Aero Elasticity
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Lecture - 12

Swept wing is basically we derive the equation for swept wing, for the static aero elastic problem.

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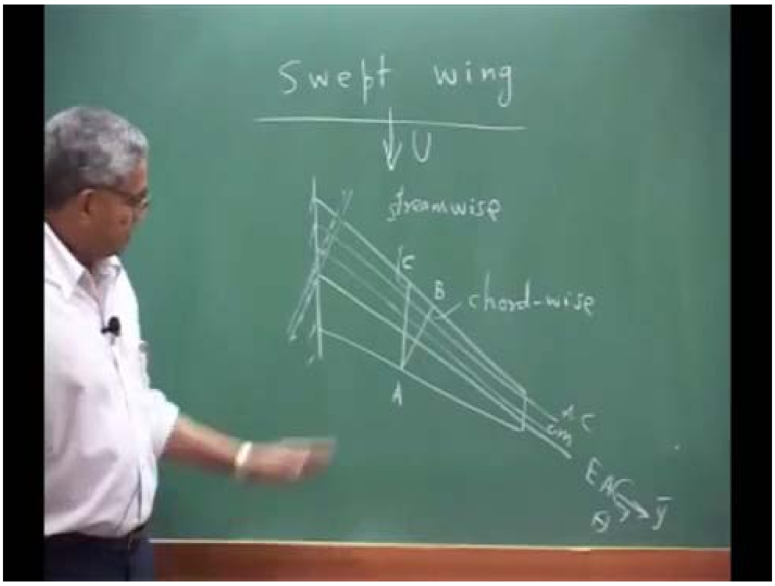

And here you will see the coupling of bending and torsional deformations, but just for understanding, if you say this is your swept wing, when the wing is bending you take bending about what axis, that is the essential part. And twist is about, in our formulation we will talk about bending of this elastic axis and twist about that elastic axis, so which is like if you take this is my some y bar. Because, I later draw a bigger picture y bar axis, my bending is w as the function of y bar, see even though the route is a little skewed, you normally you take it as though this is my root, this portion is neglected.

Essentially you take it as though this is elastic axis is perpendicular to the, and you have bending up this and a twist about the elastic axis. So, this is what is formulated when you are talking about swept wings, whereas there are straight wings we took it elastic axis, we did not bother about the bending problem. Why, because here if I take any section which is perpendicular to the elastic axis, this is my bending assume that it is only bending pure. So, you will have A, B, if it is just bending you will find A and B will have the same displacement in the plain normal to the board.

But, if I draw this stream wise, ((Refer Time: 03:34)) this is stream wise and this is chord wise, if I look at, because is my flow is in this stream wise direction, that is why I take stream wise is as though the fast wing flow how it is coming. If you look at this these two points should have the same...

Student: Deformation

Deformation out of plane, this point we have slightly less, because it is closer to the root, so the point C will be slightly having a less deformation than B. That means, if you take A C there is like a twist, but this is from the stream wise approach, no formulation here to say however, is actually formulate, there are two ways of formulating this problem in I will say approximate manner. Because, otherwise if you do influence coefficient every point take a load, what is the deformation like a plate, like a highly tip of wing.

Then, it will like a plate model you are considering, you are not considering a big model, here we are considering it as a, so the beam, now if you look at this I have one with stream wise if I stiff on my structure. That means, all the drips in the structure I place like this that means, I have stiffened my structure in the stream wise direction the other case, I can take a slightly, I do chord wise stiffening. Then you need to for the beam many problem, you treat it both of them slightly different fashion, but as it is we will say that we have two types of formulation, one chord wise, another one stream wise.

But, there is a slight difference between these two, when you take chord wise, your w data for both cases is only about the elastic axis. Only thing is how you have formulated the problem, the bending problem and the torsional problem the coupling, that will vary depending on whether you have taken it stream wise formulation, or chord wise formulation. And these two formulations will be different, the equations will be different, then we have which one is write, essentially this is all approximate method of solution. Chord wise is more approximate, chord wise you will find how we will couple that is very interesting way of couplings, but stream wise it is little bit better I would say.

So, you can take stream wise formulation, we will derive both chord wise as well as stream wise formulations. And then in the example which I gave you was the assignment, you can solve by both approaches to find out how the diversion speed varies, depending on your approximations. First will let us take the stream wise formulation, so we have to define a diagram slightly bigger diagram.

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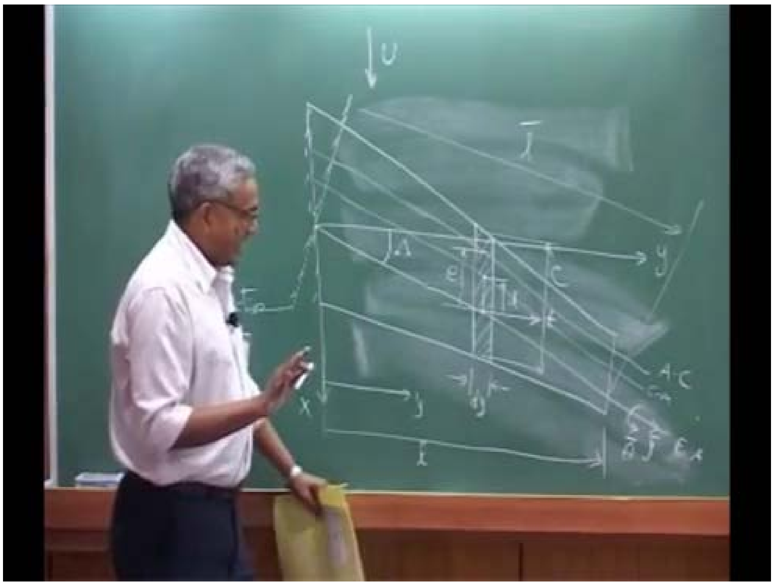

And then maybe I draw here, so that I do not have complex and this is my x axis, and let us say my aero dynamic, this is my elastic axis which I call it as y bar, and by rotation is theta bar. And aero dynamic, this is the aero dynamic centre axis and this is the centre of mass axis, and we will define this is my y axis, this angle I take it as speed, this is my origin that is the elastic axis I am taking, and the x is along the flow. Now, my various length which I have to, this is my length of the wing, and any location I go here as y and I draw a this is the effective and I go this is l bar.

So, you please note down, wherever I put a bar quantity that is defined in the y bar axis, same thing we will now describe, this is the a small section of the wing in the stream wise direction. And this is along the y axis and this distance I call it d and ((Refer Time: 12:00)) this distance call it e and the chord is c, give a mark some centre points here take the this is my chord, so c, d, e they are all measured in the stream wise definition. Now, what we need to get is, we have to get the distributed aero dynamic load and then we transfer whatever each section you say this is my aero dynamic centre.

So, I am going to get my aero dynamic load in this, transferring to elastic axis and this is of course, to gravity load it will mass centre transfer it here. So, you will have a force and the moment is along this axis, so I am going to call t is the twisting moment, which is along this direction.

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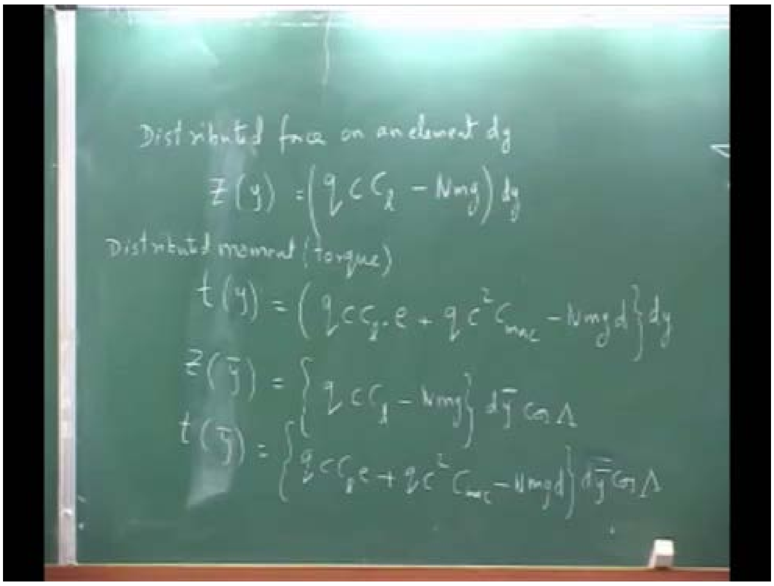

So, let us write distributed force will be, I use the symbol z y this is nothing but dynamic pressure chord C l minus N m g into d y, distributed force on a element d y basically, this is the lift, this is the gravity. And then distributed moment, but this is much for unit length, chord C l which we do not know, I am only saying C l, later quite a bit of discussion will go in how do you define C l for a swept wing.

Because, all the theory what you have learnt is, you put an aerofoil flow is coming normal to the leading exchange, that is what you take, here the flow is not coming normal to the leading exchange, so that part we will describe later. Right now, we take it there is a C l lift coefficient and m is mass, N is a load factor, now we check the t y this is distributor moment or torque, I would call it distributor torque, twisting moment. This is q c C l into half set between the aero dynamic centre and the elastic centre plus q c square C m a c minus N m g d d y.

Now, we have got I would say the forces and moments, but the geometry of this, I am having a moment along this and the force is normal to that, but I will write my equation bending and twisting above y bar axis. That means, ((Refer Time: 16:49)) this is my effective root and this is perpendicular to the elastic axis, so I have my standard bending theory. But, in doing that we have to be a little careful, in the sense what is my d y, because the element along this, along the elastic axis is d y bar, this is d y stream y, I have to get d y bar.

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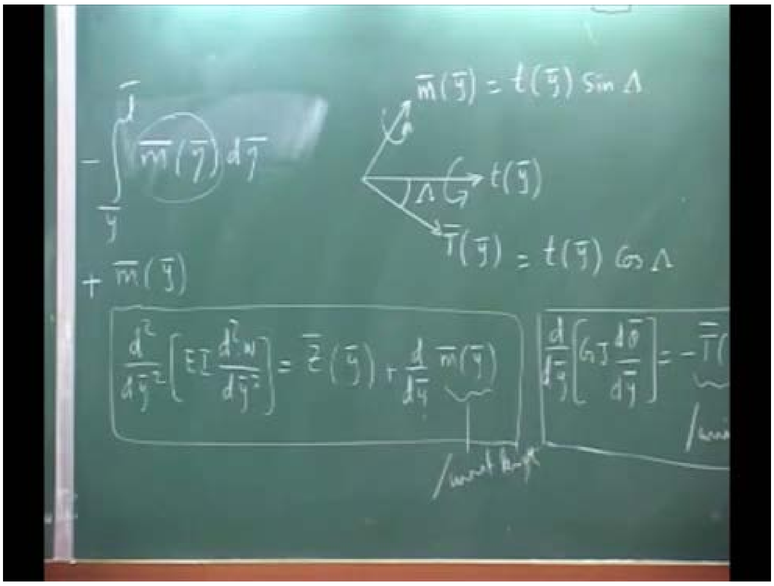

So, you will have d y is d y bar cosine lambda, because this is d y bar and so this is not, this is d y bar and you take this is d y, do not take the another way wrong, that is what I am saying the reason is this is angle is 90, there is along this I am taking this, so this is what I have take it. Now, what I go is I substitute these places, even though these quantities q, c everything is along stream wise direction, but I am now getting z I will put it y bar will be and this modifying it that is all, q c C l this will not change minus N m g d y I am putting d y bar cosine lambda, lambda is the sweep.

That means, I am getting distributed force along y bar, but if I divide by d y bar I get force for unit length along d y bar, force per unit length that is all. Similarly, I will write torque are the twisting moment. This will be q c C l e plus q c square C m a c minus N m g d again d y bar cosine, now again I am insisting that, this is d y bar, these are all equals. In the ((Refer Time: 20:28)) formulation the whole thing will become along chord, now we have to write the equation for bending and torsional.

What we have is this t is along what direction till, this is along this direction only, but only thing is I have defined my t y bar that is in the sense if I divide by d y, the twisting moment per unit line noise along the y bar axis, but direction is this. I have to resolve this t one along this direction, another one will go perpendicular to the elastic axis, what will happen in the perpendicular component is like a bending moment. Because, if I resolve t one along y bar axis, another one along this, ((Refer Time: 21:42)) this is the twisting moment, this will twist and that will also twist, the twisting will become like a distributor twisting moment in the bending direction.

And that what we have to now resolve it, so I have my this is t I will put it y bar that is all, I will get a ((Refer Time: 22:16)) this is my lambda this is my twisting moment, may be I will call t twisting moment along with this, which I may if you want to put a bar, I may put a bar also. So, that this is the twisting moment along the y bar direction and then this will go like a bending moment, which I may call it as may be m bar. Now, you know that what is this value, this value is into cosine lambda, this will be sin lambda and I have an expression here.

So, I go put them back here, then I will get m bar y bar is the moment, bending moment that is acting on the wing along y bar direction, over a small elemental length d y bar, if I divide by m bar y bar by d y bar that will give me moment per unit length, along the elastic axis. Here same thing if I divide that d y bar I will get the twisting moment per unit length along the perpendicular, now I can go back and write my equation. So, what I will do is I erase here ((Refer Time: 24:19)), I will write my two equation, the bending equation.

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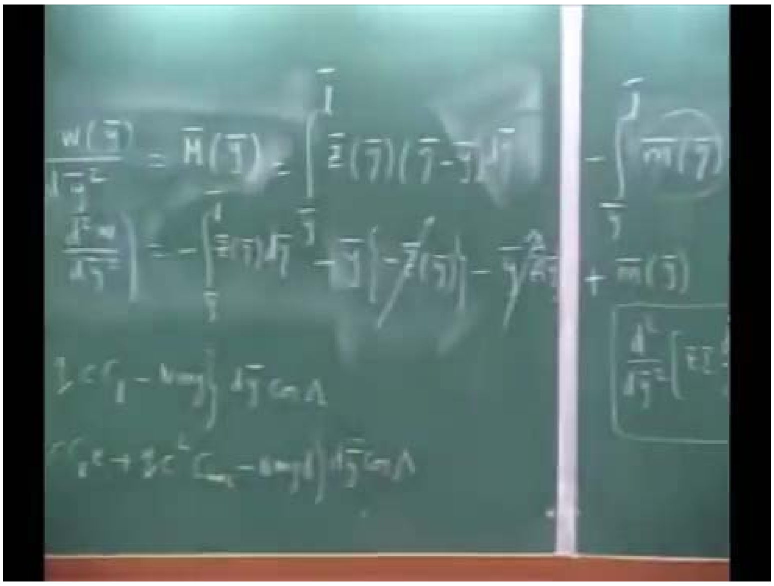

First we will derive the bending equation, bending equation is what we know that, bending moment is E I d square w y bar. W is the normal or I would say the bending displacement of the view along the elastic axis, that is what I am taking w, I am not putting the bar I am just taking the w, if you want to put the bar you can put the bar. Divided by d y bar square this is nothing but my distributor moment per unit length moment per unit length will be at any section.

The moment will be due to the aero dynamic lift, which is acting along the that is due to z plus the integral of the component of the twisting moment for the bending. So, you will have two component, so I can write this as y bar to l bar z bar, I am using now a symbol eta bar eta bar minus y bar d data bar minus integral y bar l bar m bar eta bar d eta bar. But, this is I have to take this quantity is 1 unit length that means, I have to divide here per unit length d y bar is existing everywhere, so if you want per unit length you divide by d y bar all over the place.

That is essential, that is why even though I use the symbol here it is per unit length, now what will do is this expression you already got here, and ((Refer Time: 27:34)) this z bar, I am just simply changing, because this is z y bar I put z bar that is all. So, z and y and eta these are that is eta I am taking now, this is my eta bar this is a running variable, eta is a running variable I am getting a any y bar is nothing but eta minus y bar, this force into eta bar minus y bar is the moment. Similarly I have to add all the, because I put a minus sign, the reason is this moment, this is the twisting moment which is like this.

So, when I make a component this will try to bending down whereas, the z course will try to bend it up, that is the reason I put minus sign here ((Refer Time: 28:46)), now what you can do is, you can take this quantity to differentiate twice. If you differentiate twice means first differential d by d y bar of E I, this is becoming will have two, I can write this integral into two parts. One integral will be minus y bar that will become simply y bar l bar z bar eta bar d eta bar, that is I am differentiating with respect to y bar, this is one term y bar integral z bar eta bar d eta bar.

Than second term will be minus y bar integral of differential of the integral quantity, integral quantity by differential I get the same function, but I have a minus. So, this will become minus and then this is another term integral y bar eta bar z bar d eta bar, that is the other term, if I differentiate it, I will get simply the same integrant. But, since it is a lower limit I put a minus, this will become y bar z bar y bar, then afterwards ((Refer Time: 30:57)) this will be plus m bar y bar.

This is how, these two terms correspond to the first integral, this term is plus this term, now you will find ((Refer Time: 31:22)) these two will cancel out, they will cancel we will have only this term and that term. Take one more differentiation, when you do one more differentiation you will have d square over d y bar square E I d square w by d y bar square equals, this is minus y bar, so this will become z bar y bar, this is my distributed force lift, plus d over d y bar of m bar, this is my bending equation.

And my partial equation is basically d over d y bar G J d theta bar by d y bar equals twisting moment minus this term per unit length, you will put that is nothing but minus I will put t bar, this is per unit length. So, here this is over a length of d y bar, so I have to get per unit terms, so please note this is per unit length, same thing here also this is per unit length. So, I have my bending equation, I have my partial equation, only thing is what I have to do is, I have to go substitute the respective terms.

Respective terms is that, this is t y bar I take it from here ((Refer Time: 33:49)), put it here this will be cosine square lambda, and only d y bar will go off, because that is per unit length. So, my twisting moment is essentially this term without this with cosine square lambda, and my m bar y bar will be sine lambda task completed, now this is my complete equation, what I will do that I will write that equation here, so that it becomes here.

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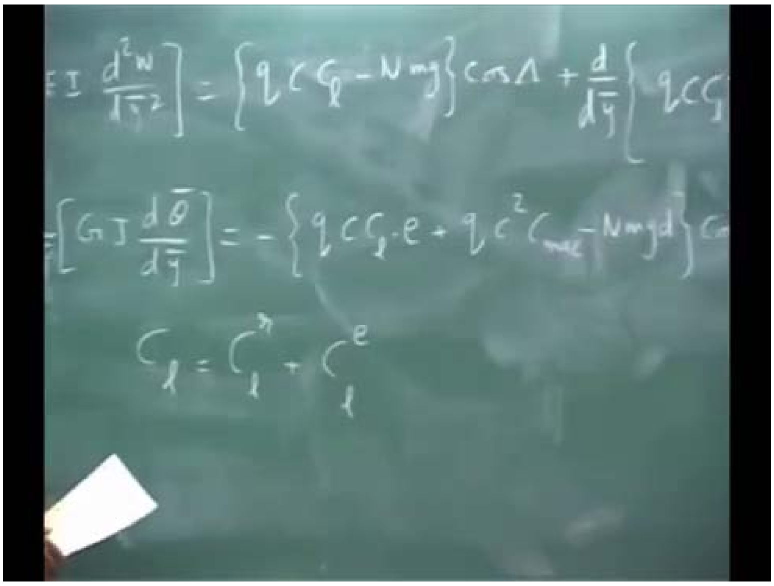

So, you will have your bending and partial equation, this is the bending equation you will have d square over d y bar square E I d y bar square equals, first term means z bar y that is nothing but this per unit length. You will have q c C l minus N m g cosine lambda, this is this term and the second term is d by d y bar of m bar thing that is nothing but sine lambda cos lambda. So, you will put q c C l into e plus q C square C m a c minus N m g d cosine lambda sine lambda.

And my torsion equation is d over d y bar G J d theta over by d y bar this, equals that is minus d by 1, so you have minus q c C l into e plus q C square C m a c minus N m g d cosine square lambda. Now, this is my complete equation, I have to split now what I do is I will erase oldest part. Now, I will be writing this will slowly you have to what is my C l, C l appears in both, that is my list co-efficient, lift co-efficient requires split it into two parts, that is one is due to rigid whatever initial settings analogue, another one is due to the elastic twist.

So, I am basically splitting my C l as C l rigid plus C l elastic I still do not know what is this, rigid is you can say C l alpha into alpha rigid. But, here I have to know what is the deformation, the change in analog attack due to deformation. Now, if I write this you go as substitute here in every place collect their C l e terms, and put it to the left hand side, and then write the rest of the terms to the right hand side. What you can do is we could write that part, because that is fairly simple only thing is I will put it in a very simplified form.

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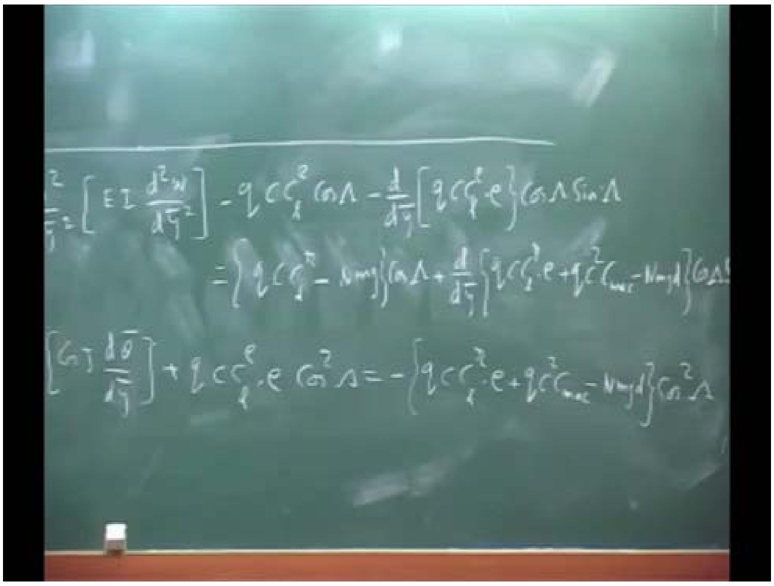

So, that this exercise is just the algebra, so you will have d square by d y bar square into E I square, and here you will get one term with the q c C l e cosine lambda, so that will come. So, you will have a minus q c C l e cosine lambda and you will have another term coming from here, that will be minus d over b y bar of q c C l e into e cosine lambda sine lambda, equals the rest of terms. You can write it as equals q c C l r minus N m g cosine lambda plus d over d y bar of q c C l r into e plus q c square C m a c minus N m g d cosine lambda sine lambda.

And torsion equation in the same way you will write it as d over d y bar G J d theta bar over d y bar plus q c C l e into e square lambda equals minus, you can have q c C l r into e plus q c square C m a c minus N m g d times cosine square lambda. Now, here what you have done is, because this term q c C l r e q c square and that appears in both the places, so you can write it as one term, but this will stay as usual. Now, you look at these are all the right hand side is independent of the deformations in both the equations, the left side we need to get still what is C l e, lift coefficient due to deformation, of that we will go a little different.

How we have defined our lift, that is why we have to go back, we wrote our equations bending and torsion, how are we defining our lift stream voice. Now, stream voice what is my angle of attack, because this is my strip, I got my lift force on this strip if the chord C and also off set e everything. Now, what is my angle of attack, because of the deformation along y bar axis I need to get what is the change in angle of attack due to the deformation of the wing along the wiper axis. So, if I erase here, so the equations are there, so we can keep this set as it is.

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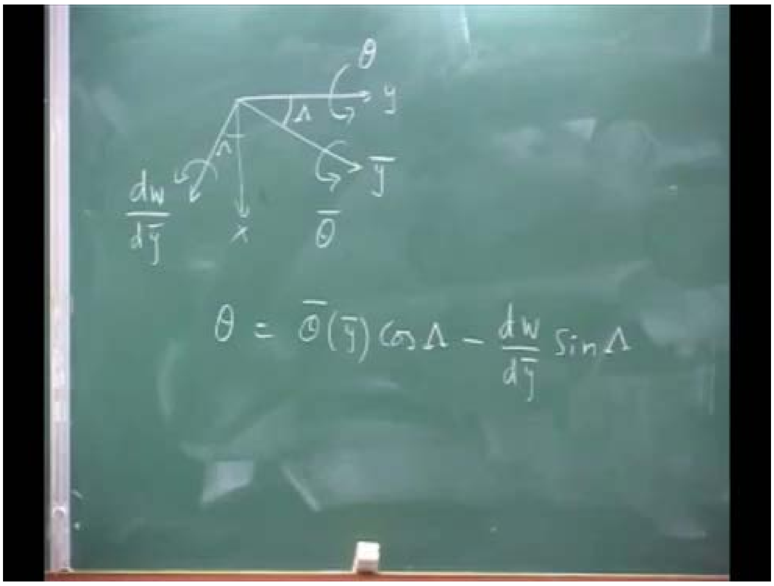

And we had our this is my y axis, this is my y bar axis and I have a zeta bar and also a bending twist, which I am taking w is coming out of plane. So, I am having a bending twist which is will perpendicular to y bar axis, I want to know what is change in angle along y axis that is my elastic; you can say change in angle attack due to these two deformations, so that will be along y I call this as theta. So, theta will be theta bar which is the function of y bar into this angle is cosine lambda this will go this is lambda this is lambda.

So, this will go sine lambda you will get minus, now this is my change in angle of attack basically this is my x axis, I cannot my change in angle of attack, but how will I get the lift. Now, this is where I do a little bit of some kind of equivalent lift formulation and using equivalent lift formulation and that is what we used because you need to know, what is my c l alpha. Using just directly multiply c l alpha here, the c l alpha is different, because of the strip this is done for a infinite wing formulation.

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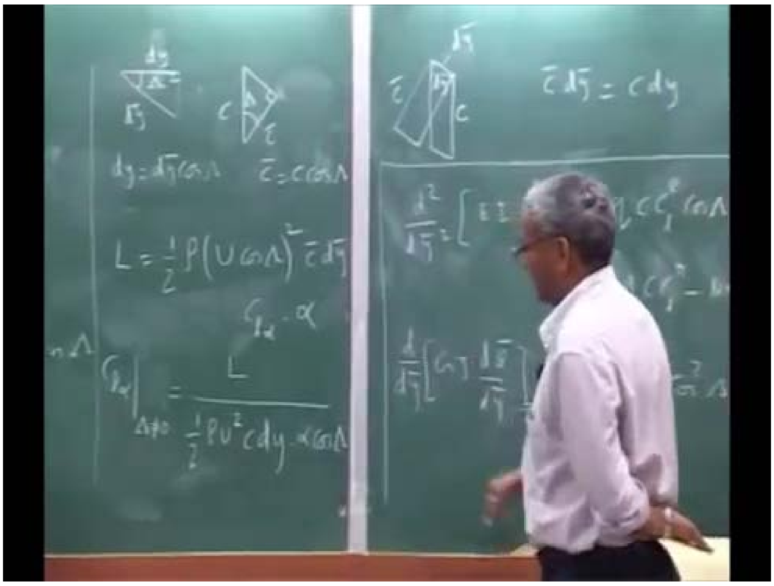

What we do is I go back and write it here we know that this is d y and this is d y bar this is lambda. And similarly this is c and this is c bar 90 degree here this is 90 degree this is again lambda I will understand d y is d y bar cosine lambda and here c bar is c cosine lambda. Now, from our basic theory two dimensional ((Refer Time: 47:36)) the list on a 2 D f y where a flow is normal to the leading edge that is lift per for a tan. I will take everything I will convert into form.

What I am having is I am having z of rho u is the un coming theta, but normally this means u cos lambda whole square. Now, what is the gain here area you know that you should check this is my strip c is this another one is I say I should check like this I put it this way draw the diagram again it is better. See I draw like this is d y bar, this is chord, this is c bar, this is my d y, this is nothing but c d y the area whether you take as stream wise or chord wise the same.

Now, I will take either I can write this c bar d y bar this is along the normal to the u cos alpha times this is my dynamic pressure area and you have to have some c l alpha times alpha. Alpha is any general language that is all it is the angular product perpendicular to this flow, now the question is I want to know what is my c l alpha? When this peak is not equal to 0, I have to lift divided by, basically what I am taking please understands c l alpha in the direction stream wise.

Please understand this is stream wise serial alpha, this is flow, normal low serial alpha this will be I will divided by half rho what is my u square, because the square is velocity what is my area, area is…

Student: c d y

C d y and my angle after attack is if this is alpha I will have cosine lambda will be my angle of attack. So, I am going to substitute here alpha cosine lambda, because alpha the angle y bar, so alpha cosine lambda, now if I substitute this slit here.

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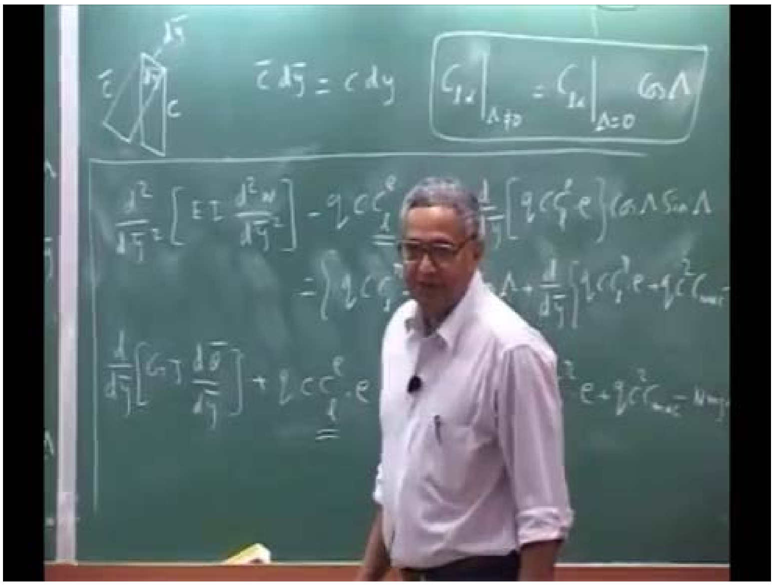

I will get an expression where I put it here c l alpha for lambda not zero stream wise, this is stream wise what is that c l alpha, this is equal to this is when the flow is normal to the leading edge; so that means, I am going to call this c l alpha of sweep zero. Now, if I substitute this expression here you will get 1 cosine lambda will cancel rest of the term will remain as it is c d y is nothing but c bar d y bar o that area is same, so you will get this into cosine lambda.

Now, please understand if this is valid if you have a stream wise lift co efficient or slope of the because this is basically a slope this is normal to the linear edge whatever if you do it will turn like an experiment normal to that you have to multiply to that like cosine lambda to get the lift of the slope with sweep. Now, along this problems we said it is a wing, wing, wing ((Refer Time: 54:26)), that is the thing; that means, it is a finite aspect ratio it is not an infinite wavelength.

So, what is my c l r form you have to correct for aspect ratio, please understand in an approximate manner you can take the elliptical loading or non elliptical also there are some expressions is it you just take the that term you check the correct c l r form; and this is with the sweep that is without the sweep. Now, what we have to do is i know my theta

Student: Sir, that 1 by cos square 1 by cos

Where this equation one more cos

Student: Cos square it will be ((Refer Time: 55:15))

No, no see this is the cos lambda this is here cosine square this is with sweep zero, please understand, this is the c l r y bar when the flow is coming normal to the leading edge this one. This is not normal to the leading edge this one that is why I have put lambda not zero this with lambda zero. Now, you have this expression you go back and write I know my theta.

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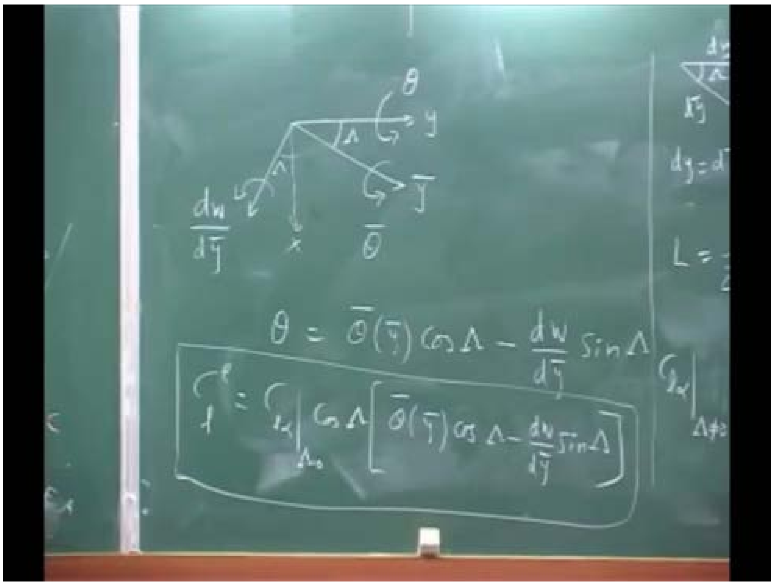

So, in this equation where ever you have C l e you will write C l e is C l alpha for lambda equal to 0 cosine lambda times this factor sine lambda minus d w by d y bar, so this is my expression. Now, we need to know the co efficient of…

Student: The cos sine bar cos lambda

This is my C l e, now total this term will go and sit here forgot now you see that C l e contains a derivative of d w by d y and also this term theta. Now, this is my coupled equations, and in this particular problem after will write the boundary condition, we will erase this part, the boundary conditions I will write it, the boundary condition at the root, figure the boundary condition is for main, which is twisting and bending with the effective root.

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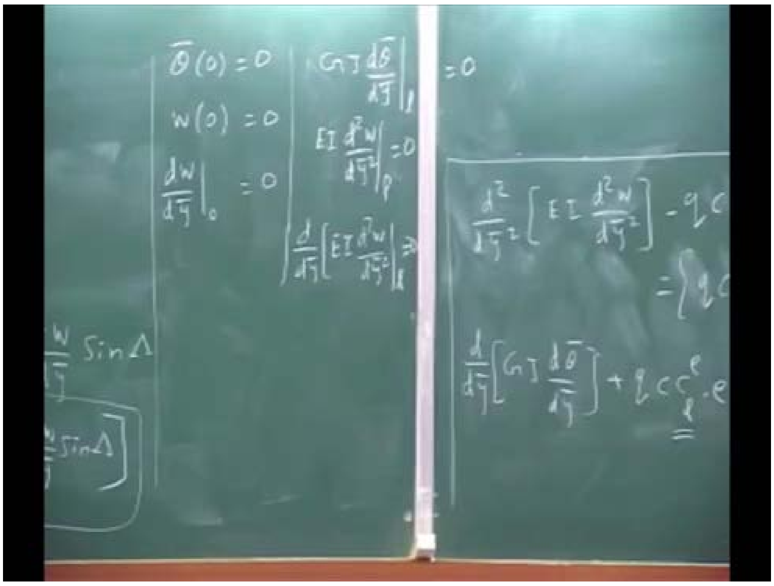

So, root you will have twist theta of 0 is 0 and w 0 is 0, and d w over d y bar at 0 is 0 the slope is 0, then at the tip you will have

Student: ((Refer Time: 59:01))

Again here, G J d theta bar by d y bar at l, that is the free edge and then you will have E I d square w by d y bar square 0 at l and d by d y bar of d y d square w, that is the shear force at the other network, now this part completes set of equation. Now, little brief description you will have here first derivative expression will be sitting, this first derivative expression and then the operators will become essentially non ((Refer Time: 01:00:25)).

So, we have done the suspect tight derivative all those solutions earlier, now this is going to become a problem with the non suspect junk operator problem. That what the solution technique, but you have to adapt this by Galerkin method, you will directly substitute this equation, assume w a function of y bar will satisfy these quantities. And similarly, you take data bar the function of y bar, and the approximate solutions it satisfies the boundary conditions, substitute them there multiply by the assumed shaped functions.

And then you apply the Galerkin approach and solve for, because you might have got equation line, it meant to you how to get the deviance problem, you will have a simplest term, you will have another term with that q dependent. Then solve that ideally and you start with zero sweep, if you put zero sweep you will see immediately the bending will be decoupled from torsion, that is why when I put lambda 0 everywhere, you will find that here, first of all data becomes y.

So, here itself data becomes independent of w, and this also you will find w will be different, theta will be different that is why for straight you have stored what of them you saw only torsion diverges. Let us then you do swept wing you have solved both of them, and they are coupled, now if you want eliminate coupling this is looking different, eliminate coupling in the sense e 0 that means, what I am saying my elastic axis is going inside with my aerodynamic centre.

The moment I say elastic axis is going inside with my aerodynamic centre, torsion equation is independent, the torsion is not in pledge by bending part. Because, this term is 0, because the C l e is only is the coupling term, the C l e which couples, now this is also, because that this term is itself 0, therefore goes up. Therefore, I will have torsion equation, but in the bending, this will have theta, bending equation will contend that, torsion will be independent.

Now, you can have bending divergences, bending divergences, bending divergences you can have no problem, I will give bending divergence to obtain the divergence speed observing, otherwise you can solve the elastic deformation also. That means, elastic deformation you put what is the rigid angle of attack, what is the dynamic pressure q, then you will have all this properties and you can pack out, if it is the level fly N is 1, and distribution of much you can find all the wing denotes.

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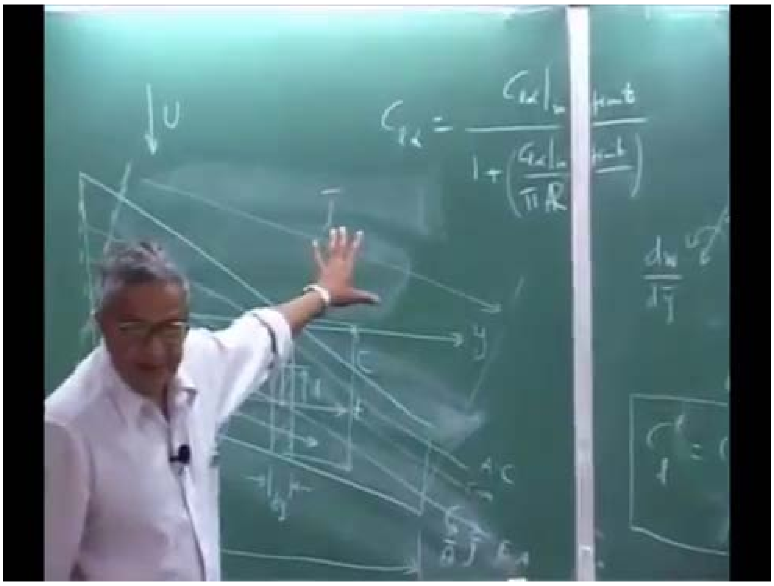

Wind, you always write C l alpha becomes C l alpha for infinite wing divided by 1 plus C l alpha infinite by pi into…

Student: Aspect.

Aspect ratio, this is the elliptic lift distribution.

Student: Otherwise, we will put e there.

Otherwise, you will put 1plus tau among e the direction factor you put it and then you take this, so this will only change your C l alpha for aspect ratio problems. If your aspect ratio is very large, you will say that this is whatever it may be otherwise, if you take this is 2 pi that means, this will become pi and pi will cancel 2 over aspect ratio will come, it will be 2 pi. Or else there is a you can take the 5.73 also sometimes, so this is only changing your correction, but this is wherever this comes you replace this part, so aspect ratio effect is also taken into consideration in some problem.