Aero Elasticity
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Lecture – 15

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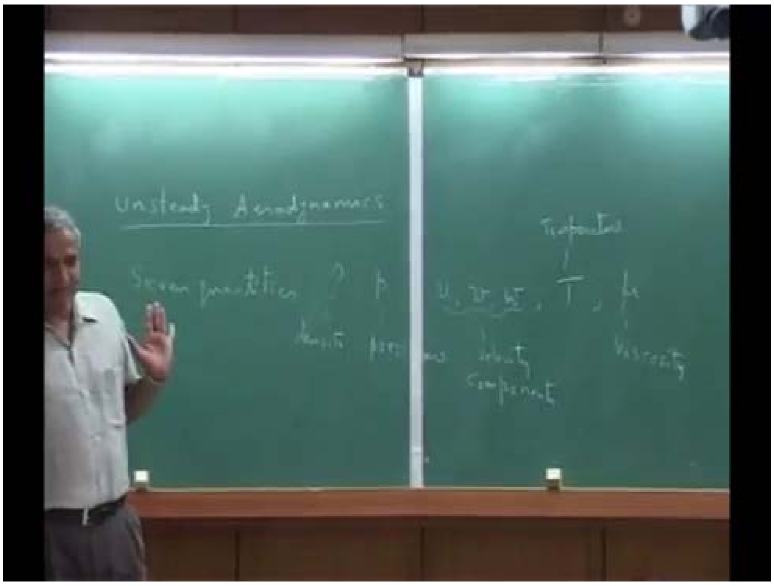

Unsteady Aerodynamics and this will go for several lectures, because we will formulate the unsteady aerodynamics and how the final expressions are applied for the dynamic aeroelastic calculation, that will come at the end of the course. Basically in aerodynamics, we need seven quantities, these seven are density then pressure and three velocity components u v w, these were velocity components then temperature and viscosity. So, essentially you have seven quantities at any point in the fluid, non reacting fluids that what we are saying.

Now, if you have seven quantities, you need to have seven equations. Now, those seven equations are, I will just write the names, you have one is, all of them are from mass conservation, which is continuity equation.

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Continuity equation, this is basically mass conservation then you have momentum equations. You will have three equations, this is one equation, which have three directions and then energy equation which is the first law of thermodynamics, this is one energy equation and you are dealing with the air that is, gas. So, you will have equation of state, this is basically the gas law, perfect gas or you may assume ideal gas, 1. So, you have six and the seventh equation is empirical relation of viscosity with temperature.

How viscosity, you forget about viscosity as the function of ((Refer Time: 04:08)) that you neglected, it only viscosity as the function of temperature you take. Now, these equations are somewhat differential equations, somewhere algebraic equations. So, you need to solve these equations with of course, that corresponding boundary condition, that is what your complete aerodynamic theory is, but if you take these seven and these seven, this set is called the Navier Stokes equation.

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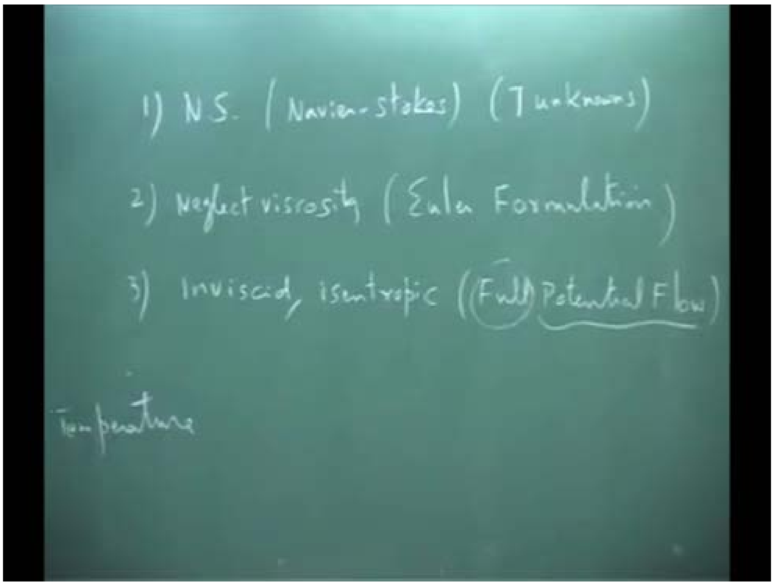

N S Navier Stokes, full 3 D problem you can solve, but then what we do is, if you take some approximation, the first level of approximation is, this is the full set, seven unknowns, that is what CFD people, direct numerical simulation or something take all the seven equation and then start solving, Navier Stokes equation. But, you make one approximation, first level of approximation, which is I say that, I neglect viscosity that is, neglect viscosity.

When you neglect viscosity, what will happen is, one of the variable drops out and you will have the Euler formulation. Now, we are not going to do this Euler formulation also, this can be further simplified, which is called the potential flow. That is, what type of approximation you make to reduce from here to the next level, this is you make, this is no viscosity that is, inviscid. You make inviscid and isentropic, you make this assumptions then you will get the full potential or full potential equation or potential flow.

You may called it potential flow then it may be fully non linear then you make further assumption, that is the small disturbance, you will get linearized potential flow equation. Now, what we developed in this course is only the potential flow and finally, we will do the linearized potential flow equation, that is what we will solve. But, of course, depending on the problem, you have to choose, because today we will have solving in the CFD. In CFD, this is also used, potential equation are also used in CFD, but now we will go for Navier Stokes or the Euler formulation, various types of formulation.

But, we want for our aero elastic formulations, whether we can get the closed form solution, even if it is approximate and use that for our aero elastic calculation for particularly unsteady conditions, that is the idea. So, we will develop the potential flow equation, so how do we start, where do we start, we will go through one by one all these equation, continuity momentum, first law of thermodynamics, equation of state, everything we will write.

And all of them later, we will reduce by using some assumption of irrotationality condition and then it will come as the potential flow means, basically my velocities are derivable from a potential. Then I will have actually only one equation for potential and of course, there will be another equation for speed of sound. These two will be there, but if I make further assumption incompressible flow then you will get further reduction in the formulation.

But, all are unsteady, that you have two apply the appropriate boundary conditions, so what we will do is, since it is going to be highly mathematical please understand, because here it is the lot of substitution and then simplification and etcetera. It will look like highly mathematical, in way people will get lost in the math to know, what is that we started with initially that is missed and you will go in algebra, jugglery, substituting this. Our aim is getting, ultimately please understand this is what we needed and we need to develop the equations, there is done, simple.

Mass conservation, momentum conservation, which is basically Newton's law and energy conservation, which is the first law of thermodynamics, and then of course, we apply this isentropic relation, isentropic condition, so always keep in mind that, our aim is to get this, because viscosity we neglect, inviscid. But then the applicability of whatever the potential flow, it is fairly good for most of the flows, except that when you have a very strong shock, where there is a heating happens.

Then, these equations are not valid, this solution is not valid, but still people use potential flow for transonic also. Please understand, the industrial course are, there is none than the last one for aero elastic analysis, is not that they do Navier Stokes, people are developing it, that is the different matter. But, it is still the field of research completely, but for aircraft wing and other thing, they develop potential flow, which is I will briefly discuss.

But, we will not be getting into that, because these are all research publications and people are develop the coach, ultimately what we want, a wing is vibrating, I want to know what is the pressure at every point on the wing. If I know the motion, what is the pressure, what is the pressure, I will get the motion, there is a relationship. So, ultimately the boundary condition will be on the aero foil you apply, because it is moving. Now, let us start with the basic equation, what are the ingredients which we required and first one is the gauss theorem.

So, it will have the mathematical for next several lecture that do not lose track, because this is invariably people lose track, our aim is ultimately to get these things. So, since the lecture will go on for, the formulation will go on for several things, because this is normally not covered in many of the aerodynamics course, they do not keep this part, that is why I thought I will cover this.

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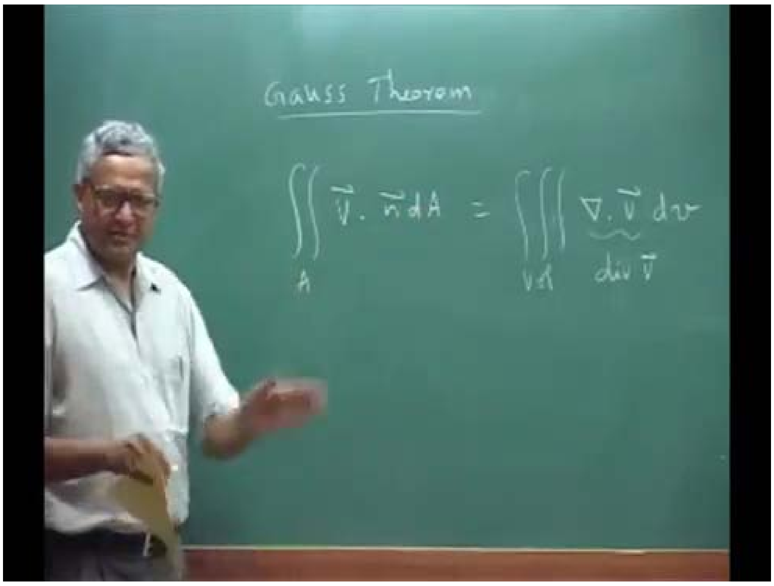

You will have Gauss theorem or it also referred as divergence theorem, this is basically states, this is the surface integral over an area, is a volume integral delta v d v or you say, del dot v is, you write it as divergent v, which is the scalar.

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This is the gauss theorem.

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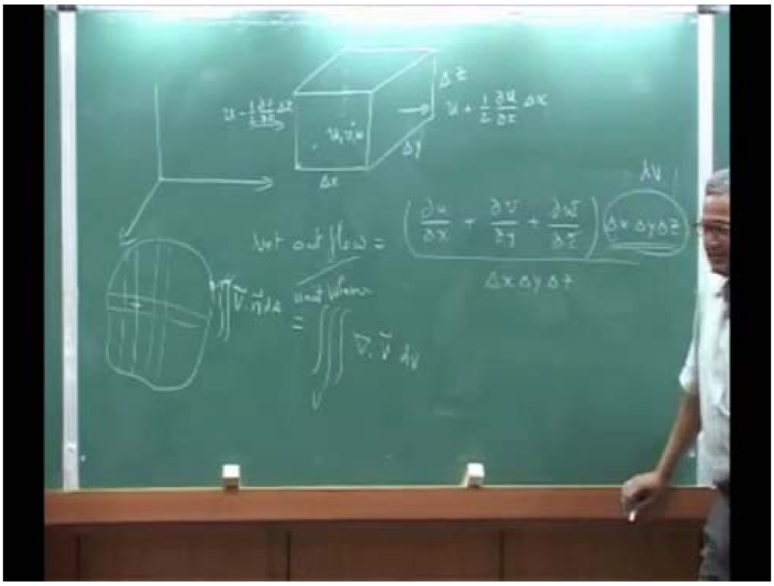

Now, the proof in the fluid mechanics, normally it is taken as, you choose the control volume that is, you define your axis system and then you take a control volume. The velocity at this point is u, v, w and you say this is delta x, this is you may say delta y and this is delta z. And you calculate what is the net outflow through this small elemental parallel low piped, the velocity here you write it as u plus half delta u by delta x delta x and on the back side, because this is at the centre point on the back space, you will have the u minus half.

Similarly, you can have all the six faces, what is the flow, into that what is coming in and what is going out. If you calculate what is going out, going out is this velocity times this area, is out minus this velocity time the same area, which is coming in. So, if you find the net outflow will be delta u over delta x, where delta w over delta z, you will have delta x delta y delta z net outflow over this elemental parallelopiped. Now, per unit volume, if you take net outflow per unit volume, this will be you divide by again delta x delta y.

That means, what we have obtained is, whatever velocity into area is nothing but del u del x del v del y from that volume. Now, if you are assumed some control volume you say and you divide this into several blocks, small small blocks in the 3 D. So, whatever is coming in here for this cube whatever is going out, that is nothing but what is coming in into the next element. So, like that if you keep the whole thing, finally it will be basically whatever is, this will become v dot n, you may say d a, d a is the small elemental area this you have to integrate over the entire surface.

The surface will be curve, this will become surface integral and then here this will be del dot, because you say whatever is coming in is nothing but this in the limit term. If you have unit volume is this, multiplied by that into the elemental volume, integrate over the full volume, you will get over the full volume, this is nothing but del dot v d v, because this is d v. So, this is your divergence theorem and that is what we have written as a d v, you can use the lower case v or capital v whatever, but do not put a vector. This is the first theorem, which we use it for divergence but then there are modifications of the divergence speed, in which you can give it as the proof.

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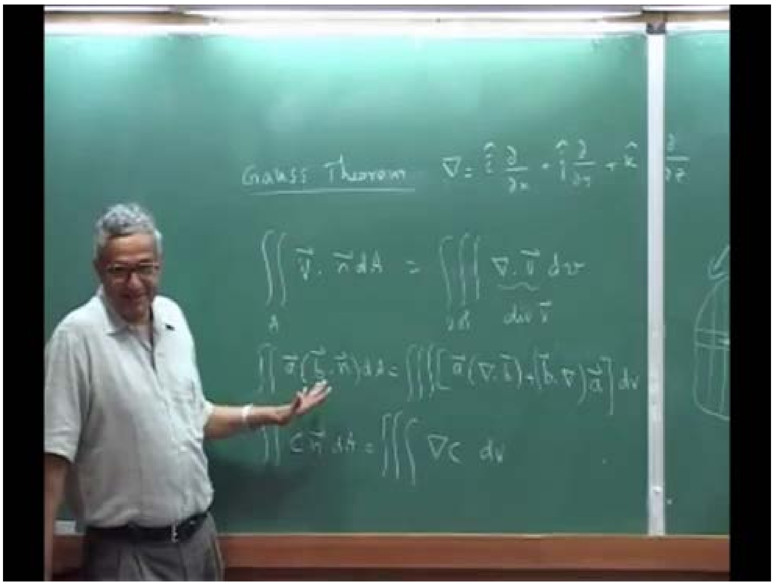

But, I am writing it only a b dot n d A, this becomes volume integral of a del dot b plus b dot del a d v. Del operator you know that, i delta by delta x i plus j plus k, these are all unit vectors and then the other one is c, which is the scalar n d A, this is gradient c d v. So, these are all modifications of the same thing, what I will say is, you prove this using this and similarly, this also c is a scalar, you prove this also, very simple. Now, we will be using all these three relations in our development of equation, that is why the very first thing is, you will develop the basic equation, which is the gauss theorem.

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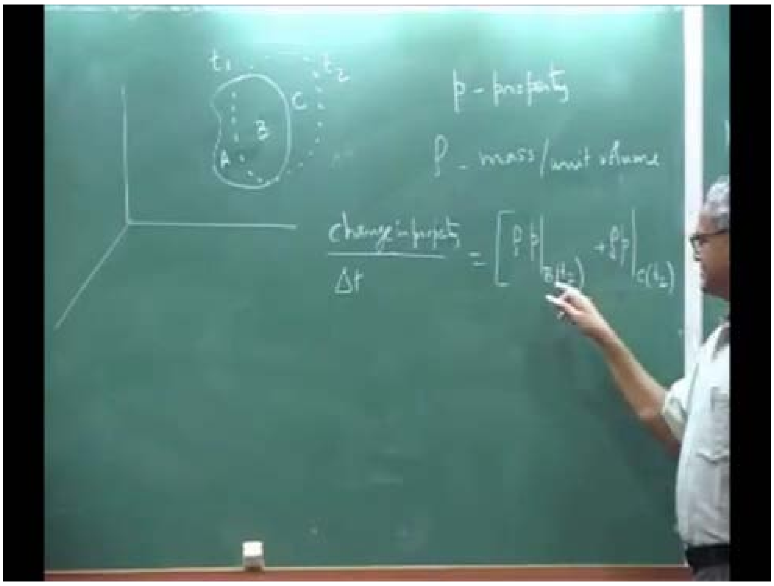

Now, let us go to the actual equation formulation, it give a very general, see mass conservation implies, that the mass if you fix, this is you look at the quantity of flowing which is here and you track this. Please understand, you are tracking the flowing that means, you are looking at the mass wherever it is going, the other one is you look at the region, you are not bothered about what will happen to your mass, you are only looking at a region.

So, the region is the control volume, but mass conservation, you can apply only to some specific mass, but how that mass conservation is related to a control volume or momentum conservation, because momentum conservation is again over. You wants the Newton's law, F is equal to m a, mass is fixed, force is apply and what is the acceleration of that mass.

Now, this is for a fixed mass, you are looking at the tracking the mass, but how that get related to a control volume later, that is what I will give you one general proof. And with that, you can write all the three equation, actually all the four, continuity as well as momentum, every equation will come out from one generic formulation. So, let us say, this is the quantity of fluid at the time t 1 and at time t 2, these are occupied this space, this is at the time t 2, the same fluid has come here in that back space.

Now, let us divide this whole thing into space A, this is space B, this is volume C, you need to know any property. Now, I am going to say the property of the fluid, which is final minus initial, the property per unit volume. Suppose, if I take density, per unit mass if you want you can take per unit mass also, rho is the density, which is mass per unit volume. So, let us say rho is, now I want to know the property of this mass, how it is changing.

If it is unity, how the mass is that is, mass per unit volume, how the densities changing, but if I say velocity then you say it is the momentum. So, I am going to write only a property which is p, p is the property, it can be 1, it can be velocity. So, when you write it as just rho p, so what is that initially the change in the, let us write change in over change in property, this is the elemental volume. Please understand, this is the elemental volume, you can integrate and you will get it. What is the initial and final property, rho p at B in time t 2, because B is this, please note B is the common volume plus rho p at C t 2 minus this is the final value. Because, what was originally in this domain that is, continues domain, it has now moved here.

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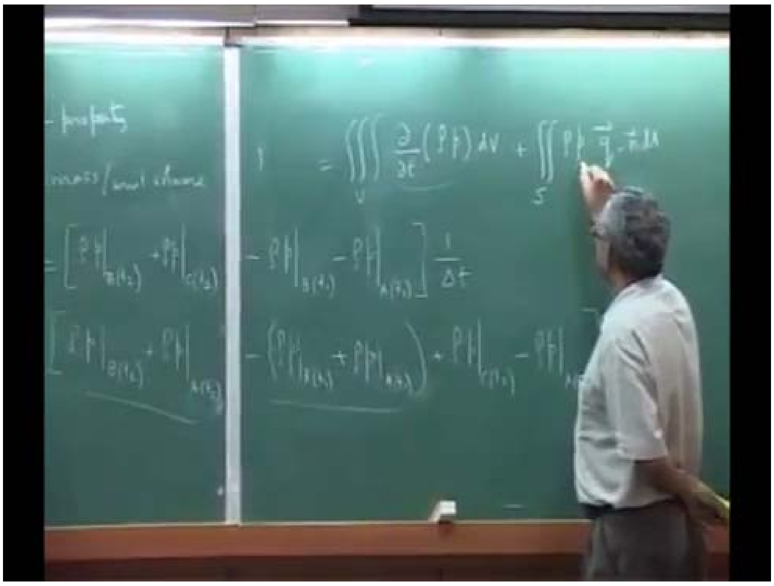

And I am at time t 2, whatever is the property is B at t 2, C at t 2 minus rho property p, B at and t 1 minus rho p A at t 1, 1 over delta t, this is the change in time. Now, what you do is, you add and subtract two quantities, rho p B t 2 plus rho p A at t 2 minus rho p B at t 1 plus rho p at A t 1 plus rho C at t 2 minus, because I will add as this rho p A 2, so I will subtract rho p at A t 2 and divided by 1 over delta t. This is over the entire full volume if I take it, this is what the small element if I take the full volume, I will not only integrate.

Now, you know that, 1 over delta t this quantity if you look at it, this is nothing but I am looking at these control volume, the continuous line, what is happening in the small elemental change in time, that is all. That means, this is the time variation of the property rho p in the limit delta t tends to 0 over this control volume. Now, what is this quantity, rho p C t 2 minus rho p A t 2 that means, this region whatever as coming, because this surface has gone out here, similarly this surface are gone out here.

So, this is like what has flown out you can say, because this is flowing out minus flowing in, so net flow out, so this change in the property, if I look at it from the control volume point of view, this is nothing but that change, this will be volume integral. Now, this is the control volume, delta rho p d v, d v is the volume plus surface integral rho p flowing in that is, velocity. And if you have q dot n d A, this is over the surface area, but please understand, I am putting delta p delta p within this, not outside, this is the property which is changing with time and this is the property that is flowing out.

So, this is what your change in property of this mass, please understand change in properties is a total, I am looking at this tracking some mass, but that is related as to the control volume here. Now, we will derive our three equations right from here, suppose if I say, my property is, can I erase this part, I erase this one, this is not necessary. If I say my property is mass, mass is what, change in mass is, so change in mass is 0, it cannot change.

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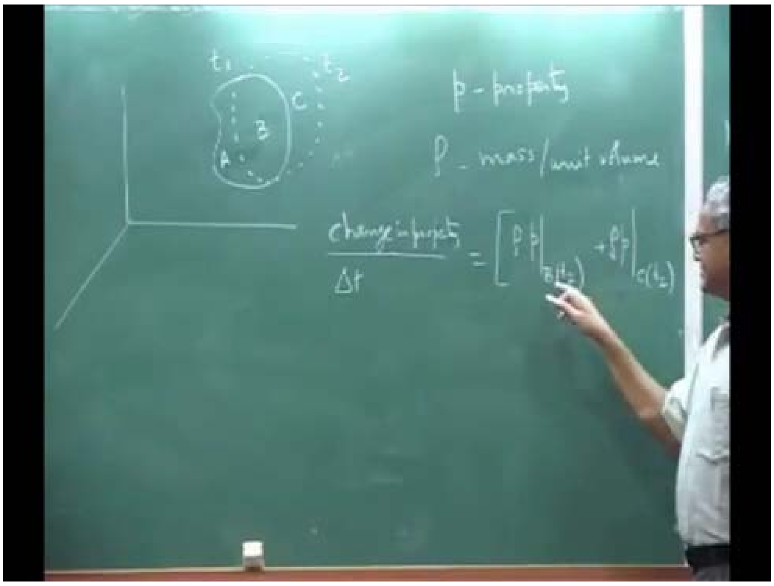

So, if it is mass, if my property p is 1, change in property is change in mass, change in mass is actually 0, because mass cannot change on the, whatever I started with.

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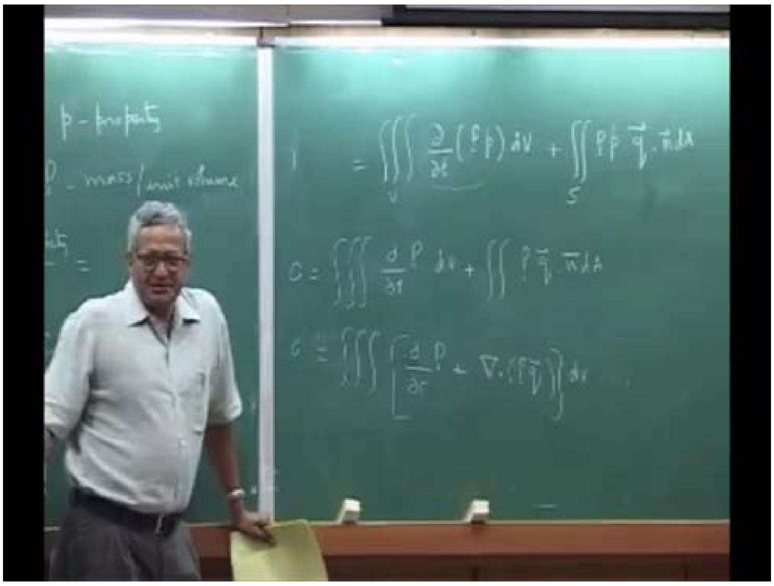

Therefore, 0 equals what, delta by delta t of rho, because property p is 1 into d v plus rho q dot n d A, this is 0, change in property. Now, you can convert this into, look at this equation, because rho is scalar and rho q is basically the vector point it is, vector dot n d A. I can convert it into volume integral, which is divergences of that, so which can be written as, 0 equals volume integral, I will write it as 1 delta over delta t rho plus this is del dot rho q d v.

Since rho q is find here, now the volume is arbitrary, because I can choose any volume I want then automatically that term within that integrate has to be 0.

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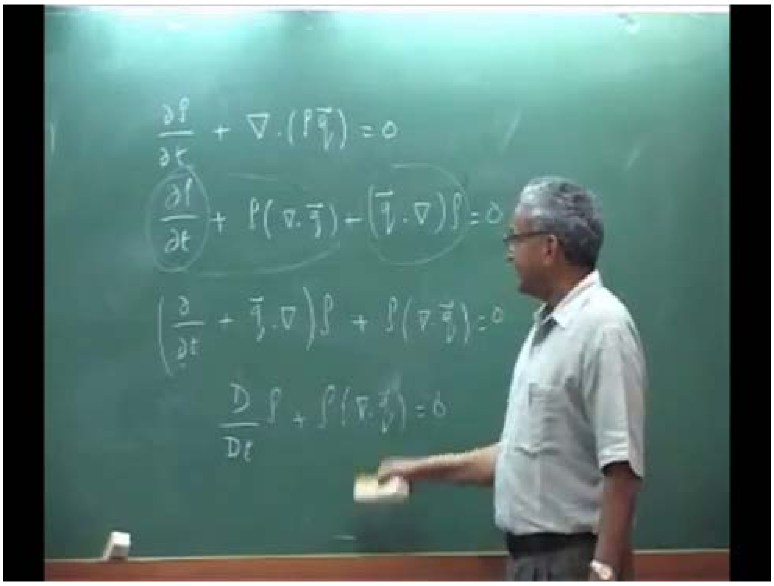

And that is my continuity equation, which I will write here as delta rho over delta t plus divergences of rho q. This is my divergence equation, it can be written in a slightly different format of, expand this, this can be written as delta rho over delta t plus rho del dot q plus q dot del. And this particular term, that is not this term, these two this and this, they combined it and write it as here, delta by delta t plus q dot. This particular term people used substantial derivative or total derivative or whatever, but basically it does not matter, you can use it this form, ((Refer Time: 35:33)) this form, this form, this form, any form, it does not matter.

Now, if my density is a constant, constant mean anyway it does not vary with time, density is a constant, it is incompressible then del dot q is 0, that is the incompressible flow. But, we do not have to use it, we will keep it as it is, compressible flow we will keep. Now, this is my continuity equation, now keep this equation, because you have to use this again. Now, possibly maybe I should write it in one corner and we will have that equation, later than we substitute.

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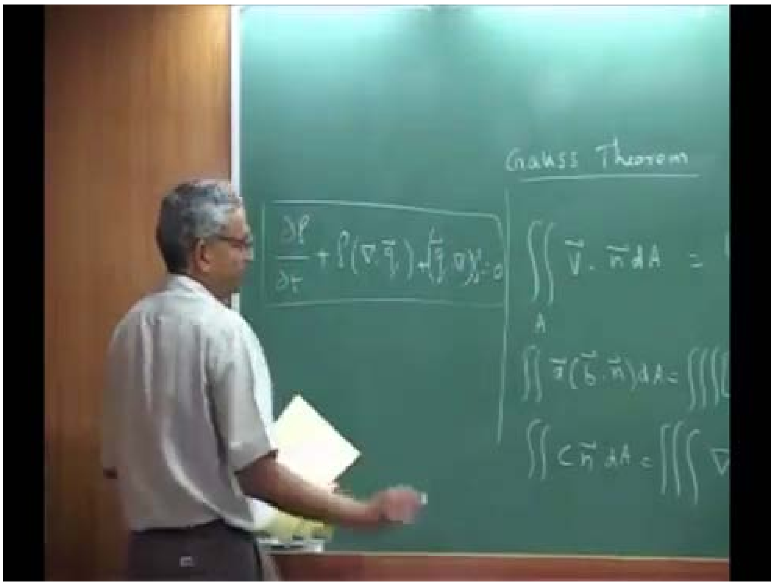

So, I can write it del rho over delta t plus rho del dot q plus q del, this is 0, this is the one equation, continuity equation. Next is, so this is written as total derivative, material derivative, substantial derivative all, basically this means this. Now, I erase this part, let us check the property p as velocity that is, conversation of momentum.

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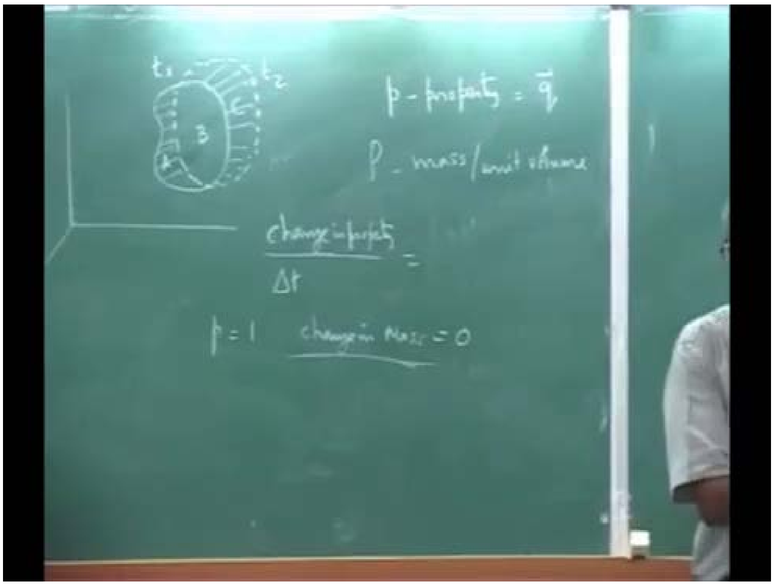

If p is my velocity p, now that equation will be the change in property whatever, because this is mass into velocity is acceleration. So, acceleration is equal to the force that is acting on that, mass times acceleration.

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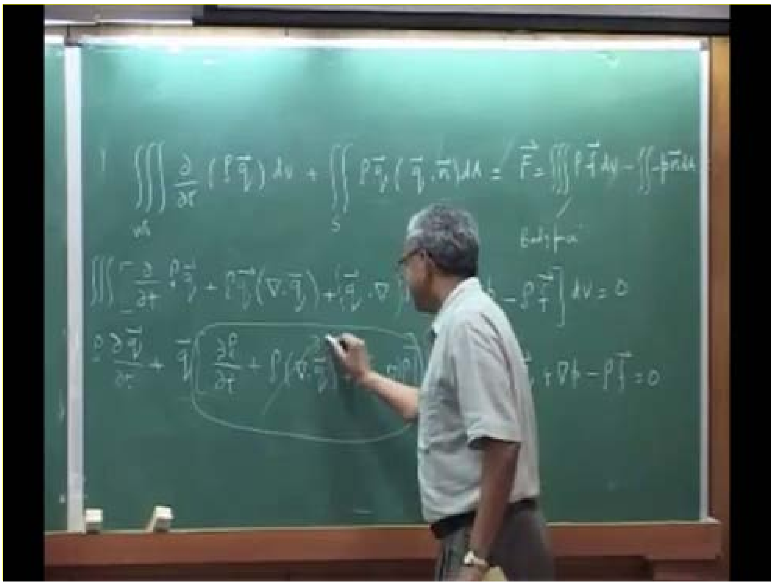

So, you will have, this is the volume integral delta by delta t of rho q d v plus the surface, this is rho q q dot n d A, this is your external force which is acting on that mass, please understand. The external force you can have it as two, one is the pressure, another one is the body force. So, F I will write it here, one can be a body force, body force means it is the volume, rho f d v, this is the body force. And then on the surface, you can have the pressure, but the pressure I am taking it, acting in the opposite to the normal, so I will put it minus p n d A.

Now, you say I do not have any surface shear force, that is why I neglect the viscosity you follow, otherwise if I have a shear, you have to put that viscous thing will come. So, I do not have any viscous throw, so there is no surface shear force acting on this. Now, this is again what will do is, you transfer it to this side, this surface integral, you converted into volume integral that is, using this relationship. The second one, because you say this as, rho q is a q dot n is B n, use that relationship.

And then this is pressure, pressure is the scalar quantity, scalar n d A, ((Refer Time: 4:21)) that last relation C n d A is gradient C d v. Now, convert all of them into volume term, if you convert all of them into volume integral, you will get like this. So, I will write the full step, delta over delta t rho q plus rho q del dot q plus q dot del rho q plus delta p gradient minus rho f d v equal to 0. Now, you say, since I have actually taken this term to the left hand side, you see this term get converted by the second.

That is why keep writing that, because you have keep that how you get everything get converted into another long expression. Now, since the value is arbitrary, therefore this quantity must be 0, this quantity you can write it in delta, you can write it in two form, what you do is, rho del q over delta t plus q, I open the bracket delta rho over delta t plus, please understand this is this term. Here, I have taken q outside, so I will get rho del dot q, here I have to split it into two parts, one is I take rho in the derivative, another one rho treated as a constant, del q as take it.

Now, when I take that, I will have q dot del rho plus, because this is I am taking two terms, the second term will be rho q dot del of q. You will have pressure term minus plus del p you can take it, minus rho f. Now, if you look at this particular term, this is nothing but my continuity equation, therefore this is identically 0, this particular term, leaving behind I now erase this term.

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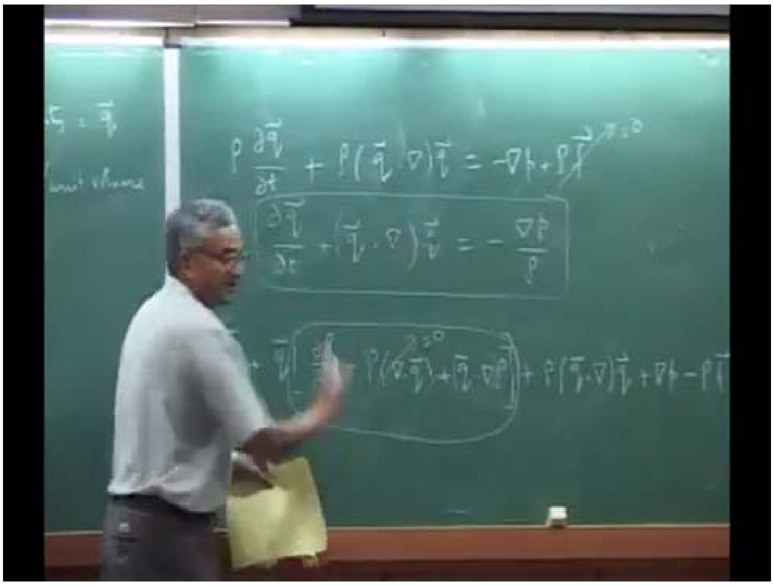

You will have rho del q over del t plus this term, rho q dot del q equals and take minus del p plus rho. Now, what you do is, if you neglect the body force, usually the gravity body force you neglect it, so this term goes to 0 and your rho is common. So, what you will get is, you will write this equation as delta q over delta t plus q dot del q equals minus delta p over rho, this is my momentum equation, that is all.

Now, this also you can write it here, so you have continuity, you have momentum, momentum you can write it as, see that is why delta by delta t q dot del, q you can take that is the total that q are substantial derivative or expression.

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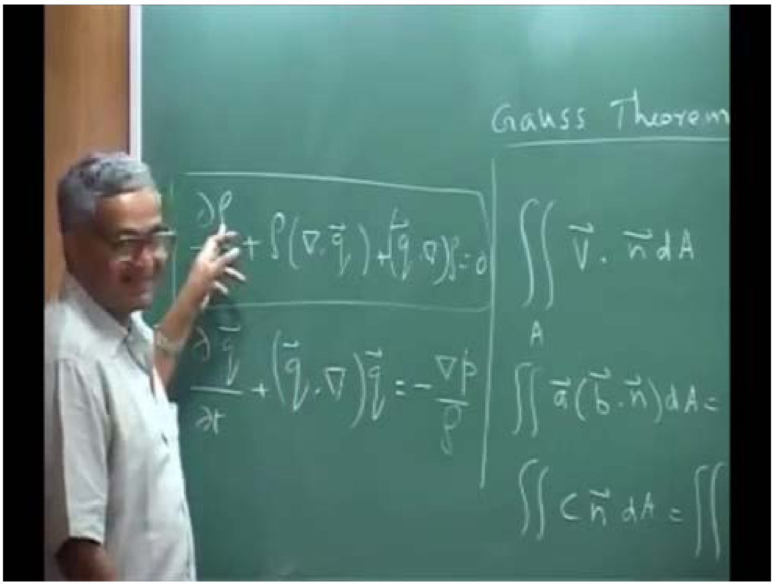

So, you will write it as delta q over delta t plus q dot delta q equals minus delta p over rho, now please understand what are all the assumptions you have made till now, because that is essential. So, do not lose track of any assumptions, which have been made in obtaining this. First is, we neglect the viscosity and we neglect the term body force. Now, you got these two equation, velocity we have see how many equation are there, how many unknowns, we have this is the vector equation, please understand because this is the gradient, gradient means i, j, k.

So, you will have three equations here, this is one equation, because this is the delta rho, delta q divided by del, that is one scalar. So, you have one equation here, three equations here, so you have four equation we are still assumed, but the variables are density, q is three velocity components, q is a vector u, v, w and then pressure, so you have five. You have five unknowns here, but four equations are over, now you have to get the fifth equation. And then from there, we will try to reduce something, first we will write the equation of state then you will use the energy equation that is, the first law thermodynamics, now I erase this part.

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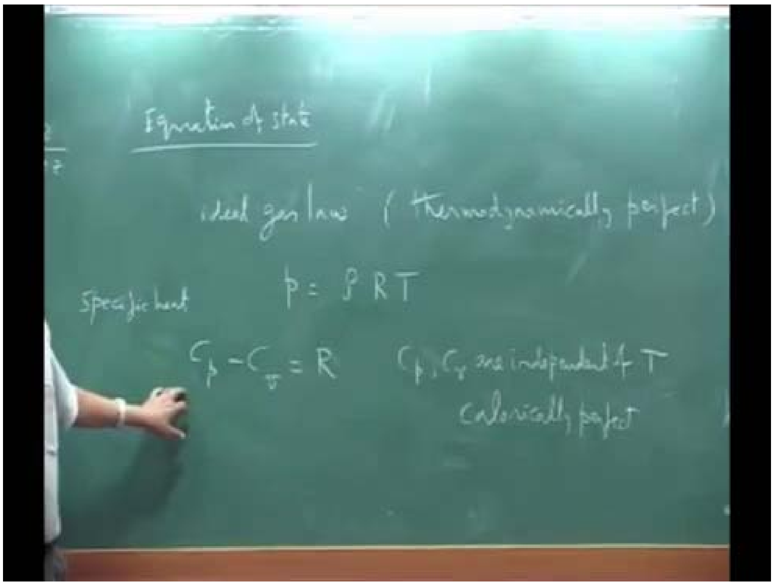

Because, the equation of state, so we have take it that, we have bothered about only f, so we say the condition is not very high, it is the cold and properties do not change. That is, the chemical composition or physical composition were same, there is no intense heating, because everything you change. So, the thermodynamics characteristics, because we are going to assume ideal gas law is valid, ideal gas law or perfect gas law P is rho R T, R is the gas constant, T is the temperature absolute.

So, you see this relates pressure, density, temperature through some gas constant, you can get it related to universally, otherwise for dry air it is 272, some number is there. Now, this is ideal gas law, you think it is applicable and that is the key, you have to assume that, this is applicable for our expression. Then you also say, this if you are assumed the gas is, this is called thermal ideal gas or thermodynamically perfect, you can say normally perfect gas, it is a perfect gas.

But, in addition you have two quantities, which are the C p and C v, which is basically specific heat. At constant pressure, at constant volume, there may be functions of temperature but then we are going to assume that, they are constant, they do not vary with temperature and then you have C p minus C v, this is a R, which is the gas constant. If you assume that, these two are independent of temperature then this is called calorically perfect that is, C p, C v are independent of temperature then calorically perfect, so we assumed both.

Now, gas law these are valid then you to first law of thermodynamics, because you know that, C v is related to internal energy, because change in internal energy is directly written as C v time delta t. In a small change in temperature, internal energy changes by C v, this is to enthalpy.

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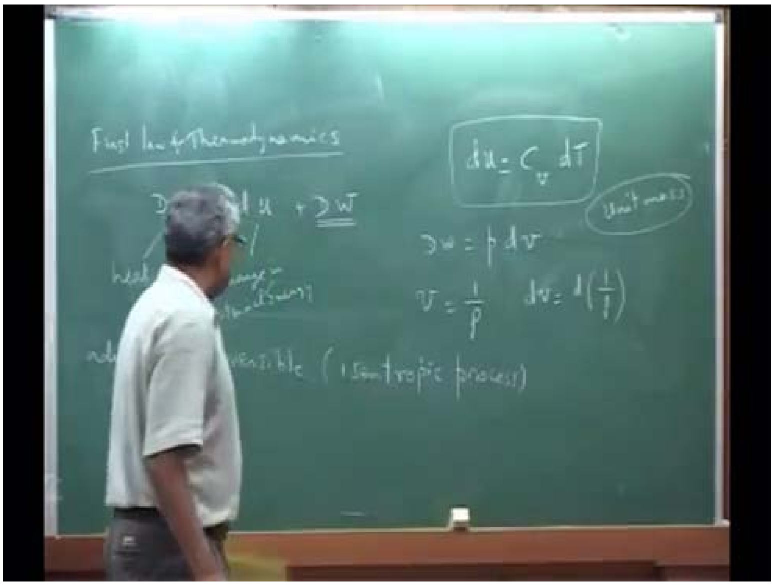

Now, I will not get into that, we will just describe very simple terms, first law of thermodynamics that is, if you add heat, the heat goes towards doing some work and some part goes towards increasing the, you can say the internal energy of the system. So, heat you may write it as D q this is the heat, but I am using the quantity q, please understand this is the heat, this q is not velocity. This I am writing as d u, this is internal energy, this is change in internal energy plus D w, because this is the you called it as a exact differential, because u 1 one is u 2 return to this.

But, this is the work, there is nothing like w 2 minus w 1 work, work is work, quantity of heat is how much you supply, some exact differential is not there, that is why this capital D is used of this. Now, here we are said that, it is calorically perfect ideal gas everything then basically d u is C v into d T and D w is P d v, this you can take per unit mass. Per unit mass if you write it then you substitute, here what we are going to assume is, then you can write v is 1 over rho, because this is per unit mass or d v becomes d over 1 over rho, unit mass of fluid.

Now, you start making assumption that, heat and want to say, I am not adding anything, this is a adiabatic process that means, D q is 0 then I also imposed condition, that is a reversible or not reversible. Because then it will only one way, if it is irreversible process, I can have an adiabatic, but it can be irreversible. But, I am going to assume, it is also a reversible process, therefore this is called the isentropic, adiabatic reversible. Please understand, I am going to assume adiabatic and reversible, so isentropic process which means, my D q will 0.

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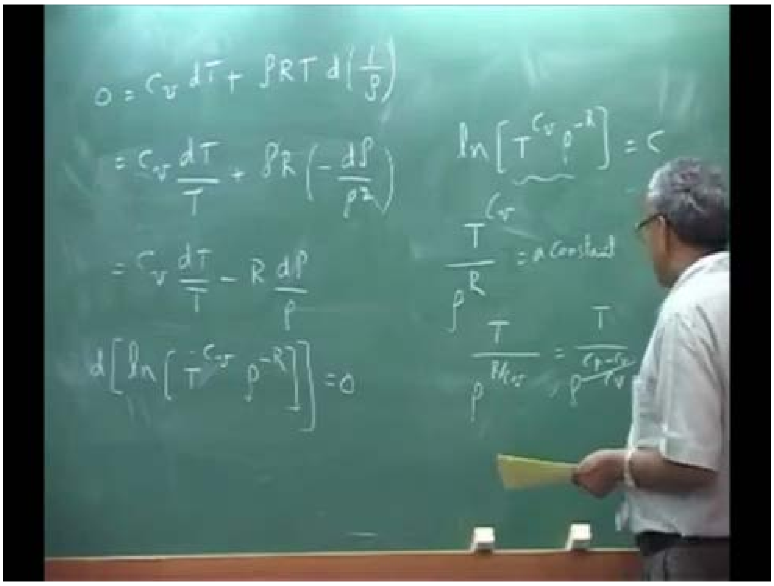

I will write the equation in this form, 0 equal C v d T plus pressure is D w is P d v, P is rho R T, d v is 1 by rho. Now, this equation I write it in this pattern, C v d T over T, because I divided by T plus rho R, d of 1 over rho will be, here minus... So, this rho will cancel out, so I will be left with C v d T over capital T minus R d rho over rho. Now, this can be you take it on your left hand side and write it as, you can put it in the simple form that is, l n something and bring that part, therefore that is it.

I will write this as differential of l n please understand, T power C v rho power minus R, this is 0, here I take it, I integrate it and then I am writing it. That means, l n of this is the constant, so I will write that l n of T power C v rho power minus R, this is some constant; that means this is the constant. So, I will write T to the power of C v divided by rho to the power R, a constant.

Now, what you do is, you again this can be, this is just algebra only, I will show that, this is T over rho to the power R over C v, because I am taking 1 over C v, which I am writing it as T over rho, R is C p minus C v, so I am writing C p minus C v over C v.

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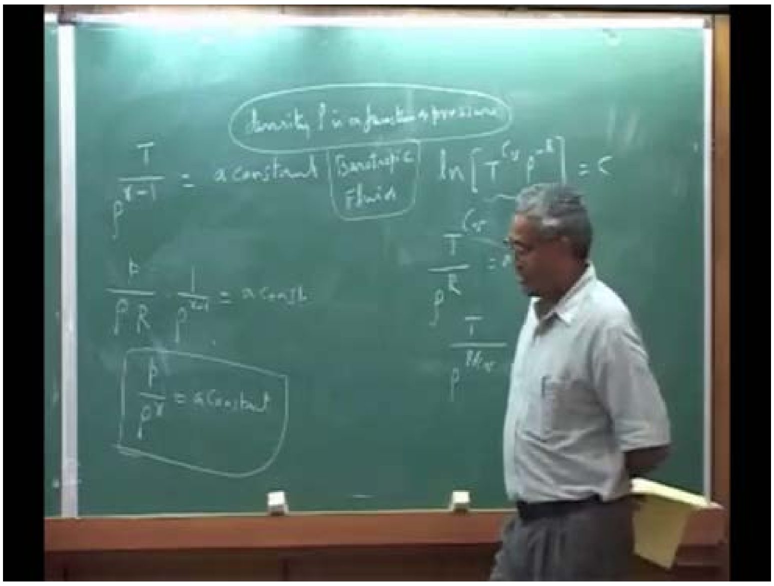

And I am going to call ratio of specific heats C p over C v as gamma and I will have T over rho to the power gamma minus 1, this is a constant and what is T, T is P by rho R. So, I am going to write P over rho R 1 over rho gamma minus 1 is a constant and R is a gas constant, therefore I am going to get P over rho to the power gamma is equal to a constant, which says pressure or you can say density is only a function of pressure.

Density rho is a function of pressure and this type of fluid is called the barotropic fluid, because that is all. Now, P by rho power gamma is a constant, now I have 1 2 3, P by rho power gamma is a constant, but this will, please understand what assumption we made. It is an isentropic flow that means, no heat is added and it is an adiabatic process and constant entropy, reversible. Usually this gets spoiled only when you have a strong shock, because shock there is a local heating, local heating it get this term, that is why I tell this will be valid even upto mach 0.3 something like that, you can use it.

Only when you go to the very high mach, very intense heating is there then this theory is fix, otherwise normal supersonic flow 1 2, you can use this potential flow theory. There is absolutely no problem in using and this is a what we used in most of the aircraft industries. Of course, CFD is progressing, I am not saying that, CFD is not used, but that is progressing, but even CFD, potential flow, because you cannot get a close form solution always for this equation.

Now, P over rho power gamma is a constant is another that means, I have a relation between pressure and density, a constant. So, essentially, you have five equations you obtained, actually these five are sufficient to solve, but once you solve this, if you want temperature, you can go back here to get a temperature, the temperature does not come fixed in these five equations. Now, even this can be further simplified, simplified in the sense, it is not that I am neglecting anything, I will reduce them. I introducing the concept of irrotational flow, so but before we go to irrotational flow, you have to define a few things.

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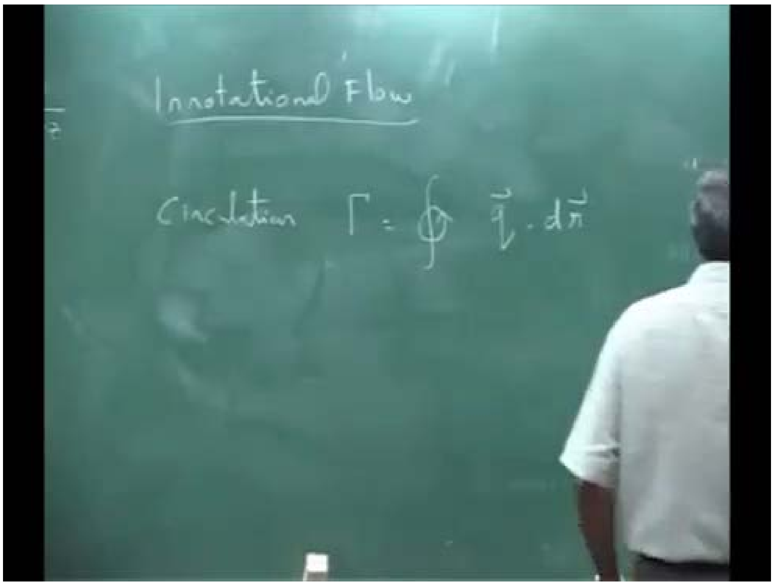

So, you say irrotational flow, because these five unknowns, five equation you can use them. But, they can be reduced if you introduce the concept of irrotationality and also the velocity potential or acceleration potential, you will define both of them later. But, right now, let us define something called as circulation, circulation gamma defined as over a closed contour q dot d r, please note this is velocity over the element, you have to integrate.

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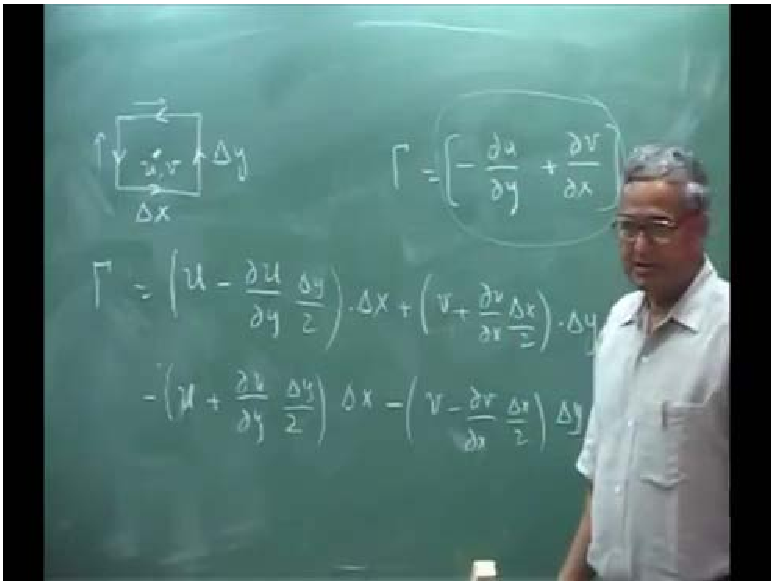

Now, let us for the sake of calculating this in the 2 D, you take a point, you take this, this is u, v, this is delta x, this is delta y. Now, you want to calculate circulation, for this element you will have this is, in this place you have to go like this in the counter clockwise. So, you will have u minus delta u over delta, because the velocity here, this is the velocity at this point, so velocity is here u along x is u delta u by delta y divided by 2 into, here you will have what is the velocity is v, we will have plus v plus delta v over delta x delta x over 2 into delta y.

Then, here the velocity will be this way, so you put a minus sign, so you will have minus of u minus ((Refer Time: 01:05:59)) u delta u over delta y into delta x and again minus of, because here the velocity is this way. So, your integration parts is different, so you put a minus sign, v minus delta over delta x, if I equate to delta y. If I take this, this quantity, you may call it d gamma or gamma over a small element, that will be what you will get, minus you will get, this is minus what, delta u by delta y, this will be plus v by delta x into del y del x, this is the area of the element.

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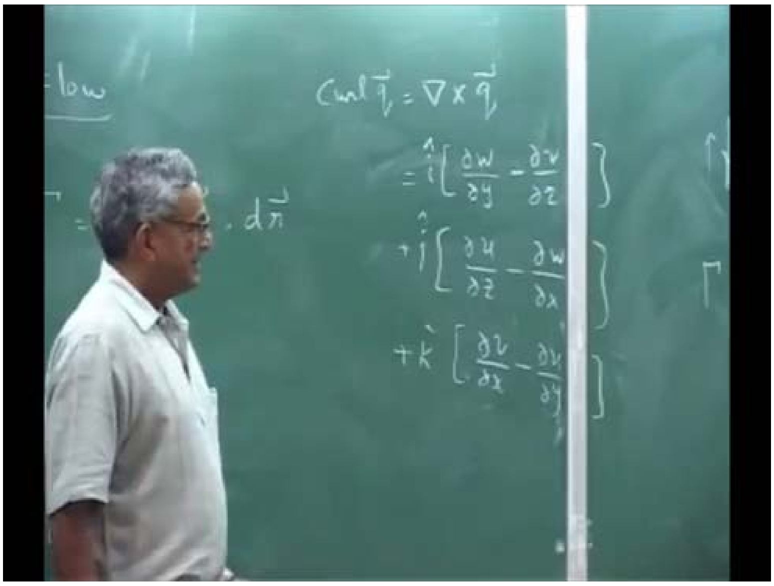

Now, this quantity is nothing but if you define curl, curl is curl q is del cross q, that is nothing but the k th component. You will have i delta w by delta y minus delta v over delta z plus j delta u over delta z minus delta w over delta x plus k into delta v over delta x minus delta u over delta y, this is curl q is this ((Refer Time: 01:08:10)). Now, if you say, this is the area d a normal to that, is the result, which is nothing but the normal component of curl q. Normal component means, you take actually to that area, whichever area you are looking at, take the normal and that normal component of that curl is basically this circulation.

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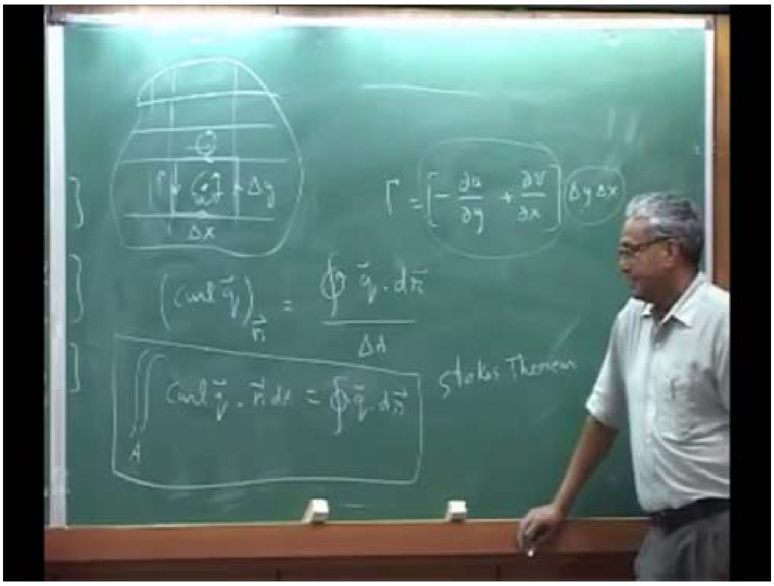

So, if you define each one of them as, you can write this as, curl q is a vector omega, which is curl q, which has omega x i omega y j omega z k. You can define now, erase this part, that is curl q n normal component, that n is you take and define a normal, that is equal to, you say q dot d r over, because I divided by delta A, this will be delta A. Because, I am dividing, this is what I am telling, this delta A I can take it here and take it as the normal, so this will become curl q dot n d A, this is nothing but integral q dot d r over a that small part, this part.

Now, if you define a larger area, you can divide it into lot of regions, every region you will have, this is going, so you will find this will cancel with that, that will cancel with that, leaving behind only the boundary part. So, boundary means, this external you can take it as a over a full surface, this is over the complete out of boundary, now this is your Stokes theorem. Now, Stokes theorem essentially relates, please understand this is the surface integral to a line integral, but the curl q is q dot d r. Here, we say divergence theorem is surface integral is getting into volume integral, now let us define one more. So, this part is over, there is some theorem, another theorem which is called the Kelvin theorem, what was the Kelvin theorem states?

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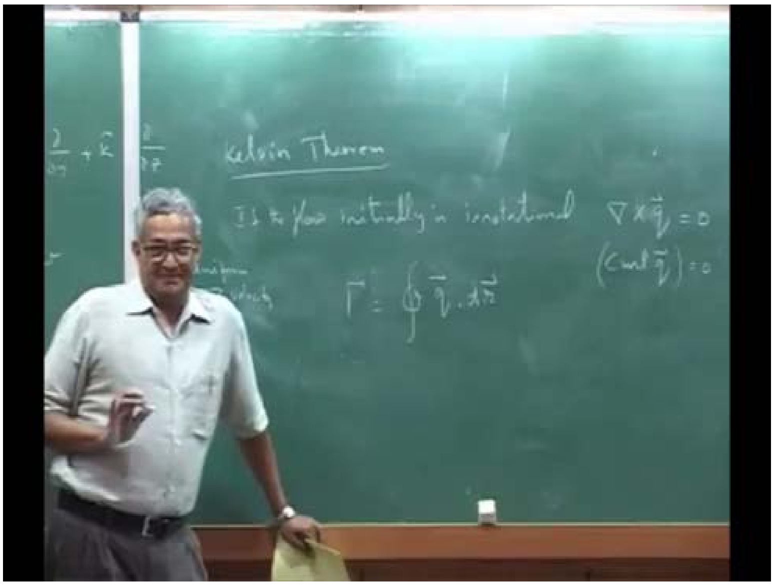

If the flow initially is irrotational that means, that is curl cube, del cross q equal to 0, which is nothing but curl q. If it is initially 0, it is 0 always, the flow remains irrotational always, now how do you justify this, is the question. Suppose, the flow starts from the reservoir, we always say from the reservoir, the flow initially far away, somewhere it is starting like a internal or anything. It comes with the uniform velocity everywhere then automatically curl q is 0.

If it is 0 initially then it is continues to be 0 always, now in such a way of course, you have to prove that, because you say, gamma is integral what, q dot d r. This is defined over a close region, now how do you say this will be 0 always. Now, this is where the proof starts, now I erase this part ((Refer Time: 01:14:23)). And maybe this we can start in the next class, may be this part I will do in the next class, because now if I start, I do not want to leave it in the middle, it will take some time.