Introduction to Launch Vehicle Analysis and Design
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Lecture - 09
Impact of Drag

Hello and welcome. In the last lecture if you recall, we had looked at the implication of gravity along with the propellant burn rate on the overall terminal performance and we also looked at the loss of energy due to gravity and its connection with the burn rate. Now we are ready to bring in the third force that we have considered in our earlier discussion that is the aerodynamic drag and consider its implications. So let us begin.

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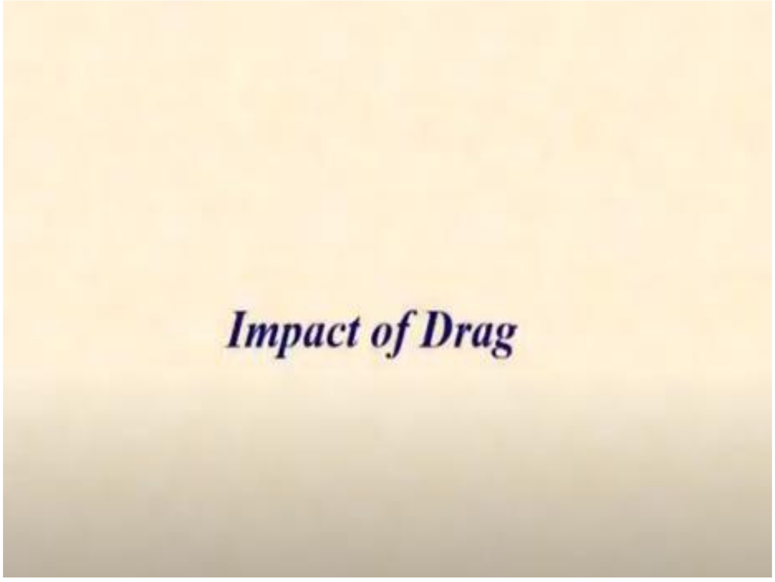

So let us begin our discussion on the impact of drag.

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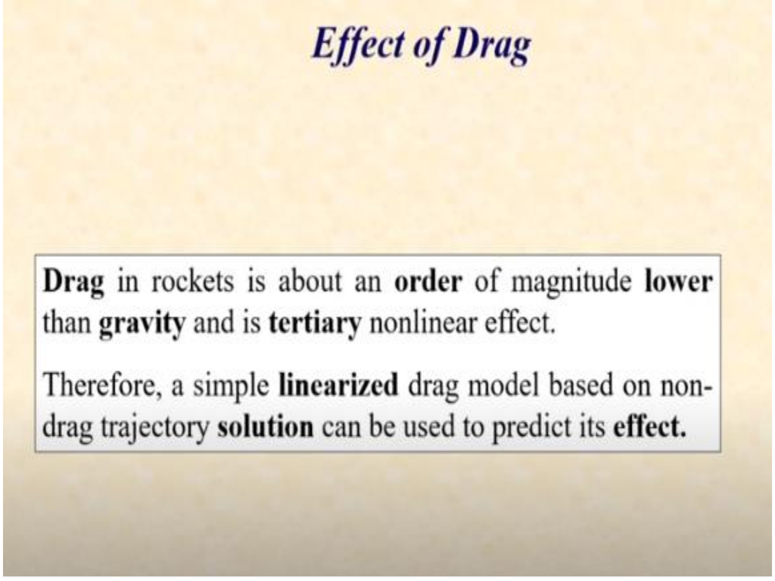

If we look at the drag as an effect which is of a reasonably smaller amount, then you find that for most missions, it will essentially be an order of magnitude lower than gravity and it is treated as a tertiary effect. While in the case of gravity, we made a simplified assumption about the value of the gravity.

Because the drag is an even smaller order effect, we can use an even simpler model to capture the effect of drag which is primarily based on energy concept which we will look at next.

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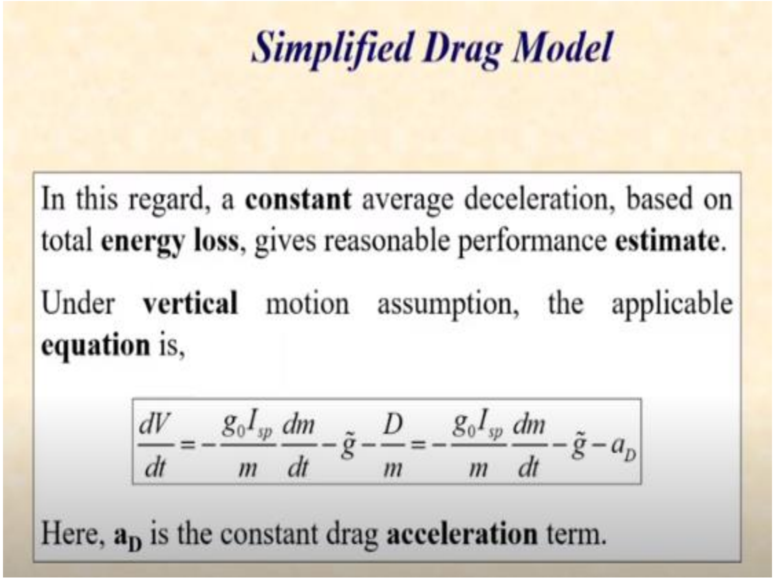

So, the simplified drag model essentially is aimed at capturing the overall loss of energy that will occur during the complete trajectory through the definition of the concept of a constant average deceleration that we can introduce in the equations of motion so that they remain linear.

And we will try to match the overall energy that gets lost because of such a constant average deceleration against the actual energy loss which is likely to happen.

And it is found that such an approach gives a reasonable performance estimate. So, we again assume that our trajectory is along a straight line that is a radial line. Under that condition, we can now rewrite our equations of motion which we borrow from our earlier discussion on equations of motion for gravity and then we just subtract one more term that is D/m on the right-hand side where D is the drag, m is the mass.

Now this $\frac{D}{m}$ is what I call an acceleration or with a negative sign we can even treat it as a deceleration to drag that the vehicle will experience while moving through atmosphere. And it is just a pure subtraction from the value of the thrust. Now in this model, the a_D term that I have introduced is going to be treated as a constant drag acceleration term similar to a constant gravitational term that we have already introduced in the last lecture.

Of course, we know that $\frac{D}{m}$ is not a constant because the drag varies in a particular manner as a function of altitude and velocity. While the mass varies as per the burn rate introduced. So obviously, $\frac{D}{m}$, which is the actual acceleration at each time instant will vary along the trajectory.

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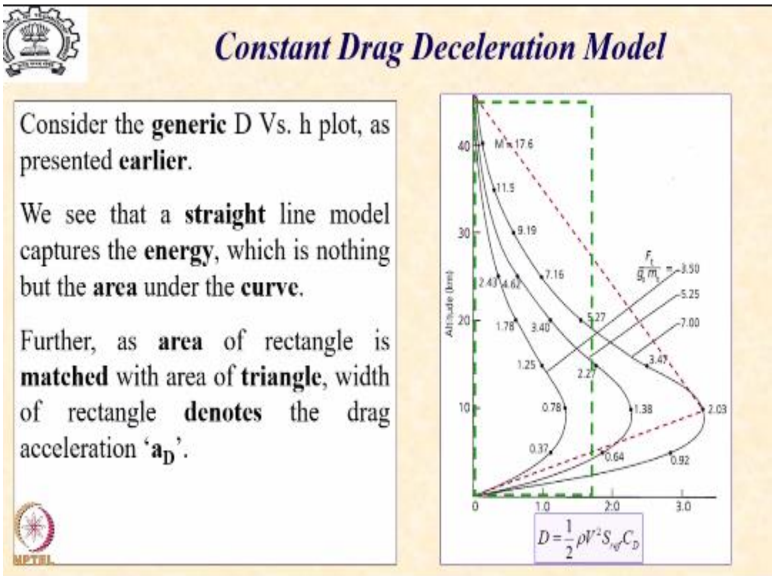

So how does it vary? And here we have something to help us which we have already seen earlier. So let us consider the generic drag versus altitude plot, which we have seen earlier as dynamic pressure versus altitude plot. But now, only it is scaled version. Here you will notice that the x axis which earlier was only the dynamic pressure has now been multiplied with the reference area and the drag coefficient so that this represents the drag variation as well.

You will realize that the variation will remain the same except that it will get scaled to the force units rather than the pressure units because the surface area at least through the atmospheric phase of the flight will remain constant. And the drag coefficient as we have discussed earlier is 1 for a bluff body assumption that we have already made. So, you will realize that the same curve can adequately represent the variation of drag with respect to altitude.

Of course, in the equation that we have written down in the previous slide, the drag is a function of time. But what you would realize is that as altitude itself is a function of time, drag versus altitude is an adequate representation in the present case. Now let us bring in the idea of the energy loss. So as drag is a force and altitude is a distance, from our basic understanding of mechanics, we realize that if I calculate the area under the curve that will represent the total energy contained in the drag term.

Because the area under the curve is nothing but the integral of D that is drag into dh taken over the applicable altitude. In this case, as we have noted earlier also, beyond 40 km altitude the actual drag is negligible so that the area under the curve can adequately represent the total energy that would be lost because of drag. Now we introduce an approximation through the red dotted triangle that is shown in the picture.

Now the reason for introducing this is not very far to seek because the shape of the curve is very nearly a triangle except that it is a curved triangle. By introducing the straight lines, I introduce an approximation. But I have another point. If you see the triangle closely you will find that the overestimate of energy in the upper part by the triangle is to some extent compensated by the underestimate of the energy or the area by the lower part of the triangle.

Which obviously means that in some manner, the triangle can reasonably capture the overall energy that might be lost. The reason for generating this triangle is that once I create this triangle, I can match this triangle with the green rectangle as the next level of energy matching. So, I can say that the area under the triangle which is same as the area under the actual curve is also the area under the rectangle.

And once I assume that the area under the rectangle is same as the area under the curve, then the width of the rectangle is nothing but my average drag acceleration that I have used in the expression. I hope this translation is clear to you. So, once we do this, then it is extremely simple to go from the actual curve to the value of average drag just by looking at certain features of the curve.

So, in this case, because we are drawing the triangle at the tip of the actual curve, which is the maximum value of the drag, by dividing this value with the instantaneous mass at that altitude it will become an instantaneous drag acceleration value which is the peak value that the system will experience.

So, once I introduce this idea of the peak value of the acceleration, which is nothing but the point at which the triangle has the apex, then we know that the area under the triangle and the area under the rectangle will be exactly matched if the width of the rectangle is half of the peak value. This is directly from our triangle relations of area calculation which is nothing but half base into height.

So, half the height is nothing but the total area. As long as I match the height the average value of acceleration is just the half of the peak value by matching the area. And you will realize that that can represent a reasonably good approximation to a smaller order effect of drag so that in the initial estimates, we can always get the impact of the drag for a given trajectory without too much of computational effort, which would otherwise be required for solving the complete nonlinear differential equation.

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Drag Loss Example – Slow Burn

m_0 = 80 T, m_p = 60 T, I_{sp} = 240 s, g_0 = 9.81m/s^2, **β = 600** kg/s, C_{D0} = 1.0, S_r = π m^2. **D** is maximum at t = 50s.

Determine the **impact** of drag on burnout **parameters**.

Non-drag Values: V_b = 2.30 km/s, h_b = 78.3 km,

$\mathbf{h_{50s}}$ = 13.2 km, $\boldsymbol{\rho_{50s}}$ = 0.267 kg/m^3, $\mathbf{V_{50s}}$ = 616 m/s

$\mathbf{D_{50s}}$ = 159.1 kN, $\mathbf{m_{50s}}$ = 50 Tons, $\mathbf{a_{D50s}}$ = 3.18 m/s^2

$\mathbf{a_D}$ = 1.59 m/s^2, $\mathbf{V_{b\text{-}drag}}$= 2.14 km/s, $\mathbf{h_{b\text{-}drag}}$= 70.3 km

With this let us rework the previous example that we have seen in our gravity last case to see what is now the order of magnitude or the impact of the drag on the terminal performance using the simplified expression that we have generated. So let us take the same problem, the burn rate of 600 kg/s and now introduce the three dynamic parameters necessary that is the $C_{D0} = 1$, the surface area assumed to be π m^2.

And one last parameter which is introduced just to kind of use the computational effort at this point is to make a stipulation that the drag will be maximum at roughly around 50 seconds, which is the half of the total burn time of 100 seconds in this case. Let us assume that it would be somewhere around this. The reason why this number is chosen will become clearer as we look at the solution.

But I just want you to keep in mind the fact that the peak of the drag is roughly around 10 km to 12 km altitude range, which means that if we take the time at around the value which results in the altitude value of around 10 to 12 km, then the drag calculated at that altitude will essentially represent the maximum value of drag for that particular trajectory.

It is an approximation, but it helps us to simplify our analysis. So let us try and determine the impact of drag. So let us first recall the non-drag values that we have already seen earlier for this case. That is the final burnout velocity is 2.3 km/s and the altitude is 78 km when there is no drag. And we would like to compare these values with the value of the same parameters when related to the drag term.

So let us now look at the analysis step by step. So let us first calculate the velocity that this particular vehicle will reach in 50 seconds. So, it is the application of the same expression that is the expression with gravity but without drag and we find that this results in the velocity of 616 m/s. The corresponding altitude that is reached is around 13 km which is not bad.

You will find that by and large the drag will peak around this altitude for most of the trajectories. And then we go to the atmospheric tables that are readily available and from that atmospheric table, we read out the value of atmospheric density at this altitude. So, which is $0.267\ \mathrm{kg/m^3}$. Now the next step is to calculate the drag which is nothing but $\frac{1}{2}\rho V^2 C_D S_r$ at 13 km altitude.

So, if you do this calculation, you will find that the drag at this altitude is 159 kN. Further because we have proceeded up to 50% of the total burn time, we have consumed half the propellant, which means about 30 tons of propellant has been consumed. So, the effective mass at this point is only 50 tons. Based on these two quantities, we now simply calculate the peak value of the drag curve as $3.18\ \mathrm{m/s^2}$ and compare this number with the gravitational acceleration, which is about 9.81.

It is one-third of that. But more importantly, because we are going to use an average value, it is not a constant it is only the peak value, the average value is just half of this. This comes out to be $1.59\ \mathrm{m/s^2}$. Now you can see that this number is practically one-sixth, almost an order of magnitude lower than the value of the gravitational acceleration. So, our original hypothesis that the drag is a tertiary effect is also justified.

And with that small term, we can also say that our approximation that we have introduced using the energy balance methodology is also justifiable. And with this drag, I suggest that you do this exercise yourself as per the equation that we have seen earlier. You will find that the velocity which was 2.30 without the drag becomes 2.14. So, this is the amount of loss. About 160 m/s is the loss in velocity due to drag.

And similarly, the altitude which was 78 km becomes 70 km, about 8 km loss in the altitude. Together you can add the potential and the kinetic energy loss together and that will effectively tell you how much of energy loss has happened over and above the gravity loss.

You will find that this energy loss is significantly lower than the energy lost due to gravity for the same case, because the ideal velocity is 3.264 km/s that represents a particular energy compared to that the gravity case gives an energy lowered by about 36%. We will find that this will add another 8 to 10% in the loss. So that the total loss is likely to be around 45% of the complete energy.

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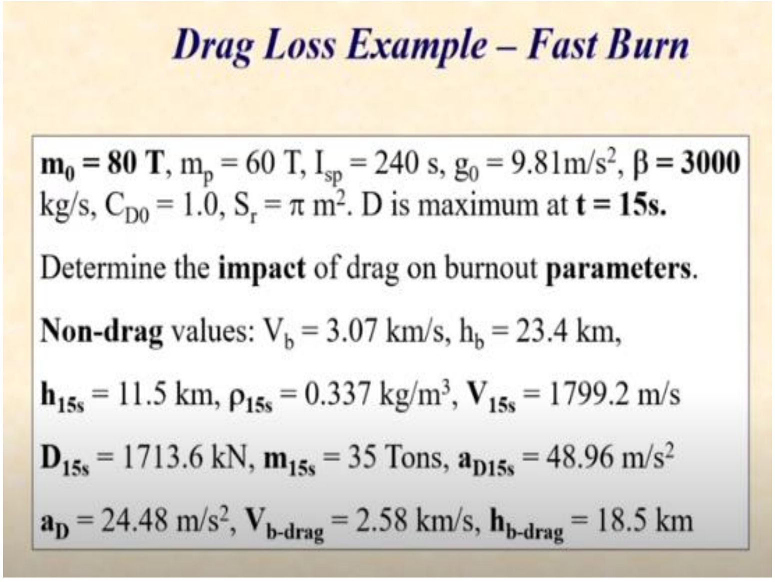

Let me now introduce an idea that we have not seen earlier. If you recall, we had given an expression for gravity energy loss as a function of β. But we had not actually characterized the impact of β directly. But we had made a mentioned that chances are that the drag loss is likely to be higher if you burn faster. Let us try and examine that idea now through the same example.

And this time, we take a significantly higher burn rate of 3000 kg/s, which is five times more than what was there in the previous case. You will immediately realize that if you are going to burn at 3000 kg/s, all the propellant will get burnt in 20 seconds itself and that the peak would be lower than what happens at 20 seconds.

So, in this case, we make a stipulation that the drag peak will occur somewhere around 15 seconds, which is in the lower atmosphere. And we are going to assume that this should again be roughly around 10 to 12 km altitude. But now because we have changed the burn rate, our non-drag values change. We have already realized that the implication of a higher burn rate is to reduce the gravity loss.

So, we get a higher burnout velocity at 3.07 km/s and a significantly lower altitude at 23.4 km. But the total energy is significantly higher, because the loss is much smaller. Now assuming that t equal to 15 seconds, we calculate the altitude which is achieved at this time and that altitude turns out to be 11.5 km. So, our assumption is justifiable. But we are still in the same ballpark of 10 to 12 km.

The velocity now at this altitude, please note, instead of 616 m/s that is what you saw in the previous case is now three times. It is almost 1800 m/s, very high velocity at nearly the same altitude. So obviously you see that your drag term which was 159 kN is practically 10 times at 1713 kN because the drag is significantly higher.

And more importantly because you have burned faster. You have consumed lot more propellant in 15 seconds. In 15 seconds, you have actually consumed 45 tons. Because you have consumed 45 tons of propellant, the residual mass is only 35 tons. So mass is also lower. So, your peak of acceleration is significantly higher, which was earlier 3.48 is now already 49 m/s^2.

That is practically five times your gravity. Which means your drag loss if you do this is going to be five times the loss that you are going to get because of gravity. Of course, we can also calculate the average drag acceleration that is half of 49. And based on that, we calculate the velocity and altitude. And now you see that the impact of this drag is significantly higher.

Now 3.07 km/s has become 2.58 which is almost 0.5 or 500 m/s lower and effectively we can say is about 20% loss, 15 to 20% loss in velocity. Similarly, you have another 5 km loss in altitude, which is also close to about we can say 20%. Together you will find that this is going to represent a very large energy loss. That brings us to the same point that we had seen earlier.

But had not this particular result in front of us to make any comments. So now let us look at it that if we burn faster, we reduce the gravity loss, but then we increase the drag loss. Similarly, if we burn slower, we increase the gravity loss, but then we reduce the drag loss. So is there going to be a golden mean of burn rate where we might be able to keep both the losses to their minimum value and thus make the mission an optimal one.

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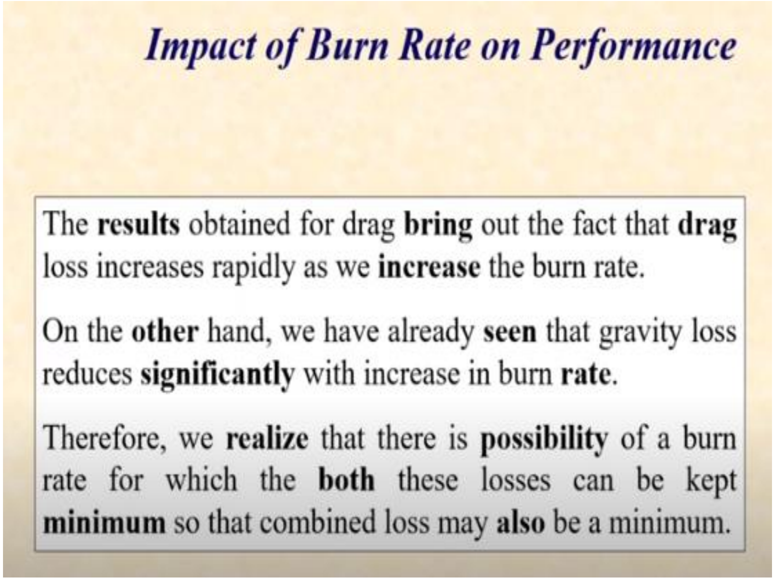

So, this is the next hypothesis that we need to examine. And we need to do this by modelling both the losses together so that we now get what is called a combined loss and see if we can get a minima for the combined loss and map it to a burn rate which is going to be applicable for that trajectory.

To say that if you burn the propellant at this rate, then this is the most efficient mission with the given propellant and the liftoff mass and the propellant that you can carry out considering the loss due to gravity and the loss due to aerodynamic drag.

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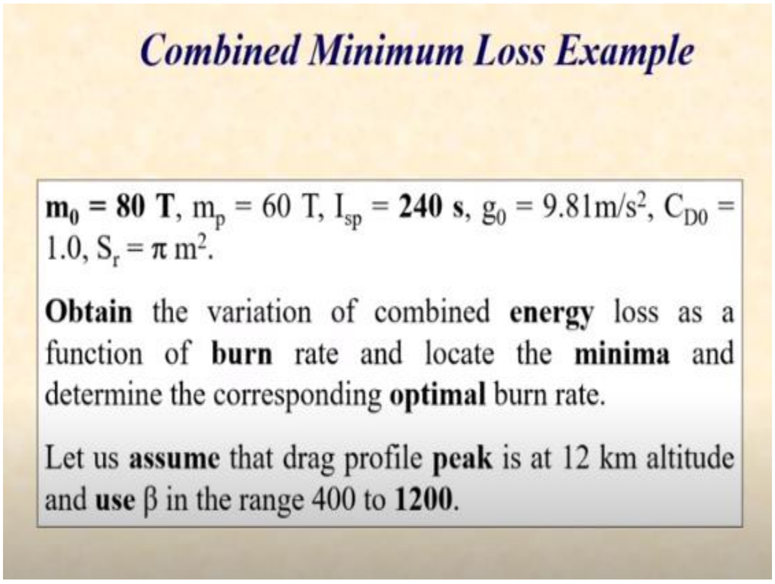

So let us take the same example. But this time we do not implement any burn rate. You are now going to look at the combined loss. So let us see if we can get combined energy loss as a function of burn rate and locate the minima and determine the corresponding optimal burn rate.

Just to simplify our analysis and the various algebraic steps, let us again assume that the drag profile peak is going to be around 12 km altitude, so that we can fix the density value. And let us also use the β in the range of 400 to 1200. Let us see if that gives us something.

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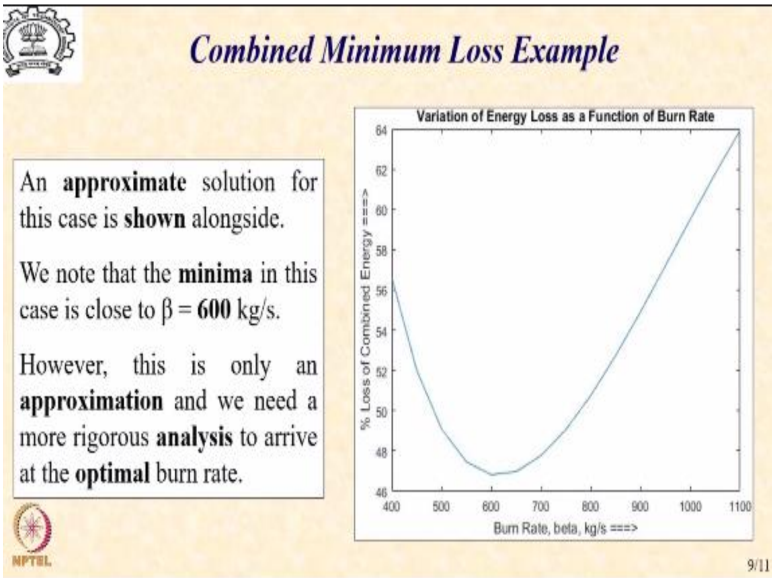

I want to show you something on the right-hand side, which is the approximate solution and then I will tell you how the solution has been obtained. So, this solution is the solution for the loss of combined energy as a function of burn rate starting from 400 kg/s to about 1200 kg/s. And you see that this is a classic concave curve, inverted parabola with its minima line very close to the 600 kg/s that we have used in our examples earlier.

Which means, the number that we had used was very close to the actual minima that you are going to get. The above curve has been obtained through MATLAB, by writing a small code. I suggest that you can also try that exercise. I will tell you the steps involved in the code that is to be developed. That is, you take the solution, because of gravity.

Based on that solution, generate the velocity at 12 km altitude and with that velocity and the density at 12 km altitude, get the value of drag. And once you get the value of drag, you can find out the time taken to reach that altitude. Multiply that time taken with β to find out what is the residual mass as that altitude as a function of β. It is not going to be a constant, it will now be a function of β.

Based on that, obtain the drag peak, which is going to be a function of β. And then go back to the expression and recalculate the velocity and the altitude under the action of this drag acceleration term as a function of β. Add this loss to the loss because of gravity and that becomes the combined loss. This is how that solution has been obtained. Of course, we need to realize that this is a highly simplified analysis.

So obviously it is an approximate representation. And you will probably need to do a more rigorous analysis to arrive at the actual optimal burn rate. But the idea that I am trying to propose is that by making reasonable assumptions, we can simplify the analytical procedure and still get a reasonable understanding of the physics involved in the process. So, you will realize that there is a possibility of arriving at an optimal solution for the burn rate by try to adjust such that the combined energy loss is a minimum.

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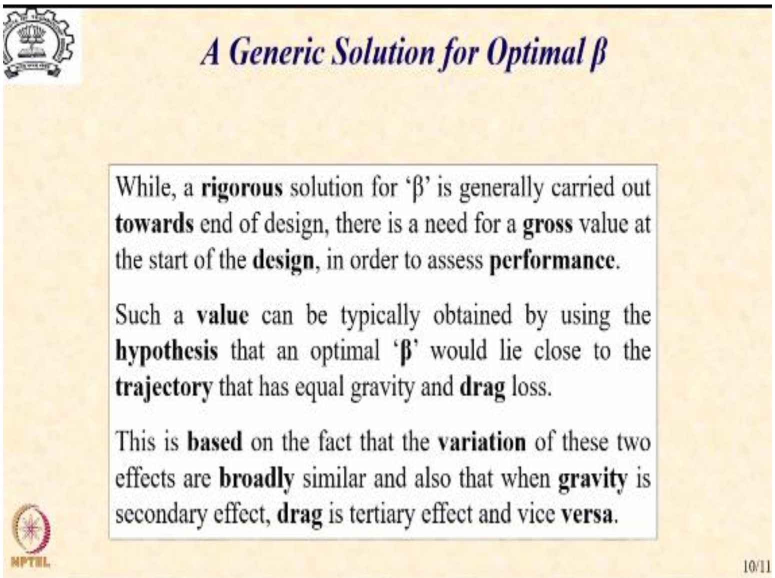

Of course, such an analysis would always be carried out even for the simplified case of a constant burn rate in a more rigorous manner towards the end of the design through complete nonlinear simulation of equations and numerical solutions. We will find that at the initial stages of design when you were trying to size the rocket and you want to understand how much loss would be there because of gravity and drag put together.

So that you can appropriately put the propellant and the I_{sp} correspondingly as part of your design solution, you need a kind of a gross estimate of the burn rate. A very crude thumb rule, which can be commonly used in such situations is that an optimal β could generally lie close to a trajectory for which the gravity and the drag loss are nearly equal in their magnitude.

This is based on the fact that variation of these two effects are broadly similar in nature but completely inverse of each other. And also, that when gravity is a secondary effect, the drag is tertiary. And when the drag is secondary the gravity is tertiary. So that they kind of complement each other and that when there are nearly of the same order of magnitude, the chances are that that would be the trajectory where you would have the minimum loss.

Just to understand this, let us go back to the previous picture. If you look at this picture, you will find that the effective combined loss is of the order of around 47% which is slightly more for more value of β beyond 600 kg/s. We already know that for this burn rate, the loss due to gravity is about 36%.

So, which means, there is another 10% which has got added because of drag and that when you start increasing the burn rate further, while the energy loss due to gravity will reduce it will not reduce at the same rate at which the loss due to drag will increase because of the square of velocity. So, you will realize that the loss because of drag will quickly climb up so that within the same ballpark it would also reach a reasonably high value and that it will overtake the loss because of gravity.

So, you can clearly see the increasing trend as you note the curve on the right-hand side.

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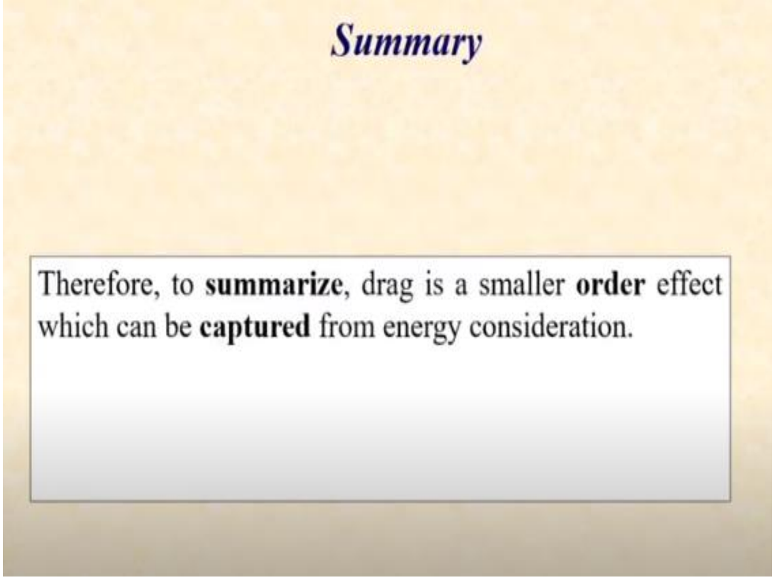

Thus, to summarize, the drag is a smaller order effect, which can be captured from energy consideration. Further, an optimal burn rate exists that results in the most efficient mission for a given vehicle from the point of view of minimizing the combined loss, Hi, so with this we have established a reasonable analytical basis for understanding the implication of various forces which are present in the trajectory solution and also a broad idea of how to estimate such a loss and find its impact on the terminal parameters.

With this we conclude our discussion on the motion along a straight line. And we will move over to a more realistic case of motion along a curvilinear path that we will look at in the next lecture. So, bye. See you in the next lecture and thank you.