you will have into gamma del rho, because I have substituted p in terms of the density, now this term if I write put it back here, this will become del, sorry I will write. Let me this is delta p over delta t, because q is delta than this is q square over 2, this term I am writing this, so you see rho and it is it delta p by rho, what is it this will happen what tell I can write this term as this is rho gamma minus 2.

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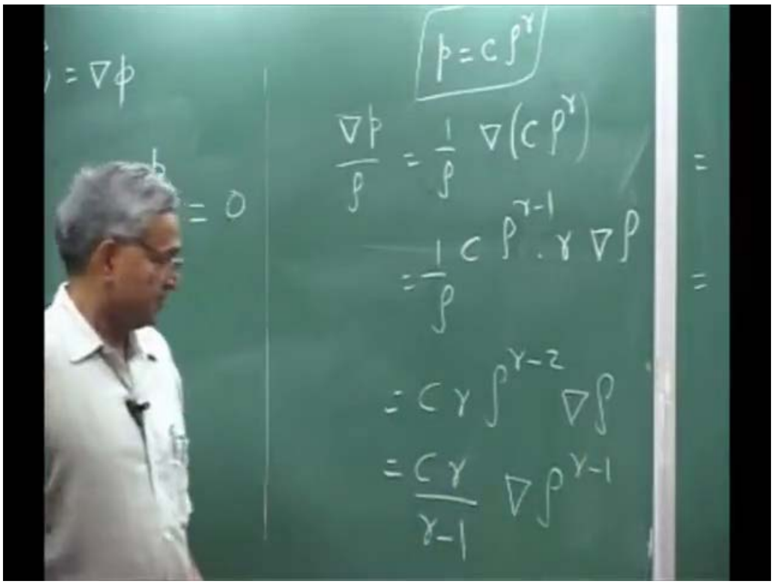

This I can write it as c gamma rho gamma minus two del rho, which is essentially c, I think gamma over gamma minus 1 del rho power, because when I differentiate that gamma minus 1 will cancel out.

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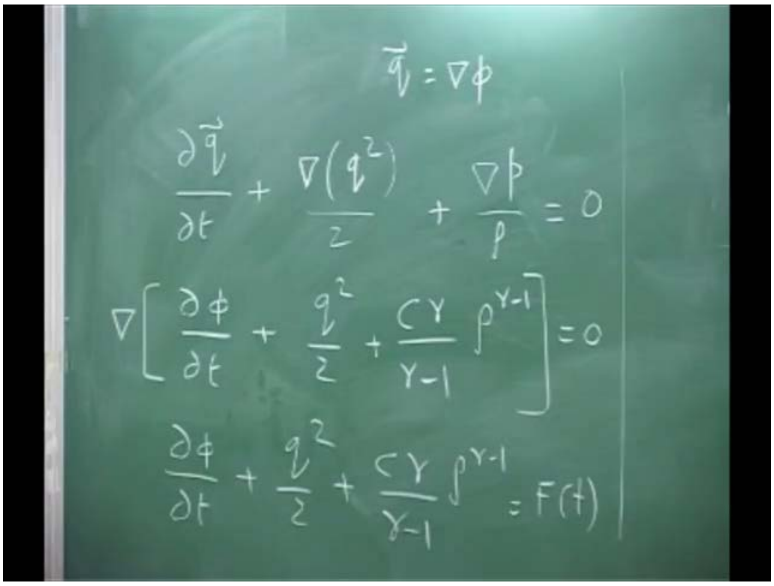

So, I am writing it in that fashion plus c gamma over gamma minus 1 del, I am taking it our rho power gamma minus 1 is 0 say not algebra. Now, you all will have lot of algebra only, now you see this is the del operator del is a i delta by delta x j delta by delta y and a delta by delta z, which means this quantity inside can only be a function of time. It cannot be a function of position, so you will write it as delta p over delta t plus q square over 2 plus c gamma over gamma minus 1 rho power gamma minus 1 equals some function of time.

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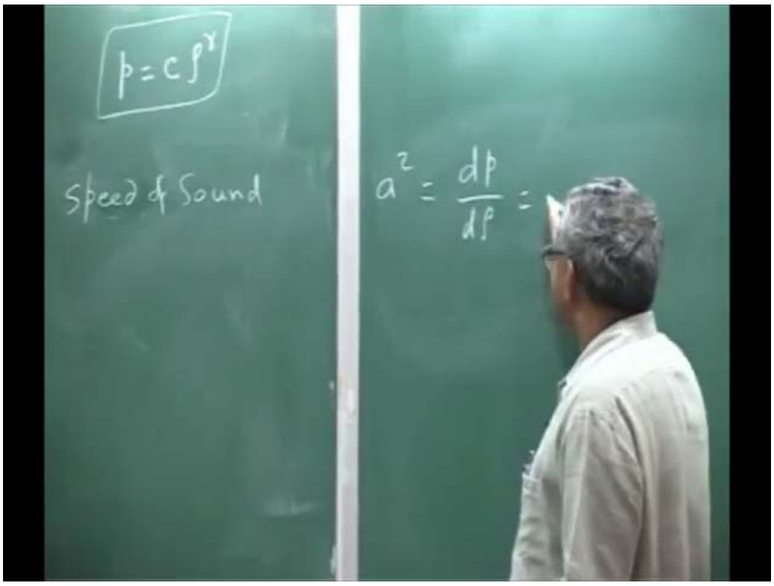

Now, you will also modify this slightly, because you will say what the speed of sound is because I am not deriving it a square is d p by d rho, now p is substitute this p c rho to the power gamma. So, you will get actually gamma c rho power.

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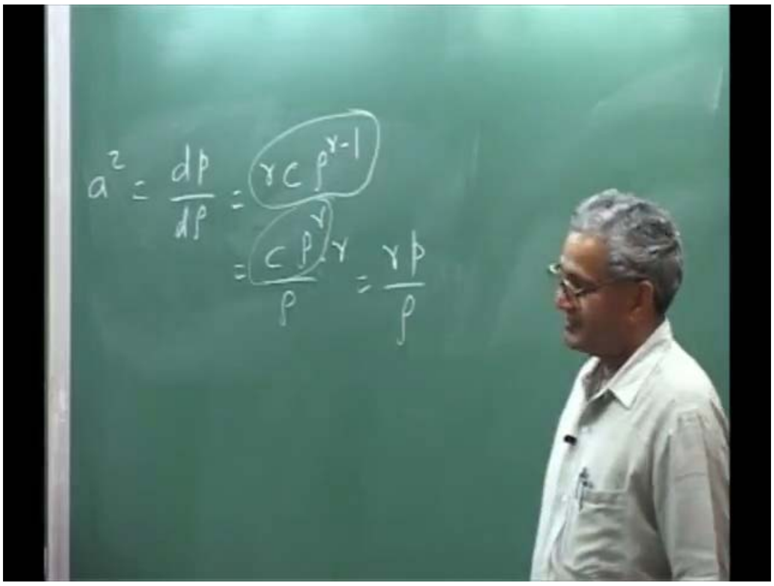

Now, you multiply this by rho and divide by rho, this will become c rho into gamma over rho and this quantity is p. So, you will get gamma p over rho, which is essentially this term a square is gamma c rho to the power gamma minus 1, now if you come back here this is nothing but speed of sound square.

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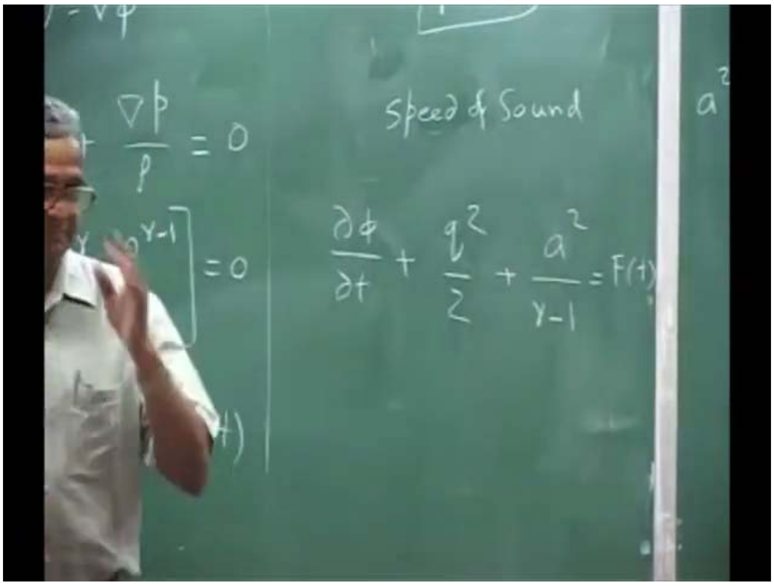

So, you will have your equation p over delta t plus q square over 2 plus a square gamma minus 1 equal f for t. That a is the speed of sound, because this part divided by the rho that is you can prove it. That is also from momentum theory and basically continuity, simple you can show the one dimensional problem, if you write it, you will get the speed of sound this.

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Now, we need to what is this f for t, because if you say my aerofoil is here aerofoil are lifting surface anything. You go too far away, here you write, because this is my i and this is my j direction j r you can have j here and k here, you can have anything you want, this can be k. This can be j this is far away and this is minus infinity, because my i always take it starting from the aerofoil. So, in the far field ahead the flow is coming steady with the speed u infinity and you know that p becomes because l p by del x is U.

So, that is what p is u infinity x and the pressure here is p infinity far field pressure, you can say atmospheric pressure and the speed of sound a is a infinity. Now, this is at a location far away in the negative infinity, I substitute that condition because p here is independent of time because p is only u infinity x. So, this term goes up q becomes u infinity u infinity, so my f of t if I apply this condition at far field, I am going to get over gamma minus 1 is nothing but f of t which is basically a constant.

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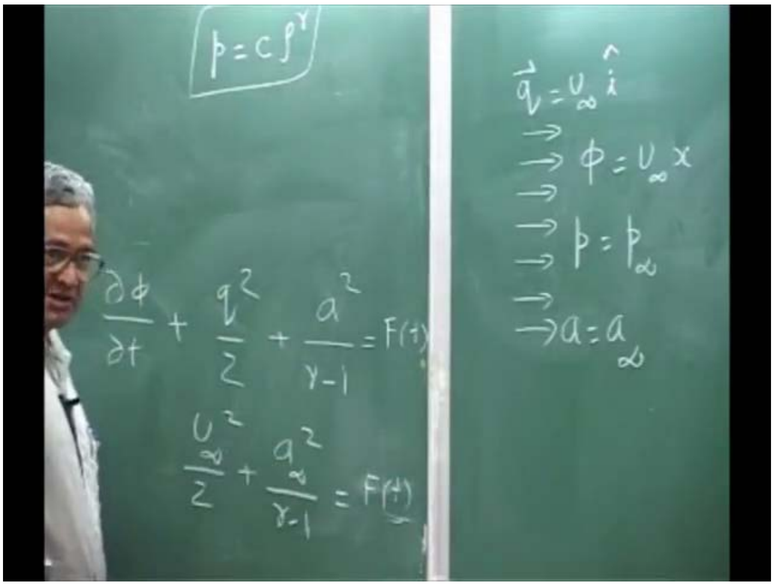

This is my f of t, but though it is a independent of time that is all, therefore my equation becomes if I summit up this one, it will be the condition, I get will be basically delta p over delta t plus q square over 2 plus a Square over gamma minus 1 equal.

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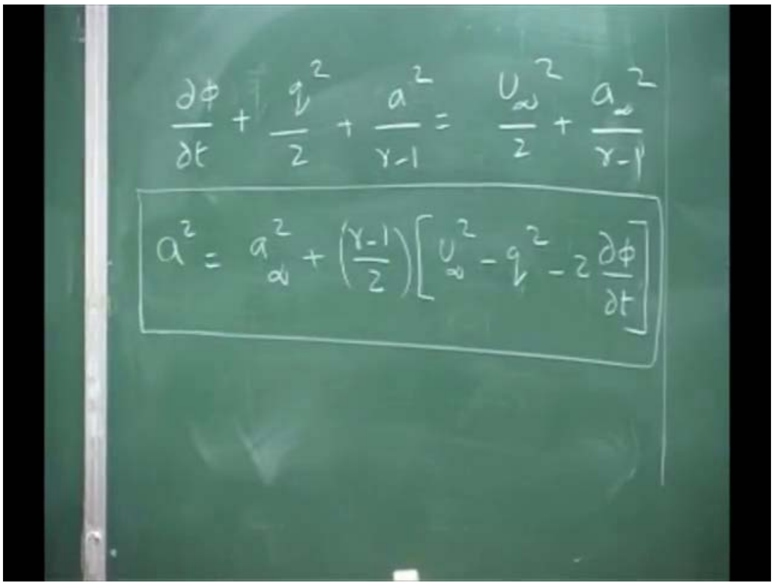

So, you rearrange this to get a is the speed of sound at any point a square becomes because gamma minus 1. You multiply everywhere, this will become a infinity square and then you multiply by gamma minus 1, you will get gamma minus 1 over 2 times u infinity square minus q square because 2 I have taken out minus 2 delta p over delta t.

Now, you see I am getting speed of sound. local speed of sound at any point. if by know p velocity potential at that point. q is Delphi, speed of sound is given in terms of velocity potential.

This is the purpose this is the speed of sound under standard atmospheric condition a infinity. So, this is the one of the important relation please note that that is local speed of sound in terms of velocity potential, now the same equation is used to get the pressure also that is what you do is, you know that a square is gamma p over rho.

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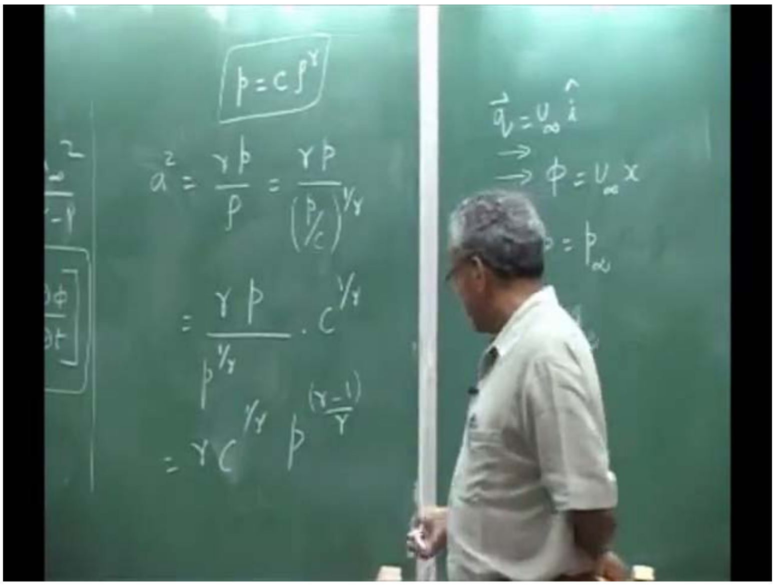

Now, this is gamma p, rho is p over c 1 over gamma, because I am replacing density by pressure. Now, I will get a square is gamma this c will go, this is not this is what one over is that correct rho is it correct rho is p by c p over c over 1 over gamma. Now, you can take the c and the p this will be p this will be p to the power 1 over gamma and the c will go to the top. I will have c to the power 1 over gamma, now if I take it up, I will get gamma c 1 over gamma p to the power 1 over 1 minus gamma, which is gamma minus 1 over gamma, which means my a square is pressure gamma minus 1 over gamma.

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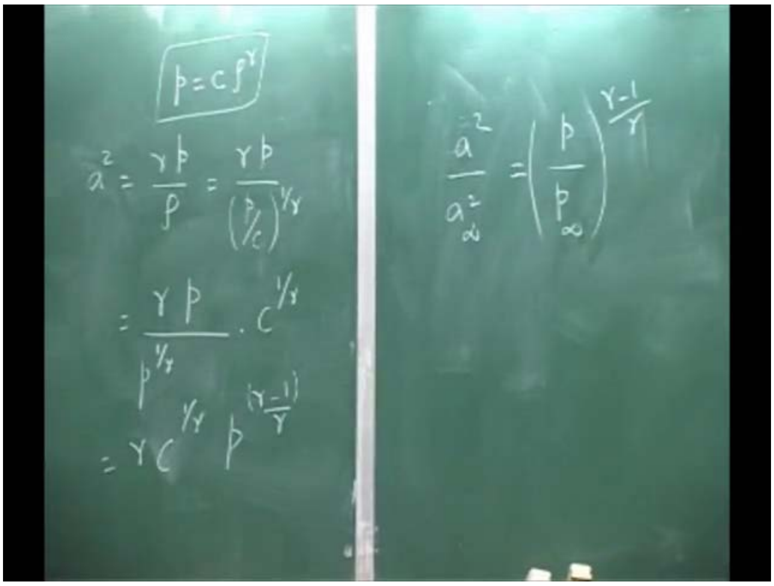

So, I can write in terms of a square over a infinity square, this is constant this will cancel out leaving behind p infinity gamma, now that is what I am going to write here.

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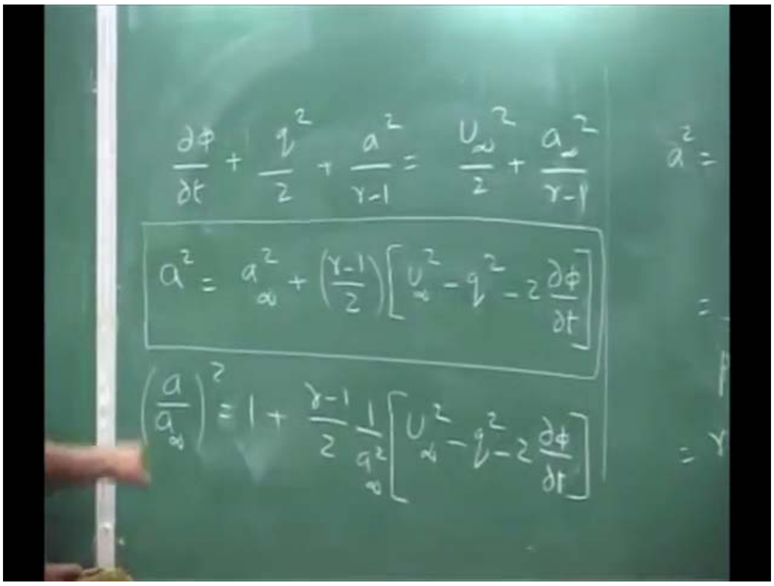

You divide by a infinity everywhere if you divide by a infinity, you are going to get a over a infinity 1, sorry equals 1 plus gamma minus 1 over 2 1 over a infinity square u infinity square minus q square 2 delta p over delta t. You just replace this by p over p infinity, then you will write your relationship I will write it here, you substitute a over a infinity square in terms of p over p t infinity and then take gamma over gamma minus 1 and multiply by p infinity here.

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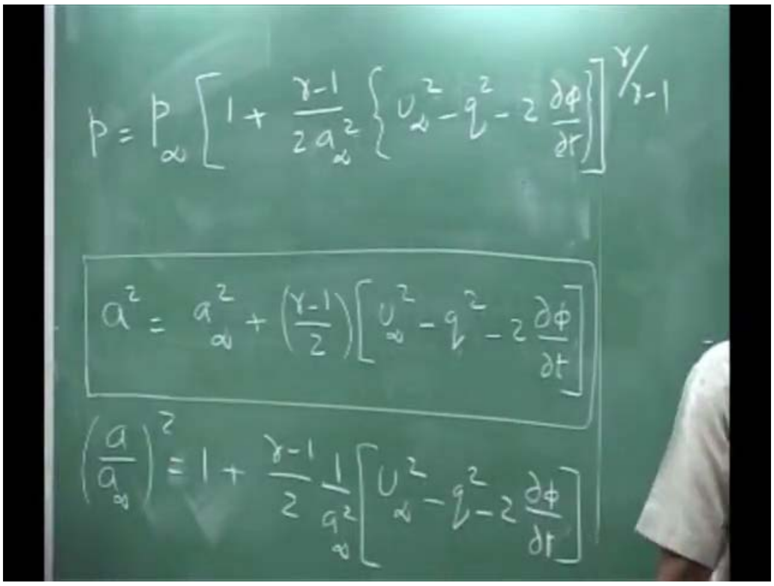

So, you will get the pressure becomes p infinity times 1 plus gamma minus 1 over 2 minus q square minus 2 del t. This is my full expression pressure in terms of again pressure in terms of potential. Now, this equation can be re written in an non dimensional form as a pressure coefficient because I will write that expression also, that we will finish that.

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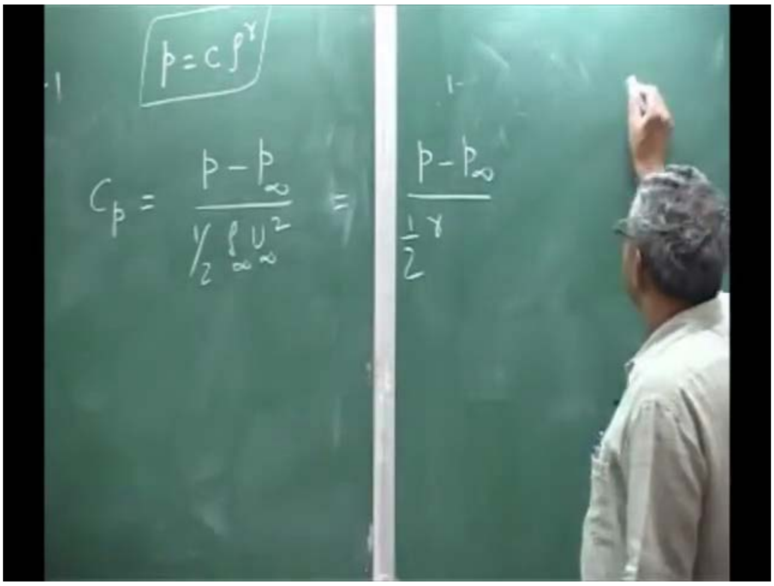

The pressure coefficient u defines C p as by half rho u infinity square, which you can write it, because you can say half rho, because this is rho infinity, which you can put it as half gamma rho infinity.

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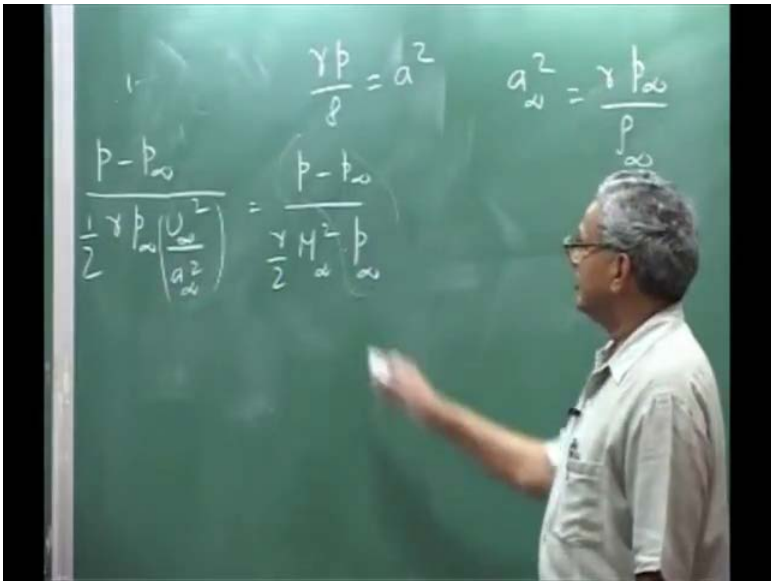

I am putting rho gamma p over rho is what, a square, another word a infinity square is gamma p infinity over rho infinity. So, rho infinity is going to be gamma p infinity by a infinity, I am going to put that substitute here gamma p infinity u infinity square. So, this is nothing but Mach number, m infinity square, so this is essentially c p is p minus p infinity over gamma over 2 m infinity square. Even though u over m infinity, because the Mach number are far field, not the local Mach number into p. Now, I can this quantity you can substitute for p in terms of the entire stop and then you will get pressure coefficient, which I am writing it finally, because p minus p infinity because I have to subtract one p infinity here.

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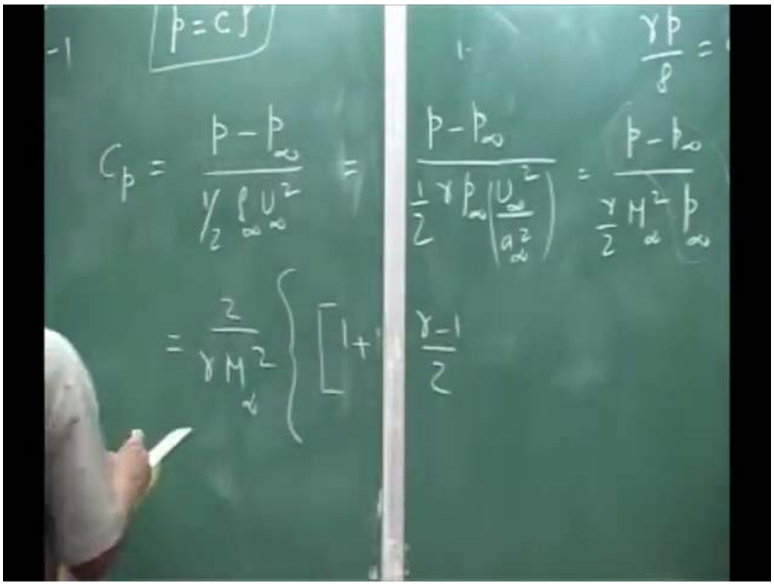

So, I will put another p infinity there minus, then divide it by p infinity that is going to be this quantity minus 1 that is all and I am simply have C p is because this is 2 over gamma m infinity square.

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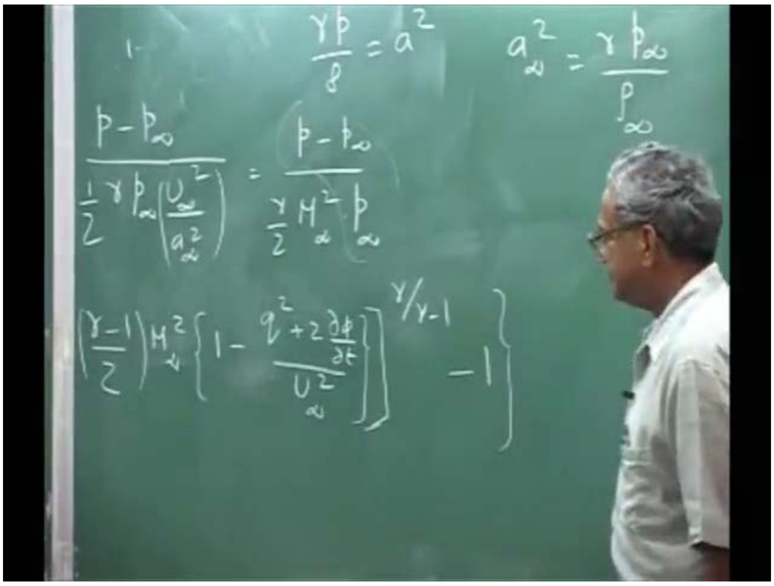

You open a bracket, open another bracket 1 plus gamma minus 1 over 2, because this term is gamma minus 1 over 2. If you take the u infinity outside this is mark number, so I will have m infinity square, you open another bracket 1 minus, you combine this two term q square plus 2 delta p over delta t divided by u infinity square. Now, the bracket this is gamma over gamma minus 1 and then I put another minus at this is my pressure coefficient C p why we write all these expression.

Finally, we may say these are very small numbers q infinity is a change is very small, and then you can do by binomial expansion and retain only the first term in sort of taking the whole power gamma over gamma minus 1. These are the approximations that are made; now you see we have the pressure expression in terms of potential. That means if I know the potential, I can get the pressure at any point and if I know the potential, I can get the speed of sound at any point. The next question is how do I get my equation for p that comes from, we have to again do some jugglery, this is a little bit because you need.

I will bring it because this is the combination of your momentum equation as well as your continuity equation. This is pure algebra if you want I can derive that entire algebra, because this is essentially some substitution back on four and at the end you will have one equation per phi. Now, we will just briefly go through that, so that you have an idea of what we have developed here as everything because we need to have this equation, because we are starting from this equation. So, let me keep please understand I will leave this also. I will erase this also, I would prefer to have and I cut it out and what you do is this equation, this is the nothing but momentum equation.

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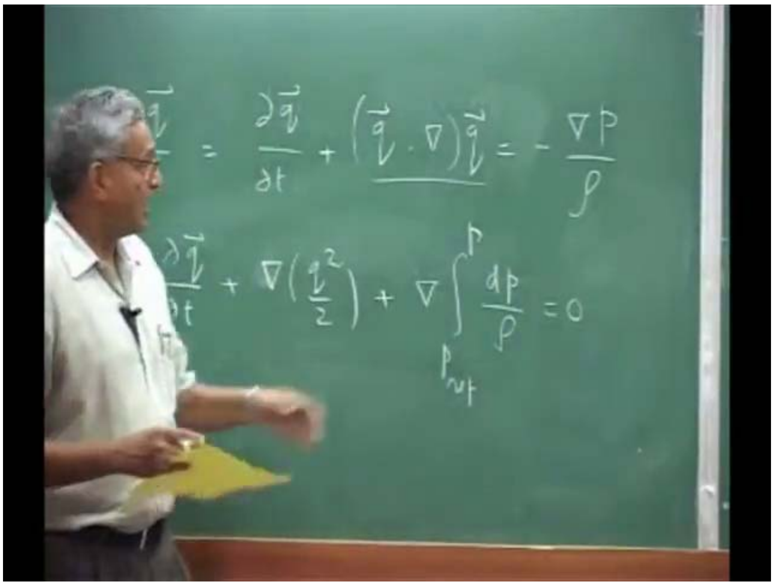

You can write in the different form that is delta t plus q, we wrote it as del of q square over 2, this term can be written because that you have to know how to write because this is again some p reference to p d p by rho. How this is written, minus del p over rho because the minus sign have brought it here, this particular exercise.

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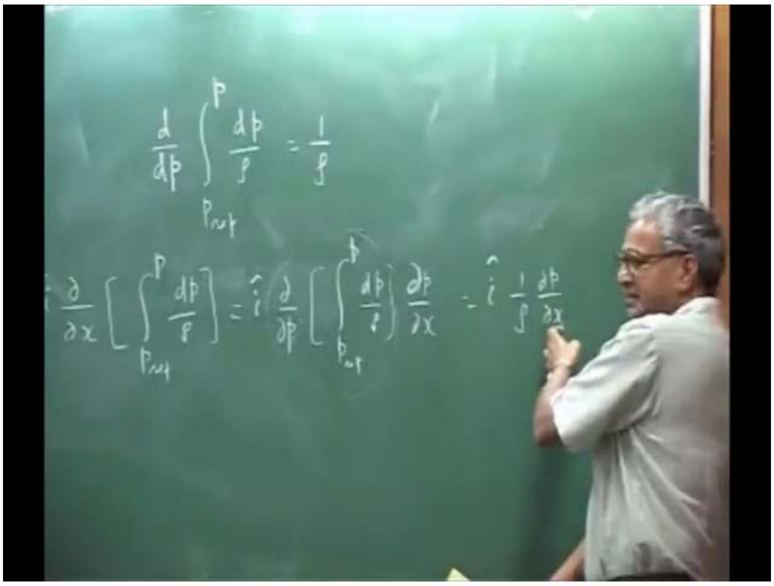

This is just some little algebra not that I will write this here and then we can erase it because you know d over d p of some p reference to any p. This is nothing but 1 over rho, because you are differentiating with respect to the integral. So, you get the same thing, now if I write like this i delta by delta x of integral p reference to p t p by rho, I am writing it as i is the unit vector delta by delta p here of integral p reference to p d p by rho into here because of delta p by delta x.

This is nothing but 1 over rho, this is 1 over rho noise delta p by delta x that means similarly i j k I can write it, then that will become i delta p by delta x is what del t del p by rho is what? I have del p over rho is nothing but del of this integral i j k, if I add that is nothing but del of this term that is what I have written here the gradient of this integral is nothing but this, now you again re adjustments comes because you know the q is del t.

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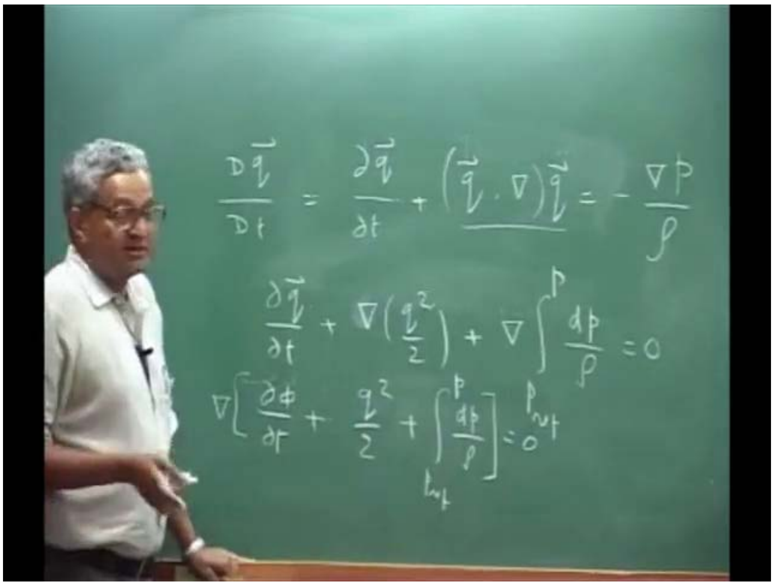

So, I can write it as del of delta p over delta t plus this is q square over 2 you can retreat retrain it as q square over 2 plus integral some p reference to p d p over rho is 0, which means this quantity is again independent of the position. That means it can only be a function of time, which you call it another f of t, then you will say what is the condition at the far field far field p reference to p that is p reference that this term goes to 0 u becomes u infinity square. So, essentially this term if you apply it to the far field, it is u infinity square over 2, so that is your equation.

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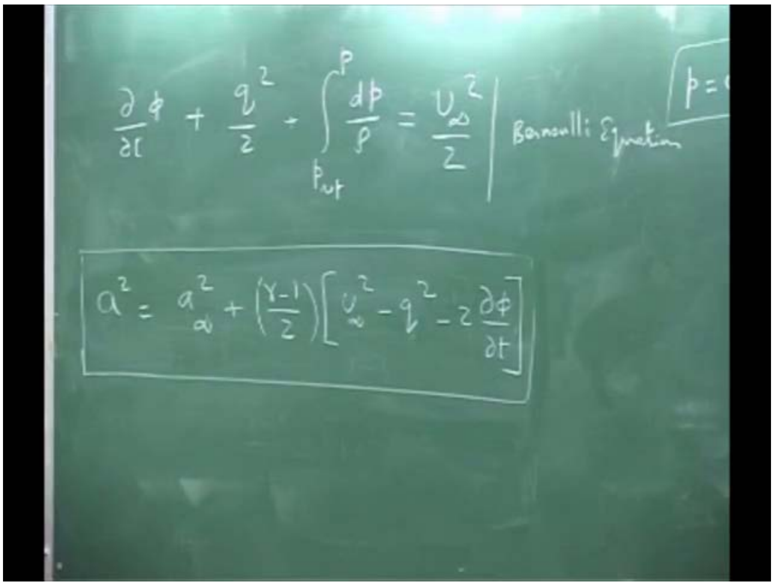

So, you will get delta over delta p plus q square over 2 plus integral p reference, which is p reference is p infinity p d p over rho is and this is called the Bernoulli equation or unsteady this is what Bernoulli, but even though it is not derived by Bernoulli. This is the Bernoulli equation, now how do we get the final equation, because this is still used please understand, this equation will be used.

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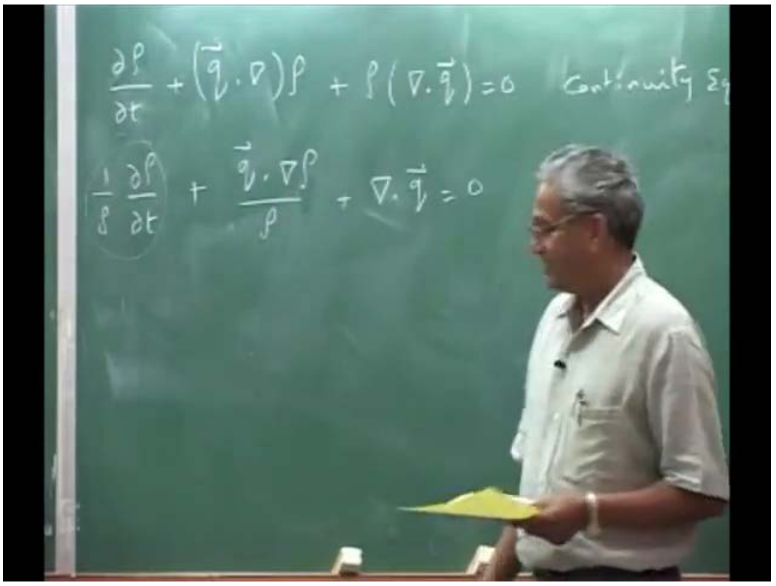

We will go back and write our continuity equation the continuity equation is delta rho over delta t plus q dot del rho plus rho. This is my continuity equation here, what I will do is I basically go on substitute every term in terms of feet and that is my final equation, but for that I need to have these things. So, you divide by rho this will have 1 over rho delta rho over delta t plus q dot del rho over rho, because this is del rho plus del dot. Now, let us take this term, what we will do is I go back here is this path cause, this is not necessary.

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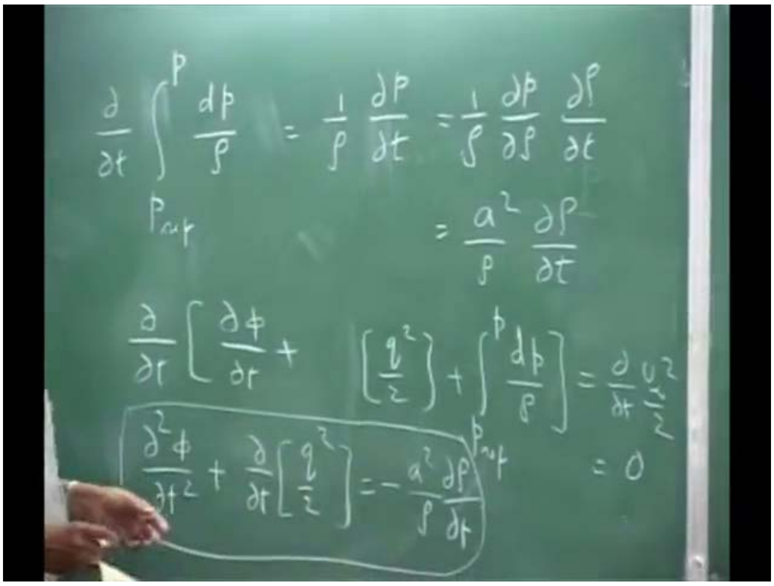

Now, we need to get, suppose you use this integral delta over delta t of integral p reference to p d p over rho, this you can write it as delta by delta p into delta p by delta t. Then, when I do delta p by delta t will be separate delta by delta p will be 1 over rho, this will be actually I can write it as 1 over rho delta p by delta t, I am just changing the, first I differentiate pressure because pressure this is only a density everything is only pressure term dependent. This I will write it as 1 over rho delta rho by delta t this is d p by d rho is speed of sound. So, I will have a square over rho delta rho by delta t, now what I will go is I will go and look at this term, I simply differentiate delta by delta t here, this is a constant rho.

So, from here I will get here from this equation if I do delta by delta t what will I get this term is going to become from here I will get delta over delta t of delta phi by delta t plus delta by delta t of you will have q square over 2 plus. Now, I should not put this inside plus integral p reference p, this is delta by delta t u infinity u square over 2 is 0 and this term I can replace a square over rho d rho by d t. Then, I will get del square phi over a t square plus del over delta t of q square over 2 is equal to I am taking this term that will be minus a square over rho delta rho over delta t. That means I have an expression for 1 over rho delta rho by delta t as this term, so I will go and write here.

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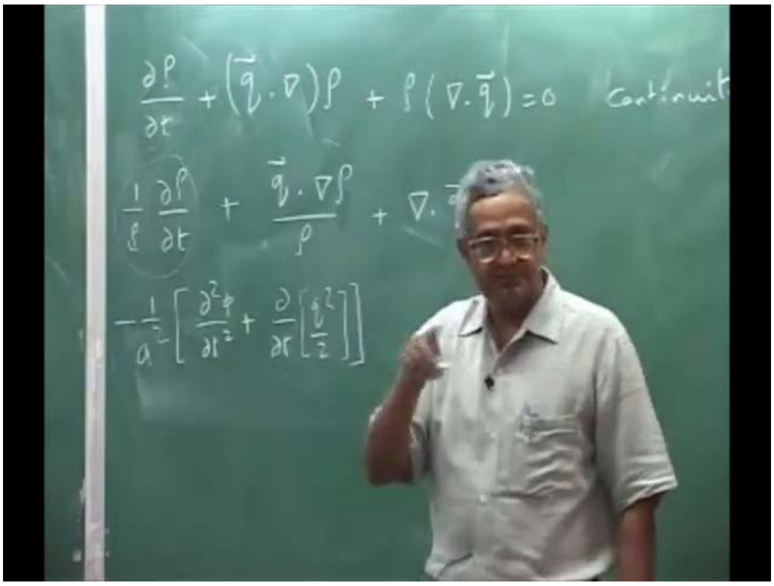

This is nothing but 1 over rho delta rho by delta t, I will write minus what 1 over a square it is not that I am getting 1 over square into del square p over delta t square plus delta by delta t of q square over 2. This is the first term, now I will go and replace this term, this is the first term is replaced everything has come rho is changing p q is del phi. So, this is also in del phi, now this term also I will go and change into phi that you do it as again you go back , I erase this part, because this is now not required.

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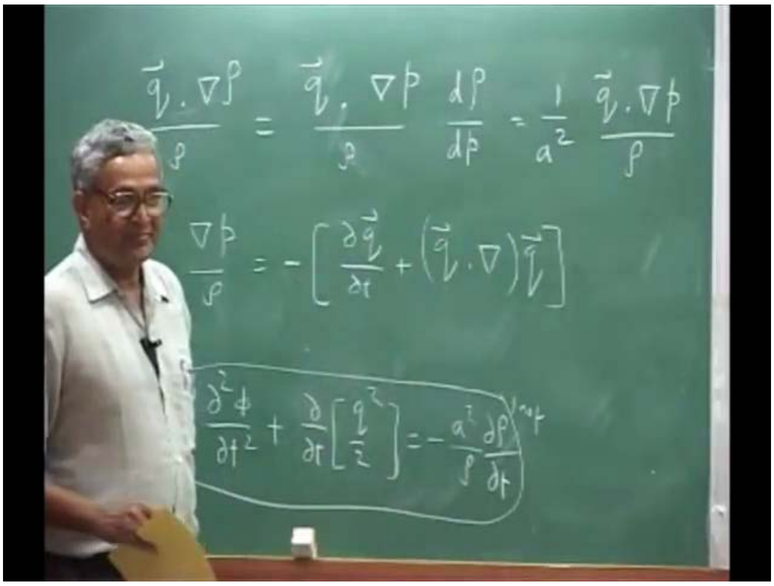

Here, q dot del rho over rho you can write it as q dot delta p into d because p is in terms of rho, now this is nothing but 1 over a square, this is 1 over a square q dot del p over rho. Now, del p over rho is what delta p over rho, this is from the momentum equation, you will have minus plus q dot. So, I can directly substitute here q dot del p over rho is this term, so I go and of course 1 over a square is there.

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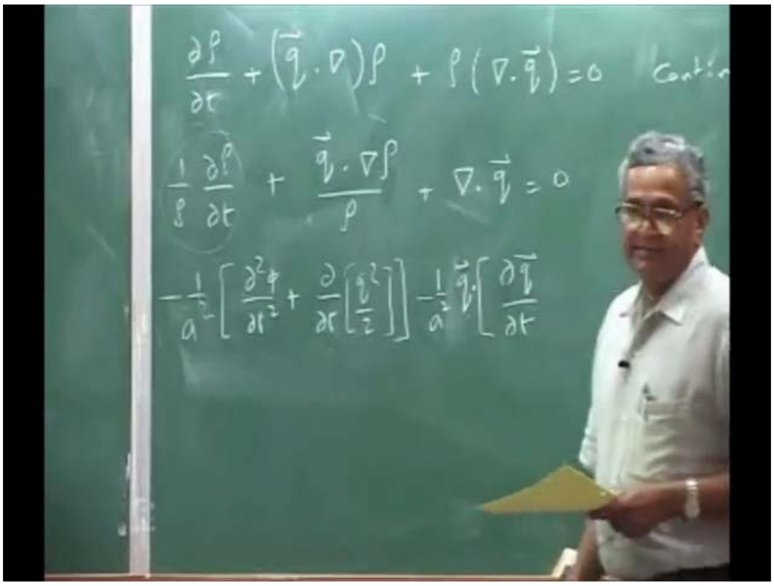

So, I will substitute this term, then this will become minus again, so minus o1ne over a square del q over delta t that is the first term, I have to get a q dot, there is a I will put the q dot 1 by a square. So, I will use a q dot q dot plus second this, now q dot of this particular term, because this term is q dot del q, how can you where is that this is nothing but del q square.

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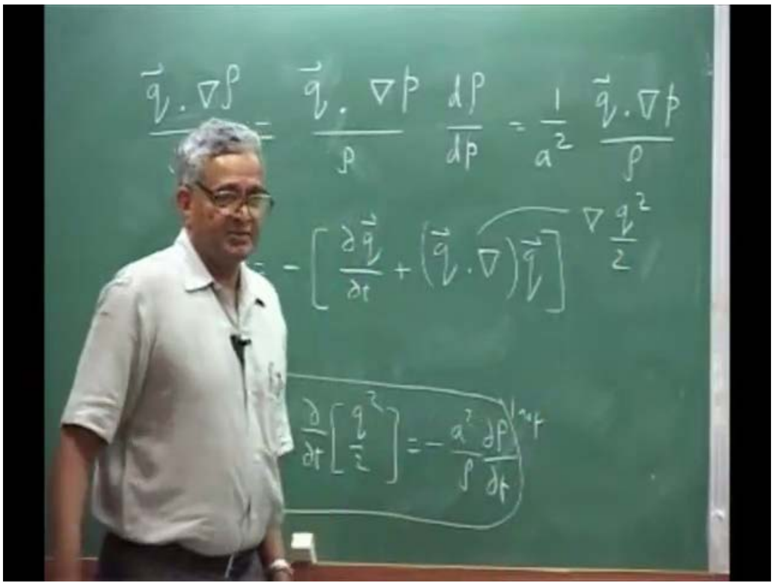

This we wrote it as we wrote it is q square over 2, this term.

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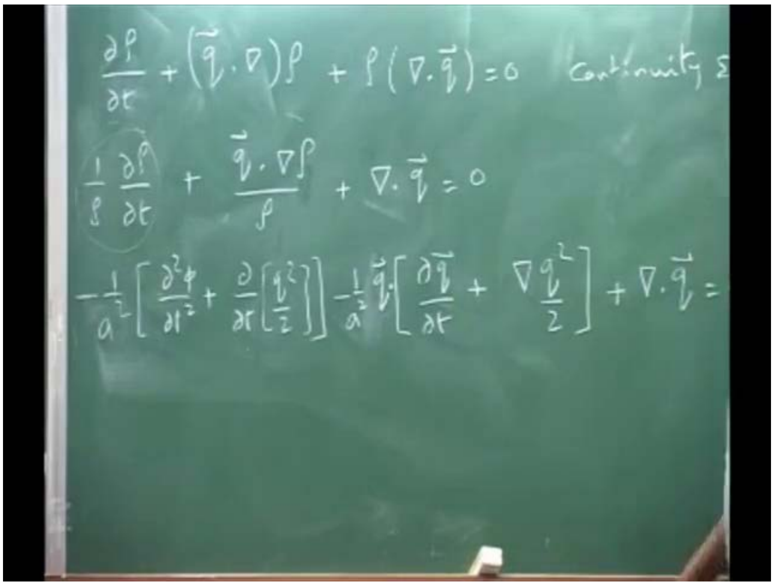

So, you will have here q dot del q by del t and then del q square over 2 plus 0, now what you do is you take this q dot this yeah diversions of rho that is what replaced here. This is replaced in terms of p this p is from momentum equation, now that is what I am doing q dot is always I have to put that is why i put a q dot of the remaining term. Now, this entire equation can be written as del q and this is nothing but q dot this will go inside, so

I will have q dot delta q by delta t, I can have q square. So, I will write it like and you look at this term this is one over a square del square this term is remaining as it is.

This will be delta by delta t q square over 2, this term is again q square over 2, you will have because this is q dot delta q by delta t is q square, you can take q dot q dot which is q square. This will add and q square over 2, it will have a q square over 2, so it will have only q square. Now, I can write my full equation like this boundary conditions I will be able to introduce a little bit today. Now, you see this is all your two equations all the algebra which we did till now, essentially boils down getting this.

Now, if you assume by flow is incompressible that means density is a constant, then speed of sound is actually infinity a will go to a very high value. This will completely drop out you will get del square p is 0 that is why even in low speed incompressible condition. If you assume del square p is 0 for unsteady flow also, but it does not matter and then you have only phi, anyway a is gone, it is infinity. So, your complete unsteady flow equation is gone by a del square p is equals to 0 that is algebra equation, whether it is steady unsteady it does not matter, you have the solution, you have this equation only thing is you have get the boundary edition.

If it is a compressible flow, once you go than you start that is why transonic when you go to transonic speed. You cannot assume my Mach number is approaching 1, you cannot make that the by speed of sound and the vehicle they are almost close. I cannot make that solution, then basically I have to use the full non linear equation, now what people do is even in transonic because we fully non we cannot get fully solution, because you have to do numeric. They use a small vertebra ion analysis that means my far field which is u infinity and my wing another thing.

They are all very thin and because of the nature of the surfaces, which we use in the aero space, we say it is a very thin surface; you make approximations to these boundary conditions. Then, essentially the problem becomes relatively simple and that is what is done, otherwise you have to solve the full equation, this equation if you solve with the respective boundary conditions. Now, we will just describe little bit about the boundary condition I will introduce and what is the nature of boundary condition, I will write that equation, but this is all algebra.

Finally, please understand one thing the algebra is that is way you get last thing, so many differentiation substitutions back and four etcetera that has nothing to do. Finally, you will need to get this two equations that is all and the whole idea is continuity equation momentum equation and we have invoked inviscid isentropic irrotational bass and they brought this equation and this is valid for any speech even 0 to Mach 0.3. So, as long as you do not have intense heating isentropic relation is invalid that is all.

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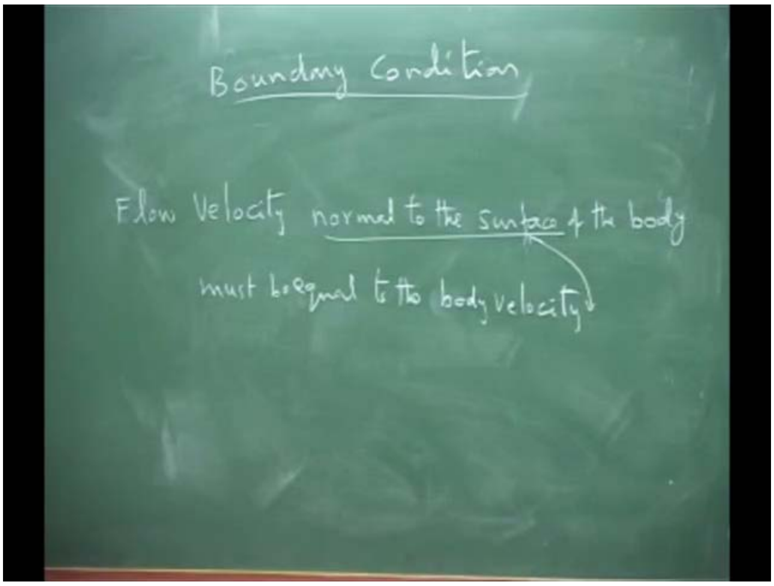

Now, let us take the boundary condition I will just briefly derive this today, then our boundary condition is flow normal to the body is should are the same velocity that is all in the sense velocity please note that velocity normal to the to the surface of the body. You may call it if you want to call it you can say the flow velocity normal to the surface of the body must be equal to the velocity of the body normal to the surface, because velocity of the body at a point you can have components u e. The normal velocity of the body at any point that is why you have two velocities, one is flow velocity, another one is must be equal to the body velocity again normal to the surface this normal to the surface is it is there in both places.

The body can move like this what if the velocity normal to the surface the time that is what you have to take at that point. So, you have two velocities one is the flow velocity and another one is body velocity you basically take the normal velocity at the surface and you say equate. The flow velocity normal to the surface must be same as the body velocity, which is normal to the surface. That means you have to firstly define the surface, then you have to find out what is the normal to the surface and then you take the velocity normal to the surface the component of velocity normal to the surface. That is what you will do, now the surface of any body is described by you say x r, y, z, t at time t.

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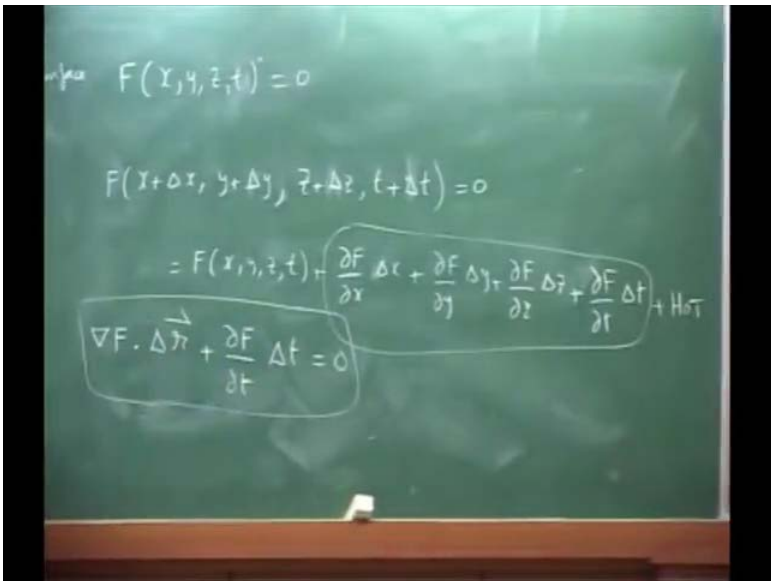

This is the equation of the surface of the body which can be a wing anything because it is a c r e dimensional body with respect to some coordinates system after some time the body is moving. That means body can deform also that it can retrain in any shape, please understand because what can be a nice circle can become an ellipse. That means your equations is also different, so what you do is you after some instant, you will have another equation that equation I mean using the same symbol. Now, x plus delta x y plus delta y comma z plus delta z comma t plus delta t is because in a small time increment, I am getting another surface. This is the delta, it should put what it is this is delta I am sorry, now this is also 0, because after some time the body surface I am describing by another equation.

Now, what you do is in a small change in time, you can expand it like a teller series with only first order term. If you expand this as a teller series of the first order term, you will get what f of x comma y comma t plus you will write delta f by delta x d del x. You will have delta f by del y plus delta f by dell g plus delta f by plus higher order terms and you know that this is 0 and this is also 0.

You say this quantity is higher order, you neglect and this quantity is 0 and this quantity is nothing but delta f by delta x this is gradient of f dot delta r plus delta t into delta t that delta t. You take it actually, what you do is you retrain it as it is equal to 0, this is you write it this. Now, I will go back here or maybe I will erase this part, this is my condition. Now, what is normal to the surface mathematically because what we need is normal to the surface.

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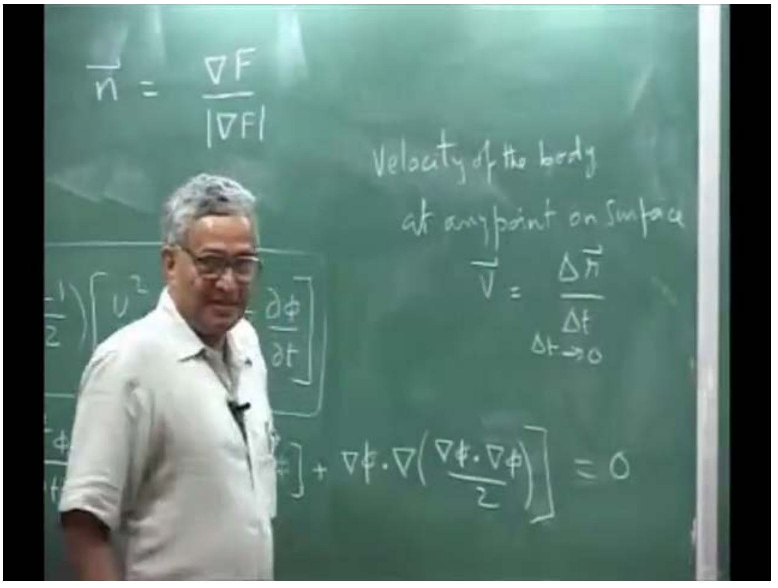

Normal to the surface is gradient of the function proportions normal, this is the unit normal. So, I put a magnitude, so gradient of a function is basically the normal and I divided by that you get the unit normal. Now, what is the velocity of the body velocity of the body at any point of the surface, any point on surface that v is what delta r by delta t delta t attending to 0. That is the velocity of the body what is the normal velocity of the body, you know the velocity of the body is delta r by delta t you take that product of that, you will get v dot n.

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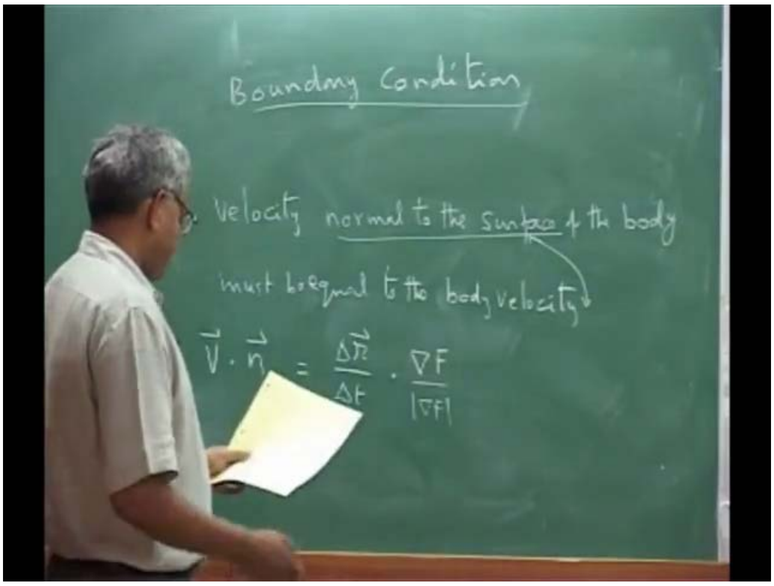

Please understand we use velocity of the body this is delta r over delta t and the normal is gradient of y mart f. Now, from that equation delta r by delta t and you divide by magnitude of delta f, what will you get you will get here from this equation del f you divide by magnitude of del f dot by delta t.

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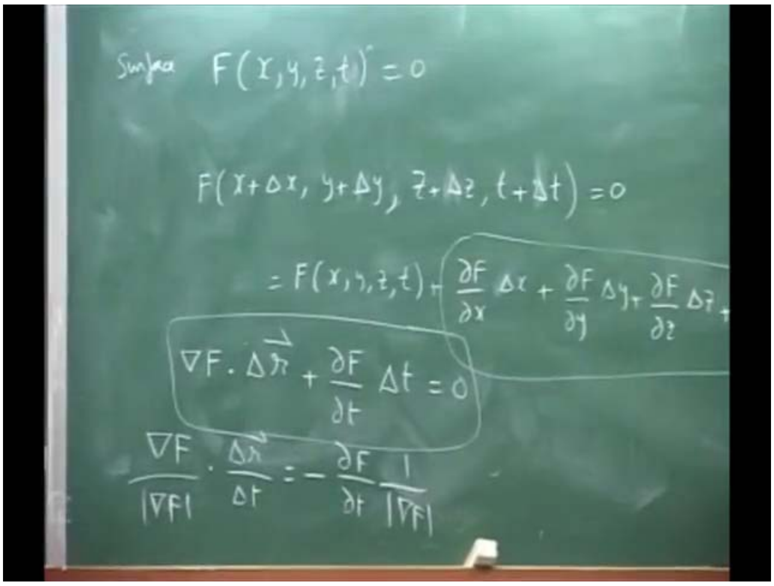

This is going to be minus delta f by delta t one by magnitude by del f that is what is going to be there.

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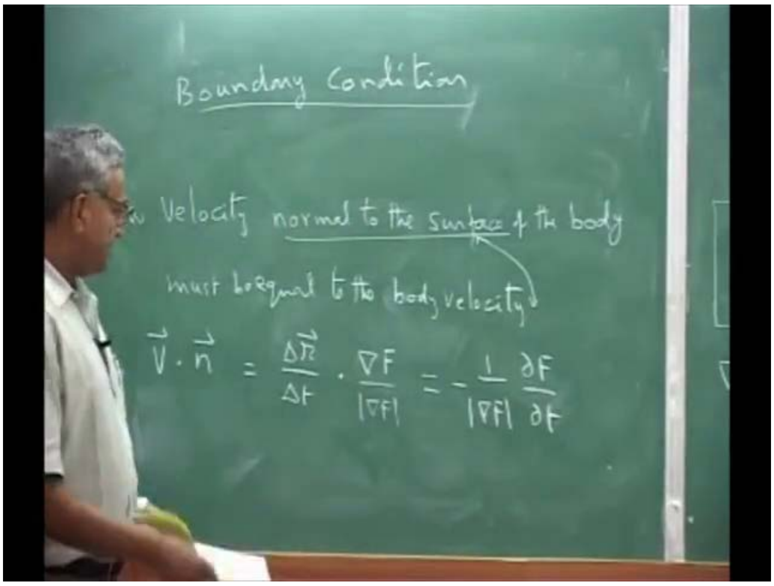

So, I am going to write this as minus one over magnitude of del f delta f by delta t, please understand this is the velocity of the body normal to its surface that is this term velocity from the fluid. Now, you go to the fluid velocity of the fluid that is flow velocity, we know that is q normal to the body means you take the same normal.

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So, you take the dot product, so this is flow velocity normal to the body, this is body velocity normal to its surface, this two are equal, this is now you cancel out, this you bring both0 of them one side. This is the condition, please understand the boundary condition is delta f plus this is this, so please understand, this is very interesting and this is nothing but it will look because del over del t q dot del is nothing but D by you can write it like it will be like this subsection theoretic. There are two velocities, one is the body velocity normal to its surface another one is the flow velocity. You have to calculate normal to the surface, where you are calculating the body velocity and this is the equation.

Now, the whole entire unsteady aero dynamics goes towards because here it is a q is nothing but del you need to simply satisfy the boundary condition and your surface. The surface is moving f is moving the most of the aerofoil, we will deal with aerofoil will take it as thin plate and the plate you can write it by equation because f surface is a straight line.

You normally take as plate equation or a if it is two d aerofoil, you draw a line for a cline, you can write it like a equation f of it and how it moves. Then, you know the boundary condition because you know that f how it is moving and then you write potential and this would be satisfied. So, this equation, now we make a person this is where a industry will code, what they do is in an aerofoil because our wing. There is a aerofoil, you have a tough surface, you have a bottom surface that means the two surfaces you have to satisfy the boundary condition top and bottom, suppose it is a very thing aerofoil you can project it onto a plain flat plate.

Now, industrial code most of the potential code, they do not even in going many of the things all the calculation. They will have treat the wing as the flat plate, you do not treat them as a three dimensional body, because three dimensional body is when your competition becomes tremendously prohibited very expensive. So, you treat it like a plate and then you satisfy that is because our wing is not very thick.

So, you can represent them by a thin plate analysis this part we will do the next lecture because industrial codes. They will try to do because the potential of course you can do c f d you can always say even Nordstrom code and many of these code, they may make their own approximations in trying to get the pressure on a wing that is the theme, I think this is enough.