Lighter Than Air Systems
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Lecture - 29
Calculation of Ambient Air Density

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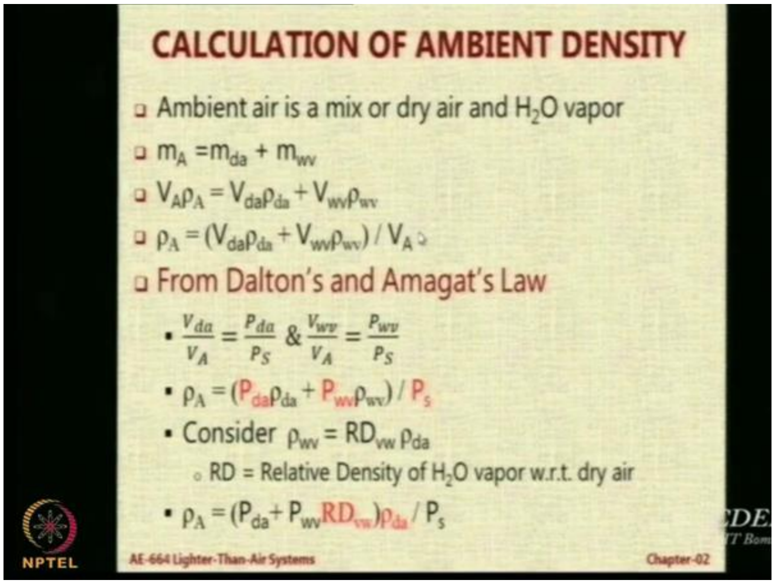

So what we do is we assume and rightly so that ambient air is the mixture of both water vapour and other vapour rather I would say which are dry. And then we say that mass of the air is going to be mass of the two components dry air and wet air. Therefore mass is equal to volume into density. Therefore volume of air into density of air this will be equal to the individual volumes of dry air and water vapour.

So, now we apply the law so just take V_A on that side just take V_A below. If you want derive this here or you can understand. Generally what do we feel, if you want here also we can do it also if you want but I have taken lot of trouble to go step by step here so that you can understand how it is being derived. And once again and saying just observing it on the screen will not help you much. It is better that you also do it down, do it as we as we proceed.

So, density of air

$$\rho_A = \frac{(V_{da} * \rho_{da} + V_{wv} * \rho_{wv})}{V_A}$$

Then we apply those 2 laws so from there, you can get the ratio of dry air volume to the total volume is equal to the pressure of dry air and total air. Similarly the ratio of volume occupied by the water vapour upon total volume is equal to the ratio of pressures of the pressure partial pressure of the water vapour divide by the total pressure of the whole system.

These two equations come from application of the previous law. So I will replaced it there now. So this is

$$\rho_A = \frac{(P_{da} * \rho_{da} + P_{wv} * \rho_{wv})}{P_s}$$

is this clear. We are just replacing applying Dalton and Amagat's law. We are able to get relationship which relates ratio of partial volume to ratio of partial pressures.

So that we put here and we get the expression for ρ_A. And going one step further now the density of water vapour can be expressed as density of air into some relative density term called as RD because density of water and density of air are fixed quantities at Standard conditions. Therefore you can say that there is something called a relative density and therefore you can replace the expression ρ_{wv} by ρ_{da} times RD water vapour.

RD is relative density, purpose of this is to use the numerical information that we have from atmosphere in arriving at the effect of humidity. So what do you get if you do this, please put the expression and tell me.

$$\rho_A = \frac{(P_{da} + P_{wv} * RD_{wv})\rho_{da}}{P_s}$$

So you have ρ_{da} outside by P_s, agreed. So this will actually become crystal clear to you only when you read it when you go back home and when you drive it yourself. So, for this expression ρ_A times this I want to just copy and paste in the next slide.

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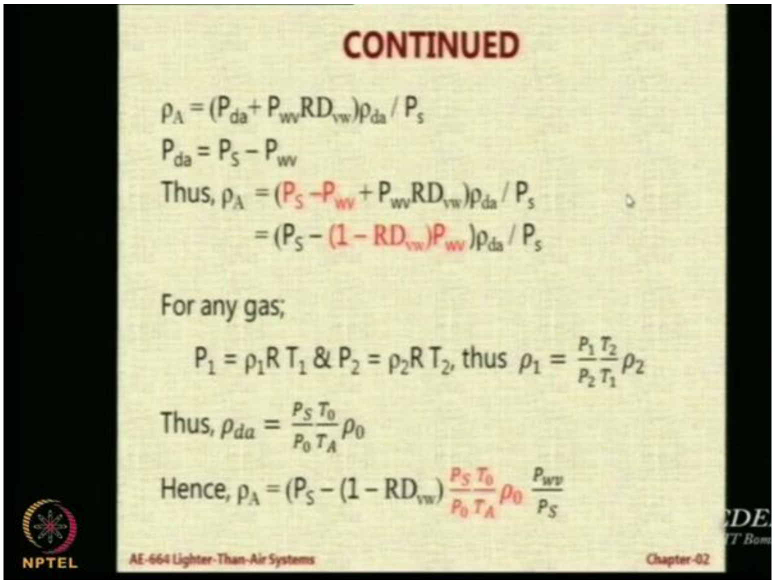

The first thing is the same expression. Now looking further P_{da} is the pressure or the partial pressure of the dry air that will be equal to the total pressure minus pressure of the water vapour that is how we define partial pressure. So you can replace for that P_{da} there. So, ρ_A will become

$$\rho_A = \frac{(P_s - P_{wv} + P_{wv} * RD_{wv})\rho_{da}}{P_s}$$

and now there is one more term that which is common that is P_{wv} water vapour is so common term in the two so you take it out and simplify the expression now.

Finally, we have to apply the simple relationship for any gas. So, $P_1 = \rho_1 R T_1$ and $P_2 = \rho_2 R T_2$ but these R is universal gas constant. So, with that for any gas ρ_1 can be taken as P_1 divide by P_2 and if you divide these two together, you will knock off R you will simply get the expression which says density at any condition is equal to density of any other condition with the pressure ratios and temperature ratios.

So applying this; now you this you should do yourself. I will not like to show it to you. So, what you do is replace ρ_{da} dry air density. So T_2, P_2 and ρ_2 will be at sea level and T_1, P_1 and ρ_1 for dry air. So replace that in the expression and tell me what you get?

Is this point clear, using the universal gas law for 2 conditions sea level condition and the condition at any operating altitude we are taking the ratio. So I want to replace this rho da which is the density of the dry air by parameters about air which I know at sea level. So therefore

$$\rho_{da} = \frac{P_s}{P_0}\frac{T_0}{T_A}\rho_0$$

you have to substitive this value. You will notice that there is P_s already in the denominator here. It will cancel out.

And what will remain will be? It will become this term in the numerator divide by T_A, P_s will knock off and what will remain here will be T_0, P_0, ρ_0. there you go. I am not knock them yet. I just written down as it is. So, all of them is I just copied and pasted the red block is the definition or the formula for ρ_{da}.

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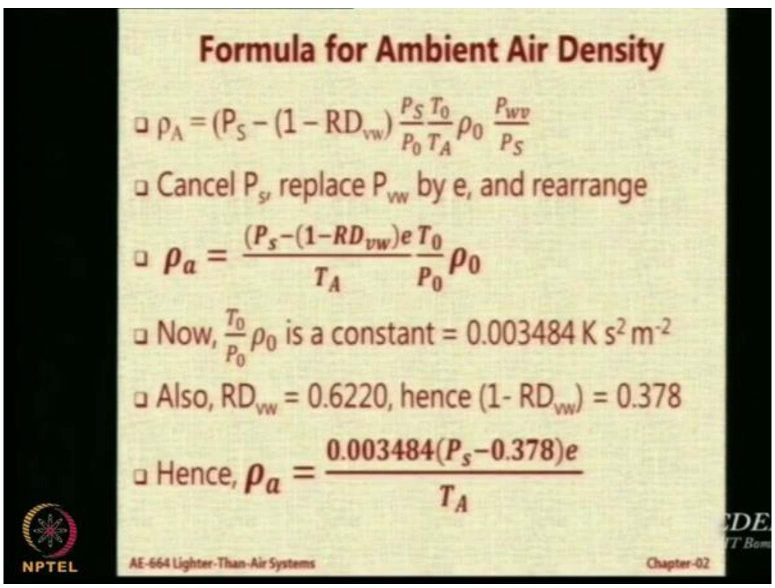

And now, you cancel when you cancel you will get use this expression to cancel because T_A will come on the denominator P_S cancel with the P_S which is below I still not cancelled it. Now P_{wv} is the pressure of water vapour that is called e in our terminology. And P_S will be cancelled so if you replace it you get now a slightly elegant formula for density of air. So the density of air is the ambient pressure P_S which is known to you because you are flying at some altitude minus a term because of the humidity.

This is basically the pressure of the; the partial pressure of the water vapour inside divide by temperature of the ambient air under ISA conditions into T_0, P_0, ρ_0. Now interestingly this T_0, P_0, ρ_0 are constants. So you can replace now these numbers by the numerical value of T_0 which is 273.16+15 = 288.16, P_0 is 101325 Newton per meter square ρ_0 is 1.256 if you multiply that you get a constant 0.003484 Kelvin second square by metre square.

So you can put that in the expression. Secondly this RD_{vw} this is the relative density of water vapour with air. And this number can be measured by experiments as 0.6220 density of water vapour as compared to the air density is 62.2%. So, (1-RD) is 0.378, so therefore what you can do is you can replaced by another simple formula which only has known parameters. Afterall when you are flying the airship at some altitude what do you know?

You know, what is the ambient pressure? Ambient temperature and this e value can be calculated by the law based on ambient temperature, any question? RD_{vw} is basically the relative density. So, density of helium is X density of air is Y ratio is constant at any altitude. It is just the question of relative density one related to the other. So, against air water vapour both of them have density, the relative density is constant.

That constant value is 0.622 so straight away you can now calculate. No, why should be inside yes you are right it is 0.378 into e. So, it has to be brought inside because that is where we went wrong. P_{wv} should come inside P_{wv} should come inside here and and therefore it will be there. Bracket is missing one more bracket is also missing I will correct this. Thank you for pointing out. I will correct this.

So then it is going to be

$$\rho_a = \frac{0.003484(P_s - 0.378 * e)}{T_A}$$

that I should correct immediately so that there is no confusion here. Other things will take sometime. Good you are looking at it very carefully and critically that is good. When I type this formally at that time I might have made mistakes in entering the data.

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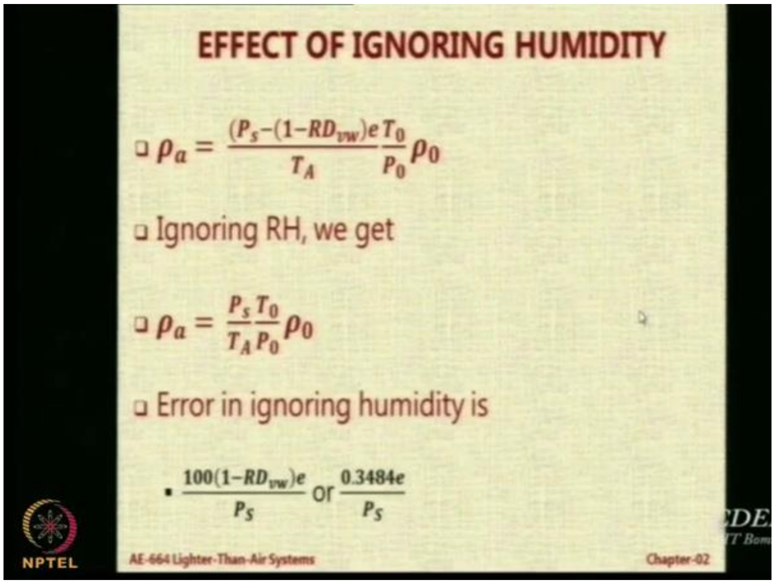

So now we are at the end of that particular place where we can take care of the humidity directly. So, once again e will go inside. So let us see what happens if you ignore humidity. You are doing all these calculations. You must know the reason and if you if there is no substantial difference then there is no point in doing all these things. You will notice that the value of e as you find the graph is very low at low temperature and it becomes very high at high temperatures.

So, this error will be actually more prominent only at high temperatures. At low temperature the value of e is around 1700 and you are multiplying by 0.38 and subtracting from P, P is 101325 so the amount that you minus is very small. So, you know it will not come out to be very different but let us see it is better to calculate the value. Suppose I ignore the value of relative humidity which means I just do not do this reduction of $(1 - RD_{wv})e$.

So I get ρ_a as

$$\rho_a = \frac{P_s}{T_A}\frac{T_0}{P_0}\rho_0$$

So, the error we are ignoring the humidity it will be just changing the pressure upon actual ambient pressure times 100.