Introduction to Launch Vehicle Analysis and Design

Dr. Ashok Joshi
Department of Aerospace Engineering
Indian Institute of Technology-Bombay

Lecture - 08

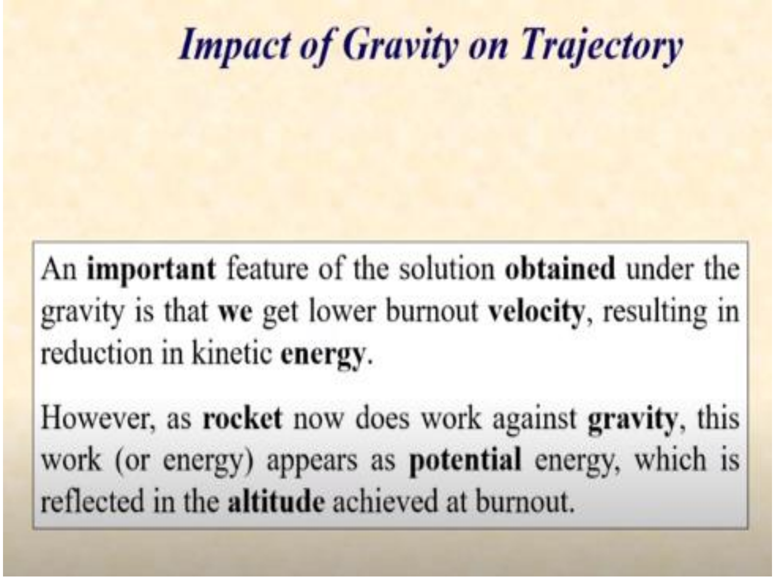
Impact of Gravity

Hello and welcome. In the last lecture we have seen two aspects of ascent mission that is force models and the performance using simplified equations. Now we will look at bringing in one of the force models that is gravity apart from the basic propulsion model that is already there as part of the idealized solution. And we will look at the implication of gravity on the ascent performance. So let us begin.

(Refer Slide Time: 01:15)

In the last lecture, if you recall we had assumed a constant gravitational model and based on that we had arrived at the solution for the performance of the typical rocket at the terminal point in terms of the velocity and the altitude. Now let us find out what are the implication of this performance.

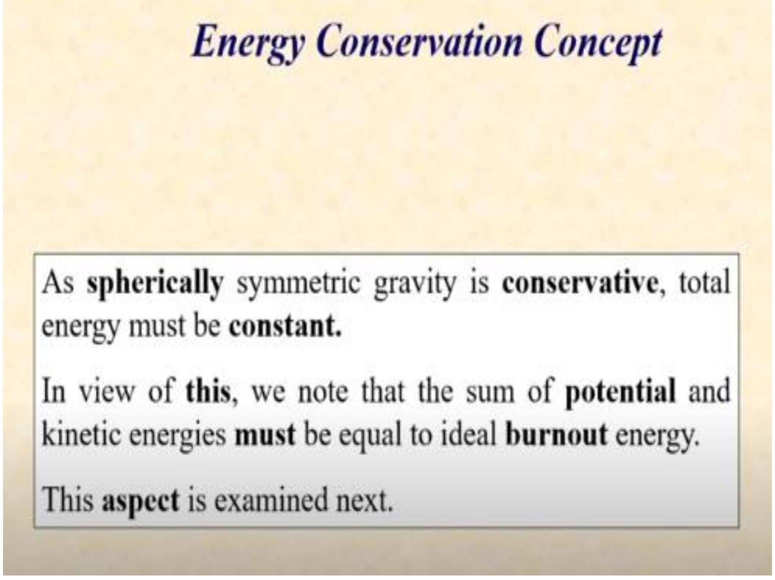
(Refer Slide Time: 02:11)



So, the first thing that we note from the solution that we have seen earlier is that the velocity is lesser than what we have obtained when we used the idealized model without the gravity. So, we immediately note that this is going to reduce the kinetic energy of the vehicle.

However, we also know that rocket is now doing work against gravity and from our understanding of work energy equivalence, the work done against the gravity will appear as an additional energy what we commonly term potential energy. And this is reflected in the altitude that we achieve at the burnout.

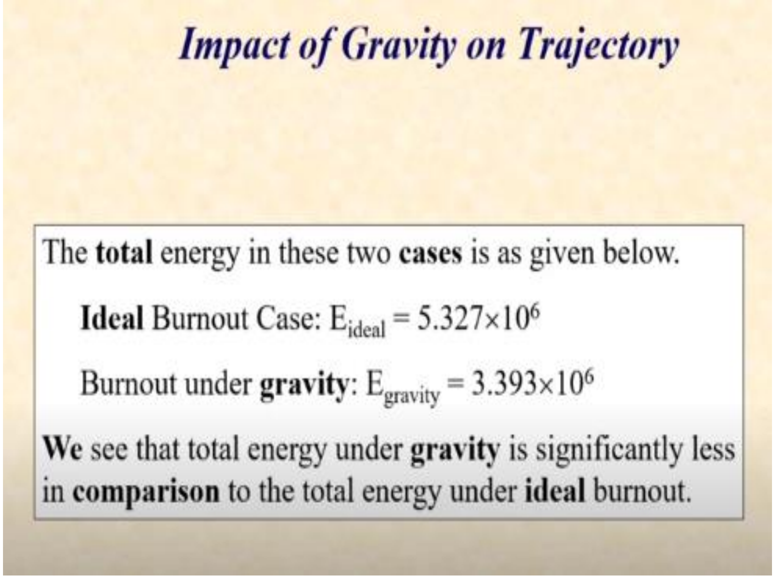
(Refer Slide Time: 03:11)



Of course, as we all know, spherically symmetric gravity model that we have assumed for the gravitational force is conservative and that means that the total energy must be constant. In view of this, we hypothesize that the effect of gravity is not really something that we should worry about.

The loss in the velocity probably appears as the gain in altitude so that the total mechanical energy is same and must be same as the ideal burnout energy in terms of the velocity that we achieved. Let us try and examine this aspect next.

(Refer Slide Time: 04:14)

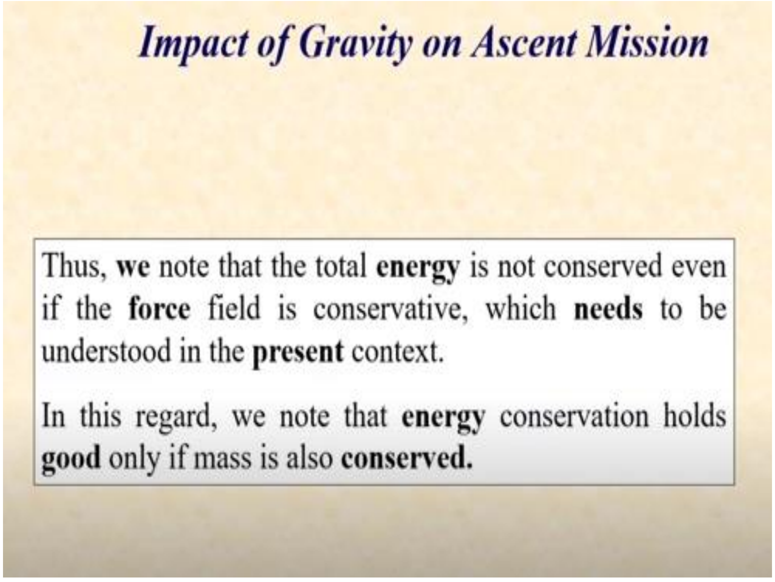


So, we take the same problem that we have solved in the previous lecture. And let us calculate the total energy in the two cases that is the ideal burnout and the burnout in the context of constant gravity model. So, in the ideal burnout case, the kinetic energy is the only energy because there is no gravity, so no work done against gravity. And that is nothing but $\frac{1}{2}V^2$ and that is per unit burnout mass.

So, we have normalized it because the burnout mass is same in both the cases and that number is 5.327×10^6 with appropriate units. In case of burnout under gravity, the total energy is going to be $\frac{1}{2}V^2$ that is kinetic energy plus gh which is going to be the potential energy part.

So, we perform that sum. I suggest that you do this exercise yourself and confirm that under gravity, the energy is going to be this value which is 3.393×10^6 and that is something which is not expected. The total energy under gravity is significantly less in comparison to the total energy under ideal burnout. So, our hypothesis that as the gravity is a conservative force field, the total energy should have remained constant is not justified, it is violated. So let us try and understand what has happened.

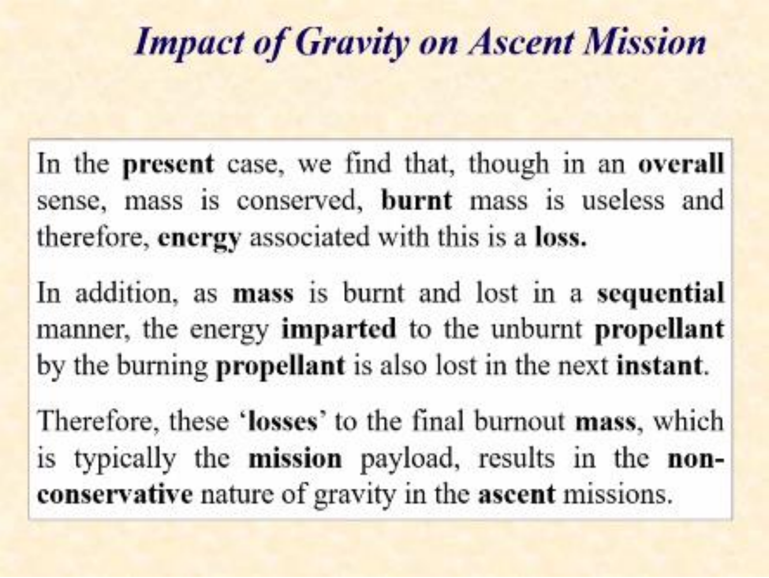
(Refer Slide Time: 06:21)



So, we know that total energy which should have been conserved is not conserved. So, we bring in a point which probably we did not specifically note while talking about energy conservation. You would realize and probably you can independently confirm that the energy conservation holds good only if the mass is also conserved.

This is something that is part of the various conservation laws that are established and you can verify this independently. But let us now bring in this idea about mass conservation to see whether there is an issue which is causing this deficiency of energy.

(Refer Slide Time: 07:23)



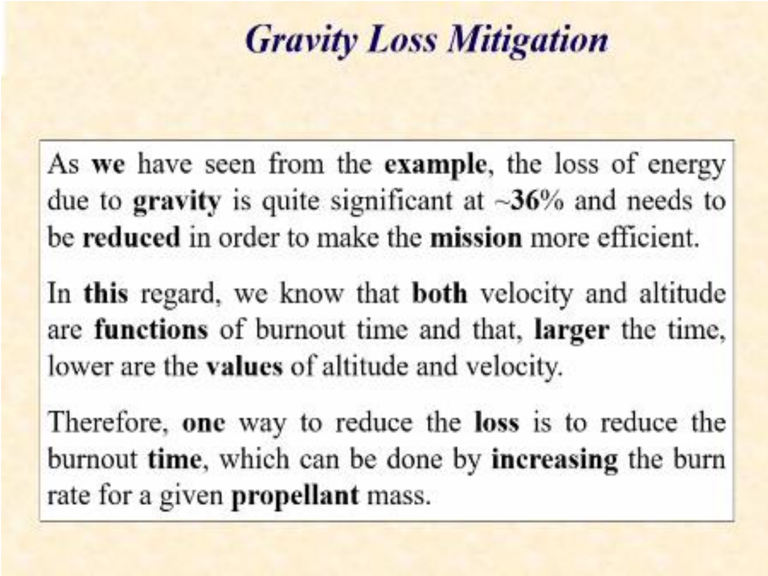
So, in the present case, we find that though in an overall sense mass is conserved, which means if we choose a control volume approach, which contains the rocket and the burnt crisis as a single control volume, the mass is conserved and so the energy is also conserved. But we realize that this burnt mass in form of a gas is absolutely useless to the ascent mission as this energy is not recoverable as far as the burnt-out mass m_b is concerned, which is the payload as far as we are concerned.

So obviously, this energy is simply a loss to us. It is released in the atmosphere at different altitudes as the ascent mission proceeds as clouds of gases, particles, which have been given velocity and altitude. But then it is all a waste. The other aspect which we need to note, that while we are burning a small portion of propellant which generates this cloud of gas, which contains particle and energy, the energy that it imparts to the rocket also goes to the unburnt propellant, which is still there, and which is going to get burnt in the next instant.

So not only we are wasting the energy which is part of the gas cloud, we are also imparting wasteful energy to the unburnt propellant, which in the next or subsequent time instance we are going to burn and throw away. So that energy also is going to be a loss. If you put all this together, we will find that this is going to represent a reasonable amount of loss and will be responsible for the non-conservative nature of gravity in the ascent mission.

So please note, we are not violating any of the fundamental conservation laws. But by defining our energy conservation in a very specific manner, that is the energy which is imparted to the burnout mass, the final unburnt mass which is left after all the propellant has been consumed, the mission is non-conservative in nature even under the action of a conservative force field such as gravity.

(Refer Slide Time: 10:28)



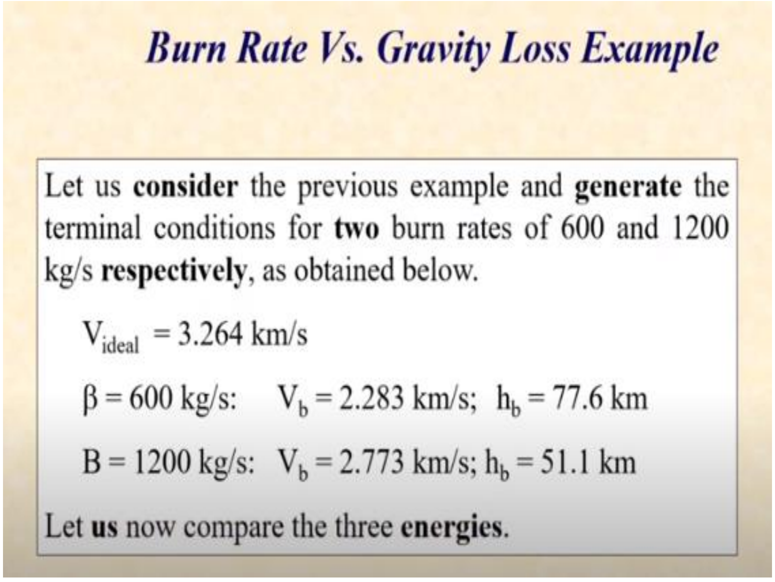
In fact, in the present case you will note that this loss is almost of the order of 36% and naturally is not tolerable. So, there must be an effort to reduce this loss to significantly lower amount. We may not be able to completely eliminate it, you do not know yet, but at least it cannot be allowed that a 36% of energy is simply lost. That is going to have a huge impact on the cost of mission itself.

So, in this regard, we know that both velocity and altitude are functions of burnout time and that larger the time lower are the values of both altitude and velocity. This you can independently confirm from the expressions that we have seen in the last lecture. So obviously, it means that an important parameter which is responsible for this loss is that burnout time, the time taken to complete this whole process.

So, if we can directly impact the time, maybe we can influence the overall loss and make the mission more efficient. And this can be done in one way by increasing the burn rate. So, if you increase the burn rate for a given propellant, then the time that it takes to finish the propellant is smaller and because the time is smaller, the loss due to gravity is likely to be smaller.

Let us see if this is justifiable and this actually turns out as what we have interpreted.

(Refer Slide Time: 12:19)

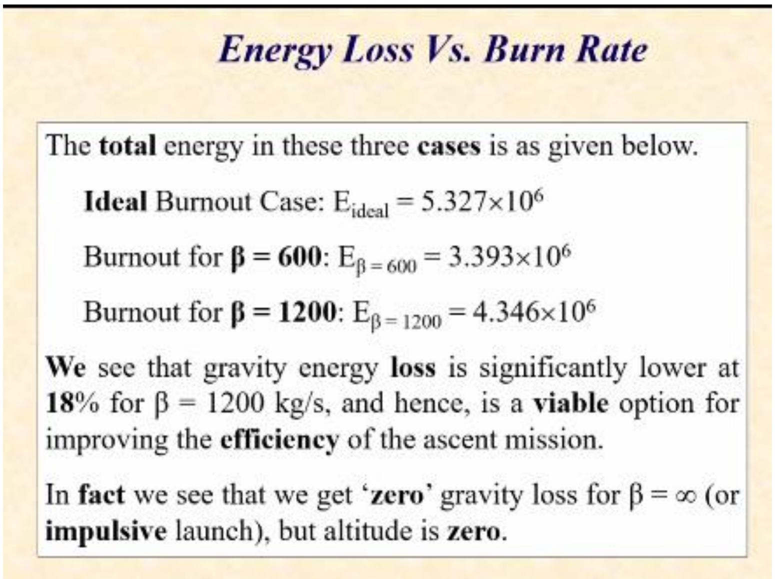


So let us now consider the same example that we have seen earlier and now generate the terminal condition for two different burn rates. One of 600 kg/s that we have already seen. And another one double, that is to 1200 kg/s. And let us look at the solution. I am not going to go through the steps of arriving at a solution. I suggest that you do that. But I am giving you the final result.

So, we already know our ideal velocity is 3.264 km/s. For 600 kg per second burn rate, our burnout velocity is 2.28 km/s with an altitude of 77.6 km and for 1200 kg/s, the velocity is 2.77 km/s with an altitude of 51 km. So, you realize that with higher burn rate, the velocity has gone significantly higher.

The altitude also has reduced but possibly not by the same percentage. And more importantly, the kinetic energy is a function of the square of the velocity. So, the increase in the kinetic energy is likely to be significantly higher than the decrease in the potential energy because of the loss of altitude. So, there is an expectation that the total energy is likely to be higher. Let us now compare the three energies and see whether our expectation is met.

(Refer Slide Time: 14:10)

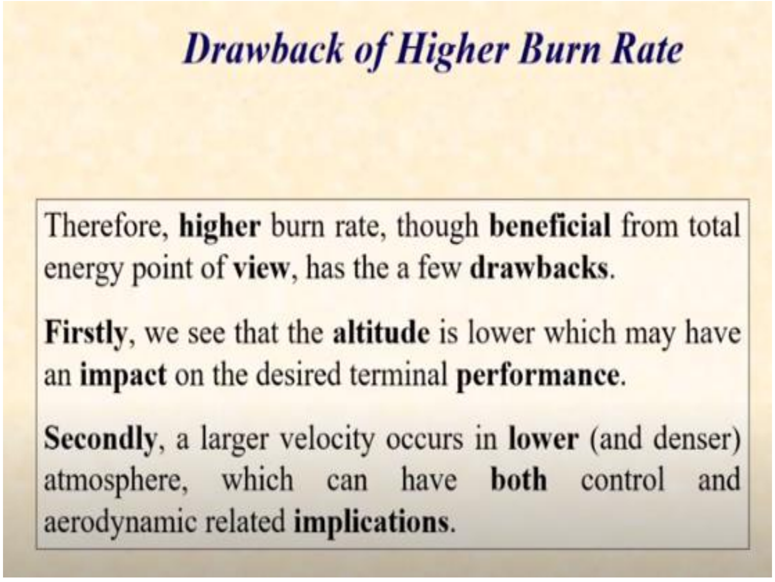


So, the total energy in the three cases is the ideal burnout is 5.327×10^6. The burnout for $\beta = 600$ is 3.393×10^6. And burnout for $\beta = 1200$ is 4.346×10^6. And immediately you realize that the energy is significantly higher for $\beta = 1200$ case as compared to $\beta = 600$ case which means that now our loss is much lower.

In fact, we can calculate this loss to be at about 18% which is half of the previous loss and immediately we realize that by increasing the burn rate, we can improve the efficiency of ascent mission from gravitational loss point of view. In fact, we see that we are going to get zero gravity loss for $\beta = \infty$ which is nothing but an impulsive launch.

Which means, if we burn all the propellant, in almost zero or a very small time we are going to get the full mechanical energy, the only thing will be that the altitude is zero. And this is something that you need to understand in the correct context.

(Refer Slide Time: 15:47)

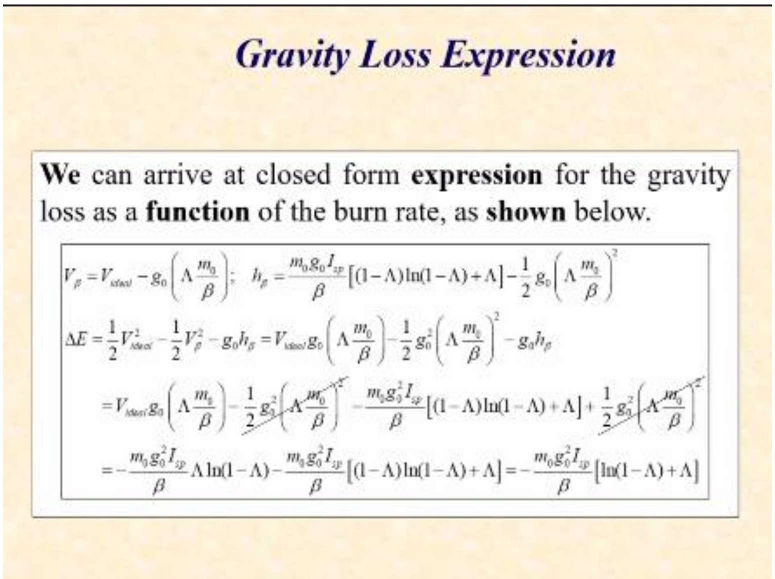


So, what is the drawback of this higher burn rate? One of the basic drawback is that you are going to get a lower altitude, which may be a requirement that you want a higher altitude. But if you burn faster rate to reduce the gravity loss, you are going to generate the lower altitude. So obviously, this is a drawback that you need to take into account.

Secondly, a larger velocity occurs in the lower atmosphere which is also denser. And it can have both control and aerodynamic related implications. As we have already seen in our aerodynamic model, a higher velocity in the denser atmosphere is going to generate higher dynamic pressure and possibility of higher drag.

So obviously, while we may introduce the loss due to gravity, we may introduce additional loss to drag and we may have to look at this issue in greater detail, which we will do later.

(Refer Slide Time: 17:03)

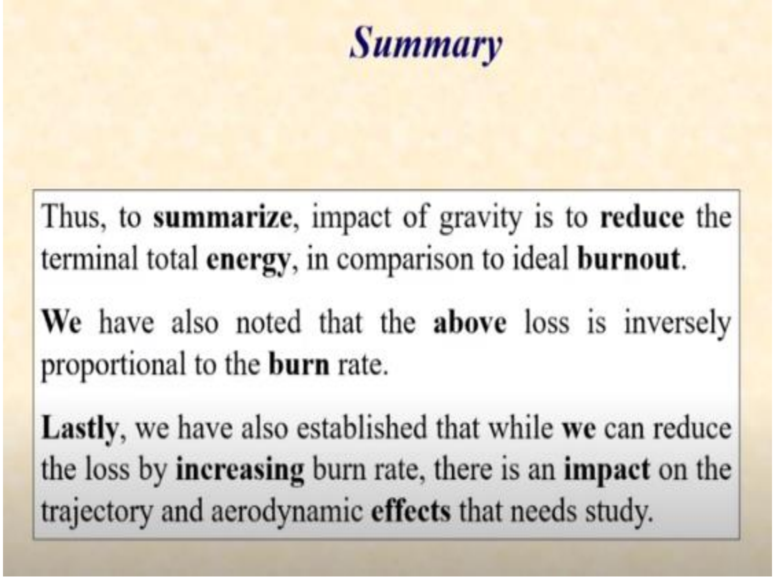


Of course, from whatever we have done so far, it is possible for us to give a closed form expression for the gravity loss with respect to the burn rate, which is a good expression and a design tool for understanding what loss is going to be due to gravity because of a particular burn rate. I am showing you the steps here. We have used the expressions that we have generated earlier.

And based on that we have modelled the ΔE which is the loss in the total energy due to gravity as a function of β. And finally, we find that this is a function of I_{sp} and is inversely proportional to β. So, we clearly see here that as β increases, the loss is going to reduce. However, we also note an interesting point that if you use a higher I_{sp} fuel or if you use a larger lift-off mass or you use a larger propellant fraction, the loss is going to increase.

And you will realize that now it is going to be a trade off in the design among these parameters for making an ascent mission as efficient as possible through in any optimization procedure.

(Refer Slide Time: 18:54)



So, to summarize, the impact of gravity is to reduce the total terminal energy in comparison to ideal burnout. We have also noted that the above loss is inversely proportional to the burn rate. And we also established that we can reduce the loss by increasing burn rate, but there is an impact on the trajectory and aerodynamic effects that needs further investigations.

Hi, so in this lecture we have seen the implication of gravity on the terminal performance. And we have noted that gravity represents a loss of total energy which is not very intuitive. And that this loss can be minimized by increasing the burn rate but will have associated drawbacks that we need to discuss and understand.

Now in the next lecture, we are going to introduce the drag as a parameter and we are going to now look at the performance of the rocket under both gravity and drag. And then we will also look at the possibility of arriving at a viable burn rate that is going to be beneficial from both drag and gravity point of view. So, bye and see you in next lecture and thank you.