Aero Elasticity
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Lecture - 14

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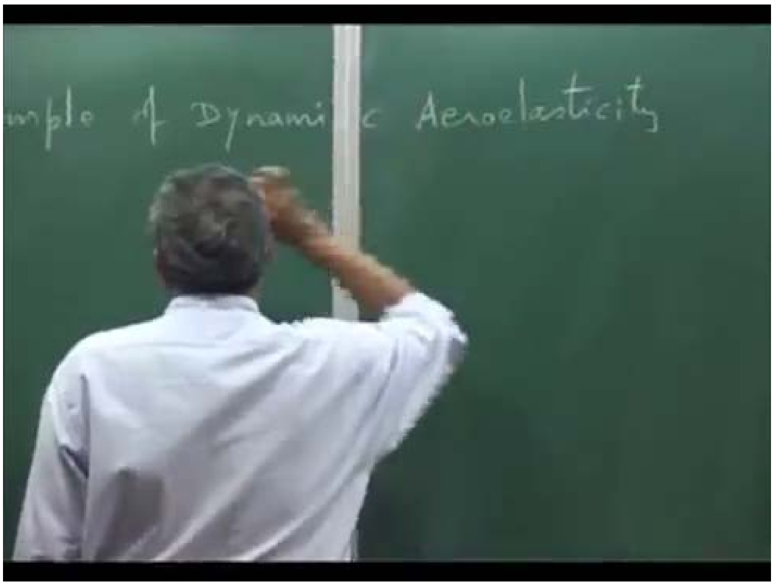

Now, we start with the very simple example of Dynamic Aero elasticity. So, we have seen the static aero elastic problem, now what dynamic aero elasticity and what are the issues in this case? So, we take a simple 2 D model, which you can say is a representative of a section of a wing and earlier in the primitive aero elastic calculations, they were doing only the 2 D analysis. And they normally take the section at the 75 percent of the span and then they just represent, the 75 percent span by a aero foil.

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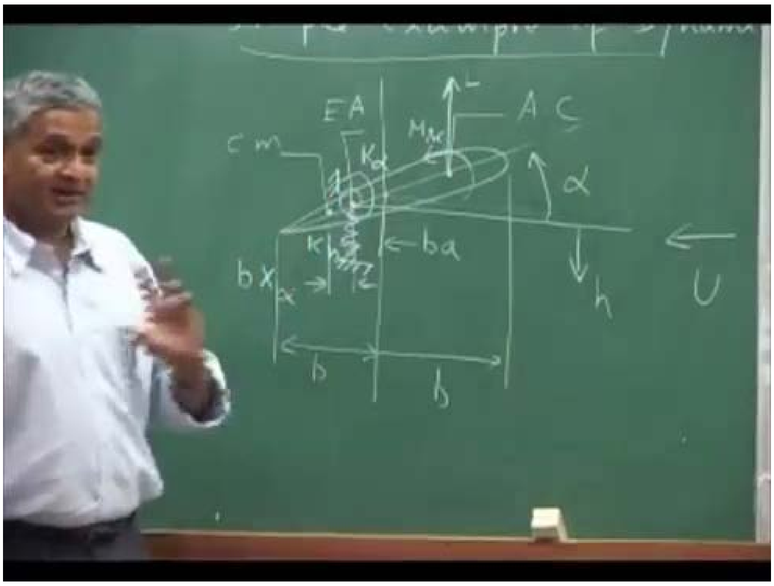

And you now start defining various, if you say this is b, b, b is semi chord and your aerodynamic center is here and your elastic axis, normally we put E A that is here and because it is a dynamic case, so you have also got the mass, so this is the center of mass you can say. Now, this is a 25 percent type, take it as the subsonic case and here these distances, this is taken as b sum a from the mid-point of the aero foil, the distance of the elastic axis is defined as b a, everything will be a is a non-dimensional number.

Then the distance of the center of mass from elastic axis is defined as b x alpha ((Refer time 03:36)). Now this is a vibrating thing you can have bending motion, so you represent this by two springs, which are we call it as k h and k alpha, h is k h is the, we can say the healing spring and k alpha is what the spring torsional motion. Now, you define this is a 2 degree of freedom system because it can the aero foil can move up and down and the aero foil can do pitch up or pitch down that is the rotary motion.

So, we define the 2 degrees of freedom as, this is h this is the downward motion, so please note our reference point is elastic axis and alpha is this rotation of the, we can say 0 lift line with respect to and on coming flow with U. Now, we have to write the equation of motion, what are the aerodynamic loads will take lift which acts as ((Refer Time: 05:35)) point normal to the slope, this is lift and the movement about the aerodynamic center M A C these are my, so please note that this is a 2 degree freedom system.

And you have get the equation of motion and the equation of motion we can get it from very simple of course, which we all know NFC method, Lagrange's equation, so Lagrange's equation we will apply to get the equation of motion for this aero foil, which is executing a plunging motion which is h and pitching motion which is alpha, h and alpha are depending quantities k h, k alpha represents the stiff nesses corresponding to the plunging motion and the pitching motion there is like plunging is bending, pitching is wing torsion option, so you see it is a very simple 2 D model.

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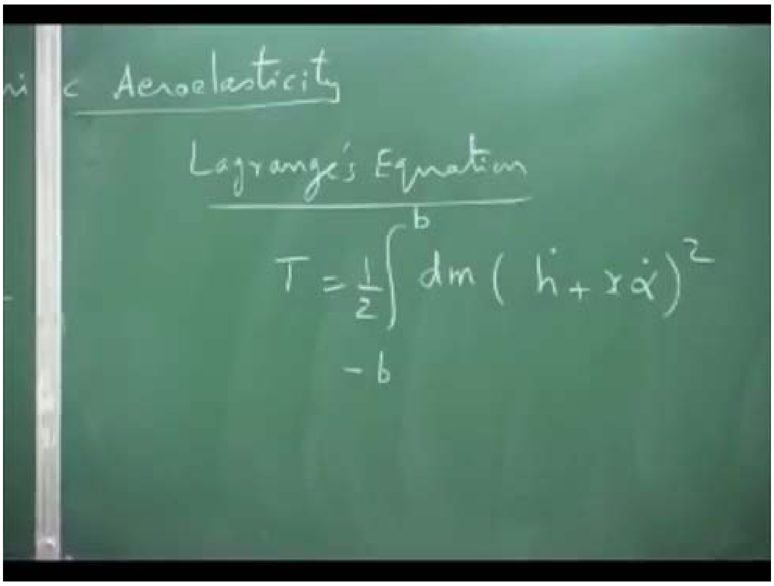

Now, we have to derive the equation of motion for this and we will apply Lagrange's equation. So, first the kinetic energy of the system, kinetic energy because you know that they will take here h is the reference point is elastic axis and we are going to define the distance from h, as we will use the symbol we may use x you want to solve it x any distance from elastic axis, but positive is better. Now, the kinetic energy will be integral they will have minus b to plus b, if you say mass per unit length of this if you call it as m or we may call it as d m, d m is the mass per unit length along the x and the velocity, velocity at any x, velocity at any positive x is h dot plus x alpha dot whole square with a vector half. So, half m velocity square, now this is split into three terms.

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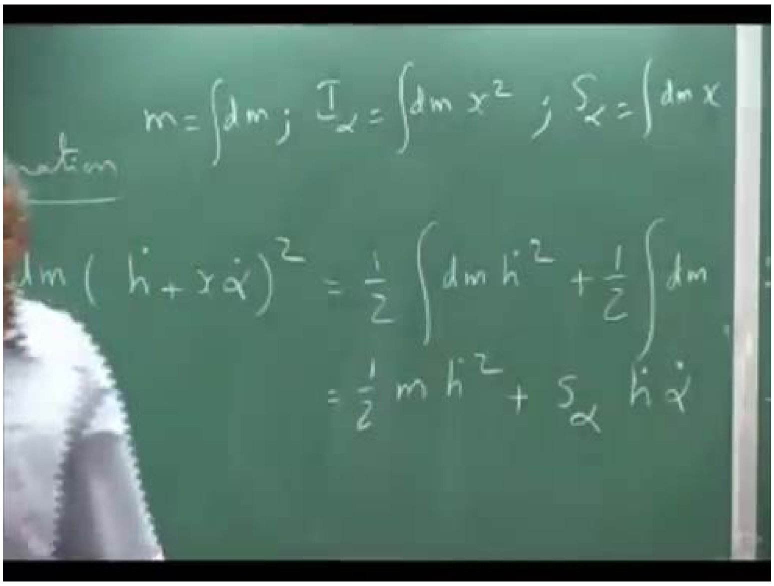

So, you will write it as half integral over there aero plane d m h dot square this is one term plus half integral d m 2 x h dot alpha dot and the third term is half integral d m x square alpha dot square and you write since h dot is constant is the independent of the d m, so this is integral d m is mass of the aero foil. So, you will get this term as half m h dot square, where m is the mass of that aero foil this term 2 and 2 will cancel of x d m integral, that is basically mass of symbol.

First movement of the mass, which we will write it as use the symbol s alpha h dot alpha dot, where s alpha is integral d m x and which is essentially I have defined here, this is at distance b x alpha from the elastic axis, so this m b x alpha this is s alpha, later we will be using the otherwise we will be using it as a s alpha, s alpha is positive moment of the mass then plus you have half x square d m, that is basically mass moment of inertia of the aero foil about the what elastic axis do that way you have to be capital, this is not about center of mass this is about elastic axis.

So, we define as I alpha dot square, so where I alpha is integral d m x square, so you see m is integral d m, so three quantities this is kinetic energy expression. Now, we have to write the strain energy, strain energy in this case you have only two springs one is the k h another one is k alpha.

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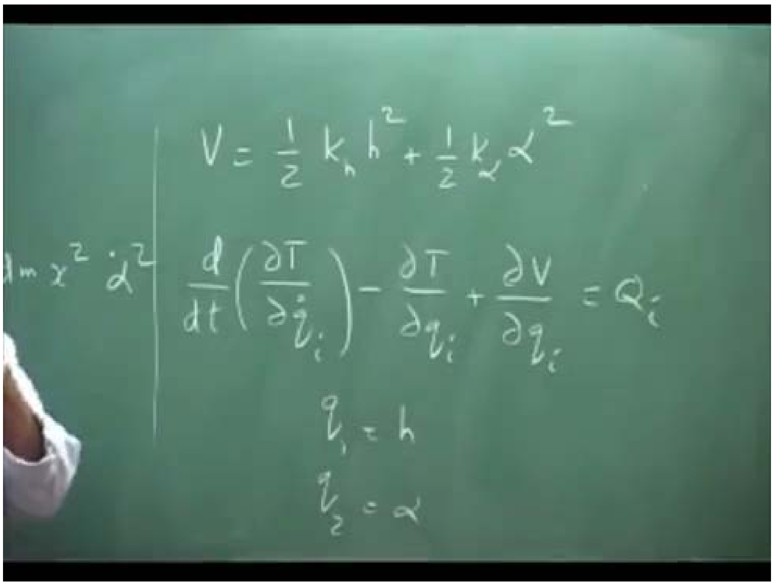

So, you can write your strain energy as the potential energy, v as half k h, h square plus half k alpha alpha square and then you also need to know that generalized force corresponding to each one, that come from basically virtual work done by the external forces. So, that spot we will write it after we get the Lagrange's equation, so what you do is apply Lagrange's equation directly, which is delta d by this d by d t of you will apply the we will apply the Lagrange's equation d by d t of delta t over delta q I dot minus delta t over delta q I plus delta v over delta q I equals q I, I varies from one and two in this case q 1 is h and q 2 is alpha.

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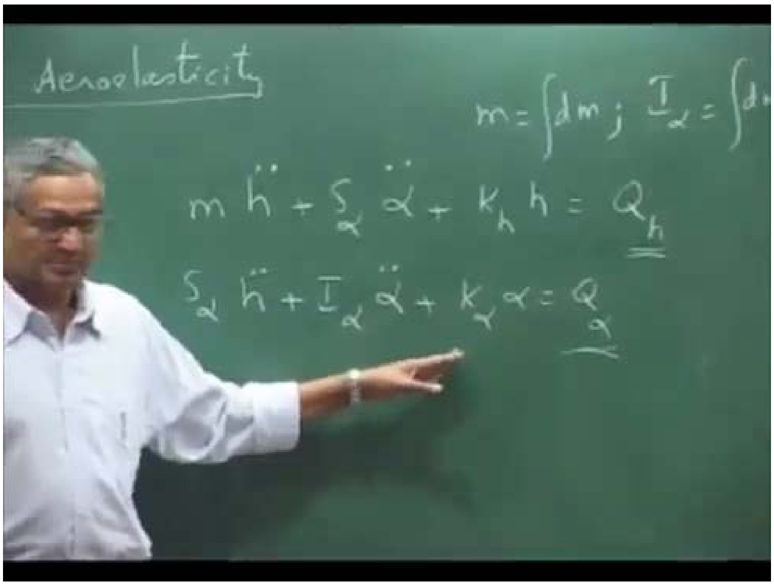

So, you just substitute that this terms and you will get your two equations, so I will write the two equations, now which is m h double dot plus s alpha alpha double dot plus k h, h equals to Q h corresponding to h. And then the second equation is s alpha h double dot plus I alpha alpha double dot plus k alpha alpha equals to q alpha, so these are the two equations.

Now, you need to get the Q alpha Q h, now this is a vibration type of problem because there is an inertia, which is a second derivative there is a spring which is here. But, there is a generalized force Q h, Q alpha the generalized force how do you get h and alpha is from because we know the movement of aerodynamic center. So, what is our virtual work done by the external load during the virtual displacement that is how you get Q h and Q alpha.

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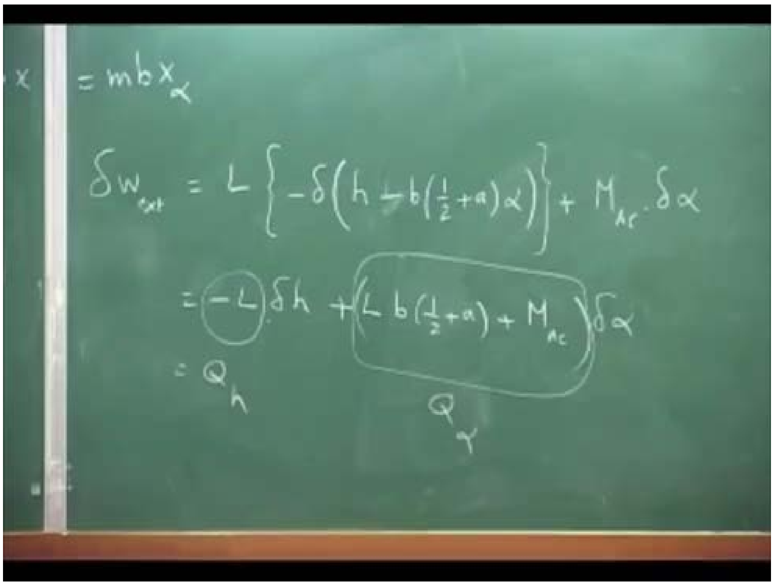

You will say delta w external is the displacement, please understand the displacement at this point virtual displacement due to a virtual motion in the each one of the degree of the freedom, that will be virtual displacement will be lift is acting up. So, your virtual displacement will be minus delta of h plus this is what b a, b by 2.

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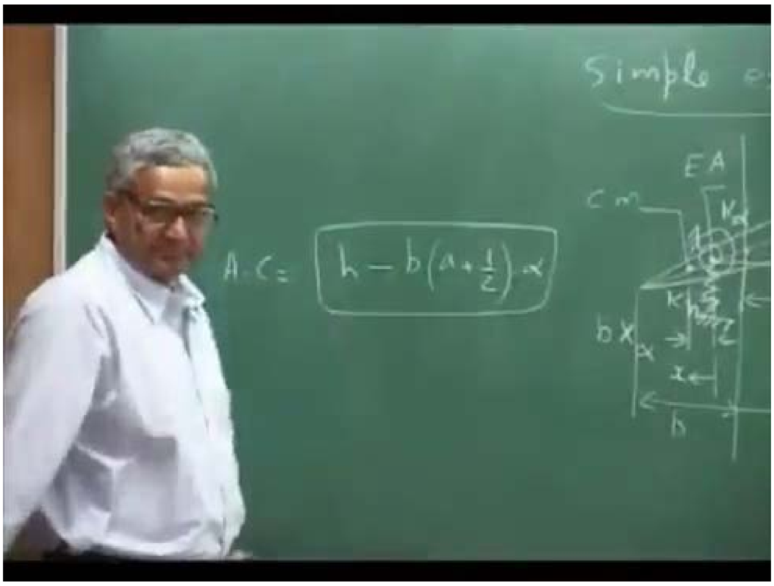

So, b into a plus half into alpha plus h, this is the displacement no minus because this is the displacement, which is up h is the downward displacement because this distance into alpha is this point is going up h is coming down. So, h minus this is the displacement of the aerodynamic center in the downward direction, so I have to put h minus b half plus a into alpha, this is my virtual work done plus of course, I will have the MAC into delta alpha. So, if we combine we get minus l delta h plus l into b plus sorry l b into half plus a plus MAC into delta alpha now this is my Q h and this is my Q alpha.

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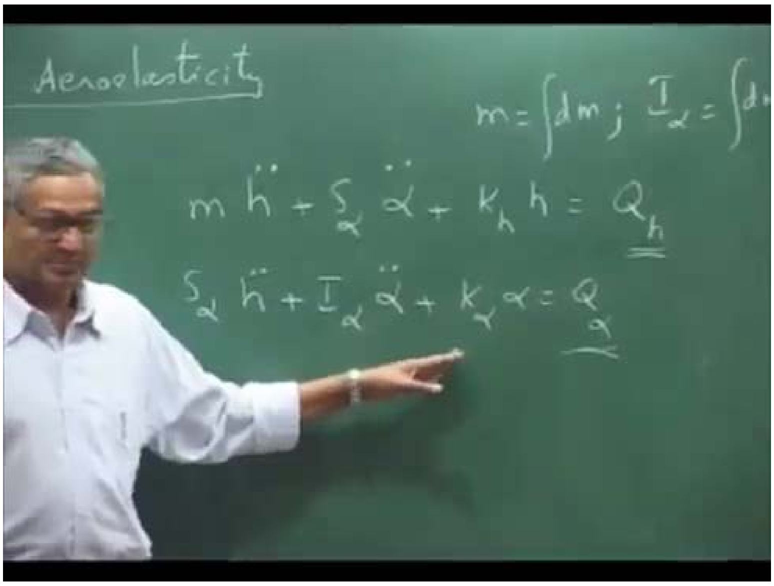

Now, I go and replace here my equation minus l, l b into half plus a plus MAC this is my set of equations, now the question is what are these lifting moment, now we write the lifting moment for an aero foil which is nut stationary. It is actually analyzing in up and down and switching this is what the unsteady aerodynamics complete to picture till that point till this everything is very clear, there is nothing difficult about this path.

Now, the question is how do you get the expression for l and moment, one is for the prelim native, because that is why it is a simple example in this case, we will make a highly simplified expression for the aerodynamic role highly simplified expression. When you make that it might be equations then you will say what is that you are solving for in this problem.

And now if you have gust which on external then you have to add that gust also which is lift due to aero foil motion, that can be an external disturbance dynamic reload that you may have to add externally, that is like a response of an aero foil to from external disturbance. Now, for the dynamic aero elasticity the first problem, the important problem you solve is the flutter then what is flutter and how do really solve the problem for flutter.

Now, that is what is we will describe that part using this simple example, after that we will go to study aerodynamics, that is why several lectures will be spend on essentially how we get this l and the moment, for a aero foil which is doing like this if it is 2 D aero foil, now it is a 3 D how we get 3 D and study aerodynamics. Now, what we will do we will take highly simplified expression for lift and moment we can set it zero. So, this is simplified model, I am going to take lift this.

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Because we know the static condition lift is dynamic pressure called into c l alpha c l lift quotient and c l, you write it as c l alpha times and sums angel of the time, what is an angle of the time at every instant. So, you are taking it although every instant the angle of the tact is I just measure, that whatever you that is an equivalent static angle of a tact, I write this as alpha then every instant will in the manner alpha plus see because this aero foil is moving down that what velocity h dot.

Now, I am going to take highly simplified thing because every point will have different velocity that is because every point is having different velocity, but I am going to take the velocity at only elastic axis, if you take three quarter cord then it has some value that will had a one more term, we will take simple I take the velocity adjust point it is nothing but h dot that is equivalent to a flow is coming up.

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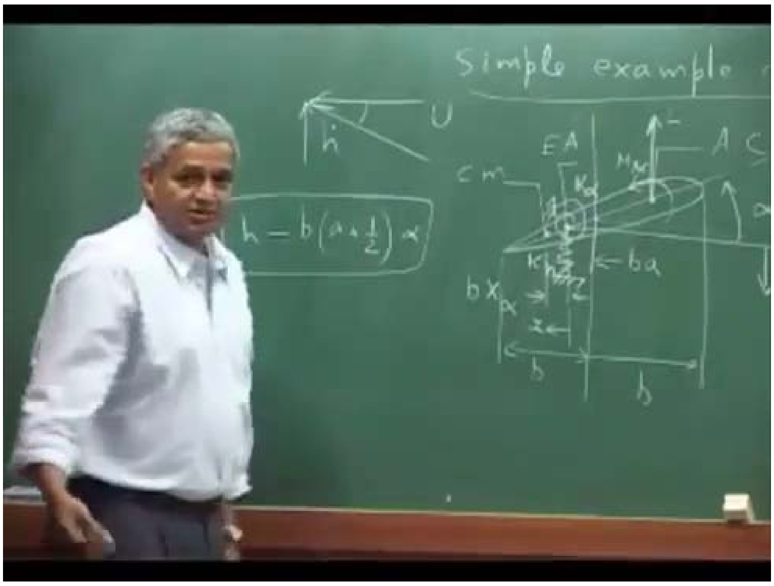

So, a flow is coming up with a h dot and there is a u, which is the oncoming feel resultant will be this angle will be h dot over u approximate.

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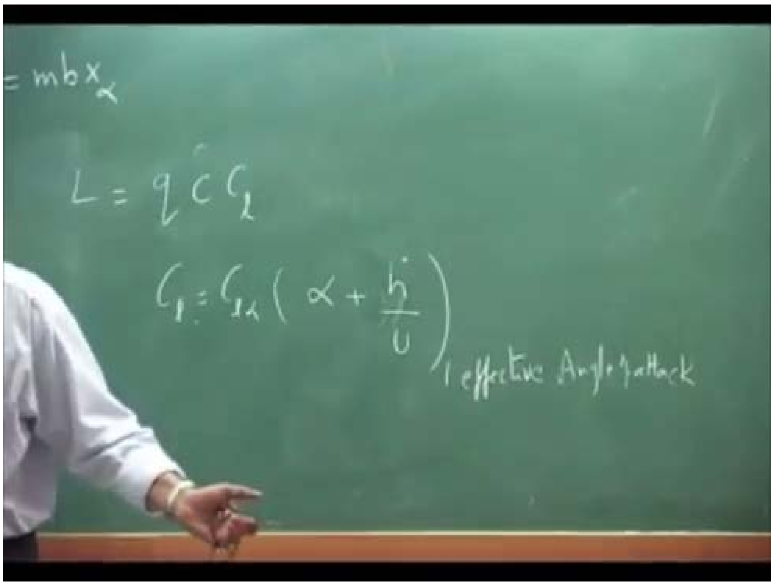

So, i am going to add here this is effective angle of attack, I call it effective angle of attack just highly simplified expression and I go back I substitute here, when I substitute here let us write this and n a c and taken it as 0.

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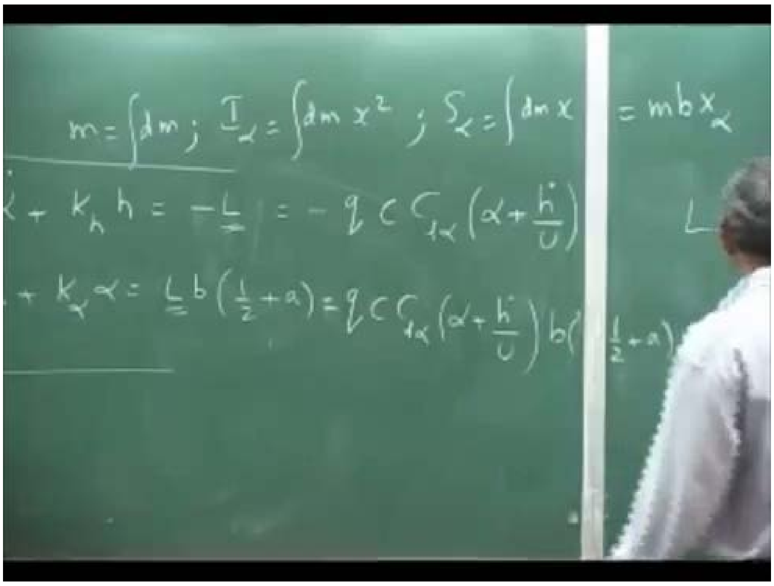

So, my equation becomes lift is minus q c, c l alpha times alpha plus h dot over u and then here it become q c, c l alpha into alpha plus h dot over u times b into half plus a. Now, you see this is not the aero elastic problem your lift is a function of alpha and h dot, but alpha and h dot are the dynamic motion therefore, my forcing function is the function of the motion of the aero foil.

And even the moment also same thing alpha h dot everything, now what you can do is you can bring all the terms to the left hand side and then write it in a matrix form, I am going to put it in a matrix form, so that it becomes easier for further analysis please understand, this is important this integral you should know then I x alpha I alpha x alpha.

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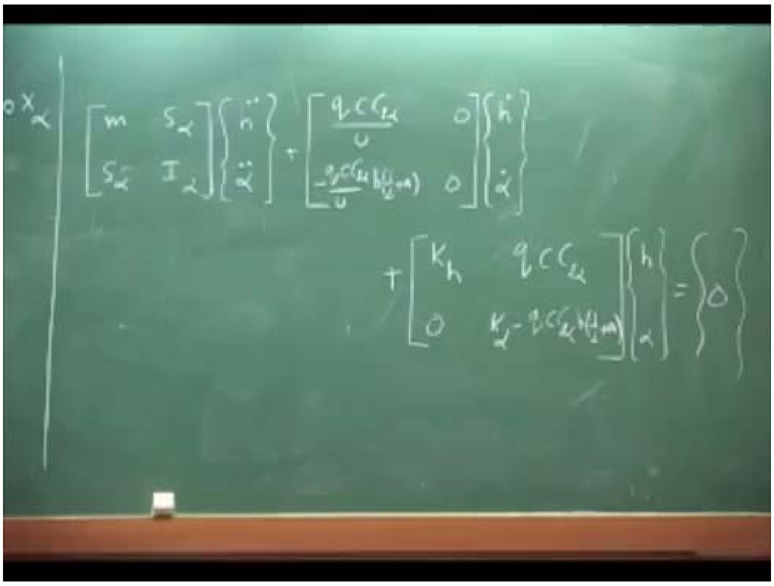

Now, let us write the matrix form m s alpha s alpha I alpha into h double dot alpha double dot, then I take this term to the left hand side here I have a h dot over u. So, I will take this as plus I am putting this q c, c l alpha over u 0, h dot alpha dot plus you will have for h is k h and you have a term for alpha. So, that will be q c, c l alpha into h alpha.

Now, we have to fill up for the alpha equation for the alpha equation you take this term q c, c l alpha b half plus a over u that will be there. So, that will become a minus sign, so I get minus q c, c l alpha over u b into half plus a and then 0 and then this is a function of alpha also.

So, you do not have any h related term, so the first term is 0 you will get k alpha you will get a minus sign minus q c, c l alpha b into half plus a equals 0, alright you check the stuff that everything is fine. So, that minus b into q c, c l alpha over u that is it q c, c l alpha b into half plus a this is my equation. Now, I erase this part now let us start analyzing this problem.

Suppose if you have an external loading; that means, some other external loading not lifted moment you will get some dynamic load, that will come on the right hand side then you can study under dynamic load for a given q, what is the response, suppose the aero craft is landing and we will say landing is equal to some dynamic force, all interest is coming that time specifying a particular fashion, we can put that as a load, how that will affect my response of the aero foil, that is one.

Second problem is my right hand side is zero, but I want to know what is the solution of the homogeneous equation, how they vary with q because q is my dynamic pressure that is forward spring dependent and you see my darting matrix, these are function of q my stiffness matrix is a function of q, which means I am varying my stiffness answer dump it with speed.

So, I need to study, how the system will respond if there is an initial condition, if there is a non zero initial condition then how many in a spot what do you say if you put an initial condition, you see how it evolves suppose if q is 0 it is nothing. But, a simple vibration problem q is 0, this is a 2 degree of freedom vibration problem, you essentially study what are the natural frequencies and the motive that all.

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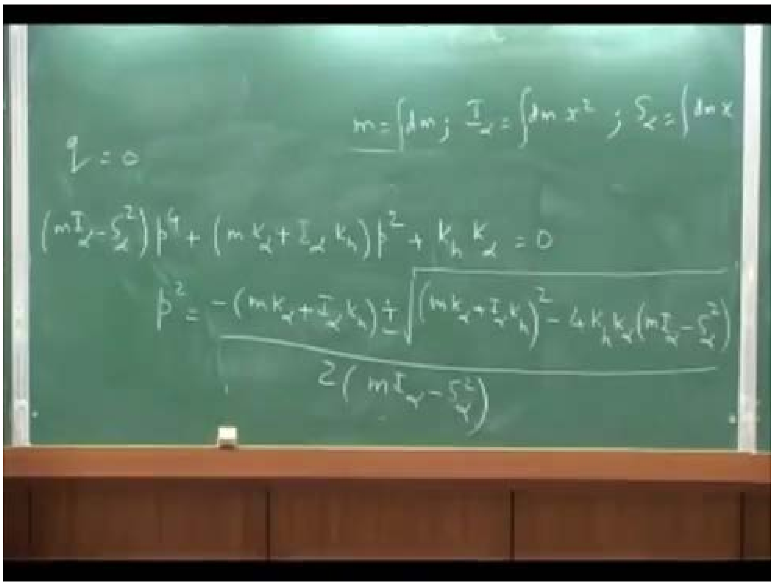

So, two degree of freedom vibration problem in which if q is 0 your equation will become m s alpha s alpha I alpha h double dot alpha double dot plus you have k h 0 0 k alpha h alpha equal to 0, you will write a solution of the type h alpha is of h bar alpha bar e power some p t, you will put it, substitute back and then solve for p, this will be like your basic Eigen value problem that is what you are solving it. So, if I substitute this p, I think now I do not think I need this I erase this part that part I need it q is 0.

But, please understand this diagram all the distances and the notation everything is referred to this because this is a very standard notation used for 2 d aero foil. Now, I substitute this what will happen it will be p square everything because h double dot will be p square.

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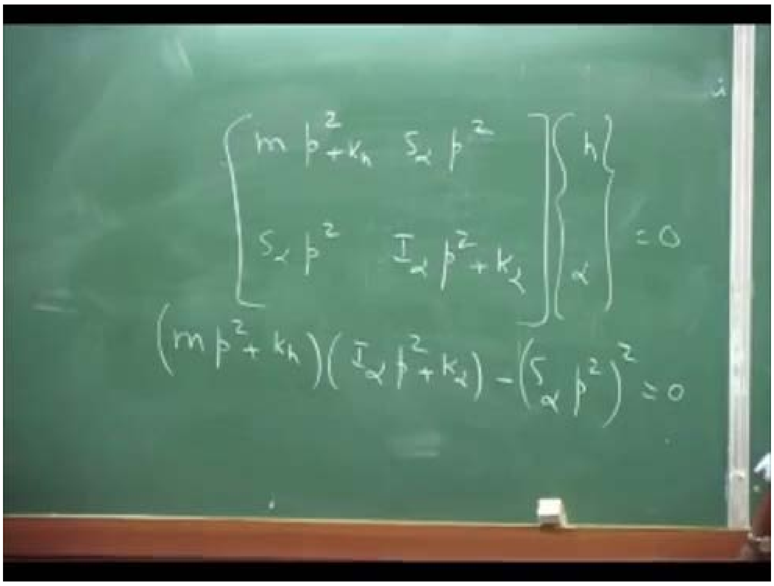

So, you will get m p square s alpha p square p square I alpha p square right, then you will get here I put it plus k h plus k alpha, this is the what you will get after you substitute you will get this, then what I have done is I have taken this secondary votive and put it here and finally, all these things will have p square p square, then the common thing is h and alpha and this is 0.

Now for a non critical solution the determinant is this 0 and that is your characteristic equation, which you will solve for p, now let us write that expression this will be m p square plus k h multiplied by I alpha p square plus k alpha minus s alpha p square whole square is 0, this is a fourth order n p are to be a second order n p square.

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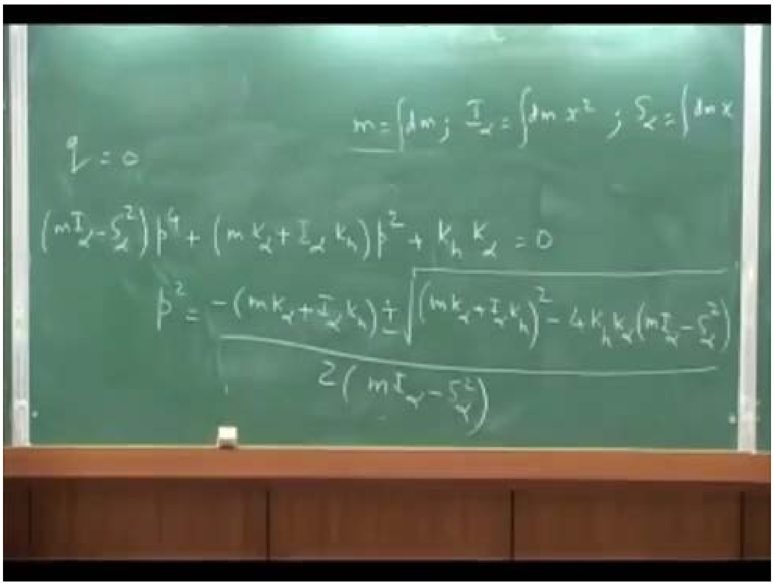

So, if you write it will be m I alpha p power 4 plus, we will have m k alpha plus I alpha k h into p square, then you will have this is minus s alpha square before, so I can write this term as m I alpha minus s alpha square and then plus k h k alpha is 0. Now, in this you can get p square if I write it a plus b minus that route, you can show that the p square you will get negative number, so p square becomes this is minus b.

So, you get minus of m k alpha plus I alpha k h plus or minus root of b square is m k alpha plus I alpha k h minus 4 k h k alpha into m I alpha minus s alpha square over two times, this is my root and all these quantities are positive quantities mass is positive spring constant is positive and s alpha can be positive can be negative. So, the whole question will go is this term positive or what happens.

Because, you can show that this term is always positive because how we have defined m is known I alpha is what d m x square x is measured from elastic axis and this is d x alpha s alpha square is b x alpha whole square. Now, if you evaluate this term you will be able to show that is nothing, but the value about the I will show you clearly it has what is this term I will erase this part it is not necessary.

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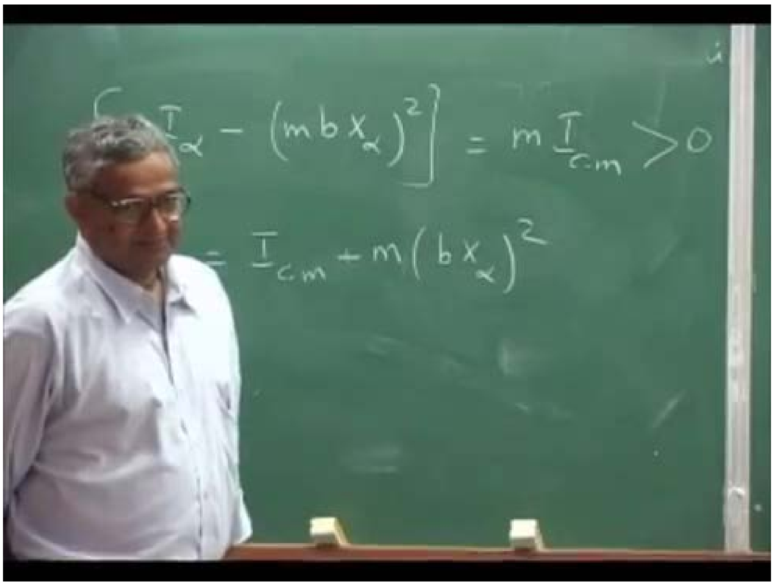

Now, m I alpha minus s alpha is m b whole square this is the first term, but you know that in any mass moment evaluation, I alpha is the mass moment of inertia is about the elastic axis this is nothing, but I about the center of mass plus shifting axis theorem, m into center of mass distance square which is nothing, but b x alpha square this is what I alpha is now if I substitute this expression there what I will get is because I center of mass into m, m that will cancel out you will get this is nothing, but m into I center of mass, this one is always positive this is always greater than 0.

Now, you look back here, you know that this term is always positive and you have these expressions, only question is you will like to know, this is plus minus whether this term the second term, what is happened whether this become a complex number, you understand you may have suppose this is a complex number see we said that this term is always positive.

But, what can we say about this entire term, now that is like that we will say that the entire term is always positive that part, we will because this is the key that is why you look at it this is happened how the roots will be…

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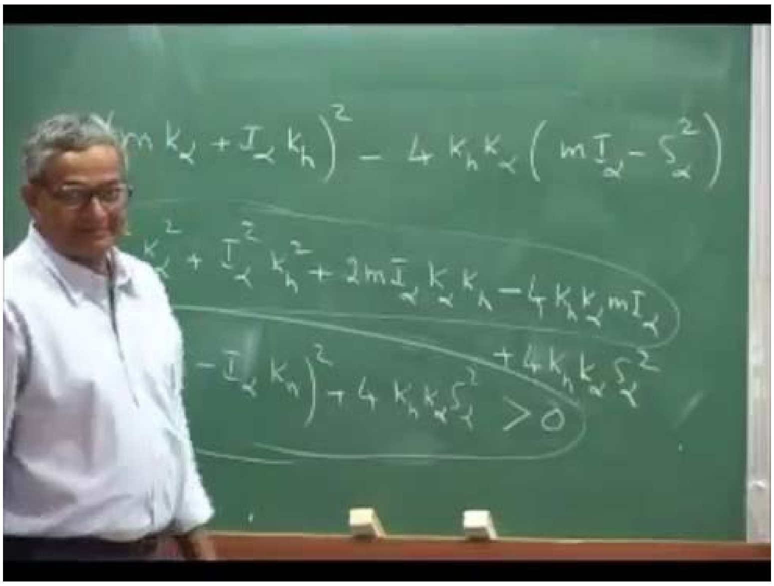

now let us erase this part and take the second term that is the basically term under square root, which will be m k alpha plus I alpha k h whole square minus 4 k h k alpha times m I alpha minus s alpha square, you expand this term this will be m square k alpha square plus I alpha square k h square plus 2 m I alpha k alpha k h, then here this will be minus 4 k h k alpha m I alpha plus 4 k h k alpha s alpha square, now this term it has 4 2 you will get a minus sign.

So, this will become m k alpha minus I alpha k h whole square plus 4 k h k alpha s alpha square, which is always greater than 0. Now, you see this term is always positive, but this term has to be less than this term because you are subtracting this therefore, this quantity is always negative because you know that denominator is always positive, so when this quantity is negative p square is negative means p is complex.

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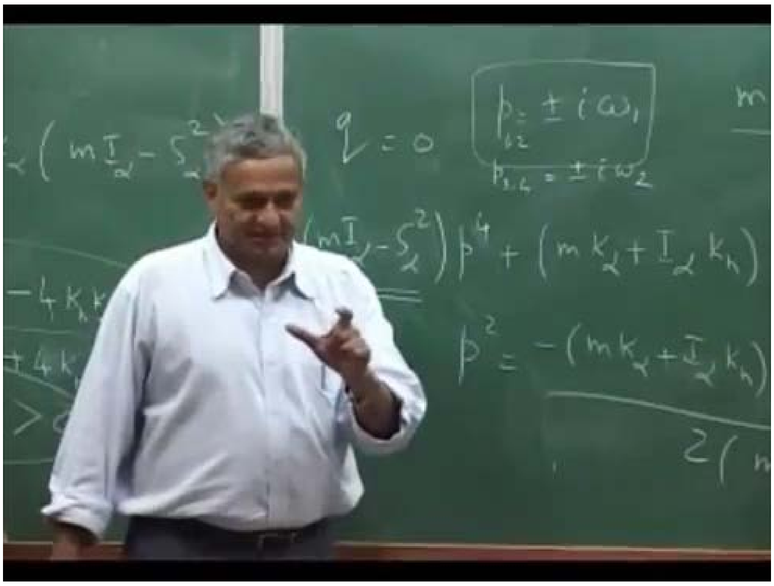

So, you will get p as plus or minus I some omega, but you will have two roots please understand you will have actually p square there is a plus sign there is a minus sign. So, you will basically have 1 2 then p 3 4 this may be more 1 I omega 2 there will be two frequencies, so you will basically have two complexions and omega 1 omega 2 and if you want you can solve actually the Eigen vector, but the idea what I wanted to say is yes

Now, it is always the vibrating system I omega 1 omega 2 and you say that, but it is a coupled please understand, it is not that omega 1 only going up and down omega 2 only missing you cannot say because in each motion you will have if you look at the Eigen vector corresponding to omega 1, you may say aero foil may go down and it may pitched up, you will have participation from h and alpha in each one of them, that depends on where your mass center is from the elastic axis, that is why the coupling this is the coupling.

Suppose, if this is zero that means, x alpha is 0 mass center is right at the elastic axis then you see you have a pitching motion, sorry pitching motion and a healing motion, which is if I neglect this q 0 h double dot, which is m h double dot k h equal to 0 that is one equation, another is I alpha double dot k alpha alpha, these two are uncoupled then you can say pitching is independent of healing.

Now, that is only when mass center is right on the elastic axis, but it is not possible you have to design it specifically on the other hand, if it is that is this coupling is called the because this happens through inertia, that is why this is called the inertia coupling you can also have some cases coupling through stiffness, that is the stiffness coupling in this problem it is a inertia coupling happens.

Now, we have shown that if q is 0 my system is if I give an initial disturbance, it will simply hustle it good that is the free vibration problem, usually you will say even though both modes participate in the sense both h and alpha participate in each frequency, you look at the relative magnitude of that Eigen vector. Because, you can solve for the Eigen vector because vector wants to know to put it back s alpha p alpha again vector.

If you find h is a big number then alpha then you say it is predominantly bending if alpha is a big number than h then you say it is predominantly missing motion. So, that is how you identify the type of motion from the Eigen vector because it is not easy to say this is bending that is torsion and anything, you look at the Eigen vectors and you check how the Eigen vectors are what are the magnitudes of the Eigen vectors.

Now, we have learnt with q 0 this is what you should have learnt in the simple dynamics problem 2 degree of freedom system, because our main thing is starting from 2 degrees here. Now, what happens to the effect of q that will be the next question if I start putting q there and introducing a damping number 1 and that is a coupling also coming. Because alpha being this equation h and my stiffness is a function of p and it is whole other than symmetric matrix and this is symmetric.

But, with aero elastic thing your problem will not be simplified whereas, you know vibration problem, you saw k h k alpha that is a symmetric mass matrix is a symmetric. So, mass matrix stiffness matrix they are same in free vibration problems whereas, the moment we go the aero elastic problem, they are not symmetric. Now, what are we really looking for in this you have to solve first, I will describe it in phases you will say even if I neglect this term, please understand I am neglecting this is for description here, how my q changes and because of the changing q how my roots are going to change.

Suppose for a particular value of q this is free vibration, there is no q effect is brought in, but now I am bringing the q into the problem simplistically I am solving without this then it will also help in 2 by 2, I can follow the same approach, which I have done right and then if I just by argument mathematically, I get some argument if I come up with expression I said that a for some value of q.

This p square here p square is always negative, please understand p square is negative if for some value of p square sorry some value of q my p square, becomes complex we will understand for some value of q, if p square becomes complex number then I write my will erase this part, this is the argument I am giving that is.

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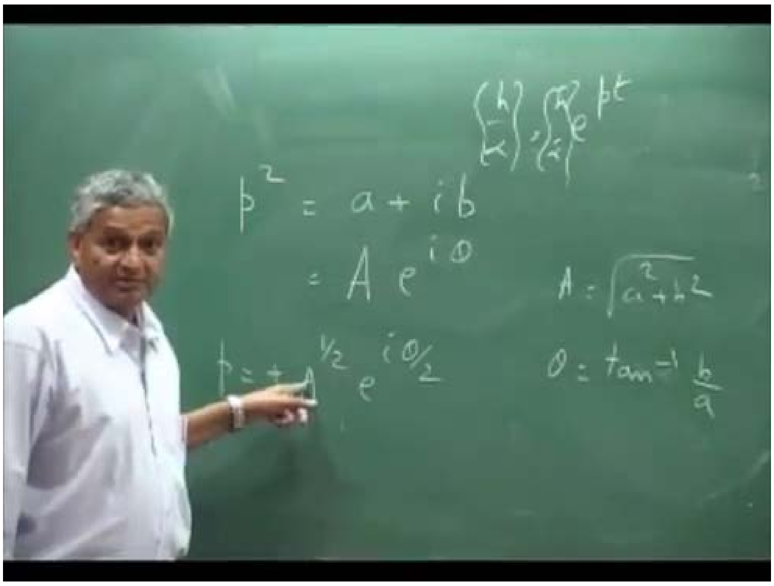

P square is complex, complex means it will be like this, which you will write it as some magnitude which is a where A is square root of a square plus b square and theta is Tan inverse b over a right that means, p square is this. Now, p will be right for some value actually p is a real number that means, my solution e power p t, what I assumed for h and alpha this I wrote it as h bar alpha bar e power p t, e as a positive number that means, this will take off in time that means, my motion if I give a small initial disturbance it will just blow off with time.

And that is what, you call it as a the point beyond which happens in the q, q 0 is only sign, but as I keep on increasing my q. Because, I do not have a damping tub that is what I am saying I am not included damping, this is just for logical description or logically you are trying to say, what is flutter for some value of q beyond that you will find my p has a real part that means, my motion is taking off it will blow up.

But, what is that boundary value q at which this is still a negative number, so that is what the condition you will try to do, so and you will see whether the condition exists and you will find yes you can exist, this is what the initial I think I have done in I would say fifty eight or something like that. And that was 1958 that was by pines and elementary explanation of flutter mechanism, that was the proceedings of the national specialist meeting on dynamic and aero elasticity 1958 paper, here you have it very simple problem very simple results.

Now, that is why this kind of explanation you have to solve a simple problem and come up with a what is flutter, what are the times to say one is automatically, you can solve and then whether you can come up with nice solutions, nice explanation of what is really happening that reference I erase this part.

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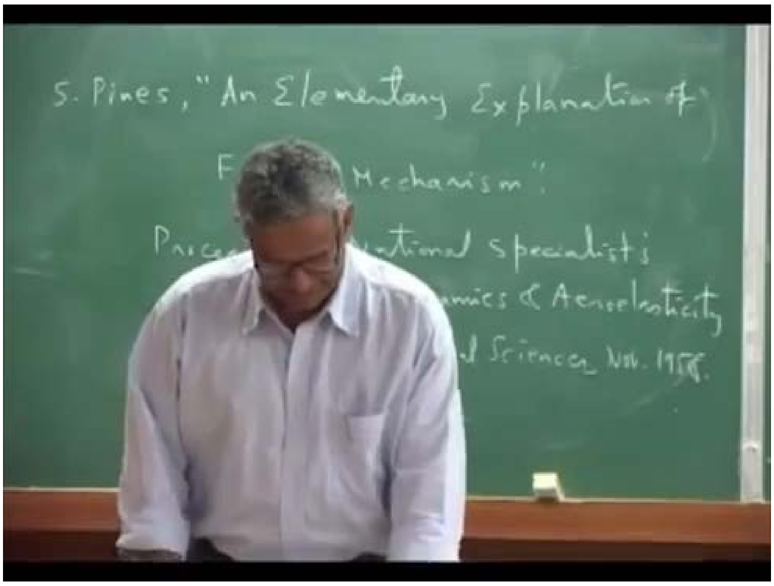

That reference is pines An elementary Explanation of Flutter Mechanism, there is proceedings of the, but I do not know whether you get it or not that proceedings of a National Specialist meeting on Dynamics and Aero elasticity institute of aeronautical sciences in November 1958, you can search the internet, check it out whether you get this.

Now, this is what our basic problem of flutter how do we understand this is if I vary my q, the condition at which this quantity becomes, a complex number and when this quantity will become a complex number means only, when this becomes a negative quantity that means, the boundary is this is equal to 0 with q. Because this is done without q, now we will have to add q in that and then put the condition, what makes it 0 at that q, that is your flutter point you should done it.

How the explanation is given you see whether like, we have shown currently this quantity is always positive we said that this quantity with q, you will say whether your m b x alpha, if b x alpha is always of this type you will find this quantities always positive you can never change it. But, if it is negative, then that is how the argument looks, but I can write the whole expression then it will be like fully writing the entire b square minus 4 a c write the full expression and then try to come up with some fluctuation actually, you can have a look at it.

Because, otherwise writing the whole thing it will be just algebra, but the explanation is getting the p square complex what is the condition for the p square to become complex; that means, the boundary will be this quantity becoming 0, but this quantity means. Because this will not change only k h k alpha this entire expression your word have few also sitting inside, so we can just write the expression, but this is without damping please understand that damping matrix is not included.

But, actually that is not correct, we are included aerodynamics and then we are putting a damping also, so that we will look at it the damping part little later, this is just I will give the neglect the damping.

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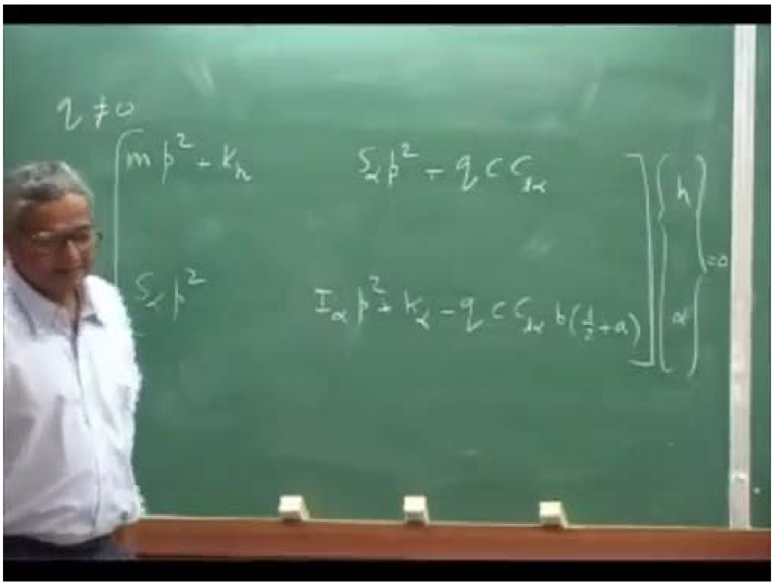

And you will write q is not 0, your equation will be m p square and then you will get that plus k h, please understand I am writing directly by looking at that equation, I am writing this expression, then you will have s alpha p square plus q c, c l alpha then you will have s alpha p square, then I alpha p square plus k alpha and then minus q c, c l alpha b into half plus a.. Now this again this is the fourth order p 4 and this you write it as a full expression s alpha p square q c, c l alpha k alpha minus q c, c l.

Now, the determinant is 0, then expand all you will get some fourth order equation and you can set the condition. Now, if you look at only the I will draw the diagram because that will not go into the arguments here how the roots will change.

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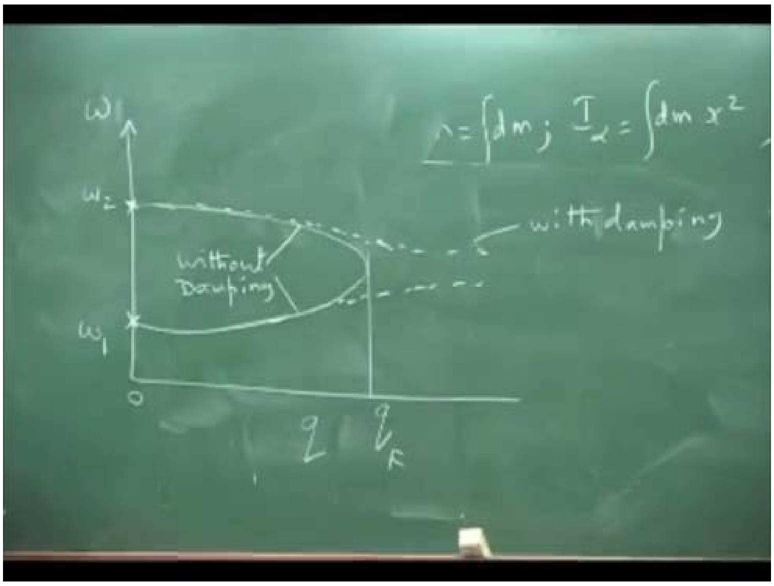

So, now you understood the logic what you look at it is, I am going to plot this is q and this is my omega. So, I have initially two values omega 1 omega 2 two values q 0 for every q, I am going to have two values. So, I will be tracking what will happen I will come like this, this is a just and then the point for this particular problem, when they meet that is my q flutter, because the radical is 0 your p square is basically the same minus that minus b term.

But, this is without the damping term, please understand without the damping term these are the Eigen values, your Eigen values will always be just plus minus I omega, but if you include damping then inner meet here, they become like this is without damping sorry this is with damping, sorry this is without damping. So, you please note that in this particular case of a highly simplified problem, I am going to give an example for you sorry homework for few on this which you have to solve.

And the natural frequency of the system, when q is 0 omega 1, omega 2 distinct, but as you keep changing the q, they keep coming closer and then when there is no damping, there is margin at that time, beyond if you go your roots will be having a real part. And that is the flutter that boundary this particular speed is the flutter speed, but if you put damping they do not merge they come close and then they can keep. But, still the flutter may remind some work here only, because beyond that it will always be you will have pass q. So, this is what and the roots how it define,

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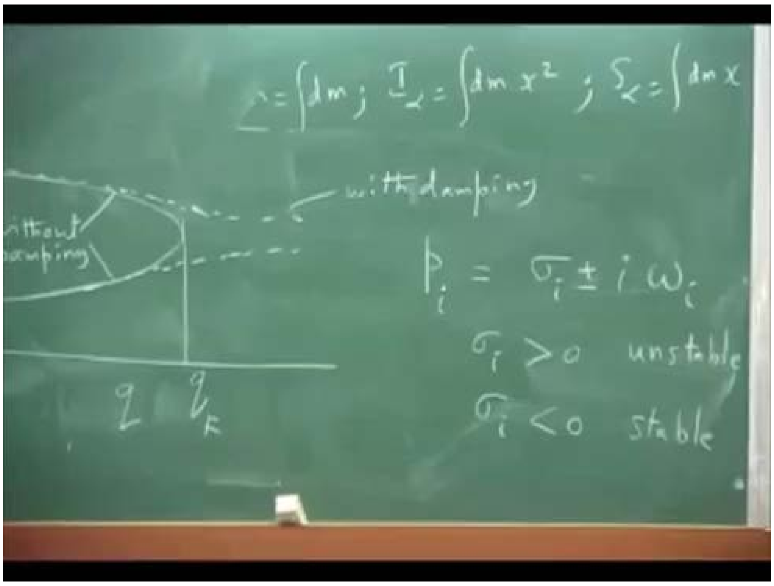

Now stability, so you always look at the roots p I as sigma plus minus I omega I, I always look at the Eigen values of the system Eigen values are nothing, but the roots of the characteristic equation. So, you see now the flutter problem is nothing, but evaluating the Eigen value of your problem, if sigma is greater than 0 system is what unstable, that is the real part of the Eigen value if it is greater than 0, this system is unstable if it is stable less than 0 means with time it will decade, equal to 0 is the flutter boundary.

And that is what you look at, always look at the Eigen value these Eigen values are complex Eigen values. And then you see how my real part is changing, that is how you find out the flutter normally, but in this problem this is just by looking at only the frequency I am trying to explain. But, if you solve the problem with damping with damping this is what you will look at it and with damping usually, the problem is you can convert it into a Eigen value problem in a much simplified way.

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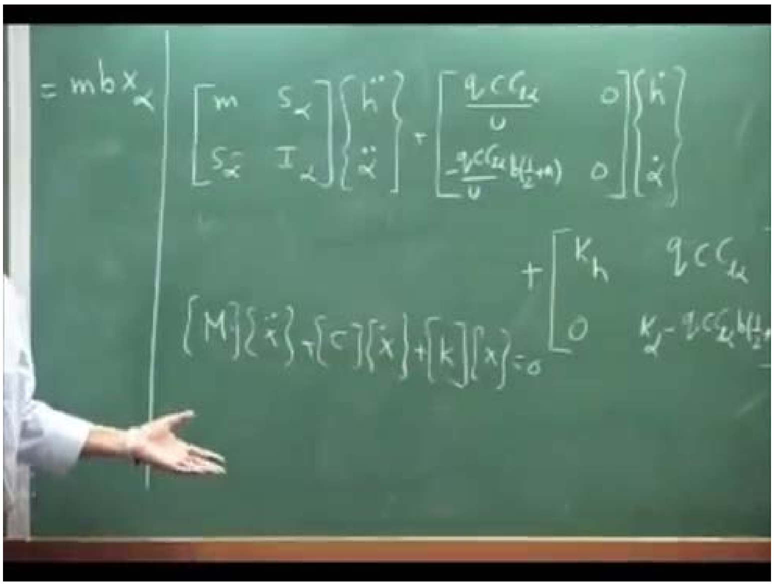

Because, this you write it M h double dot plus c x dot plus k x, convert it into a first order equation and then the Eigen values I erase this part, that is what is done with a aero elastic problem, aero elastic problem will always be of this type M c k you put it in state space form.

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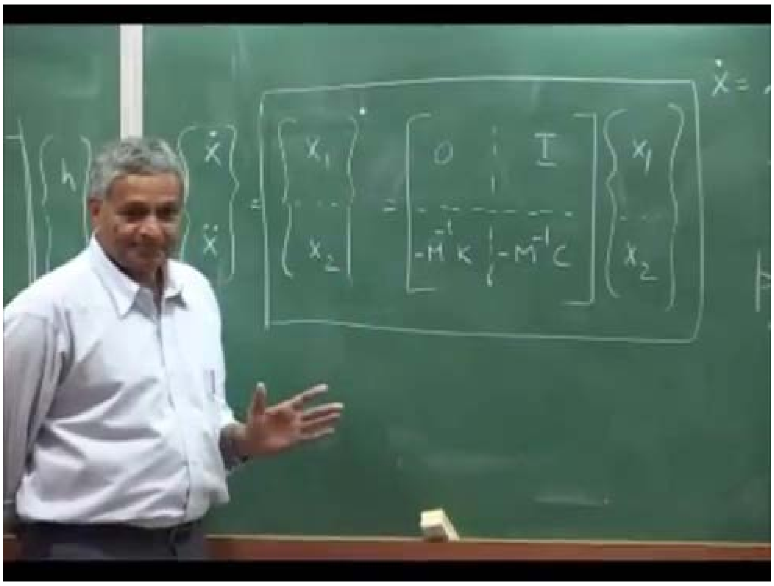

State space form, what you do is you put it first you put it x 1 x 2 two states you put them like this, you take a derivative of this that means, x 1 dot this is again x 1, x 2, x 1 dot you write it as x 1 equal to x 2 that means, this is I this is 0 x 1 dot is x 2 that means, x 2 dot, becomes x 1 double dot. Now I am writing x 1 dot I will put it in the matrix form, now x 2 dot is basically my x 1 double dot, this is minus m inverse c I take all these term to the right hand side, minus m inverse c x dot minus m inverse k x.

Now, I will write like this x 2 is x 1 dot which is basically x dot, then what is it this is nothing, but x 2 dot means this is x double dot, so I will write this as x 2 dot this as x itself and the other one is x 1. So, I will have minus k inverse sorry not k inverse m inverse k minus m inverse c. So, you see x 1 dot is x 2, x 2 is x 1 dot that is x dot, now this is x double dot, this is basically x dot x double dot that is what you are writing it.

If you x dot x double dot, that is what this is that means, state space you call x dot as x1, now what happens is no sorry there is a dot here, is there dot no there is no dot do not know this is fine I am sorry, x dot is x 1 dot. Now, this is what your state space representation that means, you have converted your second order equation into a first order equation and then you can go and solve for the Eigen values and this is not a symmetric matrix and so solve the Eigen values, usually call any of the standard Eigen value get the Eigen values for various values of q.

And then track down the Eigen values the moment, you see the Eigen value is the real part becomes positive you say that is the flutter point. And what is the omega, omega is the frequency they call it flutter frequency, the frequency corresponding to that otherwise you say each one is the damping the frequency. So, you may call it the model damping, the model frequency in this problem you have only two frequencies.

So, you will get, but you will get four roots the complex roots that is why plus minus, but you do not bother about the frequency sign, it is always sign before plus minus I omega it is always I omega is it clear, this is how you solve the I would say the simplest form of elasticity, simple example to explain what happens there, but this is with taking your unsteady aerodynamics in an approximation be used for the Aerodynamic load.

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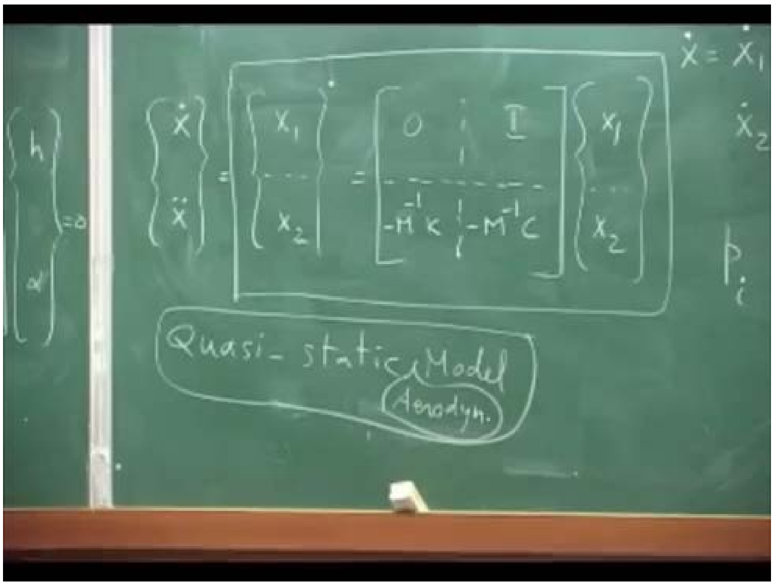

The terminology is a Quasi static model, Quasi static aerodynamic model you want aerodynamic model quasi-static ((Refer Time: 01:16:25)) then if I want static there is something else also will come later. So, this is a highly approximate representation of aerodynamic model, what you know because this is still right, but it is in the preliminary calculation to understand how to explain what really happens in the system with dynamic pressure, then this is the time this gives a you call it a basic explanation for this problem and I do not think that there is anything, I would say is this you did the Eigen values and you solve that problem.