Aero Elasticity

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Lecture – 11

The aircraft is in simple trick mode over, whether it is a level fly or dive or anything, what is the aero elastic effect during that particular condition. We have seen that, because of the twist deformation of the wing your angle of attack changes, and when the angle of attack changes again the load is changing. So, in this problem, if you consider the angle of attack as the rigid one, you will get one load distribution, the moment you consider the ultra the ring if flexible, in twist you will get a different load distribution.

And then, you need to get what is the new load distribution, due to the deformation of the wing and what is the flight condition. So, the flight condition is we say symmetric flight, and in this you need to know, suppose there are two types of problems one can always look at it, given the initial angle of attack that we call it rigid angle of attack, what is the load factor I am going to generate. The second problem is given the load factor, what should be my rigid angle of attack, because load factor is basically it is related to how much it is going to lift, lift by weight is my load factor.

Now, how much angle of attack I must give, which also takes into the account the elastic twist of the aircraft, because this is very important. We want to go in level flying, you must know what should be the angle of attack of the wing, before elastic twist, after the elastic twist the net will be equal to the weight of the aircraft in level flying. So, there are two types of problems that exists, but to solve that we consider the symmetric lift distribution.

And first we take the uniform wing, please understand only for uniform wing you can get closed form solution, if it is not that uniform wing you have to go for the Galerkin or Rayleigh-Ritz type of solution.

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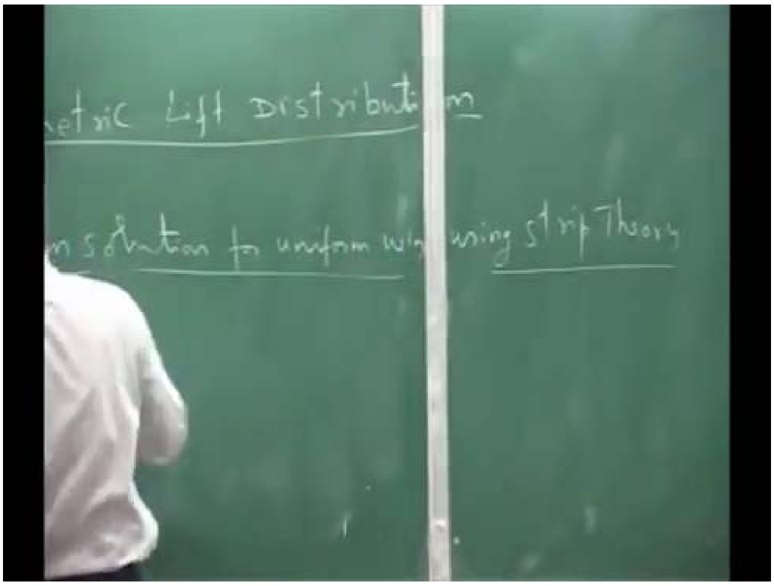

So, we will take closed form solution for uniform wing using strip theory again, now the moment I write it as a uniform wing.

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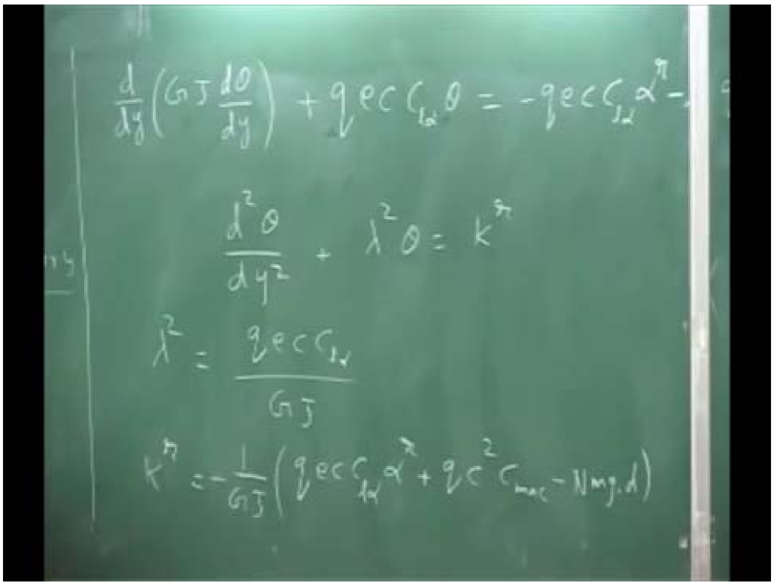

My saturate equation of motion is that, d over d y of G J and then, we know plus q e c we put C l alpha into theta equals minus q e c C l alpha alpha rigid, this is the initial angle of attack of the, ((Refer Time: 04:20)) then q c square C m a c plus N m g d. Now, if I take uniform wing, I can divide by G J everywhere, I can divide this equation in a very compact form, which is written as d square theta over d y square plus, I will write where lambda square is q e c C l alpha over G J.

And your k r is nothing but 1 over G J multiplied by q e c C l r, r in other words C l r instead of C l r including C l alpha alpha r plus q c square C m a c minus N m g d. Next short form, because uniform wing please understand, I am taking the uniform wing that is why it is coming out like this, now I have to get the solution of this. The solution is this is the second order equation with, if it is uniform wing I can write the closed form solution. If the wing is not uniform that means, all these e c and C m a c everything can change along where, even m d all these properties are functions of your spam.

Then in that case, you can get only approximate solution, that approximate solution we will have last class, either you can use Rayleigh-Ritz or Galerkin approach, you have to use one of those techniques to get the solution. Otherwise, uniform wing we take we will be able to write a closed form solution, for this problem we will use it as a uniform wing.

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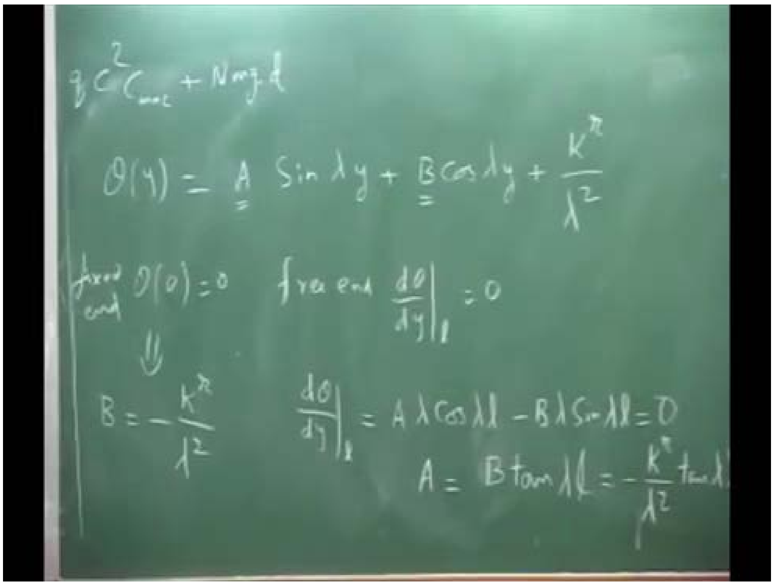

Now, my general solution for this is going to be theta, which is the function of y which will be A sin lambda y and then, B cosine lambda y, sin lambda y plus B cosine lambda y, since these are all constant because uniform wing this is not a function of y. So, your solution will be plus over lambda square, now you can find out the constants A and B by applying the boundary conditions, boundary conditions we may say theta is 0, it is a fixed end of the wing.

And at the free end we will put d theta over d y at l this is 0, now you apply this boundary condition, when I said theta 0, 0 means this sin is 0, so B k r over lambda square is 0. So, therefore, this gives me the condition B equals minus k r over lambda square, then here when I go theta prime this will be A lambda sin becomes cosine, so you will have d theta over d y at l is essentially A lambda cosine lambda l. ((Refer Time: 09:19)) This cosine is going to become sin, becomes minus sin, so minus B lambda sin lambda l equals 0, because this is a constant.

So, you will get A becomes, A is essentially B tan lambda l and B is minus k over r, so you will get minus k naught k r over lambda square tan lambda. Now, I can substitute and get my solution for theta y, which is I will write it here.

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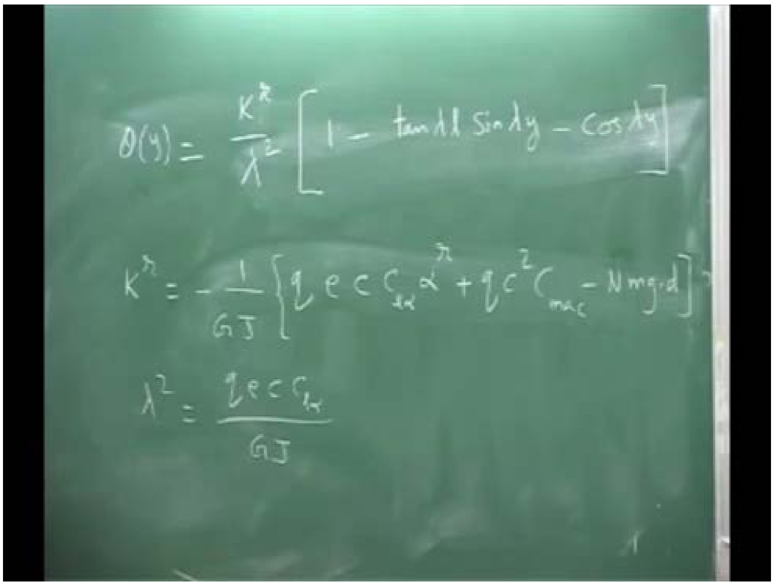

So, that this solution will be using it later, we will write your solution as theta which is a function of y, because ((Refer Time: 10:32)) k r over lambda square everything is sitting there. So, you can take out common lambda square 1 minus, because you can say A is k r over lambda square tan lambda. So, we just put that tan lambda l sin lambda y minus cosine lambda y, and what k r is given here, and lambda square is given here, so we will keep it I will write here.

So, that is k r becomes minus 1 over G J q e c C l alpha alpha rigid plus q c square C m a c minus N m g d, and your lambda square is q e c C l alpha over G J. Now, I have got the elastic twist due to the rigid and of course, the environment moment and at the load factor. Now, this where I have two types of problems and mention, I erase this part.

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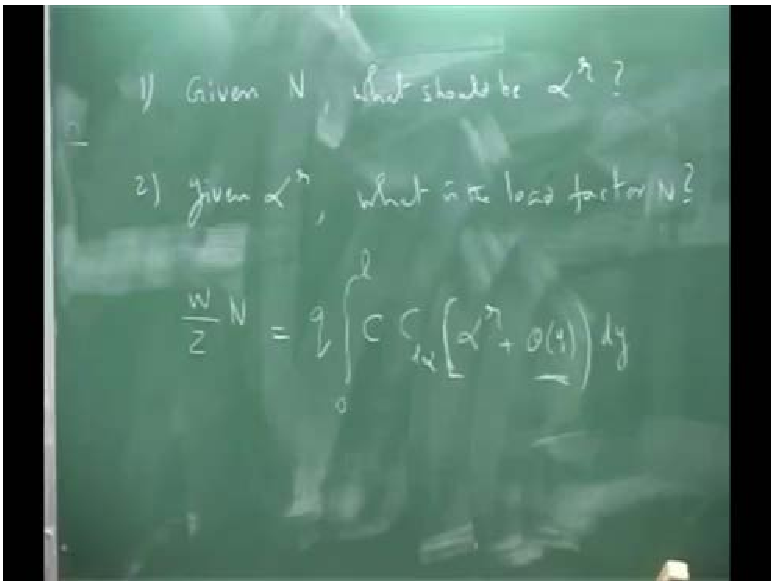

And now what we have is 1 is given N, given the load factor what should be alpha r, another one is given alpha r what is the load factor N, because this is like wind tunnel test. Because, in the wind tunnel your mount in, your model if it is an aero elastic model, not a rigid model, if it is a aero elastic model you put the alpha r given with angle. What is the load factor you generate, the other case is free fly, then you are not going to say that if you want what is the load factor, like level flying that is 1.

Because, N should be equal to the weight of the aircraft, then you must know atmosphere my alpha r, now these are the two types of problems, which will come in the aero elastic calculations. Why we need to do that is, if you set any angle as a rigid alpha r, because of the flexibility your load distribution is going to be different, because of the wing twist the lift distribution will be different from, if the wing is not elastically twisty. Now, if the load distribution, because of that changes that means, your sectional of any moment shear, everything will change.

If that changes your structural design will change, that is the reason you need to do aero elastic calculation for structural analysis, see traditionally this is a very interesting statement which is made. Aero elastic issues are treated only for to satisfying the certifying agency, because after you design your flight vehicle, it needs to get a certification like civil aviation or military certificate. There you have to demonstrate that this is fine, it does not have any aero elastic, but today the aero elastic problems are becoming a design problems.

You need to take the aero elastic effects in the design aspect itself, to have a very efficient high-performance aircraft, rather than saying I will treat, the entire aircraft as the rigid aircraft and then, design my structure. And then, I will take care of the aero elastic problem at a later stage, if there is a problem I will modify it, just to satisfy the certificate agency. On the other hand, these days you were highly flexible, highly maneuverable, various types of aircrafts are coming, and even unstable aircrafts they tried to design such that, it is stabilized using close to control.

Then your wing flexibility plays a major role, that is why the aero elastic problem becomes so importance in the design phase itself. Now, how do we solve this two problems basically, that is the basic question mark, we have obtained for a uniform wing, this is for a uniform wing. If it is a non uniform, you will again get this solution using approximate methods, I got the elastic plus distribution due to you see k r contains rigid angle of attack moment load factor, which means the twist depends on my load factor, as well as this.

Now, my load factor depends on what is the lift, so what I have to do is, I will write an expression what is my load factor, I will take half wing of the diagram half wing means, half of the aircraft. So, weight I am distributing into one half of the wing, so w over 2 into load factor, this is what w is the weight of the aircraft, this is not the weight of the wing, this is the weight of the aircraft, N is the load factor. This is what is to be supported by one half or one wing, this must be equal to you say q which is a dynamic pressure into we will put 0 to l chord, then C l alpha times we will have alpha r plus theta y plus theta which is the function of y.

So, dynamic pressure chord lift coefficient, now the lift coefficient has two parts, one is a rigid part and the another one this is the elastic twist part into d y. Now, what you have to do is, you go substitute the theta y here ((Refer Time: 19:10)), now when you substitute you know them theta y is a function of alpha r and N, do you follow. That go your right hand side, your left hand side in your N, right hand side also you have N that means, the load factor is the function of load factor.

What you do is you have to gather that term in the left, then write N as the rest of the terms and this because we have considered uniform wing, I can perform this integration. Because, C C l alpha everything is constant, so it is easy for me to integrate, but if it is not the uniform wing you have to use approximate methods, numerical integration you have to resolve. So, that is why this problem the moment you go to real problems, you need to apply numerical methods, that is why Rayleigh Ritz or Galerkin first to get this solution, after that put them here, then you have to do numerical integration.

And then, get the load factor in terms of your alpha r, so what I will do is, I will not go into the algebraic details, because the algebraic details are essentially, you take this term put it here, after that you substitute for k r, you have to substitute for k r here ((Refer Time: 21:05)), and then multiply the whole thing, is it clear. It will be a little lengthy messy expression, what I thought was I will not write that in the board, I will finally go and then, give you the closed form solution after integration.

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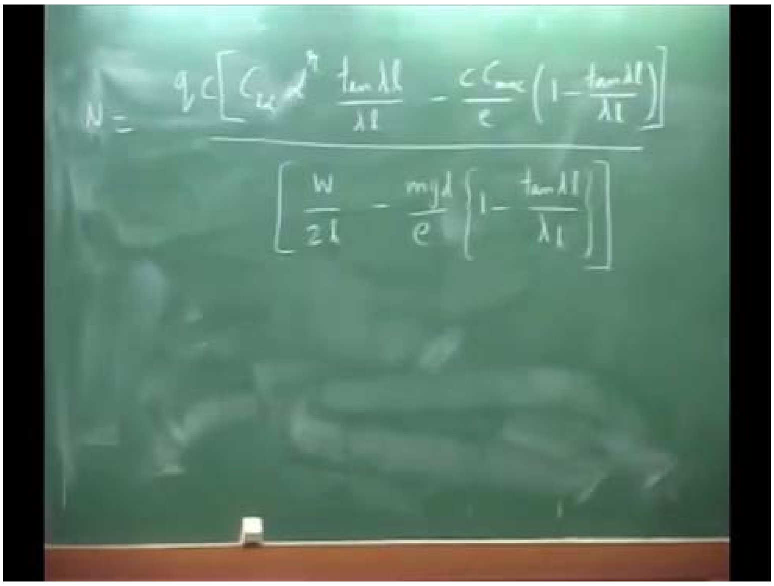

Because, this part is just a algebra, but this has an exercise you can do it this is becoming N equals q c C l alpha alpha r tan lambda l over lambda l minus C C m a c over e into 1 minus tan lambda l over lambda l divided by w over 2 l minus m g d over e 1 minus tan lambda l over lambda l. Now, this is your uniform wing solution, I have skipped basically the algebra, the steps involved in the algebra. Because, I do not want to write otherwise, I will be writing only the algebraic equations, uniform wing I take this term substitute here, substitute for k r, and whatever lambda.

And then, I integrate over 0 to l assuming uniform properties, then I gather the N terms on one side and then, I get this. Now, you see this problem you can actually solve both these things, both the problems both of them you can solve, if you are given N that means, this is here, you do not know this ((Refer Time: 24:05)), all the other quantities you know. So, given this you can get this, once you get alpha r you can go for substitute here alpha r, then you get that k r, then you can put it here, then you can get theta y that means, now I got alpha r plus theta y.

So, I can go and get the load distribution of my lift is varying along the span, that is very important for my design. On the other hand, if I specify given alpha r that means, I know this and I have to find out, because of course you have to know the flight condition, every q you have to take, that is why this will change for every dynamic pressure. The dynamic pressure is not your solving, you are valuing a given quantity, for every dynamic pressure what happens, now this is like a iterative loop problem; every dynamic pressure that means, every flight condition needs to solve this.

So, this is why this is very interesting problem, just to get load distribution on the wing and you get the deformation also, but this is the caution deformation. But, you are getting load distribution, but that is for designing for the bending problem, because you take a wing you want to design each section, your load is changed that was the bedding moment were everything will change. And then, you design your new cross-section, when you design your new cross-section your G J is changed.

Even though you may have and your location of elastic axis, location of mass centre, everything can change you follow, that is why it is an iterative process, you cannot just say one shot solution you take it. Because, once you design then you again recalculate, and this you have to do for various flight conditions that means, you basically in the design you take critical flight conditions and design further. After that rest of the flight conditions using we take whatever if the e and d and then, you get the load distribution, is it clear.

So, this is the symmetric loading, and even this simple problem is a little complicated, it is not very straightforward problem. Now, we go to the next case, which is actually the anti symmetric lift distribution, because we have done symmetric, now we need to know anti symmetric. So, what I will do is, I will again erase this part, but here I will use the slightly modified symbols, is this clear this is for uniform wing, what we get the load, but the intermediate algebra I would say you fill it up.

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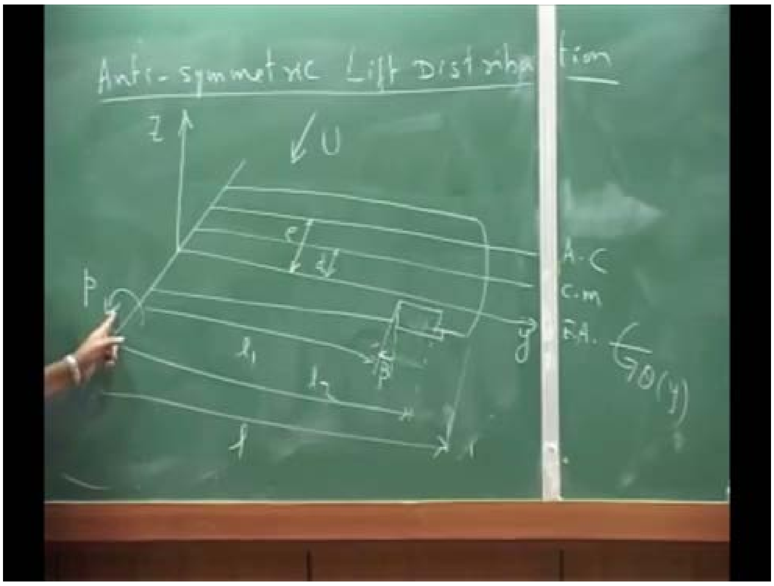

Now, let us I will erase this part, we will start a new section, which is the anti symmetric lift distribution, this is when you deflect your aileron, because what you are doing, you put one aileron down another aileron up. Basically for changing the lift term the two sides of the weight, so I will draw one simple diagram, this is my x-axis and this is my wing, I am taking uniform wing. And I have my this is my aerodynamic centre and this is my centre of mass line, and this is my elastic axis as well as my y axis, and this is the distance.

We call it e and this is the distance d and this is my y axis, this is my z axis and this is my x axis and the wind is coming in this direction U, and you have a twist which is theta function of y, now I have to put some control surface. So, I will draw a simple this angle is data, I will erase this ((Refer Time: 30:40)) part, this is the data deflection and you can take this has l 1 and wing, full length of the wing and up to this point, you have another l 2, maybe full length of the wing you can take l.

You will have the aileron is only oval, the interval only this and where it is kept that is also equally important, that is why I have plotted l 1, l 2 etcetera. Now, when you deflect the aileron, what happens you are assuming first the wing is rigid, wing is rigid means your changing the chamber of this particular section. Only this section assuming truly strict theory, there is nothing beyond we are not taking 3 D effects and other things, when you take this and changing the chamber, so my lift is changed. So, one is just the deflection of aileron is equivalent to changing my chamber line or angle of attack, that is the first one.

Next is because I am changing my lift, so I can generate a twist also, so that is a secondary effect and on the top of it, because we have deflected the other side also be deflect up. So, one side load is increasing another side load is decreasing, so the aircraft will start rolling, so the rolling will make this is a role ray I am taking, because initially you will get roll acceleration. Because, when you deflect initially it is flying what happens, you change lift on both sides, there is a role acceleration will come.

The acceleration will go and change the load factor, because the load factor what is that, we took it as actually we say lift divided by weight, we take it as the load factor. But, actually these are all related to acceleration of the whatever mass of the element, that is why the load factor when I start rotating, I am only changed my load factor on the wing. Now, how do I treat this problem and another important point is, because of this p, is the roll break, what happens the wing is going to go up, which wing I am going to get a normal component of velocity on the wing, and that will change my angle of attack.

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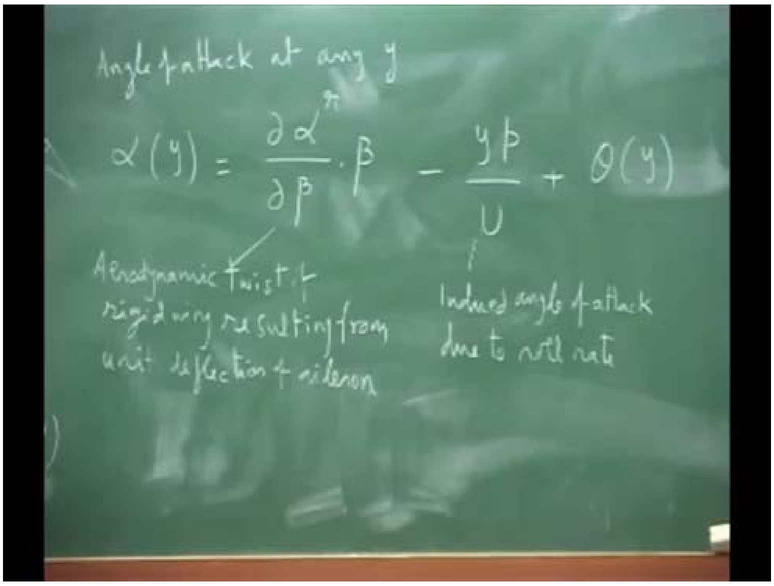

So, I will have my angle of attack, if I want to write how angle of attack at any y, i am writing alpha y is the angle of attack at any y, this will have basically three components, I am writing it as three components, one of the components can exist need not exist. That is delta, alpha, rigid, delta beta times beta, because this will exist only in this section, other sections because there is no beta here in these sections. Therefore, you cannot change the chamber, so this is basically due to the aerofoil characteristic with the control surface, wherever it exists you put it otherwise you set it 0.

So, it will be like a factor you have to take where it exists you should see, l 1, l 2 in this zone only it exists and then, the other term is because it is rolling, I am taking the rights side wing. What will happen l p will be the velocity normal velocity any l p or y p, any y into p divided by U, so I will have the change will be, because it will be like this your this velocity will be since my U. So, this is my y p, so this is the angle of attack, because this state reduce, because my aerofoil is like this that is why the reduction comes.

And then, you have the elastic twist, this is called only anti symmetric lift solution, now I have write my equation of twist equation. But, here please remember this, this is induced angle of attack due to roll break, ((Refer Time: 37:07)) this is the terminologies aerodynamic twist, this is not a geometric twist, this is the aerodynamic twist of rigid wing resulting from unit deflection of aileron. This is the unit deflection of the aileron, unit angular deflection you can take it anything, now that is due to the elastic twist.

And then, you will have a load factor also coming in, now you go back and write a start looking at our elastic twist equation, because the equation does not change, that is why the torsional equation for a straight wing that remains the same. Only thing is we have to go and then identify, which are the places where I change my terms due to this, it is very important.

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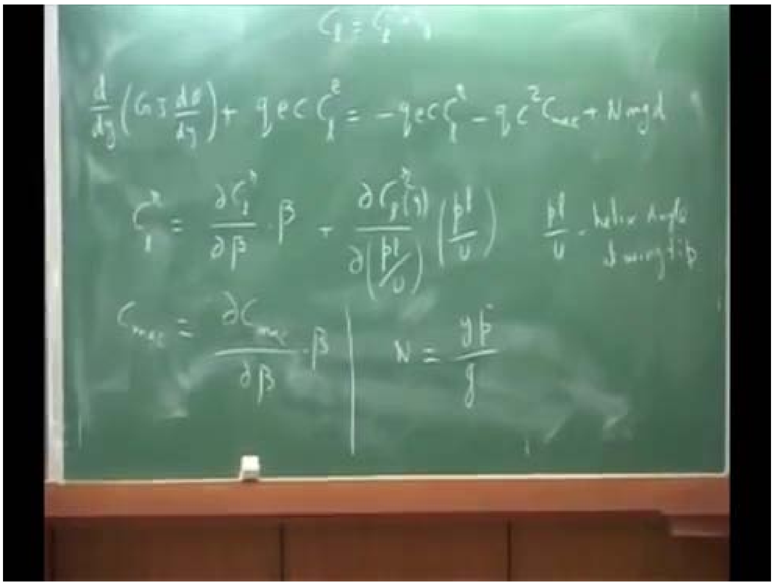

Now, your torsional equation d y G J d theta over d y, now I am going to write q e c C l alpha into theta, I will not write C l alpha into theta, I will put it just C l elastic minus q e c C l rigid. Because, my C l I am writing it as C l rigid plus C l elastic, just for my convenience only, otherwise I will put C l alpha into theta, but here it is not C l alpha into alpha, because this is a little different, that is why I put rigid minus you will have q c square C m a c plus N m g d.

And of course, with the boundary condition theta 0 root and d theta by d y at the tip is 0, now twisting moment of the tip. Now, here we have to see what are all the various terms, what is this, because anti symmetric lift distribution is a perturbation analysis, you are in a level flying, you are in a particular flight condition, you are not deflecting a control surface, what is the change that is what is more important. The change in angle of attack is due to this, the change is ((Refer Time: 40:42)) this, this you have a level steady alpha all 0 r will always be there, that is the alpha are of the entire wing will exist.

Over and above what is a change in angle of attack is this, that is why we take these two terms are contributing, because they are not functions of theta. So, these two contribute towards C l r, whereas this row is contributing to this and then, C m a c I will write the equation. Now, what we write is C l r, this is the change in lift due to reflection of the aileron, I will put delta C l r delta beta into beta this is not angle of attack, this is C l. ((Refer Time: 42:05))Here this is angle of attack, I can multiply by C l alpha, and take here and then, this will become the C l r, so this will become C l alpha alpha r, this particular factor will become the this value.

And then, plus over delta p l over U, I am taking p l over U this is a full l, I am not taking only y this is at every point I am taking, now this will become p l over U. But, this is the function of y, I am taking this variation due to a roll rate, I take that reference value as the value at the tip, actually this term is called the U is helix angle at wing tip. Every section will have change in angle here ((Refer Time: 43:52)), I am only representing it, yes my lift coefficient will change at every section, but how much it will change.

I am taking the reference p l over U, which is the change in angle of attack at the tip due to roll rate, this is for just reference p l over U, otherwise what this will if I take p y over U, p y over U it is a function of y. So, I am taking only one reference p l over U at that tip helix and then, I will calculate ((Refer Time: 44:44)) this is every section that is a because it will be what if I multiply by l and l, I will get this l I can write it this is the function, p l over U is constant.

This is just varying along the spam which is my lift is varying along the spam, similarly you write your C m a c, this is delta C m a c over this is beta, because my moment coefficient is independent of angle of attack. Because, this term is only angle of attack change, this is not due to chamber change whereas, this is chamber change beta, that is why you say C m a c is only this. Now, what you write you will go to module N my load factor, load factor because it is anti symmetric, that is why you will write this as y p dot over g.

Because, p dot is angular acceleration y into angular acceleration is the acceleration at any point divided by g, this is the load factor y p dot over g, this is the load factor at every point along the spam. Now, if I erase this part, now if you have a doubt you can ask me, now I go back and write all these things in my equation. If I write it and have to substitute of quote C l e is C l alpha into beta that is all, so you can have your equation.

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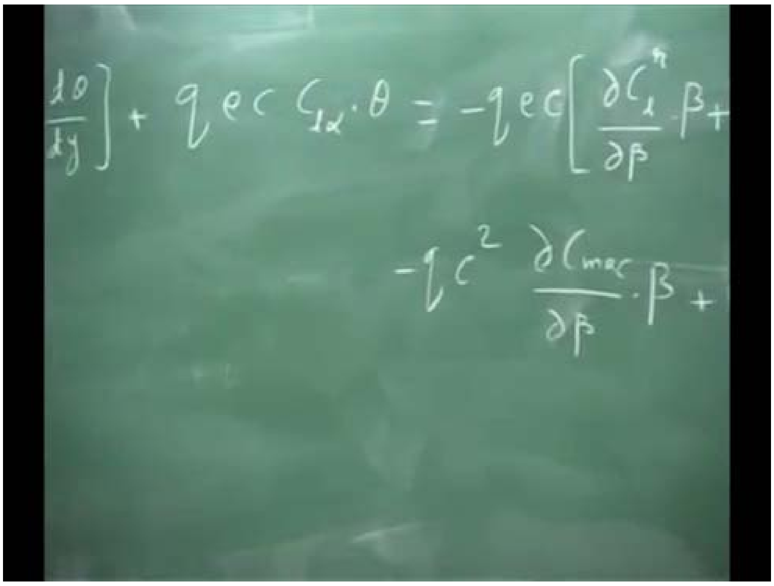

Now, let us modify it after substituting all these things, it will become d over d y G J d theta over d y plus q e c, I can write C l e as C l alpha into theta no problem, but right hand side is equal to minus q e c C l r is this term. I am going to put it delta C l r delta beta, I am going to write into beta, then you will have right plus delta C l r over delta p l over U into p l over U. And then, minus q c square delta C m a c over delta beta and beta and then, plus m g and g will cancel out, m y p dot into d.

Now, what you can do is, you can simplify those parts and then, write it that is what you are doing is your collecting the terms of beta into 1 p l over U separately and then, p dot separately.

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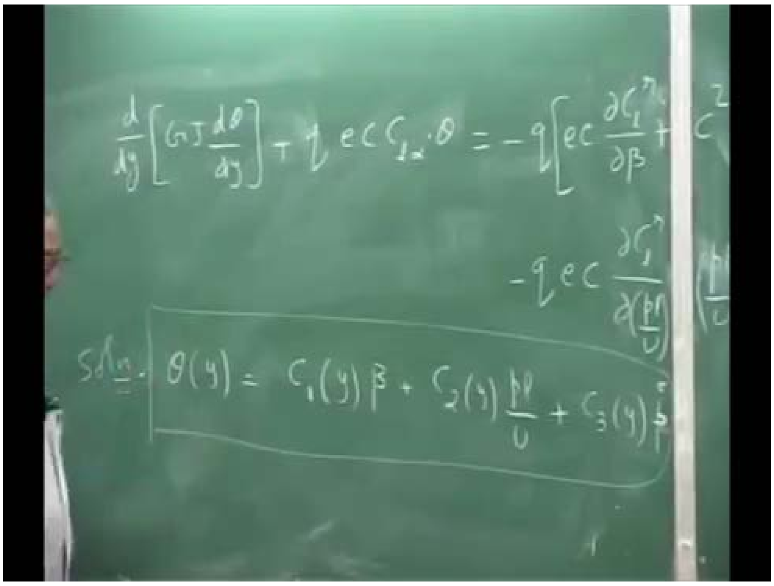

So, you will have three terms which you will write it as d over d y of G J d theta over d y plus q e c equals minus q e c delta C l r over delta beta plus C square delta C m a c over delta beta times beta minus q e c delta C l r over delta p l over U into p l over U plus my p dot d. Now, you have the equation these are all functions of y, because this exists wherever you have the control surface, other places this will not exist and this is full it will exist everywhere, because p l over U y.

So, this term is along the full spam, and this is again full what you do is you assume, now I cannot derive the equation, even if it is a uniform wing. Because, even in uniform wing what happens is this term exists only where my control surface is existing, these things I can get solution, this is one partial, yes I can write it as though it exists put it delta function and then, make it. But, I am going to write it only in symbolic form, now whatever I am deriving from now on is symbolic, just I will give you expressions, this is just for information.

Because, you need to do you have to do numerically you can do for a wing, because I got a data for a wing, we can do it as a it has to be a human problem. Now, what I do is I solve this, this is a linear differential equation with three independent quantities, one is beta that is a control surface deflection. Another one is that helix angle at the tip p l over U that is basically roll rate, third one is roll actually angular acceleration which is the rate of roll rate p dot.

Now, my solution because you can put the boundary condition, you have to put the boundary condition theta at 0 is 0 d theta by d y at tip is 0 free tip, then I can write my solution as, I am going to write it only in symbolic C 1 which is the function of y. Some C 1 which will be sum part of homogenous plus this into beta, that is all plus C 2 another function of y into p l over U plus C 3 which is another function of y into p dot, this is my solution, taking into consideration after boundary condition everything, this is my...

Now, I can go and calculate what is my C l, because C l consists of two parts, the rigid part and an elastic part, please remember that elastic part rigid part what I meant is, that is rigid part is this part. C l ((Refer Time: 54:06)) this part is the rigid part, this is a anti symmetric problem, but then my data is again a function of beta and p l over U and as well as the p dot. That means, if I substitute in my C l, this term I will substitute as it is and then, in the C l e which will become this term. So, I will write, because again I am writing it in a symbolic fashion, that is why these problems are a little involved.

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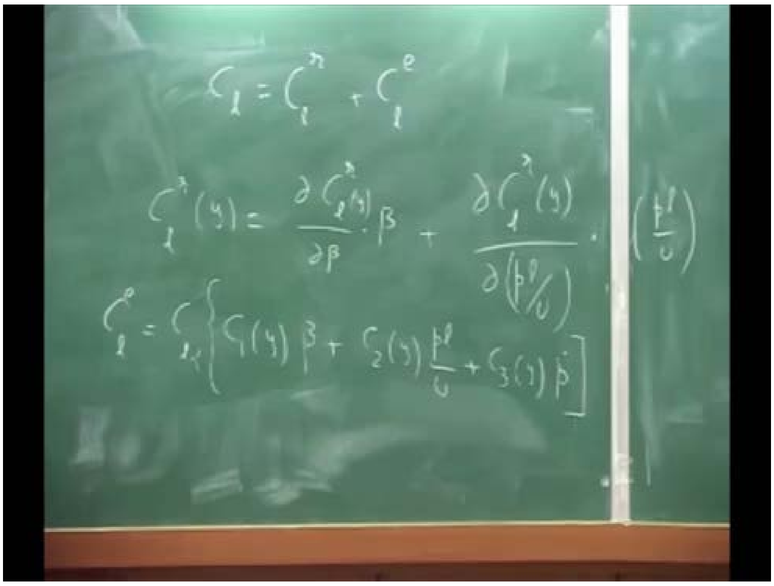

So, you have your C l consists of C l r and then, C l e is C l alpha times that, whereas your C l r is what we wrote as the earlier equation. Now, let us write our full expression and then, write it in a symbolic form C l r y is basically delta beta beta, this is again a function of y. Because, some point it will be existing some point, wherever the control surface is not there it will not exist, plus p l over U into p l over U, this is my C l r. Whereas, my C l e is C l alpha times C 1 y beta plus C 2 y p l over U plus C 3 y p dot.

Now, I have to add both, so we can add both of them, you can get a full C l expressions, now with this ((Refer Time: 57:07)) we can solve I will erase this part, three types of problems, three types you can also... Because, I will explain one is sudden deflection of the control surface, then what is the resulting effect.

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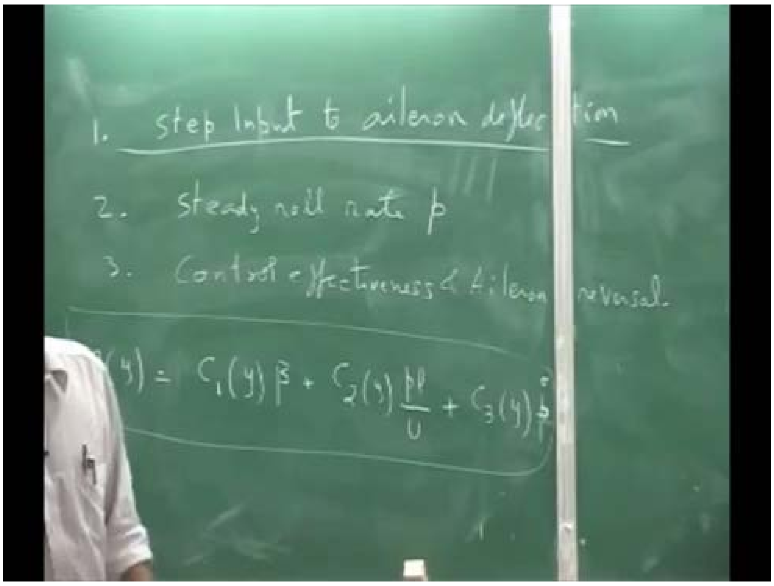

That means, sudden deflection is you can say a step input to aileron deflection, what is a resulting effect this is one problem, then because of that, what is the change in the lift distribution, I did was sudden thing what is the change in my lift distribution C l along the spam. Because, C l C into C l q dynamic pressure, I will get the lift distribution, this is one problem, second problem is I will say, if you have steady roll rate, number 2 a steady roll rate p.

What is the lift distribution, see all of them require basically what is the type of lift distribution in each case, see initially it is flying, then suddenly deflect how my word is changing. Another one is this is one role steady roll rate p and of course, third one is control effectiveness and aileron reversal, these are the three types of, you can say the problems which one analyses with this solution, now first we will go step by step.

Student: Sir I have question, sir this quantity p we are treating it as a dependent variable.

P is an independent variable.

Student: But, sir p is depend on what is…

Yes, that is what, what I am saying is I am having a roll rate, because of that roll rate, I have a angle of attack change, how I generate that is different, I have to generate by deflecting the...

Student: ((Refer Time: 01:00:46))

Yes, but if I have this much roll rate, what is the angle of attack change, now what you asked that question will be answered, now if I deflect this suddenly usually the aircraft is level flight, and deflecting here. At that time when you deflect, what happens your roll rate is 0, because you have generated the moment, you will have only roll acceleration, you will not have roll rate you are starting, that is why N f is coming to Newton's law f is m a, so that moment is i alpha.

So, I will have a roll acceleration, so what you have to solve is first problem if you take, but my roll velocity at that instant is 0 when I am starting, then you can integrate it how the roll acceleration changes with time, then you can get the velocity etcetera.

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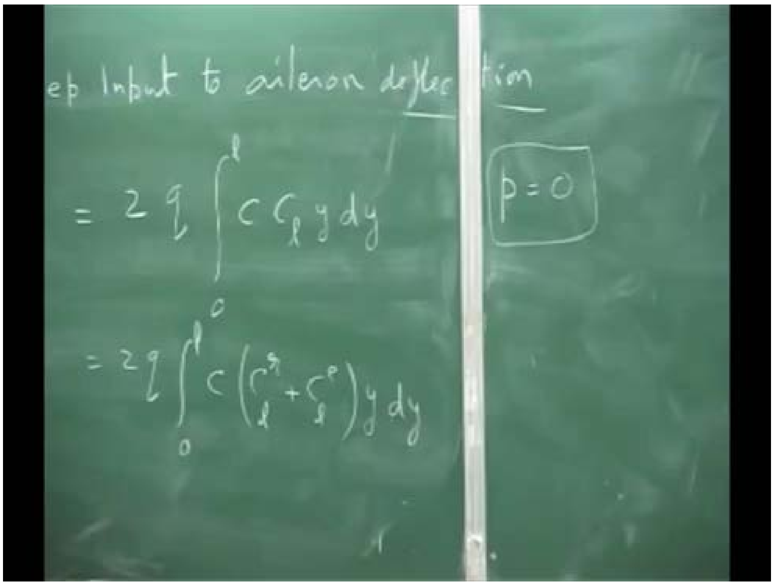

Now, we will write it in a simple form, maybe I take this lift distribution due to, I will write here lift distribution due to a step change. So, what you have done is you have suddenly put a aileron, what happens you generate actually if you say I x x is the role inertia of the vehicle. And p dot is the roll acceleration, this is what the effect, now this must be equal to what you have generate it, because the two wings you have to take, that is why it is anti symmetric.

So, I am taking 2 times q dynamic pressure 0 to l C C l y d y, but this C l is written as with what p is 0, sudden deflection there is no role weight at that time, so I can go back substitute for C l r only this term will come, because there is no roll velocity, so this is done. And I have this term is gone, but p dot is existing here, so that will come in the integration. So, what we can do is, maybe I will go to the this section, we will write it in a form which is, I am going to use some symbols, because you can put it substitute this expression there.

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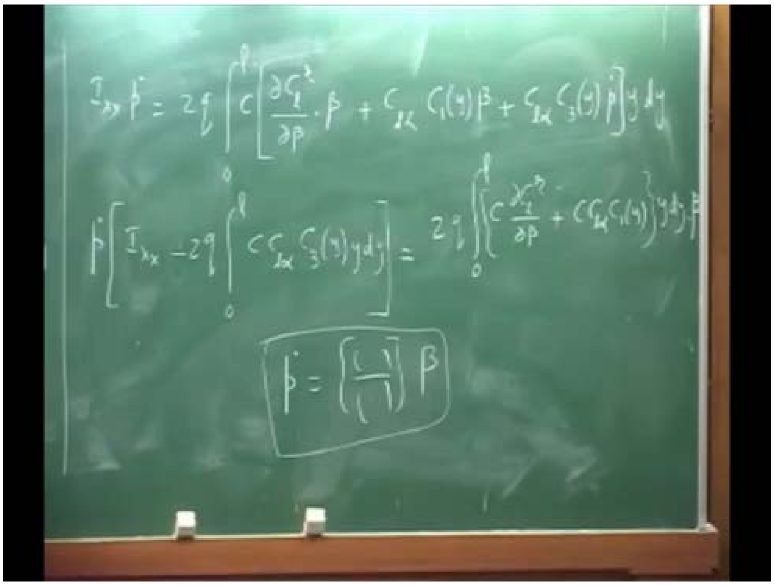

So, you will have what I x x p dot equals 2 q integral 0 to l C into C l r is into beta that is the first term, the second term will have C l e term will be plus what C l alpha C 1 y beta this is not that plus. Because, C l alpha is common, so I will have to take C l alpha C 3 y p dot into y d y, now this is p dot I can take this term on the left hand side and then, take out the p dot outside. So, I will have p dot into I x x minus 2 q integral 0 to l C C l alpha C 3 y y d y equals, you will have 2 q integral 0 to l C delta C l r over delta beta, because this is also beta dependent, plus you will have C C l alpha C 1 y into y d y times beta.

Now, you see I can get p dot in terms of beta, so this will be I just have to divide this by right hand side, I will have p dot is equal to something times, that is the numerator expression is this ((Refer Time: 01:07:26)), denominator expression is this into beta. So, this is my p dot, which is a function of beta, ones I know p dot I can go and put it p dot as a function of beta that means, I will know C l e, I know C l r I can immediately get C l which is I can get the full lift distribution, if you suddenly give a beta. Now, you see this is how the calculation is to be done, the next problem which is actually the lift distribution due to steady roll, if you take it. Steady roll means what, moment is 0, so that problem you need to write it in this fashion.

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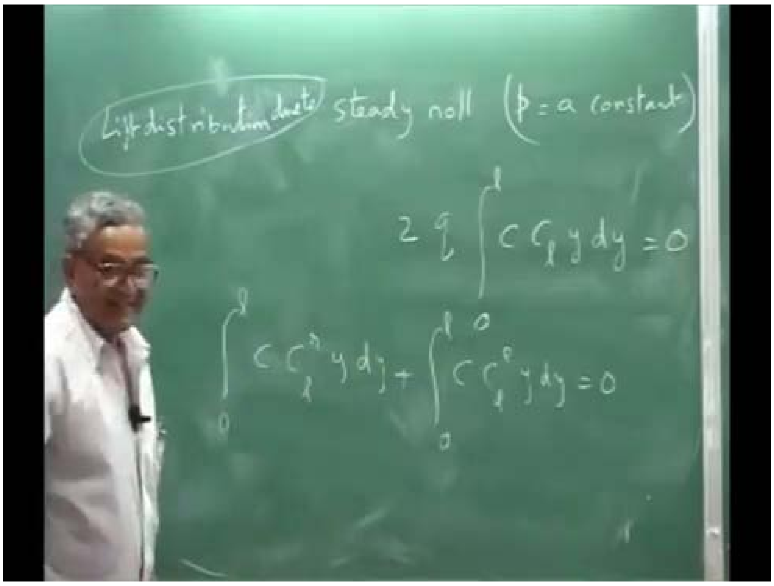

So, lift distribution due to due to steady roll steady roll that means, p a constant which means my moment, if I integrate 2, because basically the load on one side must balance the load on other side, because I have steady thing. This is basically 0 2 q, because this is my roll because lift into y, my roll moment is 0, now how do we do which means, you integrate 0 to l, 2 q anyway it is the steady dynamic pressure that can be thrown out.

You will have C C l r y d y plus integral 0 to l C C l e y d y is 0,now you substitute the two expressions here with p constant, but p dot is 0, then what will happen is I will write here. So, p is a constant there, so you will get C l r is beta and then, what you get that you will get a beta term here, you will get a p l by y, you will have a these two terms. And then, what you can do is you can take beta separately and then, p l over U separately, then you write beta equals p l over U.

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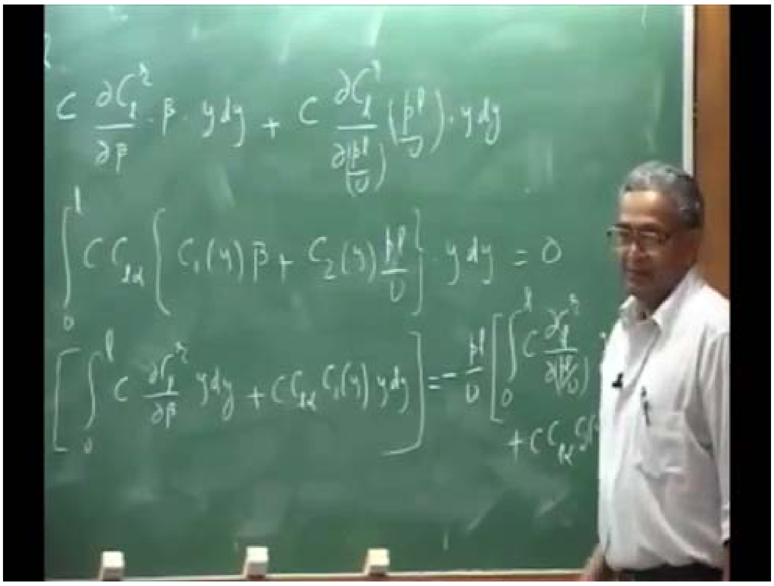

So, now we will write it what is a first-term integral 0 to l C l r is C delta C l r over delta beta into beta into y d y plus, you have any C delta C l r over delta p l over U into y d y, this is the first term. And the second term C l e will be integral 0 to l C into C l alpha I have to put C 1 y beta plus C 2 y p l over U into y 0, now what you can do is combine the beta term, combine the p l over U term and then, take two terms outside.

So, what will you have, you will have beta times, this is the first term integral 0 to l C delta C l r over delta beta y d y plus this term C C l alpha C 1 y into y d y, this integral is there everywhere y d y, y d y that is why that integral should be every time. This is equal to you put minus sign this is p l over U, again open integral 0 to l C delta C l r over delta p l over U this is the first term into y d y plus this term C C l alpha C 2 y y d y.

Now, what you can write is beta to steady roll rate, if you want to have a steady roll rate, what should be the angle of beta. Now, this you can write it in, I will write this expression, because this is the expression which is required later.

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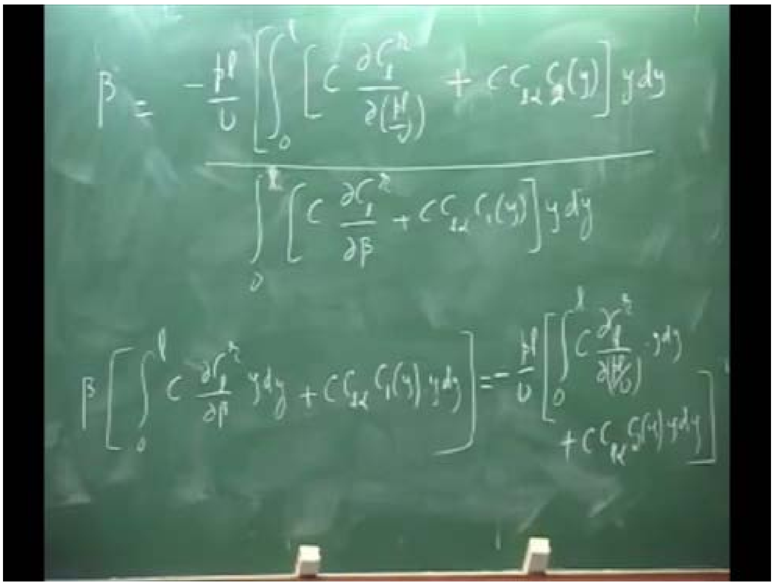

I will write here, minus 0 to l you can C delta C l r plus C C l r alpha C 2 y into 0 to l C delta C l r over delta beta plus C C l alpha C 1 y into y d y, this is my expression for if I have a steady roll rate. That is if I want to have a steady roll rate, what should be my deflection, this gives me the deflection; now the third problem which I said aileron effectiveness.

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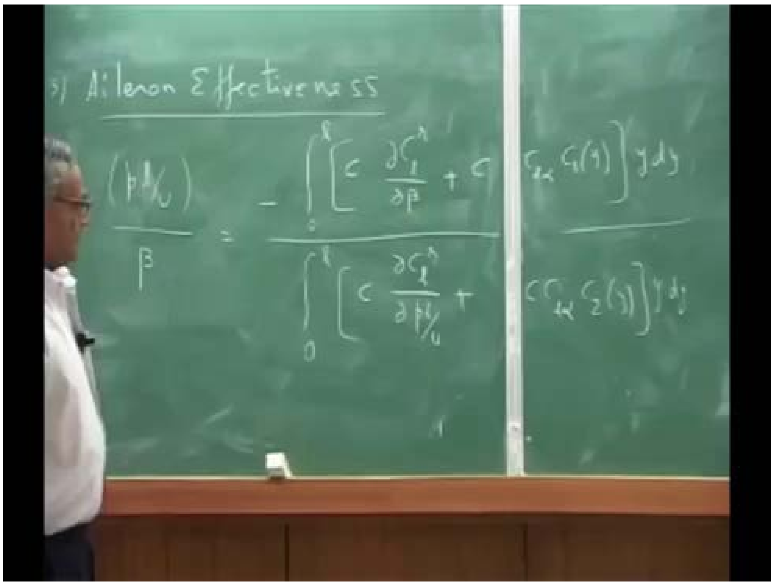

Third is aileron effectiveness, this essentially comes from that expression, because I take ((Refer Time: 01:16:19)) this term and what you do is, you divide by beta, that is our unit aileron deflection. What is the roll rate I get, in the sense I am basically writing the expression like this, p l over U, so this is basically inverse effect, that will become what, even if I take this is the linear relationship you will have minus we will put. May be the minus sign we will take it here, you will have 0 to l C delta beta plus we will have C C l alpha C 1

Student: ((Refer Time: 01:17:24))

C 1 y into…

Student: Y d y.

Y d y over 0 to l, which one the first expression, C delta over delta p l over U plus C C l alpha into C 2 y, where do they given it 1 minute, I am just checking is the, it is the lift coefficient C l r, C into lift coefficient, I have taken this q c C l. Now, you see here any deflection of this if this is 0 that means, I do not get any roll rate, which means aileron is thin effect. So, the speed at which the numeric becomes 0 is my aileron reversal speed, but other places.

Because, of if it is a rigid aircraft and if it is a flexible aircraft, rigid means you will have only ((Refer Time: 01:19:19)) this term, because of flexibility what happens this term has come into picture, because of flexibility. Not only that term, this term has also come into picture, if it is rigid you will have only these two, because it is flexible you are going to have these two terms. So, the effectiveness of aeronaut, if you consider a rigid aircraft you will get one flight for a unit deflection of this, if you get flexible aircraft for the same unit deflection of the aileron, you may not get the same roll rate.

Now, this is aileron effectiveness and the speed at which no matter what you do beta, if it is not happening at that that means, numerator 0, now when you increase the speed the dynamic pressure or whatever, because that dynamic pressure is going to change any of these values. And that speed what will happen is numerator become negative, then what will happen your aircraft will turn the opposite, so your aileron effectiveness is part of this problem itself.