Aero Elasticity
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Lecture – 13

We were talking about the formulation for the swept wings, but we will put chord wise rigid.

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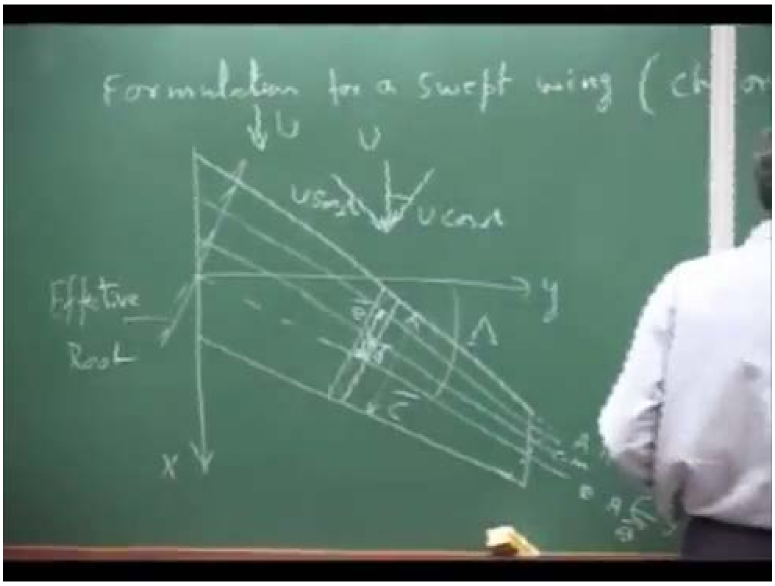

So, in this model we take is as usual the same thing and this is the elastic axis and you have your aerodynamic center and masses and this is your theta bar and this is my y bar. Of course, this is my y axis, x axis this is my wind direction and this is the effective route the same way you draw effective tip and you take now chord wise segment. And the chord you call this as c bar and this distance you call e bar and this distance you call t bar and of course, this is the sweep angle to the elastic axis. Now, essentially what we have to do is we resolve our oncoming velocity into two components u and this is lambda u cos lambda and this is u sin lambda. So, the oncoming flow normal to the leaning edge I take it as u cos alpha.

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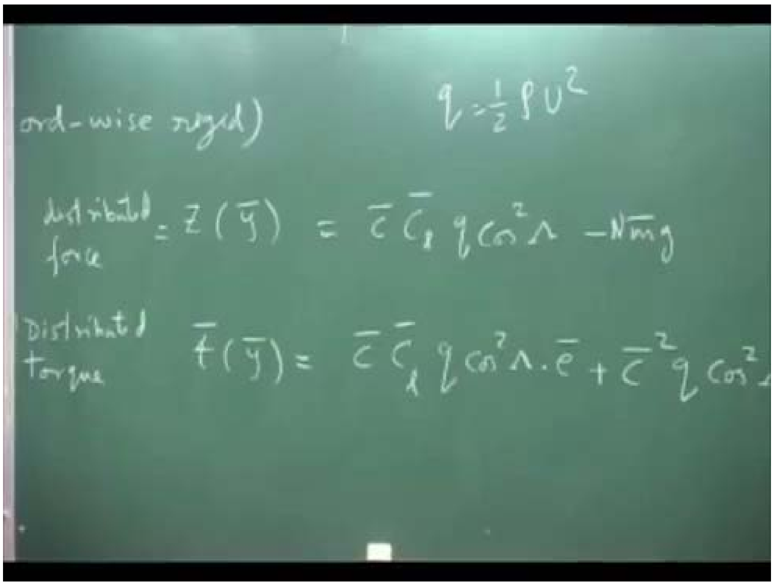

And now, I get my distributed lift distributed force is basically c bar and c l bar q and I will have actually q is half rho u square. So, I will have q cosine square lambda minus m bar g and load factor n, m bar is mass for unit length along again y bar length. Similarly, you will have your distributed twisting mode distributer torque I call it t bar y bar this is c bar c l bar q cosine square lambda into e bar because e bar is the opposite between aerodynamic center and elastic axis. And then plus c bar square q cosine square lambda c bar m a c this is the aerodynamic moment coefficient then minus n m bar g d bar, so this is my missing moment. Now, you can go and write, but please understand all this after along chord wave directions.

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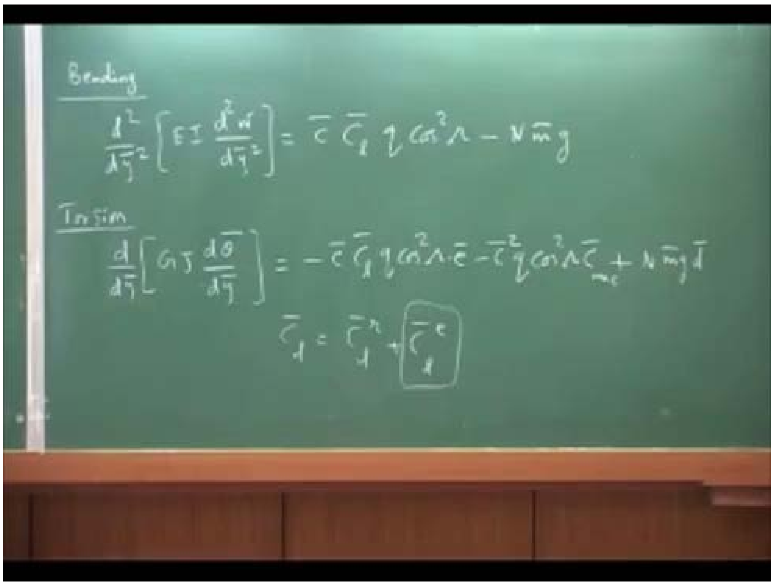

Now, your bending equation it is essentially d square and d y bar square e i d square w by d y bar square equals to a distributed load, distributed load is here we will have c bar c l bar q cosine square lambda minus n m bar g and similarly, your torsion equation becomes d by d y bar g j v d theta bar by d y bar this is equal to negative of the twisting moment. So, you will have minus c bar c l bar q cosine square lambda into e bar minus c bar square q cosine square lambda c bar n a square plus n m bar g d bar, now this is my torsion equation.

Now, what you do is as usual you split your c l bar as c l bar digit plus c l bar e elastic and then select the terms of the elastic to the left side and then write your complete equation. Now, before we shift on right what is c l this is nothing but, lift coefficient of the chord waves, but lift coefficient becomes essentially c l bar e is some c l alpha into the angle of the attack what is the angle of attack.

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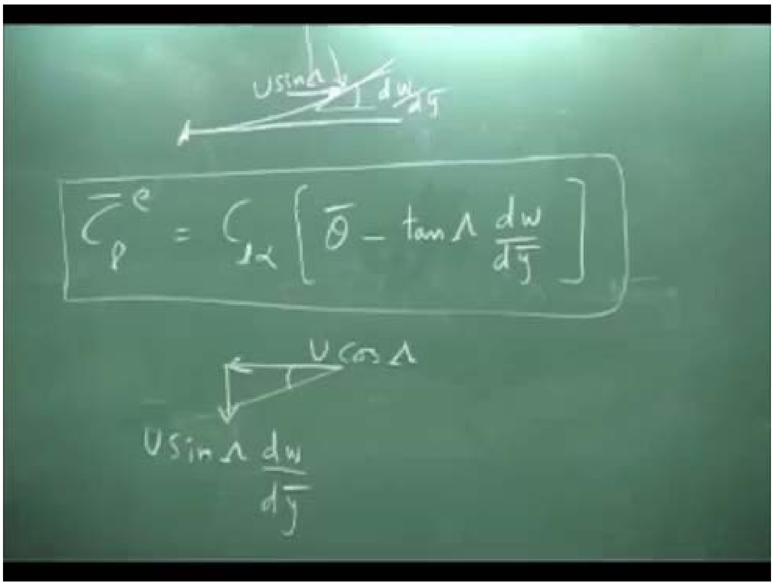

This is the question we have to be careful here one part of the angle of the attack is theta bar is basically twist. So, you will write one part is theta bar and then there is one more component which you have to take because of, bending information how that comes about is u sin lambda is going along the y bar right now, when this beam is bending u sin lambda will be like this it is like if I take the slope this is my wing u sin lambda will be like this and the normal component this is the slope.

This is the slope of the difference in bending now, you can have a normal component the normal component will be u sin lambda times the sin of that slope is very small, you take the normal component you will have something like this is u cosine lambda. The normal component will be now, this is a change in angle of attack at that section, please understand this is a little tricky in the sense this section which was originally like this which has become like this. And you have a velocity component which is coming a normal component you take the normal component is u sin lambda d w by d y the oncoming velocity is cosine lambda.

So, you divide by that this angle will be basically tan lambda d y by d y bar and this will reduce the angle of attack this is coming down so, you will have here tan. So, you see C l e will have this form so, technically if I take this term split it c l bar that is the rigid angle of attack that will stay on the right hand side left hand side that C l bar e will go right hand side and that C l bar e will go in both the terms.

And then you will have to substitute for C l alpha which is the error in lift cup slope, but the angle of attack is theta bar which is due to twist this is due to bending this is how in this chord wise rigid formulation, the coupling between bending and torsion happens. So, this form is different from stream wise rigid formulation.

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C l alpha here is directly that aerofoil because this is the aerofoil this is different this C l alpha is directly C l alpha of the because the flow is normal to the leading edge whereas, in the other case flow is not normal to the leading edge that is why there c l alpha lambda not equal to 0 whereas, here it is directly the c l alpha now, this is how the coupling of course the boundary conditions you have I will just mention it.

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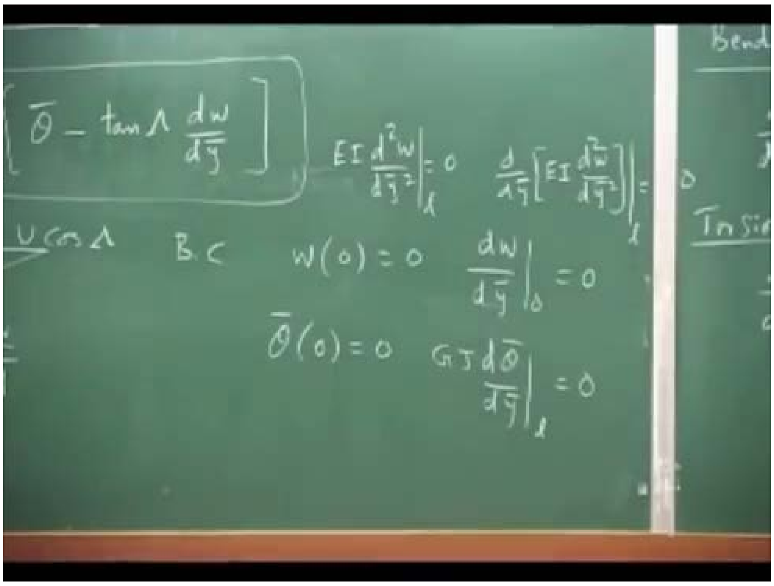

So, that you have the boundary conditions b c will be w 0 0 and d w by d y bar at 0 is 0 your slope is 0 then theta bar 0 is 0 and then g j d theta bar by d y bar at l is 0 and then you will have other two you will have e i d square w by d y bar square equal to 0 at l and d by d y bar of e i d w y at l is 0. So, these are your boundary conditions because this is the free end hear is 0 bending moment is 0.

Now you see you can solve this problem again with the boundary conditions and then apply galerkin method, but please understand this formulation looks different from stream wise formulation rigid formulation now, the question is which is right which is wrong because you look at the construction and then solve. So, these are all bending torsion type of a problem we are not taking these as a plate this is a large aspect of a wing therefore, we will calculate by both the methods.

So, assignment I will send you calculate that divergence speed for the wing by both the approaches and then compare the results. So now, we will do one example problem of how to apply galerkin method for getting the divergence of a wing, but for this I take the problem which I gave you in the earlier so, I erase this entire thing because I thought this will be good to have how to have the solve a divergence problem.

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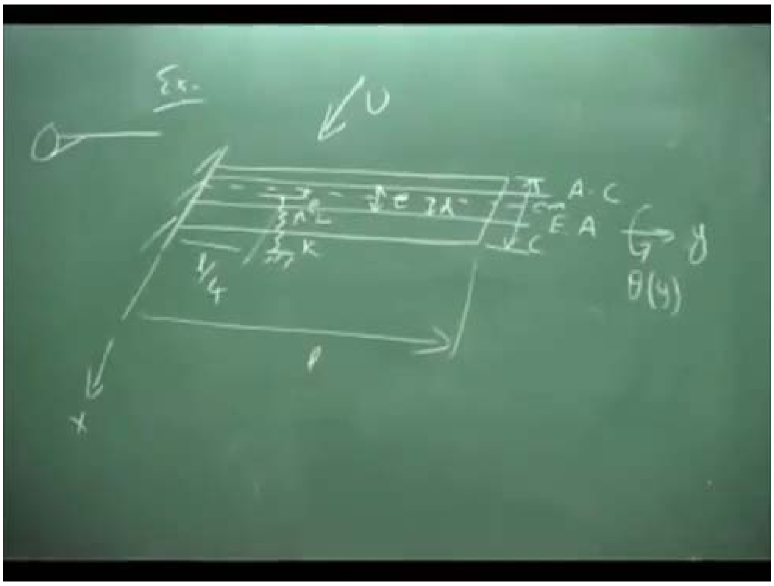

Now, we will take this thing you have a straight wing this is an example and this is the aerodynamic center and this is the elastic axis and the chord is c it is a straight wing and we put at a particular point l over 4 I have a spring. this offset I call as e l and this offset is e and this is my x axis now first we formulate the equation for this which you already you got the equation, but I thought if you have this e l this is we are formulating for basically divergence.

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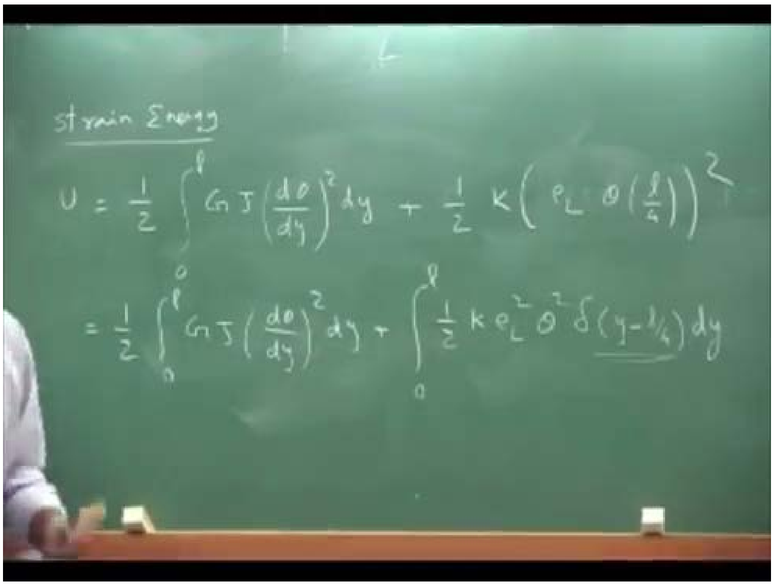

So, the gravity will only be on the right hand side it will not be on the left hand side the weight of effect therefore we will apply the simple virtual work principle and then try to get the equation you can apply Rayleigh Ritz also, please understand for this problem you can apply Rayleigh Ritz also, but I am solving by galerkin formula. The stream energy expression will be e u please understand you have to write now, the strain energy expression for the twist which is because this is the twist which is the function of this is y i have used so, let me use y only it does not matter x y and z.

This is my x axis theta is a function of, but plus I have a linear spring attached to the wing which is like what this representation means is I can have mean like this I can have some stiff structure attached here this struck is represented by a linear spring now, the spring is going to deform depending on my twisting. The twisting will be because the wing twists because this is a straight wing now, what happen this will be theta at l over 4 by the offset e l that will be the deflection at this point and then strengthen the half k into the deflection square.

So, you will add the strange energy plus half k linear strength into e l theta at l over 4 better this is an integral over 0 to l that is a point function. I can now modify this a little bit because just for convenience I am writing it half 0 l g j d theta by d y whole square by d y plus this also I am writing as k e l square theta square into delta y minus I am using a delta function alright that means, if I use delta function that means this is one when y is l by 4 already it is 0 everywhere it is 0.

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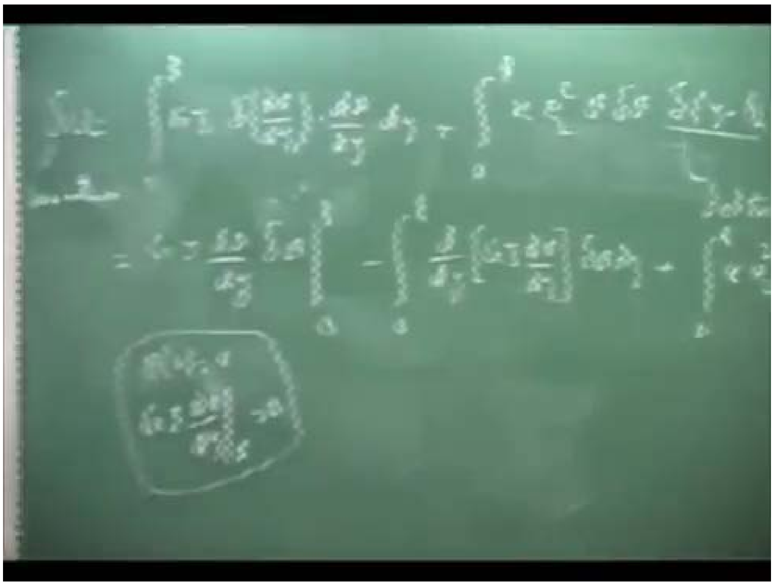

So, this is an easy way of representation now, you can write what is del u variation in strain energy is you will take the variation here and then it will be integral you will write what half will go up so, you have integral 0 to l g j del up and then d y plus integral 0 to l k e square theta delta theta delta y minus l over 4 please understand that this delta is a delta function. This is a delta function this is a variation please understand now, I can integrate these by parts if I integrate these by parts I will get g j e theta over d y delta theta at 0 l minus integral 0 l delta theta d y plus this term will remain as it is.

So, you will have integral 0 to l k e l square theta delta y minus l4 delta theta d y this is as far as the internal. Now, you have to write the external delta w external, but external for the present I am only taking only the lift force this is the virtual work external moment and gravity I am leaving it because it is not necessary for you to have gravity, but if you want to take gravity then you have to have that also whatever moments.

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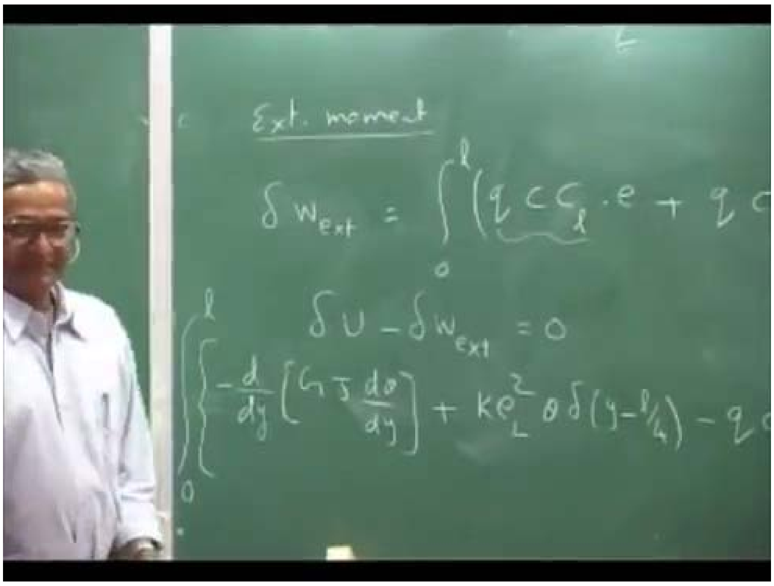

This will be delta w external which you will have 0 to l dynamic pressure chord c l into e offset because that is the lift force into offset e plus you will have dynamic pressure chord square c m a c and if you want weight you can add that term here. That will be then I will have to define all this center of mass line and with an offset some d and that will become minus phi the load factor is 1 then m g into d into d y into you have to sorry before I go let me write this is the what no, I cannot write like this wait a minute this is external force into e into delta theta this is directly delta theta I will have d y.

Whereas, for the mass I will write minus m g that distance is d into because, e into delta theta is the if you take this is the force if you take the force is acting which is normal the virtual displacement at that point is theta into e delta theta into e whereas, the force acting on center of mass lying down the displacement that comes out is d into delta theta and the minus sign because the force on the displacement are in the opposite direction. Now, combine both of them and you know the boundary condition the limit at the root is the fixed end.

So, that will have the 0 at the free end you say it a free moment, moment is 0. So, this boundary connection will go off in the sense d theta by d y this is what will come out of boundary condition that theta at 0 is 0 and g j d theta over d y at l is 0 this is the boundary condition now combine both you will have your equation will become minus because you know that del u minus del w external is 0 you can write in this fashion or you take it and put it on that side that is also fine.

So, if you write like this del u you will have minus d over d y of g j d theta over d y plus k e l square theta delta of y minus l over 4 I am going to keep delta theta term because every term has delta theta d y this a complete integral 0 to l. So, we finally, say delta theta is arbitrary therefore, the integral must be 0 that is what we are going to write this is my big expression this will be minus so, you will have minus u c c l into e minus u c square c m a c then this will become plus m g d which is the entire thing delta theta d y is 0.

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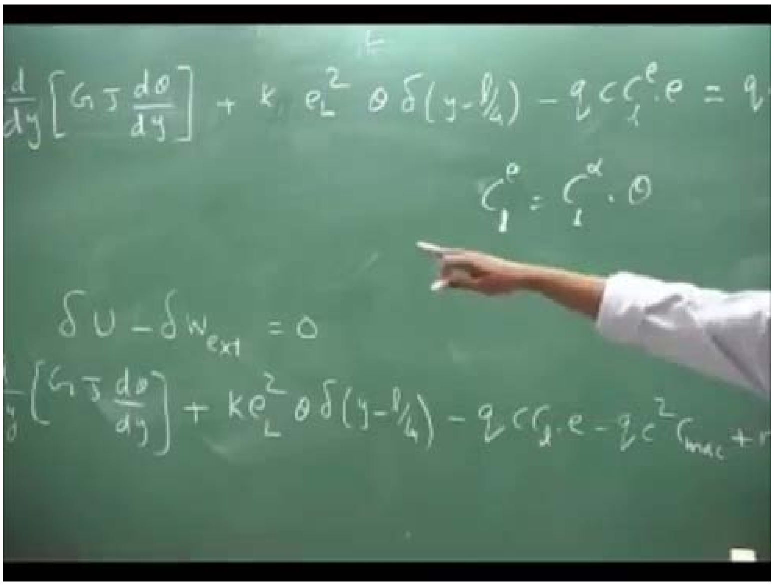

This is the integral 0 to l this is what is your final principle what you get and this you say my deltas thetas are arbitrary variation therefore, the integrant must be 0 and the integrant 0 gives you your equation of basically in the divergence problem the torsion information equation. so, we will write that equation that equation will be minus d y of g j d theta plus a l k e l square theta plus k l no k e l square theta delta of y l minus 4 this you again if you want you can split it c l rigid and c l elastic.

So, if you split c l rigid and elastic you will have minus q c c l elastic into e and then rest of the term line shifting into the right hand side if I shift it to the right hand side this will become q c r into e plus you will have this term q c square c m a c and this will be minus now, you can write q l sorry c l e is c l alpha into theta because for us that is the only difference that is not bending this is a straight wing you can substitute here this part is((Refer time 32:40)). So, I am going to write here I replace it c l alpha into theta into e I have replaced, but right hand side I am going to keep as it is.

Now, you need to get your if I rearrange my equation once again because this is just for convenience. I will have my equation like this look at the minus take it out g j d theta over d y minus k e l square delta y minus l y 4 into theta plus q c c l alpha into e theta equals I will have the minus q c c l r e minus q c square c m a c plus m g d with the boundary condition theta 0 is 0 d theta over d y at l that is g j.

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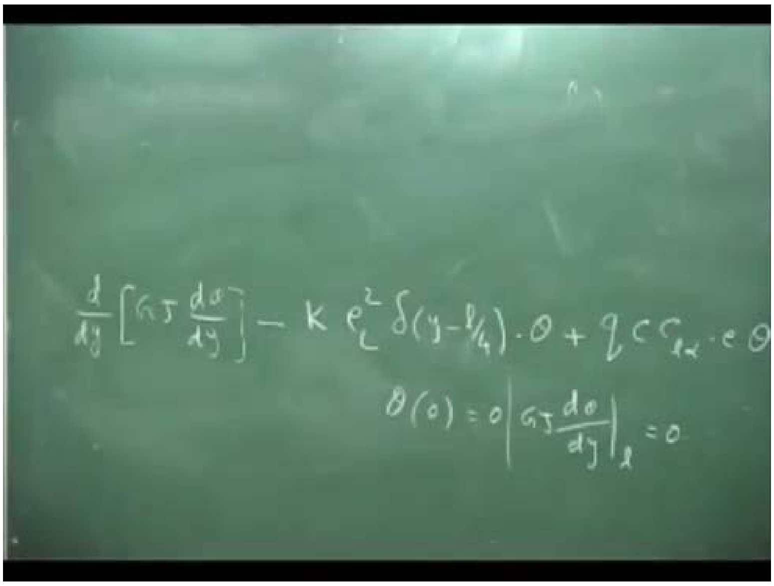

Now, this is my entire you see because this term spring is there extra spring that is the function of theta. So, this will affect your divergence speed now, here only I will apply galerkini I gave solution which is apply you check the theta which is a function of y as some constant a 1 into 2 y over l minus y over l whole square just assume that means, you see this when y is 0 theta will be 0 when you differentiate d theta by d y this will be 2 y over l sorry 2 over l and this will be 2 y over l square so, when y equal to l this is also equal to 0.

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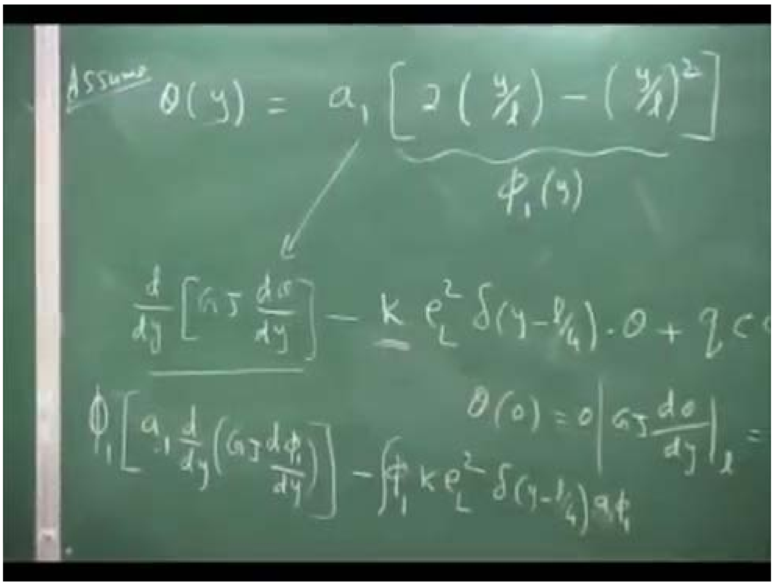

That means, this solution it satisfies both the boundary conditions now what you have to do is you have to substitute in the equation and then because when you substitute it will not satisfy the equation you will have an error that error you call it as error means everything that is this term you again bring in to the left hand side put that theta equal inside here because this is independent of theta please understand only theta exists here then you will say it will not satisfy exactly. So, you say my idea is to get basically a 1.

The galerkin formulation tells you the error which is a function of y because when I substitute here it is going to be a function of y how these regions functions on y 0 to l I call this function as phi one y. So, error is orthogonal to the assumed function so, I take this as this is what I have to do now I am taking only one function here if you take two functions then you will get two equations error is same when you substitute both now what we will do is we will go back and calculate each function separately.

Because you have to understand this is where we need lot of integrals because the error becomes essentially I can take it like this would be plus plus minus equals some error which is a function of y I have substituted here theta equals this function now this function will be sitting here d by d y what I have to do is I have to multiply phi one into this when I multiply phi one into the entire term the first term will be phi one d by d y g j d phi one by d y because this is you have to put here. So, I have to evaluate that integral.

So, what we will do is we will write like this now, let us make some assumption that it is a uniform wing uniform wing means g j is constant and all the c e everything is constant it does not vary along the span. So, we call it uniform wing then what will be my if I substitute error and the whole thing because I am going to call this term this entire term which is independent of theta you can call it by some other name.

So, that it does not matter call it some f or q whatever may be now phi one you know the first term I require integral 0 to l phi one since I have uniform wing you will have g j d square please understand the first term is going to be phi sorry phi one you will have a one will be there because a one is what you substitute here so, you will have d over d y of g j d phi one by d y do you follow this is the first term I am only taking g j constant. So, a one is anyway constant and just evaluating this integral you substitute this here and then evaluate the intergral.

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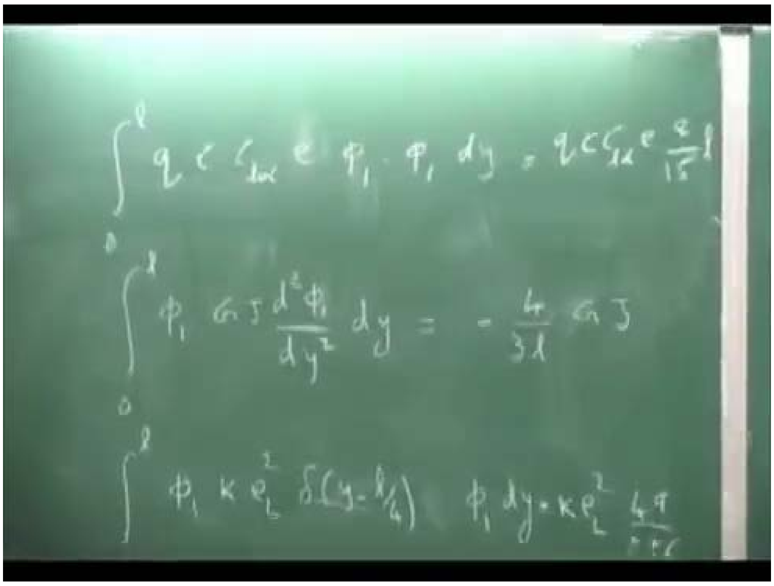

When you evaluate the integral you will get the I will write the answer this will become minus if you evaluate this integral then the second term what will happen second term is you will have minus again phi one k e l square you will get what this is an integral so, you will put all this integral that delta function will be there delta y minus l by 4 into theta right this what theta is nothing, but a e one a one phi one into you will have that d y. So, essentially this term will become integral 0 to l phi one k e l square delta of y minus l over 4 and a 1 phi 1.

Right now here you know that this is a delta function a one is a constant that I do not have to write it in this integral leave it because a one I am taking it everywhere a one a one a one that is a common constant it will come out. So, if I evaluate this integral there is nothing, but the value of the function phi one square at l by 4 so, I will get here that is phi one square l by 4 that will become k e l square this will forty nine over two fifty six you can evaluate this integral that means, phi 1 square at l over 4 so, you will get k e l square 49 by 256 then you go to the third term.

Third term will be again you will erase this boundary condition part third term will be plus you will have again q c c l alpha e theta again a one phi one you are multiplying by phi one and you are integrating again d y and that integral I will write it here this is integral 0 to l q c c l alpha e and you will have phi 1 phi 1 d y a 1 I am taking it as common a one is there everywhere here also a 1 a 1 a 1. So, that can be common term outside.

Now, if you evaluate this integral this will be phi one square integral d y that is actually 8 over 15 l. So, I will write this q c c l alpha e eight over 15 l that is this integral and other integral this is independent of theta so, what I will have I will have plus this will be some integral phi one into the entire quantity into d y this I can call it by some q or whatever.

Therefore now, I am going to write my integral my equation after it would be a 1 minus 4 over 3 l g j minus k l k e l square forty nine over to 56 plus q c c l alpha times e 8 over 15 l this is equal to sum q 1 that integral is this integral please understand you can take it to the right hand side with that minus sign will come that is what I am writing it as q now you see this is nothing, but this. So, you may say what happens to my unknowns here my unknowns are put a one of course, the rest of the quantities is q.

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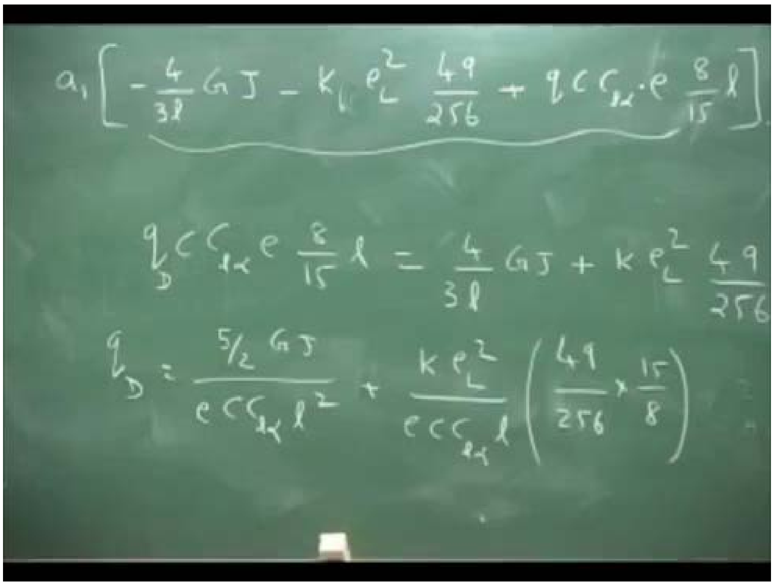

Now, you see the problem becomes like this if I am given the q that means, I know the speed of the r product that means, I know the dynamic pressure then what is my deflection because we assume that theta y is a one into what a nothing over two y over l minus whole square and you know these quantities. The weight of the wind and then this c m a c what is the moment coefficient according to the c l r you know that whatever initial angle of attack alpha r.

If they are given you know for every dynamic pressure what is my a one then calculate, but suppose you are asked the divergent speed then basically I do not know q you are looking for the value of q for which this equation in the sense basically, you say if this term is 0. Then this is a finite quantity q one is finite equals one term which is 0 multiplied by another term a one that means, you say 0 and infinity product is finite.

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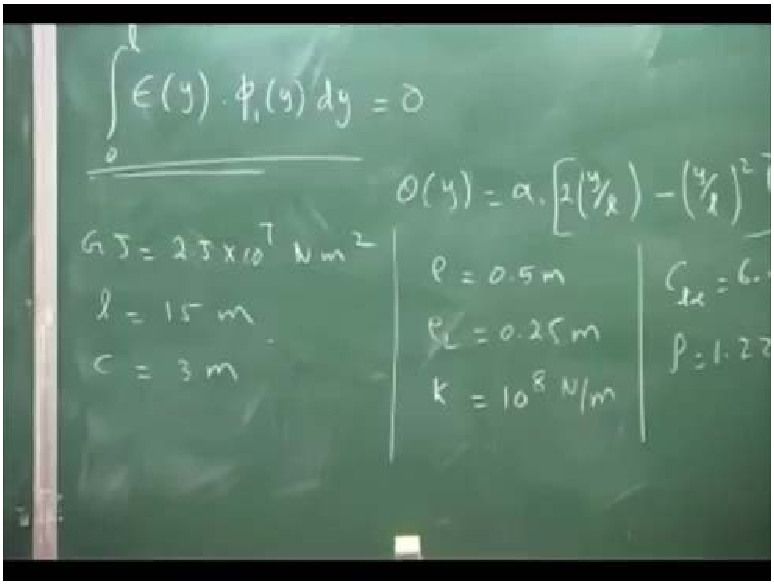

That is what, but finite what is the value you do not bother therefore, the condition of divergence is you say that setting this term equals to 0 that is a divergence. So, now, you know Galerkin method or Rayleigh Ritz now, they will all lead to an equation like this now if you say that now let me write the values here so, please understand what is the kind of problem you are solving if you are solving divergence speed you say this term 0 because if this is 0 multiplied to a very large value that means, any value of very large value is a finite quantity.

So, you really do not have to evaluate this also you are only setting this term is equal to 0 you will have q c c l alpha e eight by fifteen over l equals 4 over three l g j plus k e l square forty nine over two forty six this is q I am going to call it q d because q d only this one will be satisfied not at every other q. So, you can find out given the value of c what are the values I specified or I given you the values.

Let me write the values here g j is 2.510 to the power of 7 u turn meter square and the length is 15 meters and car three meters and offset e is 0.5 meters and e l is 0.25 meters and the spring k is ten to the power 8 u turn per meter and c l alpha is six point 0 and density is 1.225 k d per meter cube. So, I have given all the values what you do is you go back substitute if you look at this entire expression I am going to give you two d becomes because this is 15.

So, it will become 5 over 2 g j over e c c l alpha l square plus k e l square over e c c l alpha into l49 over 256 multiplied by 15 over 8 this is the value which you have now if you substitute all those quantities you can see the effect of this term and this term. If you substitute the values I erased here if you substitute you simplify all the values and I am just writing the final answer which is 3.0864 210 to the power of 4 plus 1.66 151.

Because I am keeping both of them separately just to see the effect of that spring alone now, q d is you know q d is half rho u d square. So, substitute the value you will get u d becomes essentially 2 seventy eight point five meter per second with the spring with k whereas, without k u d becomes.

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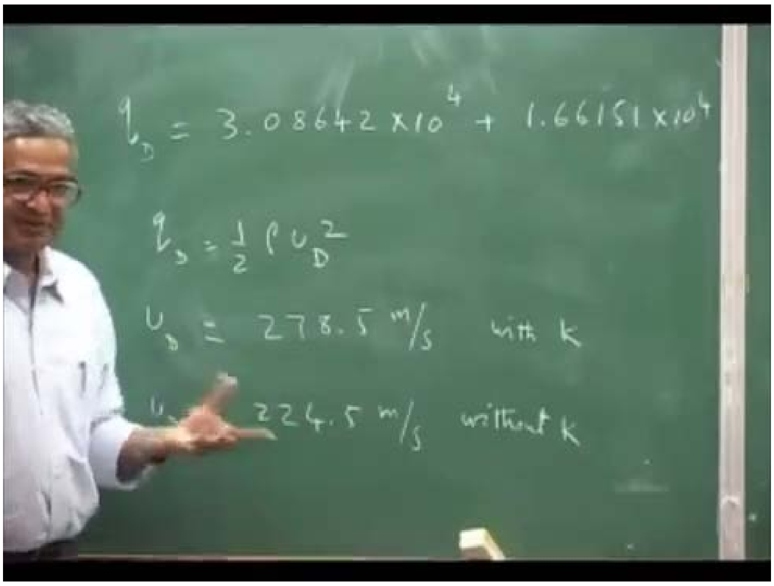

Actually 224.5 meter per second without k that means, I do not have this term I am having only this term now you see this is the difference is the order of about 50 meters per second 2422 percent can be 20 plus percent. So, the effect of straining at one location you can increase you r divergence speed substantially now, the question is this is with the approximate method you seen galerkin, but the key step is I should choose the function which satisfies my boundary condition.

That is the essence all the boundary conditions it must satisfy and we have chosen only one function like that you can have several functions now, this is where the entire problem is what is the choice of the boundary function which satisfies my boundary conditions. In this problem so, that is why I have given you the boundary condition and you are able to evaluate all these things if your function is not given then you have to go and identify that function its satisfied so, this is the problem in using galerkin method.

Please understand, but galerkin method starts with the equation the field equation and the boundary conditions field equations when you substitute it gives you an error always multiplied by the assumed function if you have a series of functions each function you put it you will have. So, many equations here I have used only one to get the solution now , I say you are going to use this technique for this swipe technique problems, but you will have please understand a bending equation and torsion equation.

That means, you have two equations now, two equations you will have two errors right now, what do you do what you have to do is bending equation error you apply the bending assumed function that will give you a set of bending equations. Torsion equation will have its own set of errors you multiply by the assumed function for the torsion that you write finally you will have two sets of algebraic equations and which you solve that way you do not mix all the error everywhere.

Please understand bending equation error multiplied by bending assumed shaped function torsion equation error multiplied by torsion assumed function. Pardon

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You will not have no because you will have bending deformation and torsion deformation two equations are there right what will happen is that will have some a 1 a 2 a 3 b 1 b 2 b 3, but these two are coupled equations the equations are coupled. So, you will find the bending deformation sitting in torsional equation torsional equation sitting in bending equation so, there will be a coupled equation and you will have a two by two or whatever depending upon the terms you assume you will get that form.

That is why solving that problem is a little different form straight wing problem. Please understand that is why when you apply galerkin understand thoroughly you will have bending equation error you will have torsion equation error multiply the bending equation error by the bending assumed, but you will the find constant what is it finally, landing up with a 1 a 2 like that what we have set here is because this is only 1 equation.

So, this is set equal to 0 in that it will come as a matrix something into some a 1 b 1 something etc. This is your external anyway you are not bothered about this you say that the determinant of this going to 0 and you get only one q 1 q means you will have a q square or a quadratic equation, but finally, you will choose the lowest value for your divergence.