Aero Elasticity

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Lecture – 10

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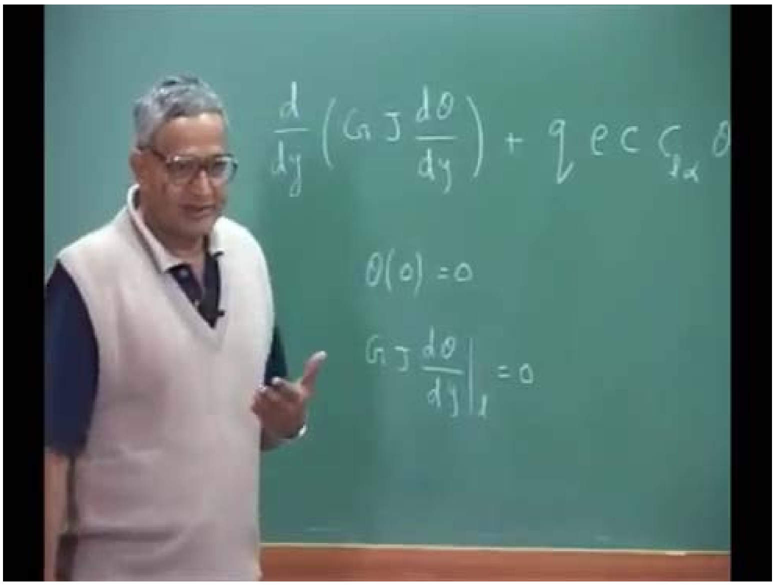

Let us look at the torsional deformation equation of a straight wing, which we wrote it as d y plus q e c C l alpha theta equals minus q e c C l alpha alpha r minus q c square C m a c plus N m g d with the boundary conditions theta at 0 that is, the root, it is 0. And since there is no moment, it will be d theta over d y at l is 0, these are the boundary conditions, this is we have used trip theory. That means, every section of the wing is treated as a 2 D aero foil, we applied the aerodynamic movement and we just wrote the torsional deformation equation.

Now, alpha r is the initial rigid angle of attack you may call it, but this can be a function of y that is, along the span and m can be a function of span, all these things. Now, our aim is to here establish, what is the deformation of this wing, and under what condition that the deformation will become very large, so for that we look at the, first we take a very simple case of uniform wing, then if my wing is not uniform that means, my G J can be function of the span y axis, all these things e, chord, everything can be a function of y then how do we solve for that type of equation, that is why approximate method.

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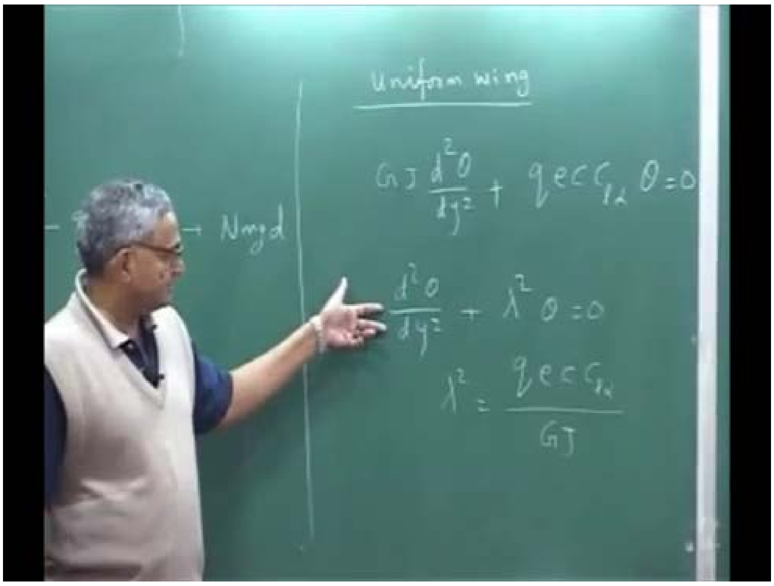

First let us say, it is a uniform wing when I that means, G J e c C l alpha, they are all constant and I look at the only the homogeneous part of the differential equation. That is not the particular solution, because I want to see how the homogeneous solution changes, what is the form of the solution and what condition it will be towards the divergence speed. Because, the key parameter is the q, which is the dynamic pressure, which is half rho U square, which can vary with forward speed.

Now, if it is a uniform wing, you will have G J d square theta by d y square plus q e c equals 0, this is the homogeneous part. And this will be like a second-order equation, you will have d square theta over d y square, I am putting it as some lambda square theta equals 0, where lambda square I call it as q e c C l alpha over G J. Now, this is the like a simple second order differential equation.

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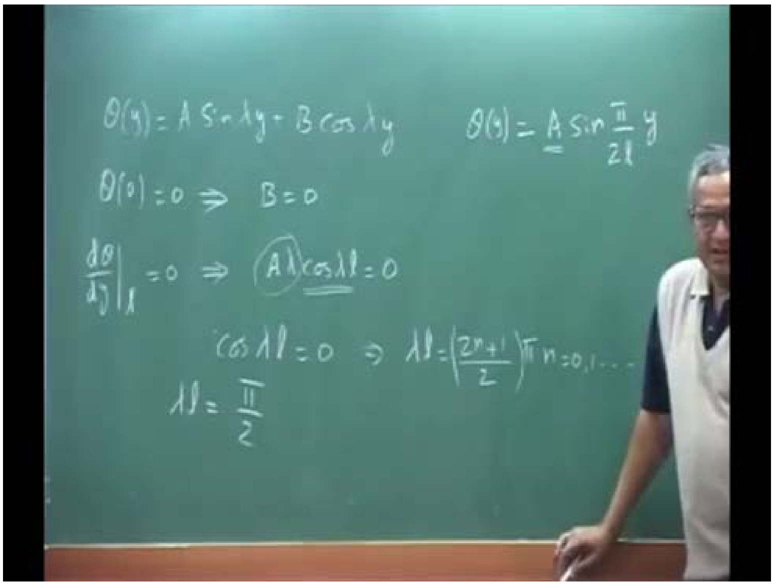

The solution of this, you can write it as theta y as A sin lambda y plus B cosine lambda y. Now, you have to get this constant A and B from the boundary condition, boundary condition is theta 0 which means, this implies when you put y is 0, this term is 0, this is 1, therefore this is B equals 0. Then you put the second condition that is, d theta over d y at l, this is 0, this implies you will have, when I differentiate this is going to become A lambda cosine lambda l.

So, you will have A lambda cosine lambda l and if you say my lambda is not 0, A is not 0, I should have the condition cosine lambda l this is 0 which means, my lambda l is 2 n plus 1 by 2, n can go from 0, 1, etcetera into pi. Now, this is the condition for your, now what does it mean is, if lambda l takes the value, let us say for the lowest value is pi over 2. If it takes this value then your solution is theta y equals A sin, because lambda is pi by 2 l y.

Now, A can take any value, because this condition essentially says, I have satisfied my boundary condition and I have satisfied my differential equation. This is an exact solution, the condition for it to be satisfied is this and you call this lambda l equals pi A by 2, this is actually the lowest value that become your critical speed, which you call it as, I erase this part, your lambda square is there.

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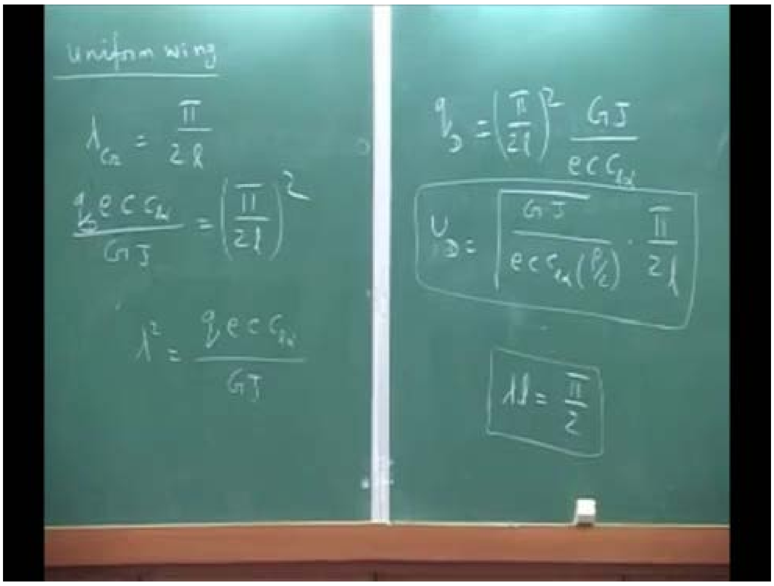

So, lambda critical will become pi over 2 l, that is the first value and you know lambda, so if you take lambda square, you will get that q e c C l alpha over G J equals whole square, because lambda square is pi over 2 l. You can find out, this is my q divergence you call it, because that is my dynamic pressure at the diversification, for which I satisfy my equation, I satisfy my boundary conditions. Now, if we write our solution, now you say q d becomes pi over 2 l whole square G J over e c C l alpha.

Now, you can find out, what is the divergence speed, divergence speed becomes, because q is half rho U D square, you will put U D becomes square route of G J over e c C l alpha into rho over to density and then pi over 2 l. So, this is my divergence speed, because you find this solution, theta y equals whatever value of A you choose, it satisfies your differential equation, that is ((Refer Time: 10:21)). That means, basically you say, this is like a buckling problem of column, column buckling, exactly the lowest value.

That is why you say, the lowest the first value, you take it as the divergence speed, at that speed, any A is the solution. That means, any amplitude that means, any of the fixed value of the wing, but you cannot find out what is that value, because it exactly satisfies that equation and this is for a uniform wing divergence speed.

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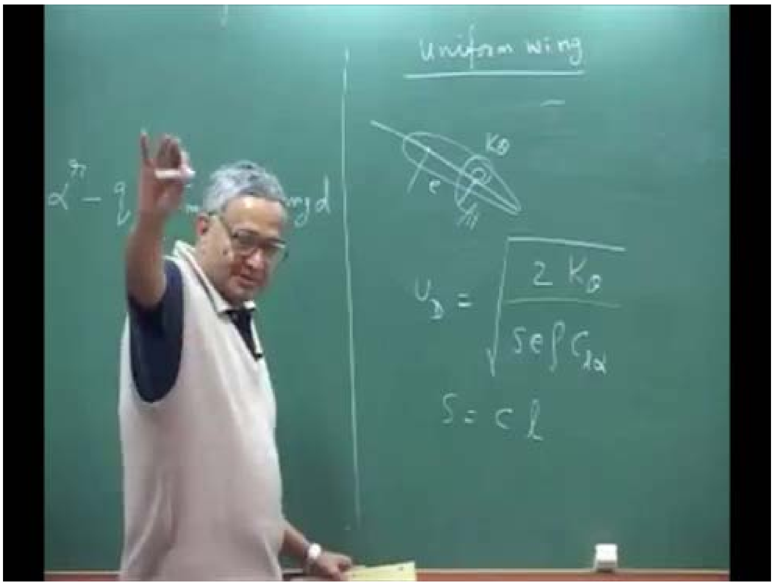

Now, if you look back your notes in the sense, if you look at your for the 2 D aero foil, we wrote the divergence speed as, let this equation be there, because we had the 2 D aero foil and there we got, we prove this with the spring. Here theta, this problem for this we got, because this is the offset e, the divergence speed we got as U D is 2 k theta over S e rho C l alpha under root. Now, what is S, S is area of the ((Refer Time: 12:19)) wing what you will get, area of the wing is C into l that is, chord into length. So, if I substitute here that value and you say, that is what my U D if I consider the full wing then may I erase here, comparing these two expressions then you can find out what should be my k theta.

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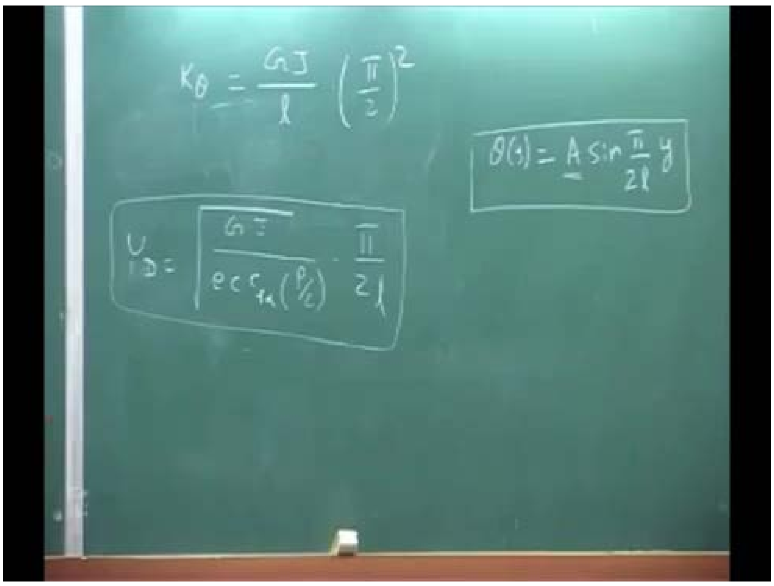

Because, there is a, if I take this l here what will happen, this will become l square and you will have G J over l square. And let us write it, k theta become G J over l into that means, it is the equivalent torsional spring, which you can use in the 2 D model that is, essentially G J over l, because you match this, so you will have G J over l pi over 2 whole square. Now, you see this is by comparing the divergence speed, which we obtained with the 2 D model, very simplistic model and for a uniform wing.

Please understand this is for a uniform wing, because G J is constant along with span, suppose if G J is varying then it is the different issue. Now, there is a another way of getting the torsional spring, if I want to get for a wing an equivalent k theta, you follow what I am saying then I can directly go ahead and use this formula for divergence speed. Now, how do I get that that is, the question is, how do I get an equivalent torsional spring for a straight wing even if G J is varying.

This is like, I will show the approach, later we will solve the full problem with G J variation, what is the approximate method. This is the another way of approximation, I think let it be there, I keep this, what I do is, what is the strain energy of that spring due to a deformation, whose amplitude is A, because this is my elastic twist.

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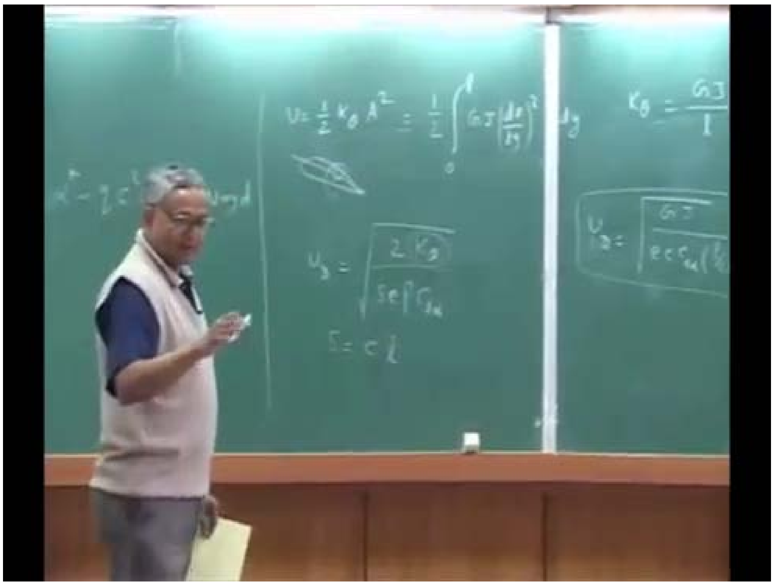

And I say that, the maximum value when y equal to l, that is 1, theta is just A, suppose if my deformation of this 2 D model, this elastic deformation is A, my spring energy will be half k theta A square, strain energy in the spring u. Now, I say equivalent fashion, I want to know if that is my deformation, what is my strain energy in the entire wing, that will be 0 to l G J d theta by d y, G J pre prime whole square d y. That means, if I have this deformation and then the strain energy in torsion is half G J d theta by d y whole square into d y.

If I substitute that expressions here and then do the complete integration, because that will become cosine then you will have pi by 2 l term will be there. That part you integrate it then what you will get is, you will get here half G J over l pi over 2 whole square A square.

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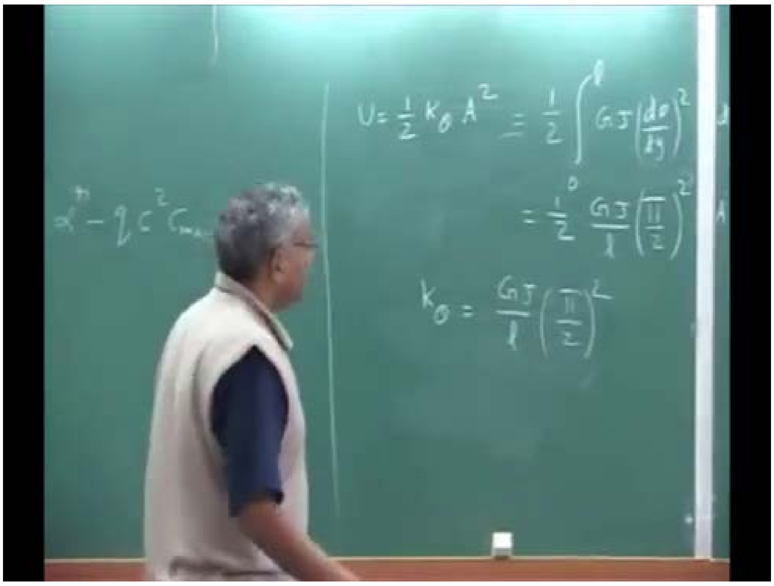

Now, if you compare k theta, you will get G J over l pi over 2 whole square, if G J is constant please understand, because I am taking G J constant in this integration. If I take G J constant then I get k theta is this, which is exactly same as matching the divergences speed itself. Now, you know, you have a simplistic approach, quick calculation, you want know what is the divergence speed of a wing. If you are given the G J, which is ((Refer Time: 19:21)) this is a quick calculation only, G J can be a function of y then what you do is, you are making a highly approximate method.

You are going to assume, even though this may not be valid, you are assuming this is my deformation or the solution or you say that is what I am going to have as a theta. You simply put that here, G J is the function of y, you can integrate 0 to l, you will get some quantity and A square will cancel out and you can get an equivalent torsional spring. The moment you get an equal and torsional spring, you can go on and then put it in your divergence speed expression and area of the wing, you can take the excess area of the wing.

So, this is one very quick approximate way of calculating the divergence speed of a wing. Now, let us look at the case, where if my wing is not uniform, if my wing is basically non uniform then how do I solve the problem. Because, this is our, there are two approaches, one is Rayleigh Ritz, another one you can use Galerkin method. So, what I am going to do is, now you have ((Refer Time: 21:07)) for a uniform wing, how to get the divergence speed.

Now, let us take and in an approximate way, non uniform wing also you got here, this is the very crude approximate. Now, let us look at the case, if you have a non uniform wing, how do we get the divergence.

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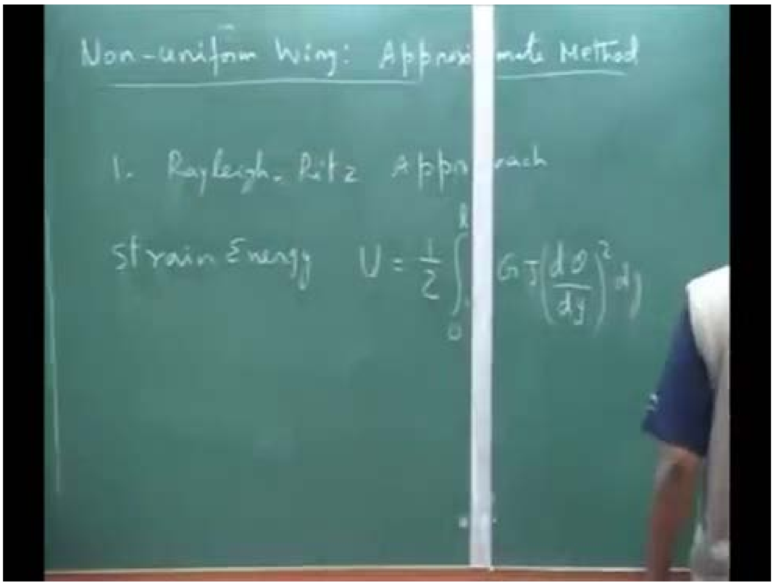

So, we will say, non uniform wing and approximate methods, the first one is you can take, this is the Rayleigh Ritz approach. What is that Rayleigh Ritz approach, this is essentially from principal of virtual work we will apply, because you have done one example earlier. What you will do is, what is my strain energy of the wing during the deformation, strain energy u become half integral 0 to the length, G J d theta over d y whole square d y, this is my strain energy expression.

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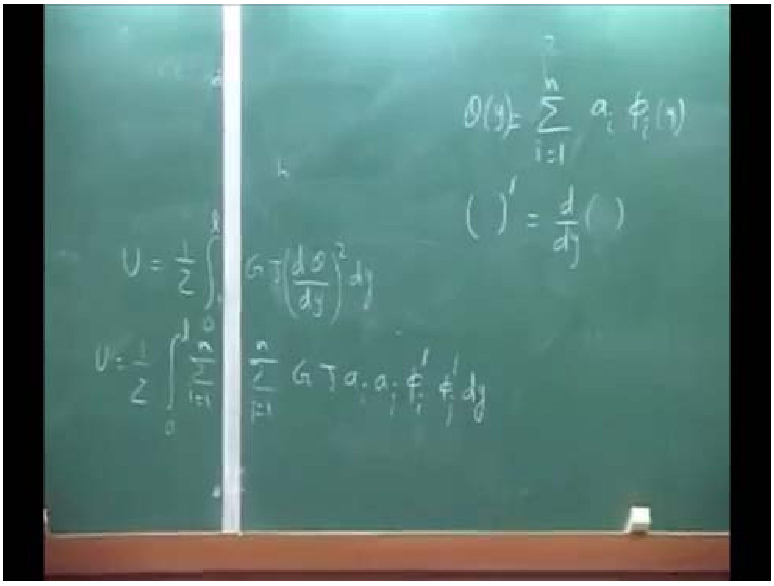

Now, I am going to make approximation in the sense, my theta which is the function of y, this is the elastic twist, which is the function of y. I am going to write it in terms of i running from 1 to n, from a i phi i y. Phi i y condition is, they must at least satisfy the geometric boundary condition, which is theta equal to 0, at least they must satisfy that geometric boundary conditions. Suppose, if it satisfies this also it is good that is, natural boundary condition also, that is why the choice of the functions d y is very very important.

And you substitute this here and you write your strain energy expression as U becomes half 0 to l, you will have double summation i running from 1 to n and then j running from 1 to n G J a i a j, I am going to solve phi i prime and phi j prime d y, where the prime symbol is basically differentiation with respect to y. Prime, I am using just the prime symbol, easy for things. Now, what I can do is, take the del U that is, the variation, principal of virtual work you have to apply.

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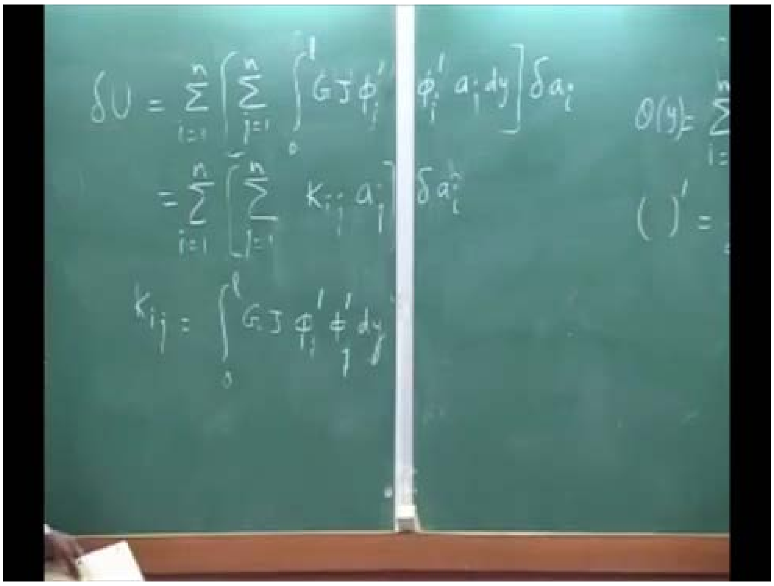

So, your del U is, your half will go up, you will have G J can have summation i running from 1 to n and summation j running from 1 to n, integral 0 to l, G J phi i prime phi j prime a j d y, I will take this into delta a i, I am writing like this. Since you know that, these are basically the d y, this a j is independent of y, because a j is all constant, only phi is the function of y. So, you can write this whole thing as some summation i running from 1 to n, summation j running from 1 to n, I am calling this has k i j a j delta a i, where k i j is my integral 0 to l, G J phi i prime phi j prime d y. Now, you see this is symmetric in the sense, again change i and j, k i j, now this is as far as delta U, you have to get the expression for delta W external.

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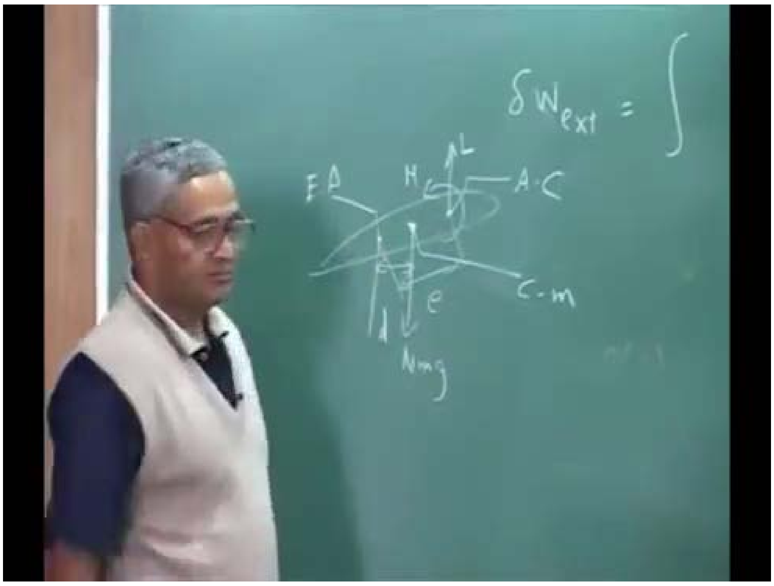

So, when we go and write the delta W external, I erase this part now, you have to know what is my delta W integral? I have only a twisting moment, I am not writing the, because I am considering only torsional deformation, I am not considering bending deformation. Therefore, if you take the cross section, this is our elastic action and our lift is here and we search the moment is here and this is the offset e and then there is the gravity, which is acting.

This distance we took as d, this is N m g, so this is the center of mass, this is the aerodynamic centre, this is centre of mass. We need to get the moments, so what is ((Refer Time: 29:06)) the expression for the virtual work, virtual work moment into delta theta, because theta will be my deformation, so I give a virtual rotation in theta.

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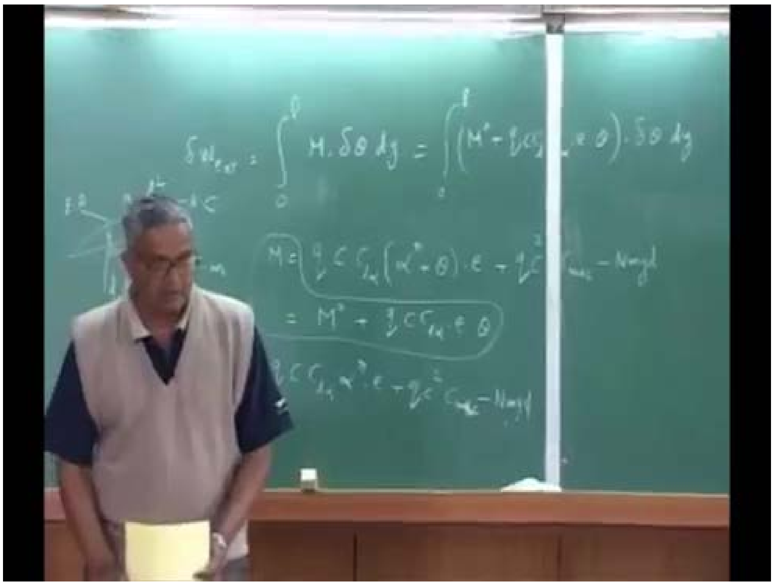

Now, my moment expression is, lift this moment and then due to gravity, lift is e q c C l alpha into I will have, because alpha rigid plus there is an elastic deformation please understand, into e ((Refer Time: 29:59)) distance. And then you will have your other momentum, which is q c square c m a c, this is the aerodynamic moment minus N m g d, this is their moment due to gravity ((Refer Time: 30:26)). Now, this is my moment expression, I substitute here then I will write my external.

If you look at this moment itself, because I am going to now split it, because I do not want to write every time long long expressions, you see terms which are independent of theta and the terms which are dependent on theta, so I am going to write this as M 0. The term which is dependent on theta is only this term, q c C l alpha into e theta, the rest of the terms are M 0 is q c C l alpha alpha r into e plus q c square C m a c minus N m g d.

That means, I have split my moment expression like this, because this is a constant moment in the sense, this is not function of theta, it will go to the right hand side of my differentially question. Now, I substitute here, I will have integral 0 to l, M 0 plus q c C l alpha into e theta, times delta theta d y. Now, what I have to do is, I have to go, substitute for theta and this expression from my assumed solution, I erase this part.

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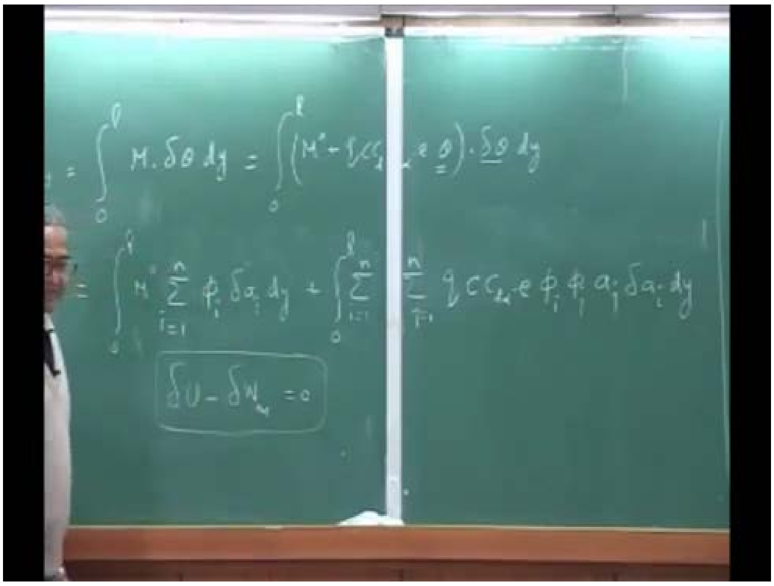

You will have, this will be integral 0 to l, M 0 summation i running from 1 to n, phi i delta a i d y plus integral 0 to l, summation i running from 1 to n, summation j running from 1 to n, because theta delta theta, that is why two summation, q c C l alpha into e, you will have phi i phi j a j delta a i d y. This is my delta W external, ((Refer Time: 33:51)) you know that delta a i term separately and here, I have a delta a i. So, you apply your principle of virtual work, which is delta U minus delta W is 0, you apply this, take out the delta a i, so we will write that expression, now I erase here.

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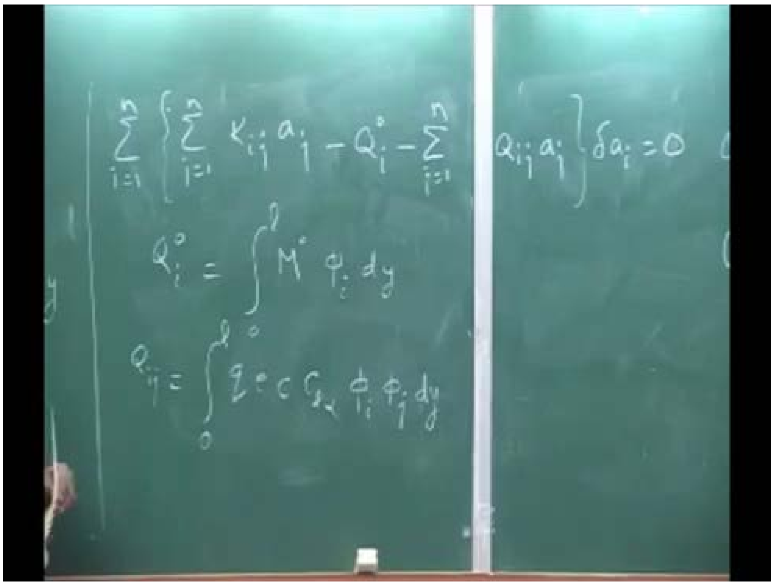

You will have the whole thing as summation i running from 1 to n, you open the bracket then summation j running from 1 to n k i j a j into delta a i, that I keep it as it is. Then you will have minus, I am going to call this term M 0 phi i d y and Q 0 i, I will defined that part later. Then I will have this, because this minus will come and I will have minus, again I am going to define another term which is summation j running from 1 to n, Q i j a j.

Now, I will define what is Q i 0, Q i 0 is nothing but, integral M 0 phi i phi i d y 0 to l and then Q i j is nothing but, integral 0 to l, q e c C l alpha phi i phi j d y. This is the Q i j that is, this expression q c C l alpha e phi i phi j d y. Now, what you immediately go and say is l, my delta a i's independent, because there all the a i's are independent. Therefore, the terms within the brackets, that must be 0, now if you write that equation, what you will be writing is, I go and erase this part this side.

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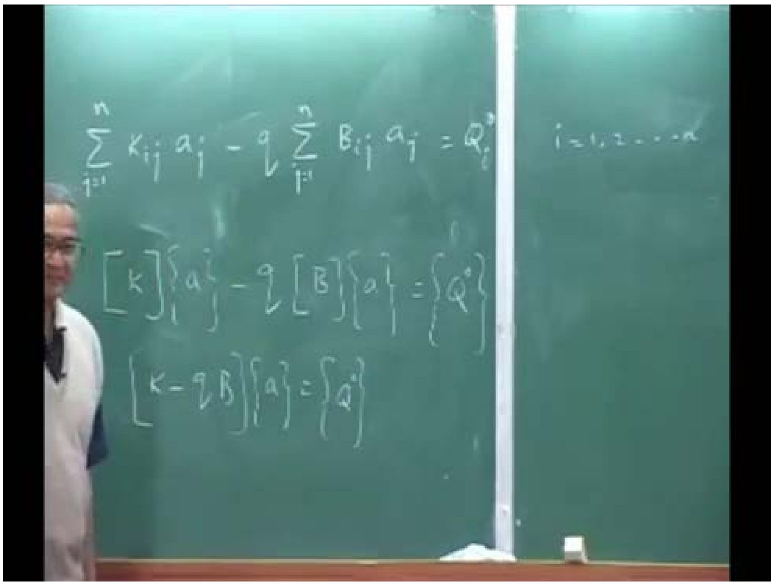

You will get a matrix equations, which is summation k i j a j minus, I am going to now take q outside, because this quantity in capital Q i j, this q is a variable, because q is dependent on forward speed. So, I would like to call this as q B i j, q is I am keeping it as a separate quantity. So, I will have q summation B i j a j, again j running from 1 to n, here j running from 1 to n, equals Q i 0 with i equals 1, 2, etcetera. Because, I have delta a i, n equation, so this can be put in a matrix form has k a minus q B a equals Q 0.

Now, you can put it also in this fashion, k minus q B into a, but Q naught is the external that is, the others loading, which is independent of a or your deformation, that is what we have said. Now, if you want to get the divergence speed, basically what happens is, as q increases, now what happens to this, the q can have wonderful word in the sense, this is like if I want a what you do, I take inverse of this matrix. The inverse will exist only when the determinant is existing. If the determinant goes to 0 then inverse is infinity matter the in word infinity.

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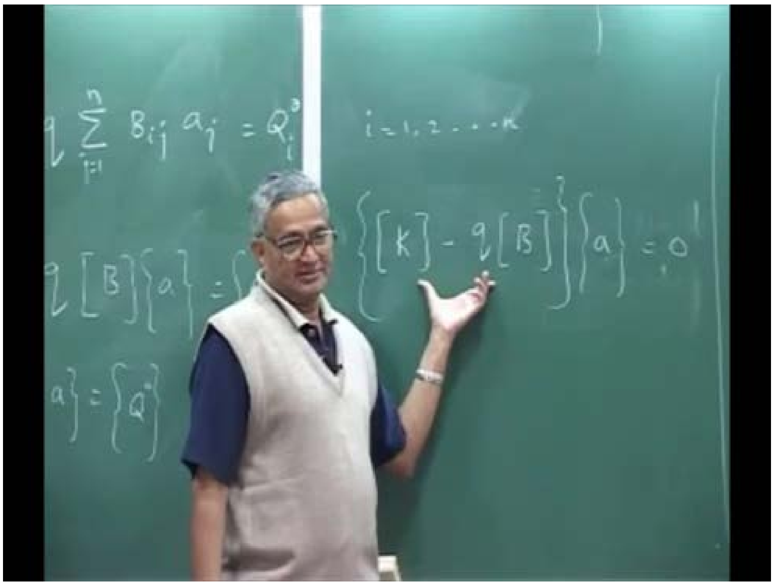

So, from the large if you say, when this determinant goes to 0 then this is like a Eigen value problem, where k minus q into a 0. Now, the determinant of this if it is 0 then this is like a Eigen value problem if it satisfies equal to 0. Now, for what value for q, I can have this entire thing go through 0 that means, k minus q B goes to 0. So, you get the Eigen value problem now follow, now the lowest Eigen values is your divergences speed.

Now, you can have, this is a any size you can choose depending on the number of functions you have chosen. i equal to 1 to n, you can choose one function two, three, four, like that you can have several function. And then you see, how the fundamental Eigen value have it convergence with increasing number of phi i. Then you will be able to say, this is my divergences speed, so this approach is your Rayleigh Ritz approach, but the interesting part of Rayleigh Ritz is, matrix k is symmetric and matrix B is also symmetric.

So, k is symmetric, B is symmetric, so this is the symmetric Eigen value problem you will get. Now, we can use this method and get the divergence speed for a wing, the same we got a uniform wing, we use the same technique, this is just for a example, I erase all these things, because they are not necessarily now. So, evaluation of divergences speed is converted into a Eigen value problem, suppose you say, if you take one mode approximation.

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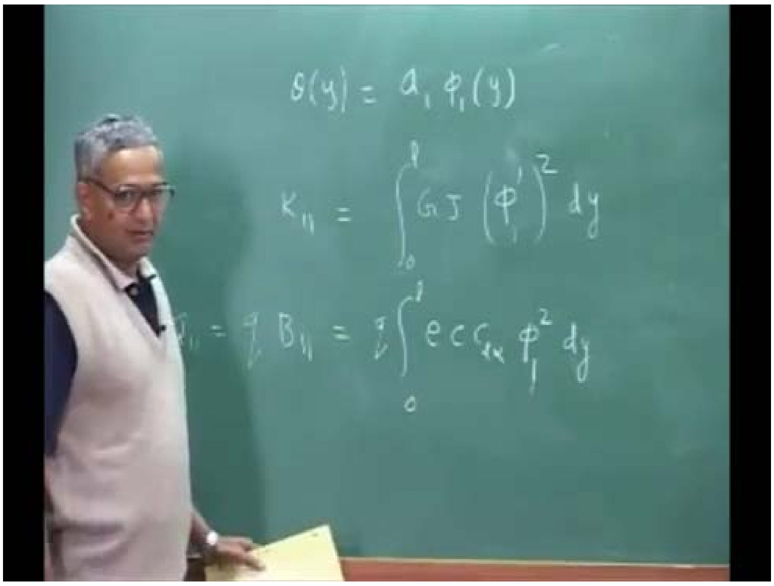

Only one mode that means, my theta y, I am choosing only a 1, that is all, nothing else or in other words, I am taking only one mode. Now, what is your k 1 1, k 1 1 is integral G J phi 1 prime, again phi 1 prime, so you will get phi 1 prime square d y, what is B 1 1, which is actually Q 1 1 is, this is the dynamic expression term we have taken as B 1 1. This is integral 0 to l, I take a q term outside, because q is independent of the integration e c C l alpha times phi i phi j. So, you will have phi i square or in other words, I will have phi 1 square and d y, now as per this, I am having only one equation. That means, I am taking only one mode, one mode means, what I get q is, k by phi that is, k 1 1 by B 1 1 is my q divergence.

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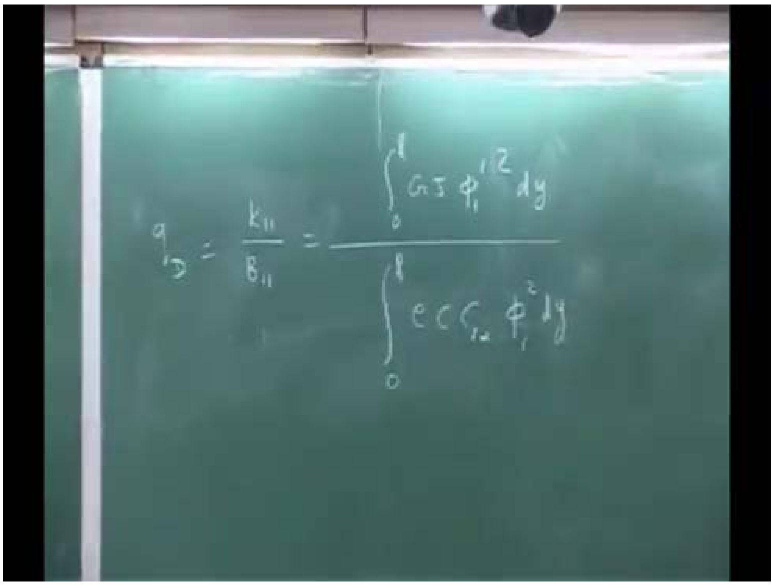

So, I am going to write my divergence speed as or q D becomes k 1 1 by B 1 1, which is nothing but, integral 0 to l, G J phi prime square d y over integral 0 to l, e c C l alpha phi 1 square d y. That is very simple, I got just divergence speed and I can have, but this is with only one mode approximation please understand, if I have two mode approximation then I will have 2 by 2. Then I have to solve for the Eigen values, this just the single mode approximation.

Now, I will show you this result, if your choice is very good, for the same problem which we solved, we will take it phi 1 as, we will apply this for a uniform wing. Please understand, for a uniform wing I am solving in this fashion.

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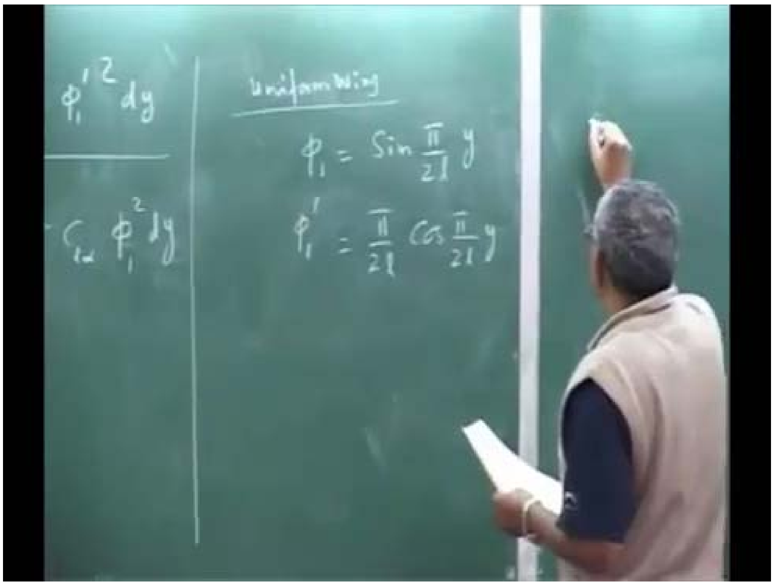

This is just an example, if I say my phi 1 is sin pi by 2 l y and just assuming, because I have to assume some phi 1. Now, what will happen phi1 prime is what, phi 1 prime is pi over 2 l cosine pi over 2 l y. Now, q divergence you will get, substitute that uniform wing, therefore G J is constant e c C l alpha, everything constant.

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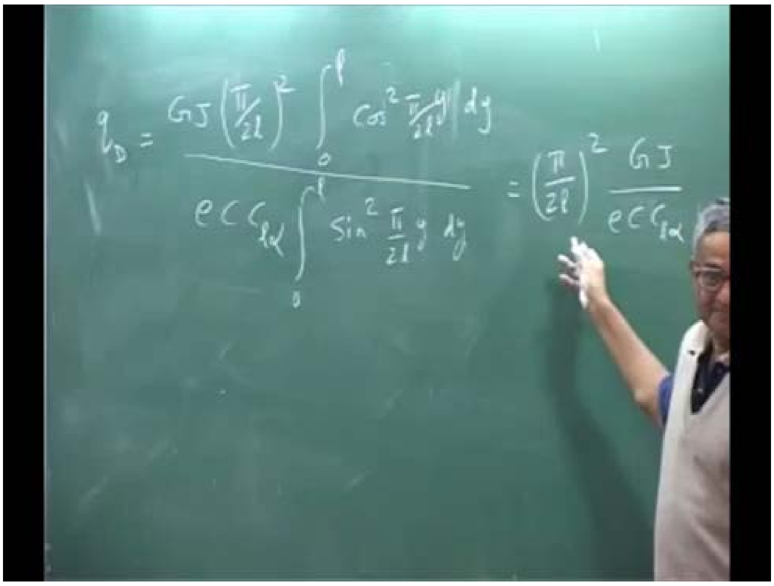

So, you will have your q divergences will becomes G J and this is phi 1 prime, so pi over 2 l whole square integral 0 to l, cosine square pi over 2 l y d y divided by e c C l alpha. Because, this is the uniform wing, everything is taken outside, so 0 to l this is sin square pi over 2 l y. Actually, if we integrate these two, they will be same, you will get q D becomes whole square G J over e c C l alpha. Now, you see, this is exact for a uniform wing, what we obtained earlier solution, the divergences dynamic pressure nothing but, G J by e c C l alpha pie over 2 l whole square, this is exact solution for a uniform wing.

That means, my Rayleigh Ritz techniques even with one mode, please understand with one mode gives me exact solution. But, please understand, this mode really they satisfied both the boundary conditions, because why we got a exact solution is, because this is the exact solution of the differential equation as well as they satisfy the boundary condition, that is why you got the same thing. Suppose, just for the sake of exercise, I am going to choose a function, which will satisfy my boundary condition.

That is, phi 0 is 0, phi prime l is also 0, let us say slightly different functions, so you see this technique gives me exact.

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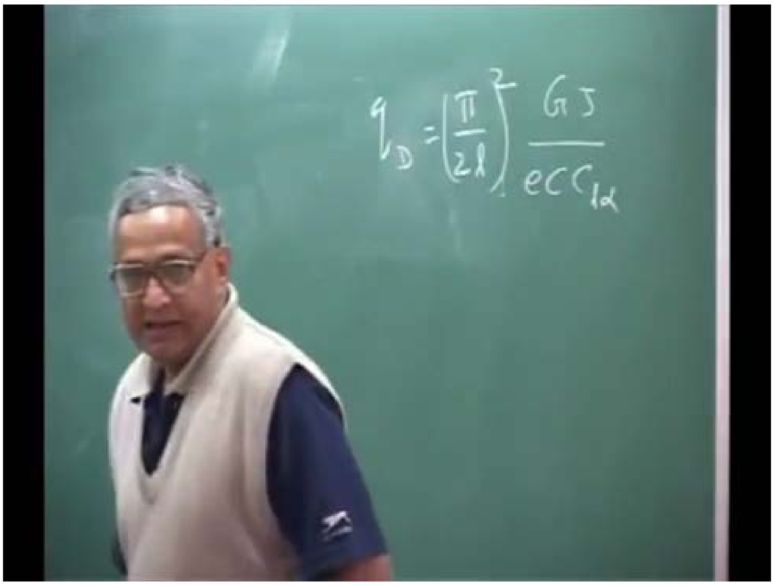

I am going to choose slightly different function, which is I just want to show that, here I write this result keep it, this is pi over 2 l whole square G J over e c C l alpha.

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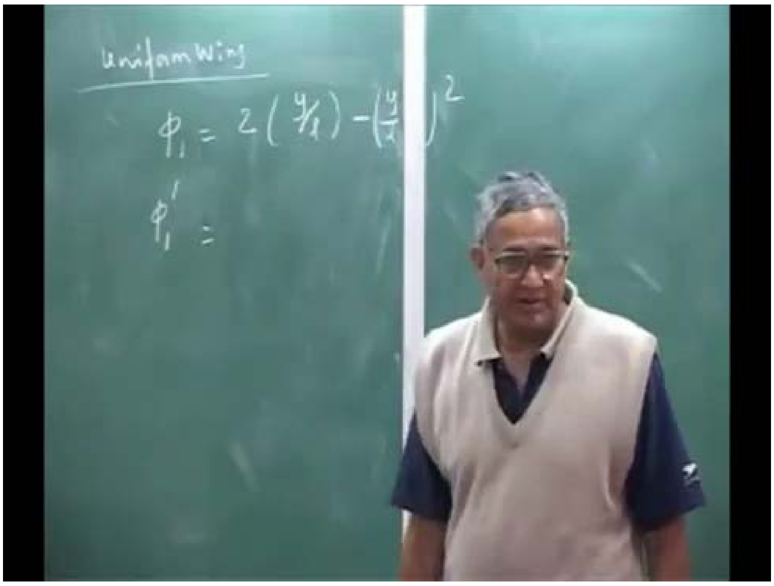

Now, I am choosing a different function, which is 2 y over l minus y over l whole square. Now, you see, this is satisfied both boundary, because the function is 0 that is, it satisfied the boundary condition.

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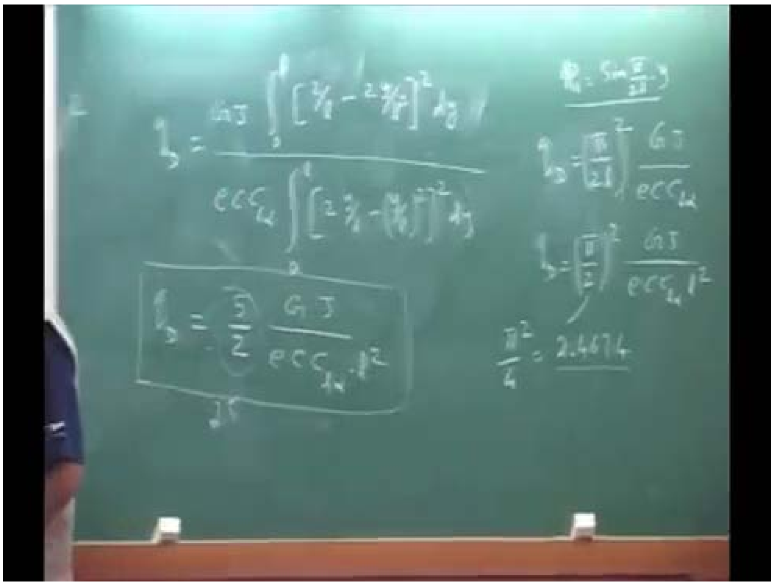

Then, the d phi l by d y at l, this is first term is 2 over l, second term is 2 y over l square that is, 2 y over l square at l this is also 0. That means, I have chosen a function, which exactly satisfy my boundary condition, both the boundary conditions. But, this is not going to satisfy my equation, so let us see, if I use this function, how my result q D will be. So, my q D is what, because uniform wing, I am taking same uniform wing only, you will have G J integral 0 to l.

We want prime, this is nothing but, 2 over l minus 2 y over l square whole square d y divided by e c C l alpha 0 to l and the function, you will have c 1 square, which is 2 y over l minus y over l whole square square d y. Now, you integrate all these things, I leave it to you for integration, I will write the final answer, which is essentially pi over 2 G J over e c C l alpha into l square, this is pi over 2 value. Now, let us look at the other results, which I assumed sin pi over 2 l that is, here I assumed, the sin pi over 2 l into y.

For that, I got this answer, this answer if I write it, this is pi over 2 whole square G J by e c C l alpha l square. If you compare these two results, pi square over 4, this value is actually 2.4674, this is 2.5. Now, ((Refer Time: 54:59)) approximate solution, but of course they satisfy the boundary condition and able to get the divergence dynamic pressure, which is very close to exact solution. So, please understand, this is the approach which is adopted for Rayleigh Ritz method.

Rayleigh Ritz approach, you first choose your function, the approximate function, make sure this satisfy the boundary condition. If they satisfy the boundary condition then you will find the results are particular, you will get a divergence, that is why you choose several function, keep on show, calculate the same value, but this is with one mode only I consider. Because, if you really solve the actual wing problem, you need to take more number of modes, you cannot do only with one mode.

But, you can use sin in that 2 n plus 1 over 2 pi, that value you can choose, many sin functions you can choose. And that is the approach you have to use, because I will give a actual wing theta, which you have to calculate a diverging speed for that wing. Yes, this is of bending divergence, there is a bending divergence, that will come to swap wing. When you do swept wing, but here it is purely a torsion problem please understand, whereas when you couple, bending and torsional problem that is, that will happen only when you sweep the wing, that part we will do later.

Because, swept wing, how do you treat the diverging problem for the swept wing, because here they will be independent. Bending deformation is, you can split it from torsional, because there is no bending component coming into picture in your torsional equation. The deformation of bending does not come into picture, only the angle of attack, angle of attack is purely torsional. Whereas, when you go to the swept wing, the bending deformation will come as the change in the angle of attack then these two are bending and torsional are getting coupled.

Right now, for this problem, that is why we took I wrote in the beginning, it is a straight wing, not a swept wing, swept wing problem we can study in next. What we have till now seen is the approach using Rayleigh Ritz, now we see how do you treat apply the Galerkin method. So, I am going to describe Galerkin approach for this same torsional divergence problem.

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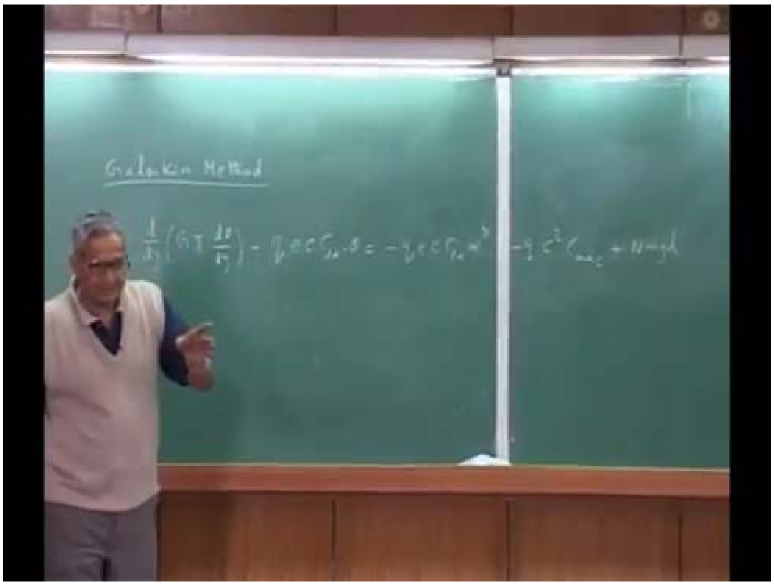

And have we know our torsional motion equation d by d y G J d theta d y plus q e c C l alpha theta equals minus q c C l alpha alpha r minus q c square c m e c plus N m g d, this is my torsional equation. I am going to, how we use Galerkin method to solve our diverging problem, because we taught Rayleigh Ritz. Please understand, Rayleigh Ritz you start from strain energy pressure load done by the external load, whereas in Galerkin approach, you directly start from ((Refer Time: 01:00:27)) in different situation. Now, what we do is, we have to do some non dimensionalization here, because that makes it relatively simpler problem in the sense, compact form.

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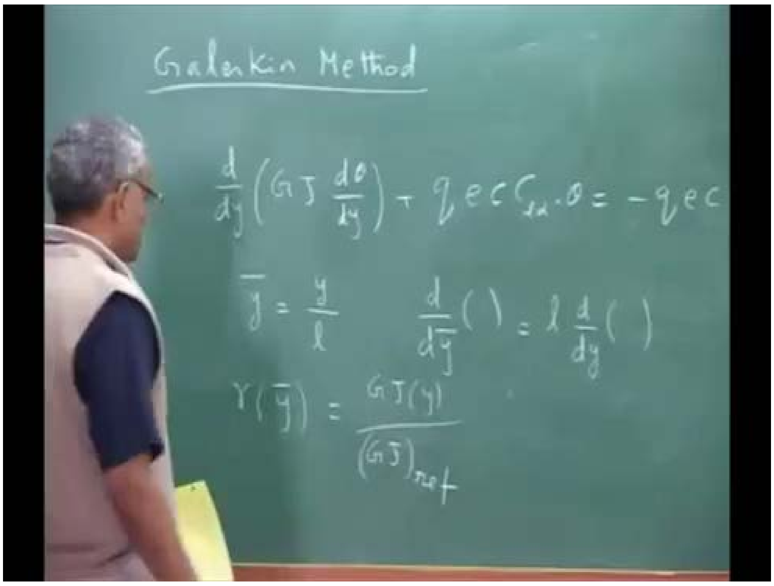

So, I am going to use a non dimensional y bar is y over l, where l is the length of the span, span of the wing. Then you will have d over d y bar of any quantity is nothing but, l d by d y of any quantity, because y bar is y over l. And then I am going to use gamma y bar as G J, which is the function of y by some G J reference, I am going to use a reference quantity.

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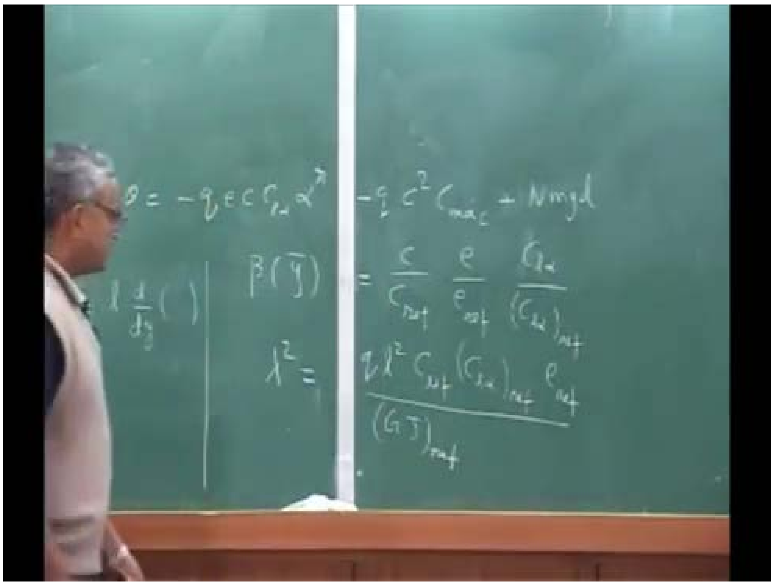

Similarly, I am going to use another beta y bar c over c reference e over e reference C l alpha by C l alpha some reference. Then lambda square equals q l square c reference C l alpha reference e reference over G J reference. Please understand, I am defining this type of quantity, now I simply go wherever d over d y I will put it, 1 over l d of that function d y bar, I will replace.

And then I will divide by some reference quantities, this entire equation is converted into, I will write full expression. Then you will know what I have done then I will write it in a similar form, this is for just a little algebra, going to be little long.

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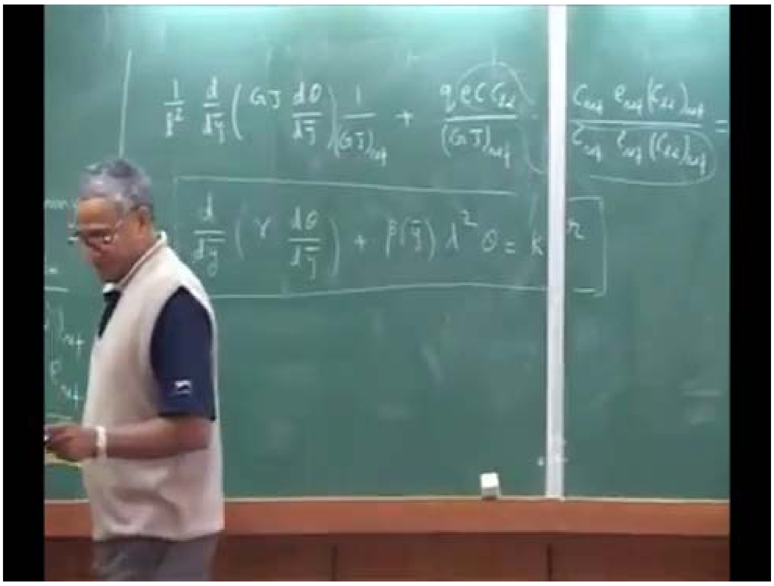

So, first term will be 1 over l square d over d y bar of G J d beta over y bar, I am dividing by 1 G J reference, the entire equation I will divide plus q e c C l alpha over G J reference. But, here I multiply c reference e reference C l alpha reference, again divide by same, c reference e reference C l alpha reference equals minus q c G J reference into e C l alpha alpha r, this is the first term. Then second term q c square c m a c over G J reference then plus N m g d over G J reference.

Now, what you are basically do is, take this G J reference inside then multiply by l square completely then what will happen, G J by G J reference I defined as gamma y. And then here I have defined lambda square q l square c reference C l alpha reference e reference over G J reference. So, I will take q, there is a l square will come, all this quantities divided by this I will write it as lambda square then the theta y c e C l alpha c reference is this, so that is this term.

And whatever on the right hand side, I put it there some other loading, so the equation will become in a compact form, please understand I am writing it only for convenient, gamma d theta d y bar plus beta y lambda square. Beta y means it will be y bar lambda square theta k r, where the term k r which is the right hand side, I am putting thus q C l square, I would say that the entire term is written as the k r, I think I do not have to repeat it again, I do not have to repeat please understand. This entire quantity multiplied by l square that is, the k r, so for convenient, now you will have this equation as your torsional deformation equation. So, we will use this, I erase all the other things, so please understand k r is nothing but, square times the right hand side.

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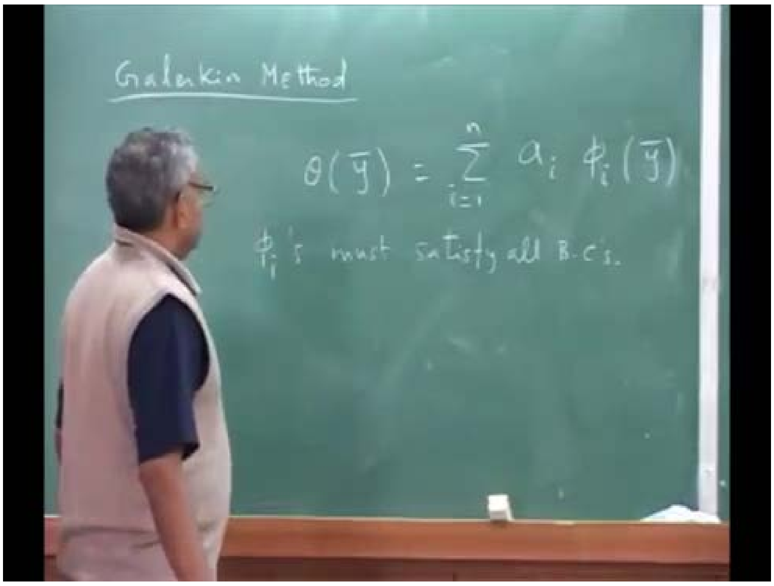

Now, how do we do the Galerkin method, you have to assume theta is the function of y bar, because y is non dimensionalized. This you are writing in as summation i is running from 1 to n, a i phi i y bar, very similar to Rayleigh Ritz, but only thing is, here phi i, they might satisfy all the boundary conditions. So, phi i must satisfy all the B Cs, all boundary conditions, now what you do is, you take this, substitute here and then I erase this one.

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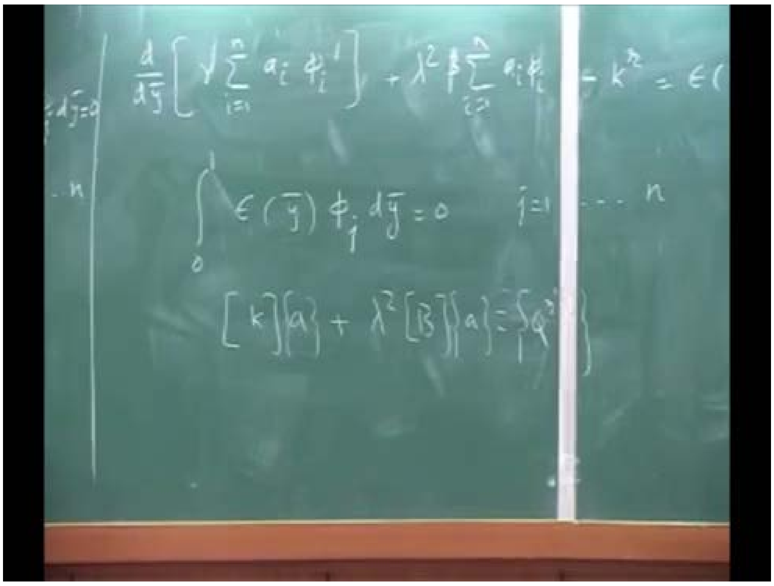

When I substitute that function, I will get d over d y bar gamma summation a i phi i prime, because please understand I am using prime for differentiation with respect to y bar, plus lambda square beta summation theta is, again a i phi i, i running from 1 to n and here, k r. Now, you do not know, this is not going to satisfy my equations, so what I will put it, I take the k r to the left hand side and then say this has an error, which is the function of y bar, k r on the left side.

Now, I say the Galerkin technique says that, my error is orthogonal to ((Refer Time: 01:10:19)). So, I hope that, I substitute this condition, because I have to apply this condition that is, integral 0 to 1, because my non dimensionalized. So, I will have epsilon y bar phi, you can take j d y bar is 0 for j running from 1 to n. So, you are going to have n equation and I am substituting this in that. So, what you do is, you multiply here, put this function in the epsilon y bar, you put this entire thing inside and then integrate what you will get is, I write that part here.

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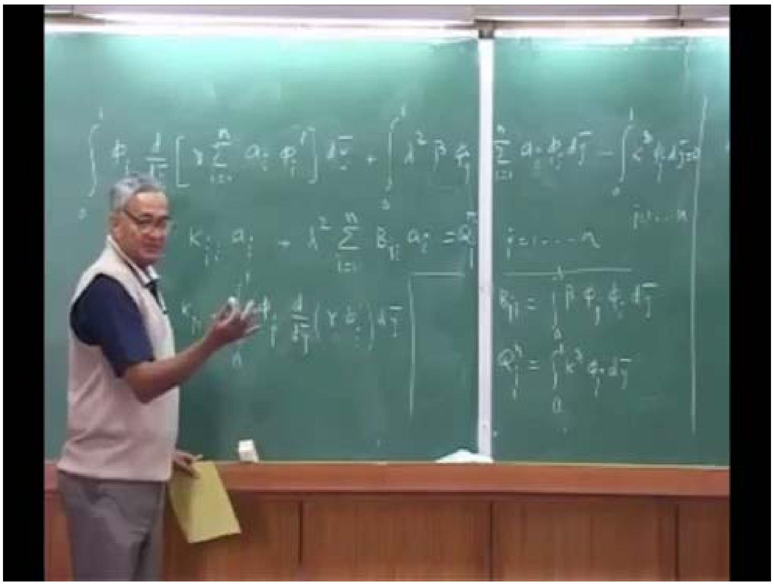

You will have integral 0 to 1 phi j d over d y bar gamma summation i running from 1 to n a i phi i prime d y bar plus integral 0 to l, lambda square beta phi j summation i is running from 1 to n, a i phi I d y bar then minus 0 to 1, k r phi j d y bar equal 0. You will have j again 1 to n, this particulate thing you can write it as in matrix form. The matrix form I am putting it as, summation i running from 1 to n, k j i a i plus lambda square summation i running from 1 to n B j i a i equals Q j r, j running from 1 to n.

Now, this entire thing I can put it in a, you know that k j i is nothing but, this integral 0 1 phi j d over d y bar gamma phi i prime d y bar and then B j i, this is integral beta phi j phi i d y bar that is, this term phi j phi i d y bar into beta. And then the last term is Q j r is, k r phi j d y bar again 0 to 1, now you can put this in completely matrix notation, that will become k a plus lambda square B a is Q r. Here again, you solve the homogeneous part as a Eigen value problem that means, that the right hand side 0.

Because, you have to look for a Eigen value problem and you will get here the lambda, the lowest Eigen value in your divergence speed. But now, I would like to compare this equation with the earlier one, one interesting thing which you will immediately issue is my k j i, please understand phi j d over d y bar of gamma phi i. If I use it in this form that is, phi j gamma phi i prime, there is a d by d y bar, if I change the index, it is not same. That means, my stiffness matrix k is not symmetric, whereas here it is does not matter in the D, that is why the choice of function if you had chosen properly, I can do integration by parts here. Because, this d y if I transfer its here then I will have a, this is basically you u d v, you will get a u v minus v d u. Unless the u v is satisfied at the both the boundaries exactly 0 then only, they will be equal. So, you have to be, that is why this form, Galerkin method of course, if it is a self adjoined operator everything then both Rayleigh Ritz as well as Galerkin will give same.

Because, when I integrate by parts what will happen, I will get this will go to phi j prime and phi j prime phi i prime I can change then there is a same thing. But, you will leave it as it is, it should be non symmetric but then non symmetric matrix you cannot guarantee the Eigen value. See, the real symmetric matrix will have real Eigen values, but if you say real matrix, you can have real Eigen value, you can have complex Eigen value, you do not know, you cannot guarantee.

That is why when you use galerkin approach, you have to proceed in a proper way, this is the way you will get it and you make guarantee. Because, in the swept wing, you will see slowly, but you will get the solution, it is not it is not possible, but these are the fundamental differences between the two approaches. Suppose, if the operators is not self adjoined, that is the reason why we did the self adjoined and all those things in the procession problem in the beginning of the course, two lectures earlier.

This is in the form of, what kind of final matrix I will land up with and just out of the mathematical approach and the beauty, what will happen if the k is non symmetric, how the Eigen value will come. Only thing is, you cannot guarantee, does not mean that you will not get the real Eigen value, you will get, but the only thing is, mathematically you cannot guarantee. So now, you see the difference between Rayleigh Ritz and Galerkin, both lead to the same kind of form, only thing is plus sign and minus sign, that etc, but that will be taken care of with the formulation itself.