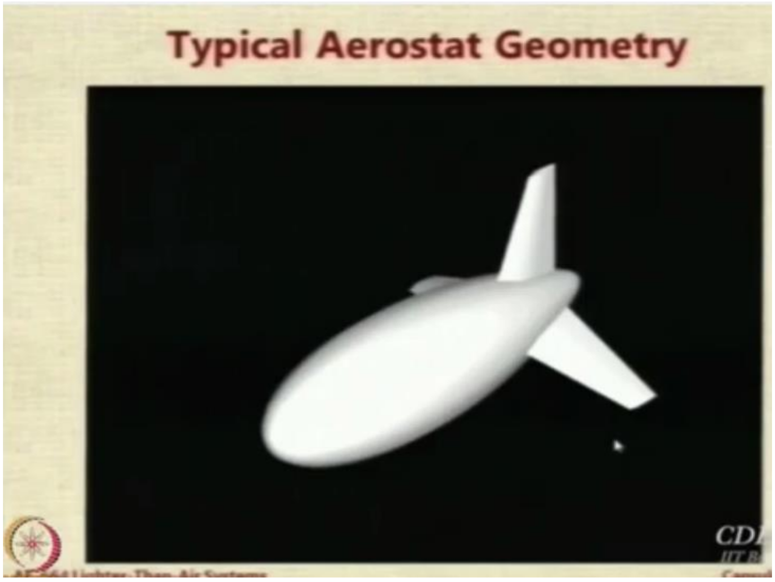
Lighter-Than-Air Systems
Prof. Rajkumar S. Pant
Department of Aerospace Engineering
Indian Institute of Technology - Bombay

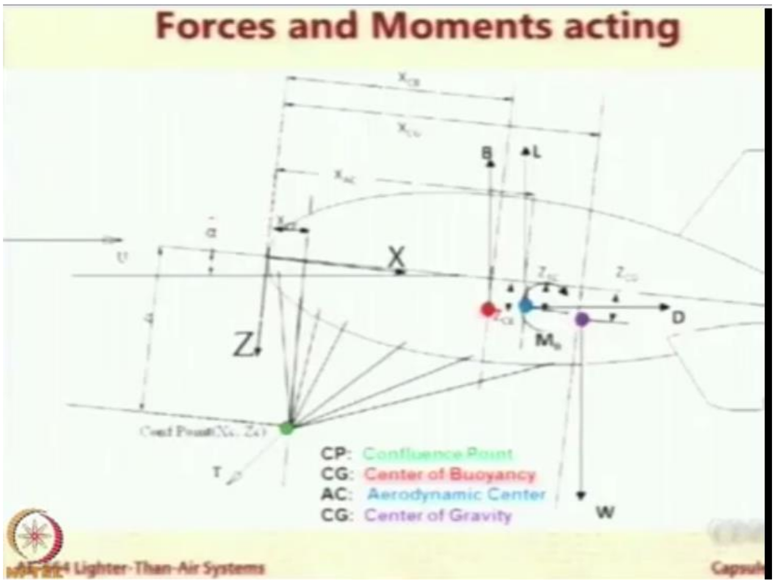
Lecture - 97
Equilibrium Analysis of Aerostats – Part I

(Refer Slide Time: 00:15)



So now let us assume that this is the typical geometry of the aerostat that we will be using, which means we are having one envelope and we are having 3 fins on the back. And I want to now explain to you the steps required for arriving.

(Refer Slide Time: 01:18)



First thing that we have to do is we have to do the equilibrium analysis. And this equilibrium analysis is done to arrive at the desired location for the confluence point. If you want to bypass this, you simply say confluence point is decided is equal to diameter ahead, behind and diameter below the envelope. You will get a reasonable answer, but it will not be the optimum.

But by doing the analysis, which I am going to show you, you can arrive at a much better location. For this, we need to work out the forces and the moments which are acting on the balloon. So do not worry about this picture. I am going to upload a few papers where the whole thing has been derived very nicely. In the class today, we will just look at the basic principles.

So, this is picture of an aerostat, I admit it looks very imposing because there are so many things there. But let look at it. There is an envelope with fins on the back and there are these confluence lines and then there is a confluence point at which tension T is acting on the tether which are very important. Now under the action of the ambient wind and depending on the confluence point location and the tension, this aerostat is going to automatically acquire some equilibrium angle of attack.

It may not be 0 degrees like when you fly a kite it trims at a particular angle and remains like that, correct. So, under the action of external forces, the aerostat will automatically acquire a equilibrium position. So, what is true or what is the condition which gives you the equilibrium? What should happen for it to be called as under equilibrium? Which means there should be no unbalanced moments, which means that the moments should balance.

So there will be no net moment because if there is a net moment, then it will keep on deforming or it will keep on displacing. So, one important point in our analysis is the confluence point, I call it as CP. Please do not mix up with center of pressure, this is confidence point that is shown by the green point. The next important point is the center of buoyancy. The center of buoyancy basically is the geometric center of the location where the buoyancy force acts.

It is equivalent to the center of volume of the whole envelope. If that is the case my question to you is why is it not along the center line? Because this is axisymmetric body, why is it below? Fins, because the fins are also assumed in this case to be filled with the LTA gas. Now here I must tell you something some aerostat manufacturers have told me that they do not put gas in the fins, they put air in the fins.

One reason is that they tend to fly off if you put gas, it has its own buoyancy. So they say that it is constantly under tension. Whatever the reason, generally I have heard from many people that we do not fill the fins with LTA gas, we fill it with air. If that is the case still the center of buoyancy will be below because if there are no fins, center of buoyancy will go to the center of volume. But anyway fins whether they are air or gas or anything, it will be off center.

So in this figure we have shown them to be off center because there are two on the bottom, one the top. Recall the figure I showed you one on the top, two the bottom, so it will move slightly below. Because if they are filled with air, air is going to exert some weight. So, if you consider the whole system including fins and the envelope as one system having some buoyancy it will not be at the center.

"Professor – student conversation starts." the equations are equal. Equations are equal but on top we have one and on the bottom we have two. If it go bottom and top would it work like when the top length this picture. Yeah, the volume of the fins on the bottom is twice that of the one on the top. **"Professor – student conversation ends."** All three fins are identical, same volume, two are below the center line, one is along the center line. So, in our general case let us assume they are filled with gas.

So therefore, there will be an off center center of buoyancy. And you know it is a point and its location from center of buoyancy is called Z_{CB}, Z_{CB} could be zero, it could be negative, it could be positive depending on the geometry. The next important point is the aerodynamic center. This is the point at which the net aerodynamic force and the aerodynamic moment will act. In general, we have shown it as the blue dot.

Notice that there are two forces L and D, lift and drag acting at this particular point and there is also a moment M_0 which is acting. But this is a classical aerodynamic which you have studied, any aerodynamic body subjected to alpha, angle of attack and ambient wind U which is coming, it will have net reaction. This net reaction can be replaced by a moment and lift and drag forces in such a way that at any angle of attack, the location does not change.

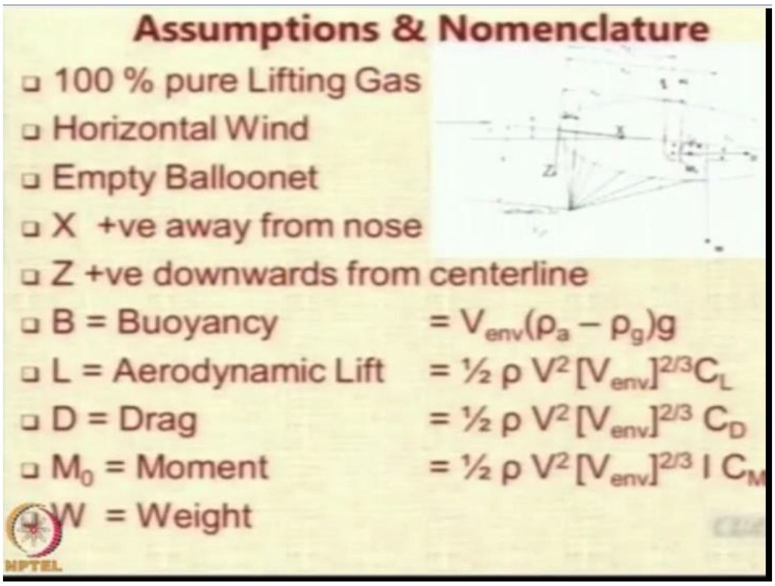
Otherwise, if you take only the net reaction, its location will change as the alpha changes. So, to take care of that we say there is something called an aerodynamic center where the forces will change but there is a moment and that takes care of the change in its magnitude with angle of attack. So, we have this aerodynamic center and then there is center of gravity which is the center at which the weight acts or the location at which net weight can be supposed to act.

In this case, we have seen the center of gravity is not only below but also behind the central point because there are fins mounted behind. But you can easily move the CG forward or backward or upward by putting other things by ballasting. Suppose you want the center of gravity to come exactly below center of buoyancy, what will you do? You want the center of gravity shown by the purple circle to be exactly below the red circle.

Yes, you can ballast by adding something in the front. If possible, you can put payload in the front, you can hang it in the front if possible or you can put any other items somewhere, even put some batteries, etc. In many aerostats the user is able to attach the payload or items directly on the balloon by local strengthening. In most aerostats or I should say in most small aerostats we do not do that.

We mount the payload at the confluence point so that you are not causing any great change in the moment because of the payload. So in most aerostats that you will see the payload will mount at the confluence point.

(Refer Slide Time: 09:06)



So if this is the case, I am just taking we are making some assumptions in our analysis which I will show you. We assume that the lifting gas is pure. We assume that the wind is horizontal, coming at an angle but we are assuming dynamically changing wind, etc. We are assuming an empty ballonet, so we do not want to look at the ballonet dynamics. There is one paper which talks about ballonet dynamics, dynamics because of the air in the ballonet changing with temperature.

So the same figure that I have just copy pasted here in the small. So sign convention is that X axis is positive away from the nose and it is 0 at the nose. Z is downward, positive from the center line. And Y we do not worry, we are not worried too much about the Y. So why is that? Correct, it is symmetric about that plane and we are not too much worried because we know that the system will automatically weather cocked to the changing wind direction.

By putting a kind of a bearing at the confluence point as the wind direction changes, the aerostat can be expected using fins to automatically align. So, we assume that the aerostat is automatically going to be in equilibrium along the Y axis, so we do not care and it will never have a yaw angle and remain constant. It will automatically rotate, but it can have a pitch angle which can change. So, that is what we want to calculate.

The buoyancy is nothing but $(\rho_A - \rho_g)Vg$. Aerodynamic lift is going to be

$$L = \frac{1}{2}\rho v^2 C_L S = \frac{1}{2}\rho v^2 C_L V^{2/3}$$

S in LTA systems is normally $V^{2/3}$. And similarly D will be

$$D = \frac{1}{2}\rho v^2 C_D V^{2/3}$$

Now, do you understand the meaning of this $V^{2/3}$ why is it used?

This is a very important aspect about aerodynamics of LTA systems. It will give a term which is a theoretical term, but units are meter square, correct. See you can use the cross sectional area of the envelope also as an area. You could use surface area of the envelope also as an area. But LTA systems, we know that the buoyancy is a function of volume not area, not cross sectional area.

Therefore to bring into volume as the important parameter, we need to bring in volume, but if you bring volume it becomes V^3. So, this is a very interesting manipularion. You say $V^{2/3}$. So, the term is meter square, but volume is present. So, as volume is doubled it is as if the area is increasing. So as long as we understand that we are doing this, you understand, I understand there is no communication gap.

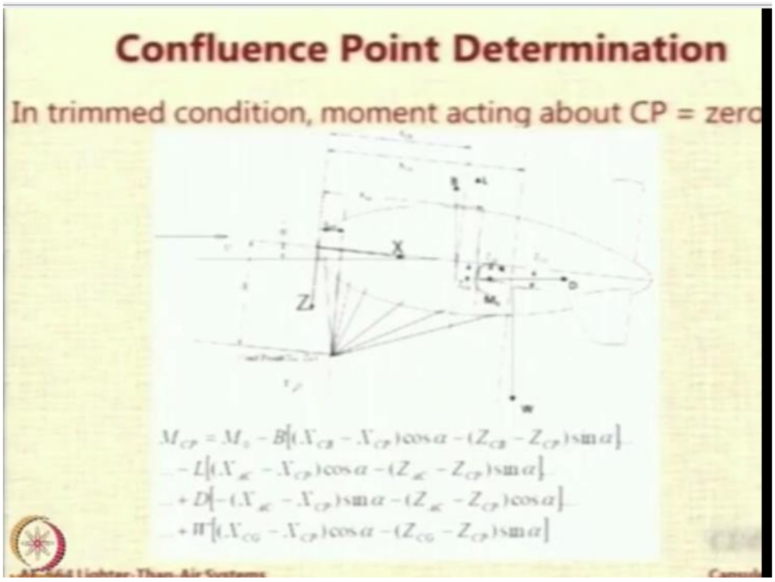
But if you do not understand and you start calculating and you say area, surface area. So what we have done is this the drag coefficient using $V^{2/3}$ as the reference area is called as C_{D_V} volumetric drag coefficient. And if somebody says no I will use surface area it is called a C_{D_S}. For example, I will tell you when students do calculation in Fluent or any other CFD software, they want to calculate drag of the aerostat or airship.

They will put the grades around the envelope, fins, etc., they will run the analysis. Fluent will give you drag. Now, Fluent inherently assumes cross sectional area as the reference term for area, but when students go quote that number and use for aerostat, they will see hardly any match. So, we have to be very careful in defining these terms. So, this C_L and C_D are volumetric C_L and volumetric drag coefficient, when we use $V^{2/3}$.

$$M = \frac{1}{2}\rho v^2 C_{M_0} V^{2/3} l$$

where l is the aerostat length. So, if the only forces acting are B, L and D; buoyancy, lift, drag and tension in tether that is also a force which has two components vertical T_X and T_Z will also be two components of T and weight of the of the whole system.

(Refer Slide Time: 14:04)



So, we know that when this aerostat comes to a trimmed condition at that time the net moment acting about CP will be equal to 0, only then it can be called as trimmed. So, what we can do is take this particular picture, all the forces L, B, W with the distances X_{AD}, X_{CG}, X_{CB}, etc., and

I have taken from one of my papers you can show that the moment will be equal to that moment minus buoyancy times this $\cos\alpha$, $\sin\alpha$ all these terms can be easily obtained.

Do not worry about it right now. When you go back and look at the paper, you will automatically be able to derive these terms. So what we can do is if you have the formula for M_{CP} as mentioned here, so who will give you M_0? What is M_0? M_0 is basically the aerodynamic moment. It will come from the aerodynamic data of the envelope and fins. X_{CB}, X_{CP}, Z_{CB}, Z_{CP}, etc., are all geometrical distances.

Two of them are important for you because they are the confluence point location that is X_C and Z_C. Others are all available in geometry of the aerostat.