Lighter-Than-Air Systems
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Lecture – 98
Equilibrium Analysis of Aerostats

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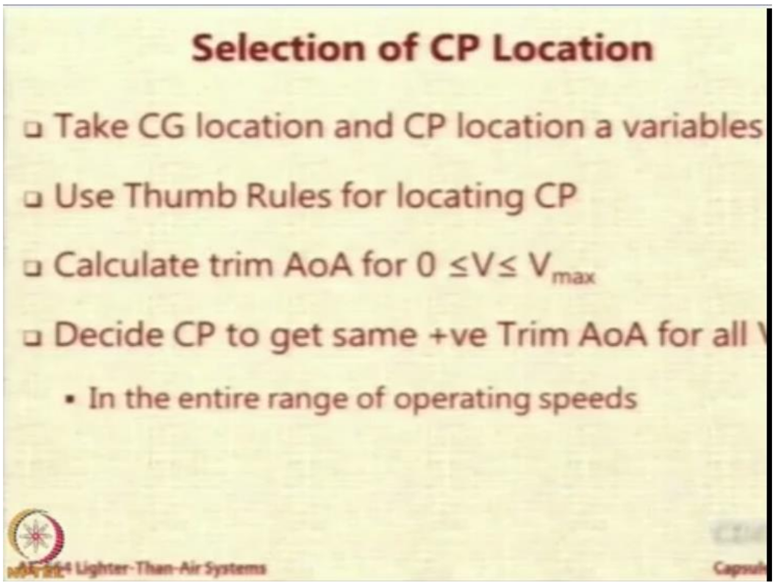

So, by putting the moment, so what we do is you take the CG location and CP as variables that means the centre of gravity is not fixed, by balasting you can change it. So, first you assume at someplace, no ballast, and assume the CP location to be equal to diameter for x and z.

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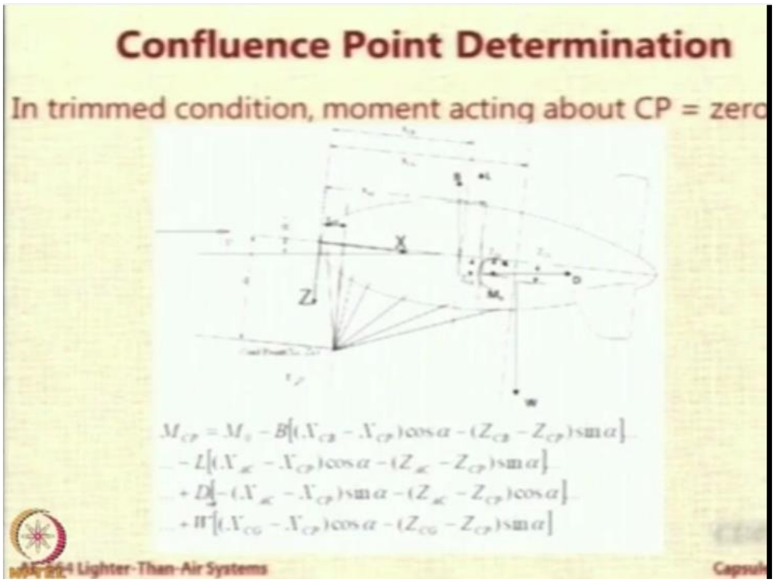

Then what you do is in this calculation you will see that you have these forces L and DSo, what you can do is vary the velocity from 0 to V_{max} which is expected and calculate the α at which it trims. For every velocity, there will be a different α at which it trims.

So, I will show you in the next slide it is possible for you to invade the relationship in such a way that can take a derivative $\frac{\partial C_M}{\partial U} = 0$. That means whatever be the velocity my α remains the same because my CM remains the same it is possible.

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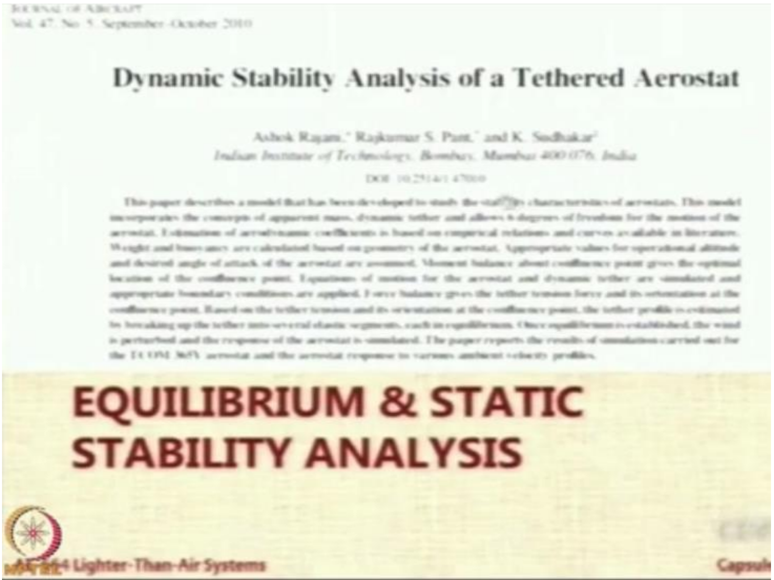

So, to answer this question in detail there is a very interesting paper written by a B. Tech student called Ashok Rajani. Interestingly, last week he got married and I got a card from him. So, Ashok Rajani did his B. Tech here in 2008 and 9 I think and with also Sudhakar as the co author we did some work on the dynamic stability analysis of a tethered aerostats system and it appeared in Journal of Aircraft. It is a big achievement for a B. Tech student to publish a paper in Journal of Aircraft. So, I am very proud of this guy.

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II. Historical Development of Modeling and Simulation of Aerostats

So, you can see what he has done. You will see many things which are already discussed with you. So, he has done a very good work on the historical developments in modeling and simulation of aerostats. A very nice piece of work where all papers were reviewed very nicely.

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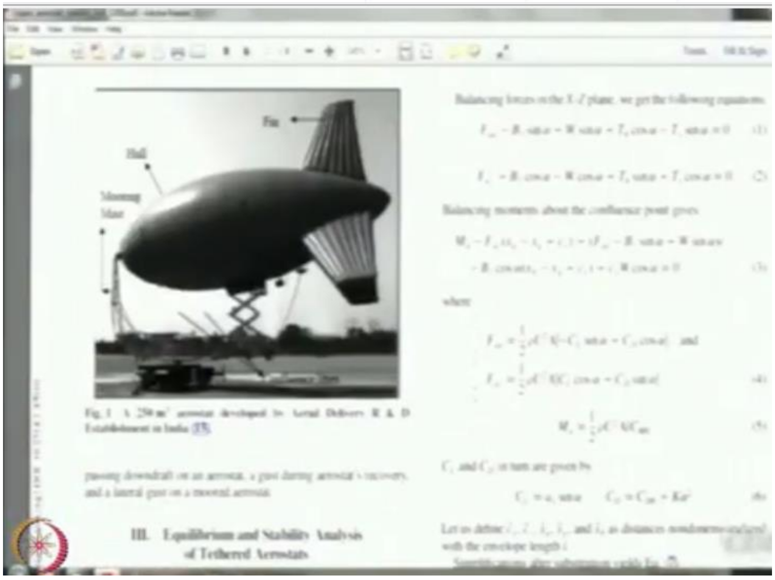

III. Equilibrium and Stability Analysis of Tethered Aerostats

Then he looked at the ADRDE Aerostat.

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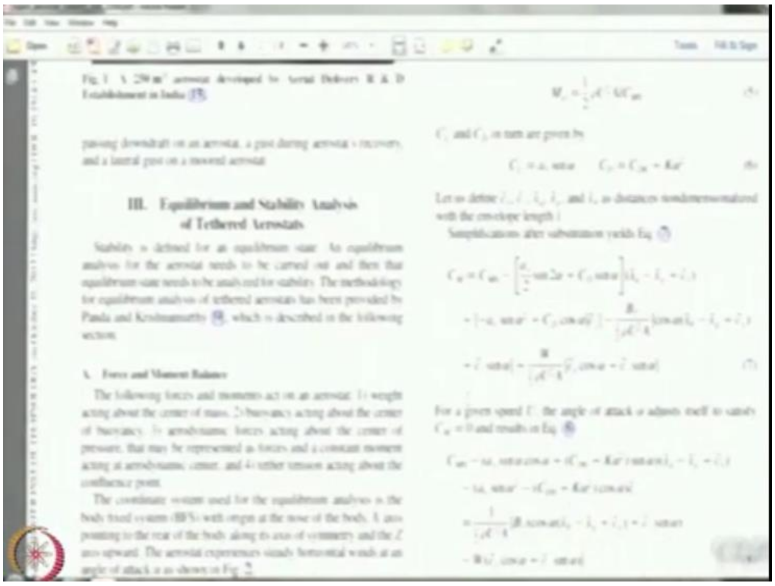

And he did the equilibrium analysis. But this analysis was done based on a method given by a professor Krishnamurthy and Panda. This paper has come from NAL Bangalore. Professor Krishnamurthy is a professor of Aerospace Engineering at IIT Kanpur and then towards the end of his career he shifted to Bangalore to NAL and there he supervised the some scientist called Panda to work to derive the expression which I just told you.

So, Krishnamurthy and Panda's reports is a very famous report which gives basically the picture which I showed you. Then it gives equations for the forces, the moment equation and the force equation.

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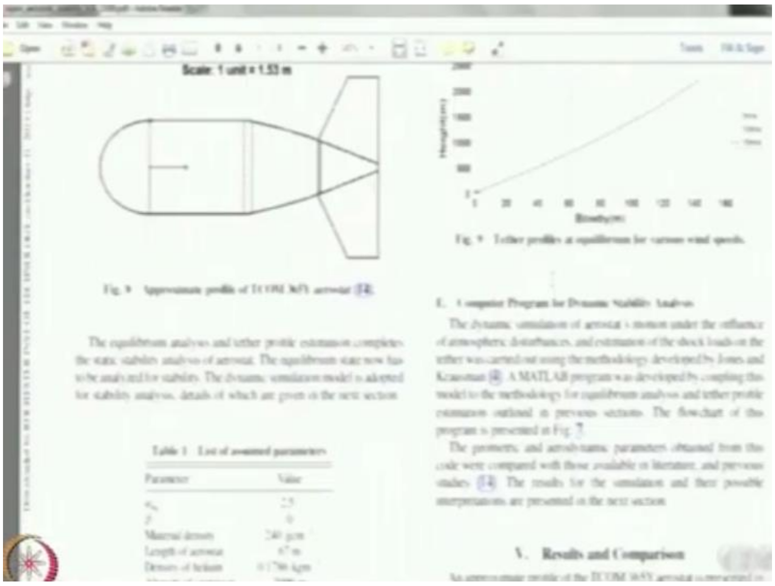

And finally, if I just save some time in the end you will see there is a very interesting chart which he gives. So, I will upload this paper and you can read it and derive the expressions and use them.

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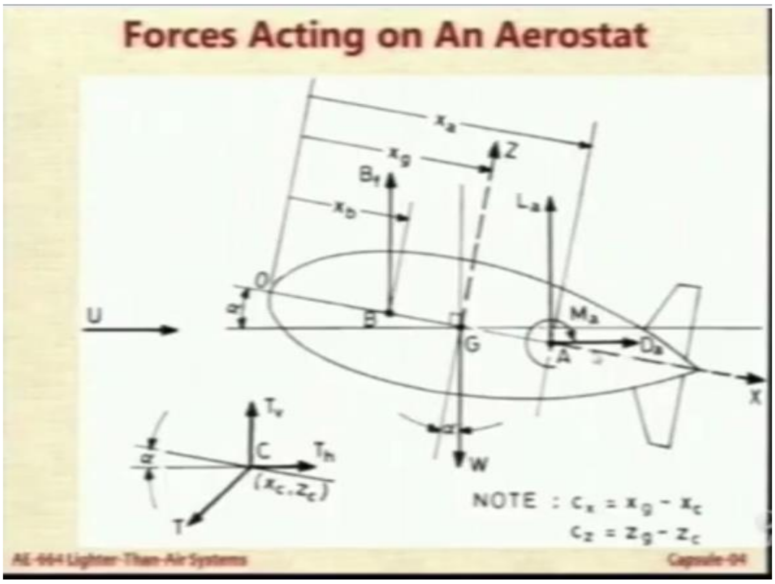

So, this is a picture from that particular paper which is very similar to what we saw, but there are some minor changes. Now, the locations B, G, etc are all variable, do not worry about them. By looking at the value of X_g and X_c you can always change the location.

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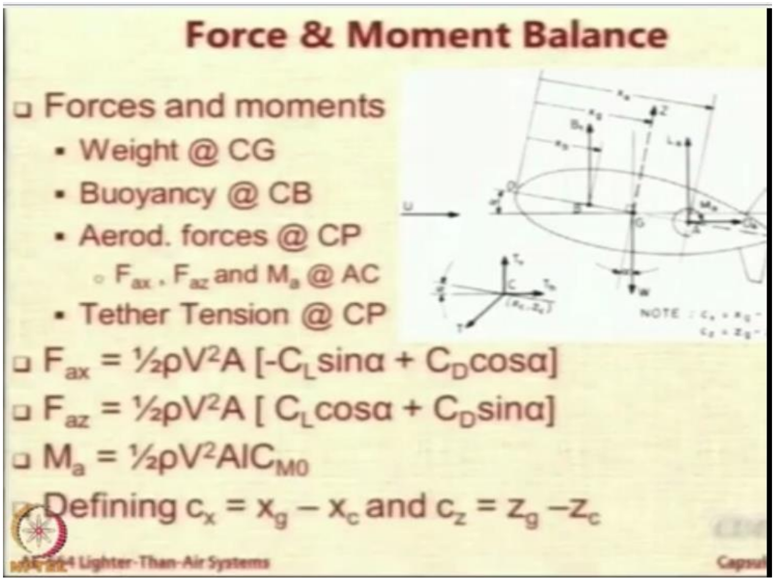

So, the forces are weight, buoyancy and aerodynamic forces. These forces can be converted into x force, y force and a moment and the tension. So, if you take the balance of the forces along the x direction, you will see that F_{ax} will be equal to this force L_A and this will be α. So the horizontal force will be basically L C_L sin α and C_D cos α.

And if you take the vertical force will be $(C_L \cos\alpha + C_D \sin\alpha)\frac{1}{2}\rho v^2 A$. And moment as I already showed you. And then he defines the differences as C_x as C_z which are the locations.

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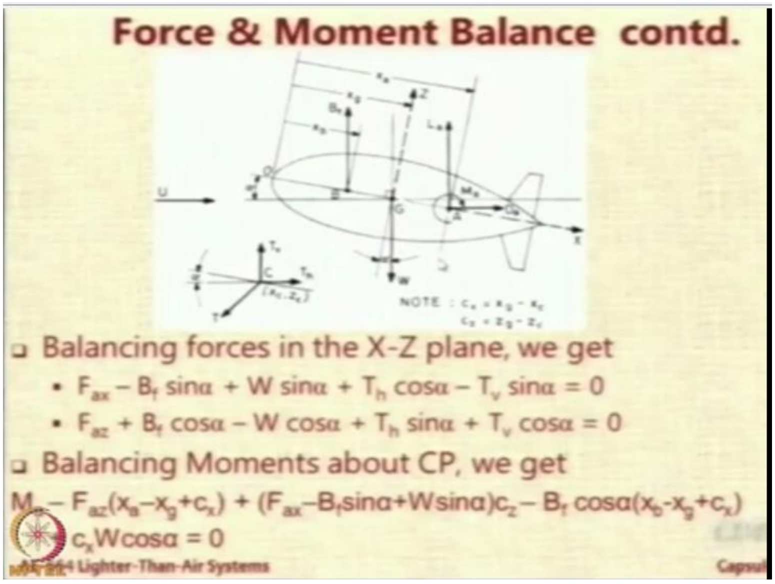

So coming back to the same equation let us do this derivation ourselves. I think it is important for us to get the feeling. So, can you look at the figure and help me in force balance in the X-Z plane? So, can you get this expression please? Now do not copy it down blindly, do the force balance. So on your notebooks I want you to carry out the force balance. There you go. Are you able to see? Now you can see?

So, take the balance of forces along the x axis or along the center line of the envelope. So, remember the forces that you will consider will be B, B f or the buoyancy force. It will have 2 components. Similarly, you have W, $W\cos\alpha$, $W\sin\alpha$. Then you have L again $\sin\alpha$ and $\cos\alpha$ and then you have D $\sin\alpha$ and $\cos\alpha$. And then you have this T_v and T_h which are the vertical components of the tension and horizontal component.

This is also at an angle θ. So, along x for example you will have $T_h \cos\theta$ acting along the right side which is the positive side and you will have $T_V \sin\theta$ acting in the opposite direction. Similarly, you will have $L_A \sin\theta$ acting on the left direction $B_f \sin\theta$ acting this way and $W\sin\alpha$ but $D\cos\theta$ will be along the direction. So, please take this moments and get the expression.

Similarly, do it for the vertical components. In the vertical components, remember the downward force is positive, upward is negative downward is positive because from center line down is the positive axis. So, you will have negative component of B_f, $B_f \cos\theta$, negative of $L_A \cos\theta$. You will have positive of $D \sin\theta$, positive of the $W \cos\alpha$. You will have negative of $T_h \sin\theta$, sorry positive of $T_h \sin\theta$ and negative $T_V \cos\theta$.

So, with this please get me the expressions for the two forces. The moments are to be taken along the x direction and normal to it. So, along the x direction you will have F_{ax} and vertical will be F_{az}. Those of you have finished, you must start the moment balance now. So, for the moment balance there are these distances X_a, X_g, X_b and for the tether we have X_c and Z_c.

The moments are to be taken about the confluence point for which you will need the distance between the confluence point and B, confluence points and G, confluence point and A. So for that what they have done is they have given you a term called as for example you have a term $X_g – X_c$ which is C_x. Similarly for the gravity you have $Z_g – Z_c$, yeah that is X_c and Z_c. From nose, the location is X and Z for the confluence point. So it is X_c below and Z_c ahead.

Here you do not seem to be ahead, but their distance is, so Z_c will be draw a vertical line from here up to here and different between that and the nose will Z_c. Ready, you have done it? Let me show it to you. So these are the two force equations you will get. You can see them in the figure now. F_{ax} is the force minus $B_f \sin\alpha$, it will go on that side to become positive. So, please notice that F_{ax} is the net force which will be made equal to 0.

So, you take all the terms other than F_{ax} on that side you will get the correct signs. For $W \cos\theta$ is shown negative here because it is brought on this side. F_{ax} and F_{az} is actually lift and drag. These are basically the aerodynamic forces acting which are being equated to other forces for equilibrium. So, F_{ax} is actually your D and F_{az} is basically L only. **“Professor - student conversation starts.”** Yes. instead an angle okay.

So, if you see what we are doing now we are basically you know along this direction you have F_{ax}, actually this direction is F_{az}, this direction is F_{ax} along the line and normal to the line. **“Professor - student conversation ends.”**. Now, let us balance the moment about the center of confluence point. So, balance the moment about CP. I will just go ahead because you will be able to get these numbers only and only if you do the calculations.

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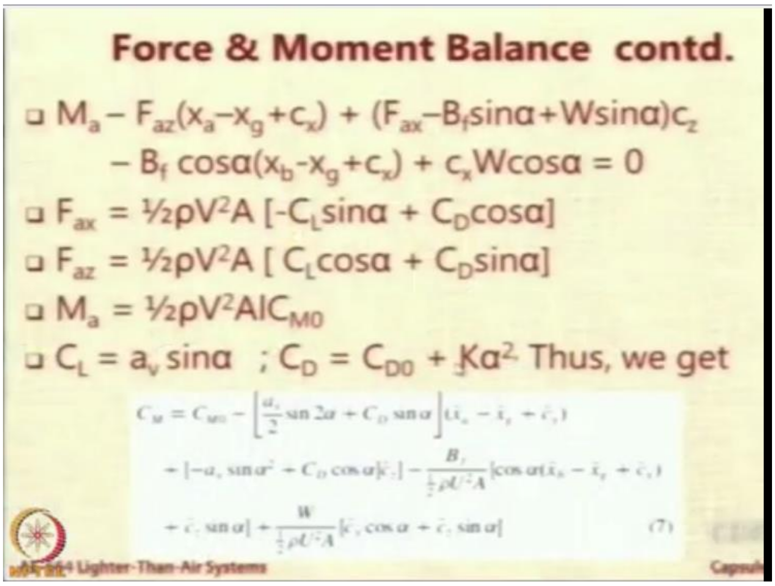

From this point, we will do a small hand calculation. So, you have the notes withyou so you can derive it yourself. Important point is I have copy the equation from the previous slide here to give you the moment balance and then I always bring in the fact that F_{ax} is this, F_{az} is this, M_a is this. C_L is basically lift curves slope and C_D is $C_{D_0} + KC_L^2$, but C_L and α.

So, this K is not $\frac{1}{\pi Ae}$, this K is a constant which gives you the link between C_D and α^2. Understand that C_L versus α is a straight line. So for every alpha there is a corresponding C_L. So, when I have KC_L^2 I can have K\`$\alpha^2$ where K\` takes care of K and the relationship between $\frac{dC_L}{d\alpha}$. So, if you put all the expressions for example if you replace F_{az} with this expression.

I have done this myself, you get expression even in my paper. So right now at the moment those of you were interested to verify go home comfortably, paper is going to be uploaded, look at the equations. See what happens now. Now, let us see how we use this expression to get the value of the optimal location. Now if you look at this expression, C_M is the net moment coefficient, which should be 0 at equilibrium.

C_L now takes the aerodynamic moment of the body which is a fixed number based on the geometry of the body. It is available to you from aerodynamic data, a_v is the lift curve slope which is fixed on a variable. Then you have C_D drag coefficient, which is known from aerodynamics. $\overline{x_a}$, $\overline{x_g}$, $\overline{x_{cs}}$ are basically $\frac{x_a}{L}$, $\frac{x_g}{L}$, the momentum will have an L term when you divide it becomes bar, this is all geometry it is known to you.

So the only variable in this are u that is the velocity and alpha. So what you can do is you can solve this expression C_M equal to zero. For various velocities, you will get various alphas. But if you take a differeciation of this, and if you say that is equal to 0 that will give you an expression which is invariant with velocity.

So then at every velocity, you will have the same value of alpha, which is the equilibrium or trim alpha. And that is a good thing for you because you do not care about velocity as you are flying, you want airship to keep changing the alpha with time. So, the same expression I copied here and I want to show you the variation of the equilibrium angle of attack alpha with velocity for two conditions.

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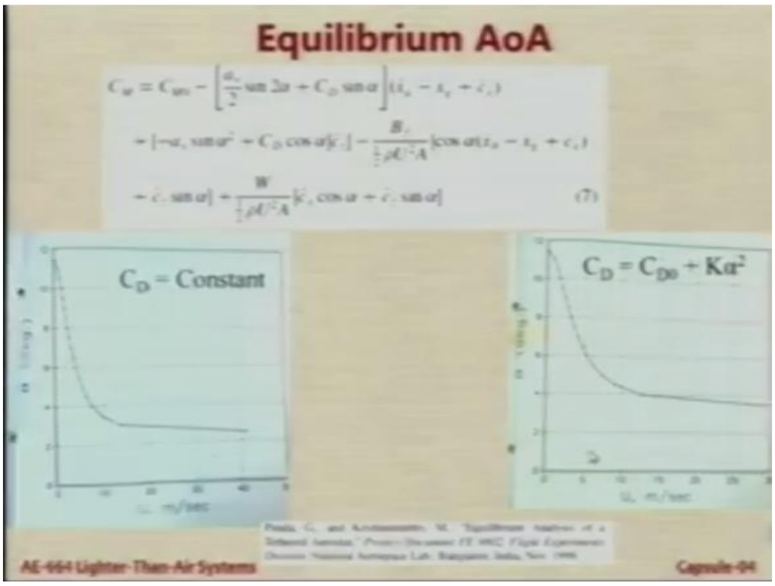

One is a constant C_D condition. That means in this expression you assume C_D to be constant basically K is 0. So, therefore, there is no change in alpha. So, you get this. What we see is at 0 velocity or low velocities, the aerostat will trim at a high angle and as the velocity increases this angle will keep reducing and a time will come when there will be hardly any change.

So, at around 10-12 meters per second onwards the angle is not changing much. So, the aerostat will trim at this angle at low speeds and then come and then it becomes almost fixed. Now, this particular information has come from this Krishnamurthy and Panda's report.

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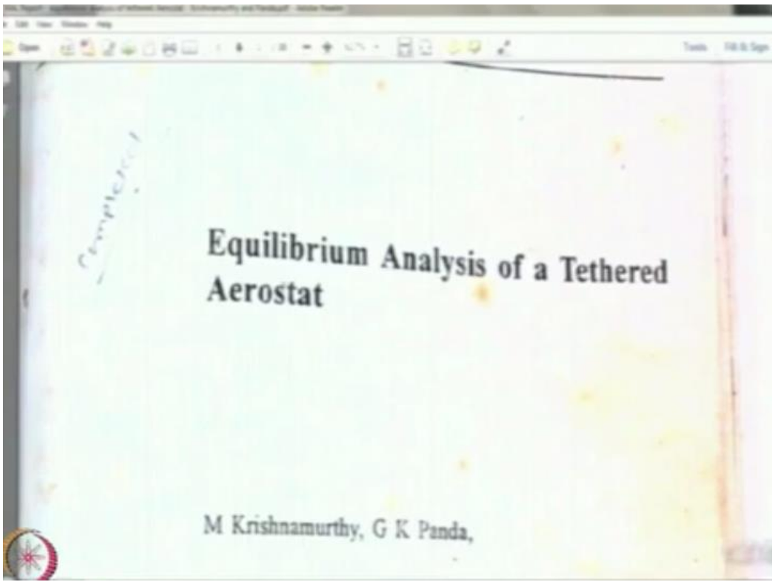

I will just show you I have scanned this report because it is an old report from NAL and in this they have done the analysis. This particular work was sponsored by ADRDE Agra. So, in the end they have actually done the analysis for one ADRDE aerostat and they have given the pictures which I showed you. These two figures are from the same report only.