Introduction to Launch Vehicle Analysis and Design

Dr. Ashok Joshi
Department of Aerospace Engineering
Indian Institute of Technology-Bombay

Lecture - 05
Force and Geometry Models - 2

Hello and welcome. We have already seen in the last lecture the models for propulsion and gravity. Let us now proceed further with the remaining two force models that is the aerodynamics and the earth geometrical model.

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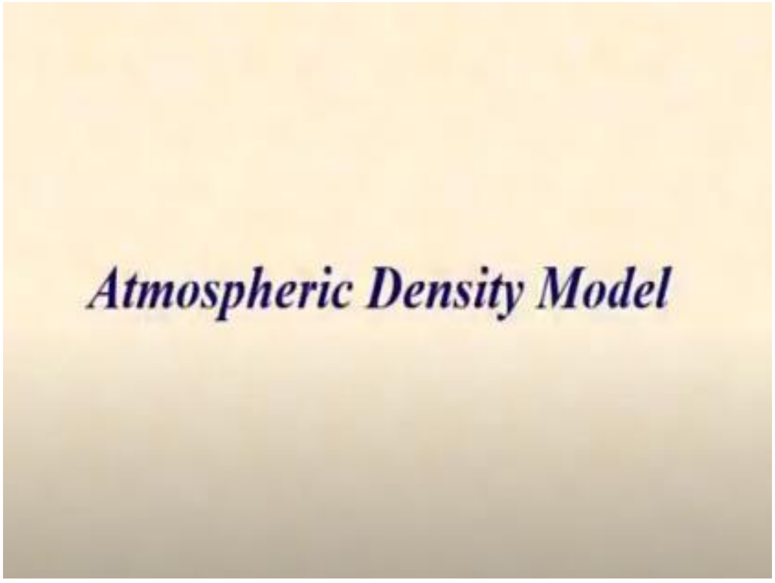

In this regard, let us first consider the atmospheric density model.

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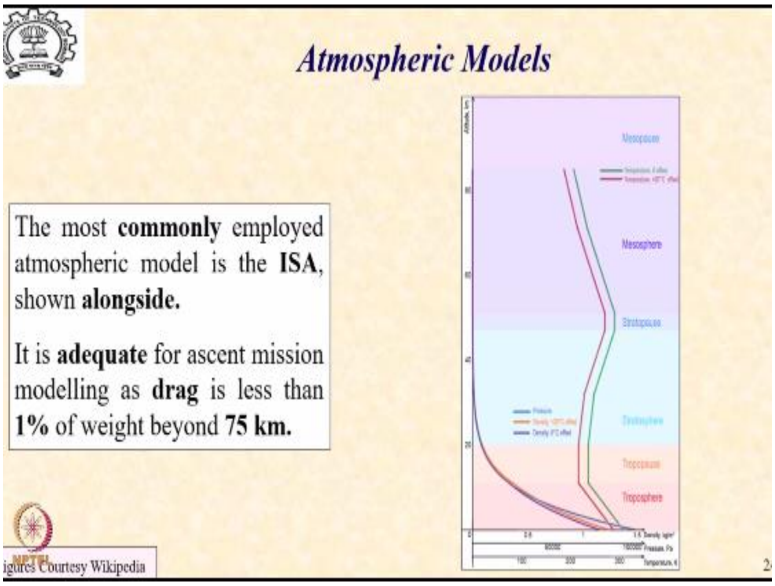

The most commonly employed atmospheric model is the international standard atmosphere, which is shown alongside. This model of atmosphere contains three basic parameters that is temperature, pressure and density as a function of the altitude above the surface of the earth. Of course, as all of us know, the atmosphere is composed of various layers.

And the schematic also reflects the various layers which are part of it starting with troposphere going to stratosphere, Mesosphere and then we know that beyond mesosphere we start getting the ionosphere. We realize that our ascent mission which typically ends between 200 to 400 kilometers will be crossing all these domains.

But what we also realize is that the parameters which are going to impact the performance of a rocket while it is going through the atmosphere are going to be two primary parameters that is pressure which is going to impact the I_{sp} or the thrust of the rocket and the density which is going to define the aerodynamic drag.

In the present case, we are going to focus more on the drag modeling assuming that the static pressure will be taken care by suitably calculating the I_{sp} of the rocket motor. So, in the context of drag it is found that beyond 75 kilometers, drag is generally less than 1% of the weight and obviously can be ignored, the motion being practically in vacuum as far as the ascent mission is concerned.

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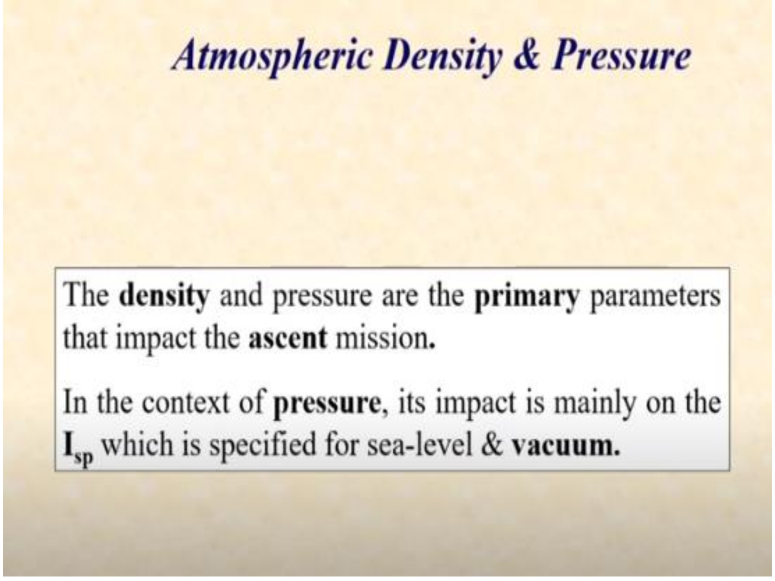

As I have mentioned density and pressure are the primary parameters which are going to impact the mission. And we have also seen that the pressure will directly impact the I_{sp}.

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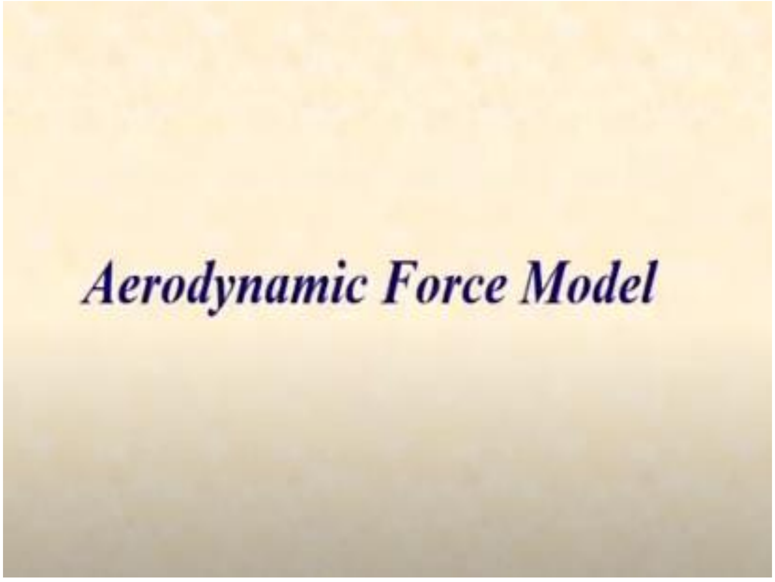

So let us focus more on the drag model. I have already mentioned that beyond 75 km we are not going to consider drag at all as a force for ascent mission calculations. The question is what about lower altitudes?

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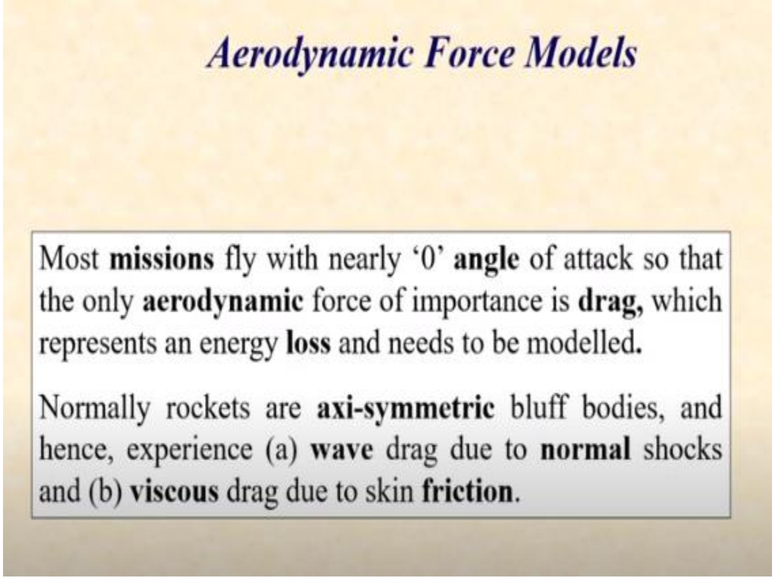

Now in order to understand what happens at lower altitudes, let us also try and understand what kind of mission could typically be flown in the lower atmosphere. And an important point which comes out strongly is the fact that most launch vehicles fly with zero angle of attack through the atmosphere so that the aerodynamic forces are kept to a minimum because aerodynamic forces represent energy loss and we would like the mission to be as efficient as possible.

In this regard, we note the following. Normally rockets are axi-symmetric bluff bodies and hence are subject to the following two drag components, that is the wave drag due to normal shocks and viscous drag due to skin friction.

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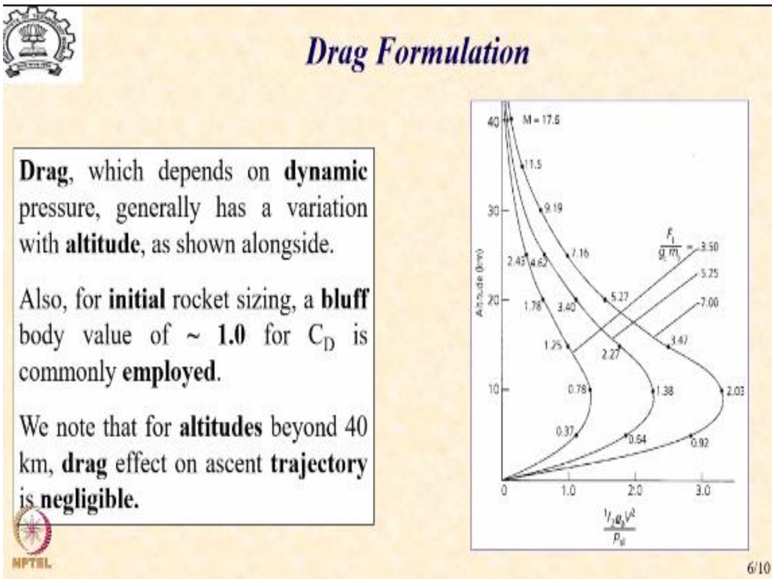

Typically, the drag is driven by a parameter called the dynamic pressure, which is nothing but $\frac{1}{2}\rho V^2$, an aerodynamic parameter which determines the extent of aerodynamic force which will get generated. So, we look at the variation of this quantity as a function of altitude, which is shown alongside. And we notice two important parameters.

One that starts with zero at the sea level where the mission begins, which is a maximum value somewhere around 10 km altitude and then decreases continuously until it crosses the 40 km altitude beyond which the value of dynamic pressure is practically negligible. The second point that we note is that the magnitude of the peak is a function of the amount of thrust to weight ratio that the rocket generates.

Which means, that higher the thrust to weight ratio higher is the peak which is generated even though we see that the overall shape remains more or less the same. Here it is worth noting that higher thrust to weight ratio indirectly reflects in a higher velocity in the lower atmosphere so that a high velocity in dense atmosphere generates higher dynamic pressure.

Now for actual drag force calculation, we need three quantities. The first one is the dynamic pressure. The second one is the surface area, which is a given for a given rocket. And the third one is what is commonly called a non-dimensional parameter drag coefficient. Now this is a function of the launch vehicle external geometry and the flow regime like subsonic or supersonic.

But for initial sizing purposes, particularly as the rocket is treated as an axi-symmetric bluff body moving in axial direction, generally a bluff body drag coefficient value of 1 is employed, which is considered to be adequate for initial sizing purposes.

And then we note that even though the actual atmosphere is ignored only beyond 75 km from aerodynamic drag calculations, maybe an altitude of about 40 km is considered adequate for considering drag to be absent beyond that altitude so that the motion can be considered in vacuum.

And that is why you would realize that in the ascent mission segment that we had seen in one of the earlier lectures the atmospheric flight was treated between 0 to 50 km altitude beyond which we were assuming that the motion is in vacuum.

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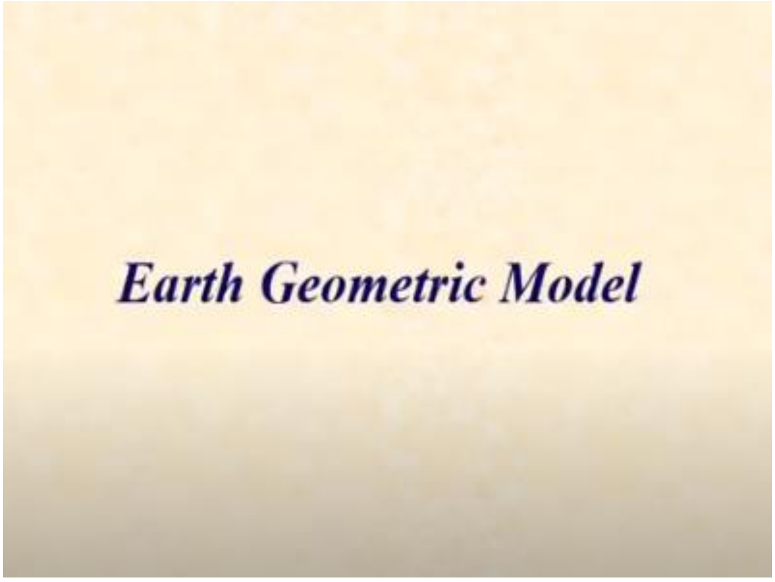

Let us now move over to the last modeling requirement that is earth geometric model.

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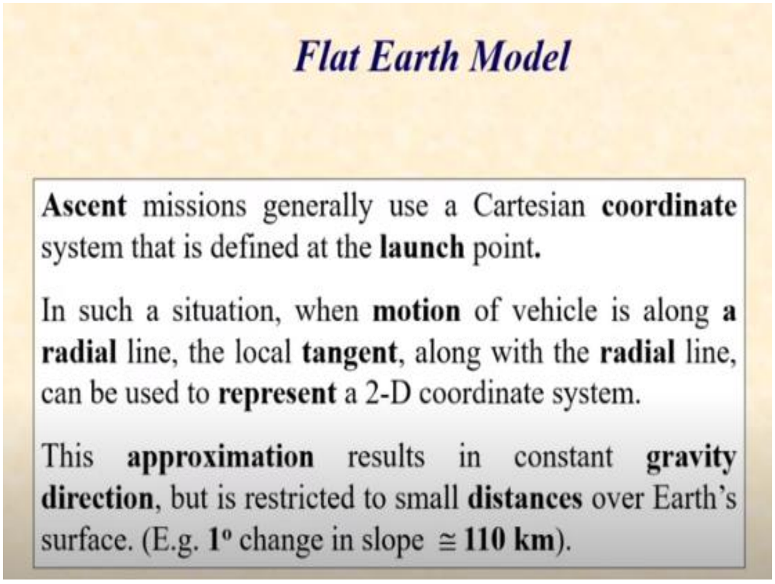

Ascent missions generally use a Cartesian coordinate system that is defined at the launch point, we have seen this fact. So, in such a situation, if the motion of vehicle is along a radial line, then the local tangent along with the radial line can be used to represent a 2-D coordinate system for describing the mathematical equations governing the ascent mission.

Another benefit of such a model is that it results in a constant gravitational direction, which is along the radial direction as per our universal law of gravitation model. Of course, we realize that this is going to be restricted to small distances traveled over earth's surface.

And a very crude thumb rule can be used to assess the impact of this assumption through the simple equivalence relation, where we say that 1 degree change in the direction of the gravity is equivalent to traveling 110 km over the surface of the earth.

Which means, that as long as your distances traveled over surface remain less than this quantity, the change in the direction of gravity would be only about 1 degree or less. So, if you can ignore this, you can assume that the rocket is moving along a straight line or a radial line.

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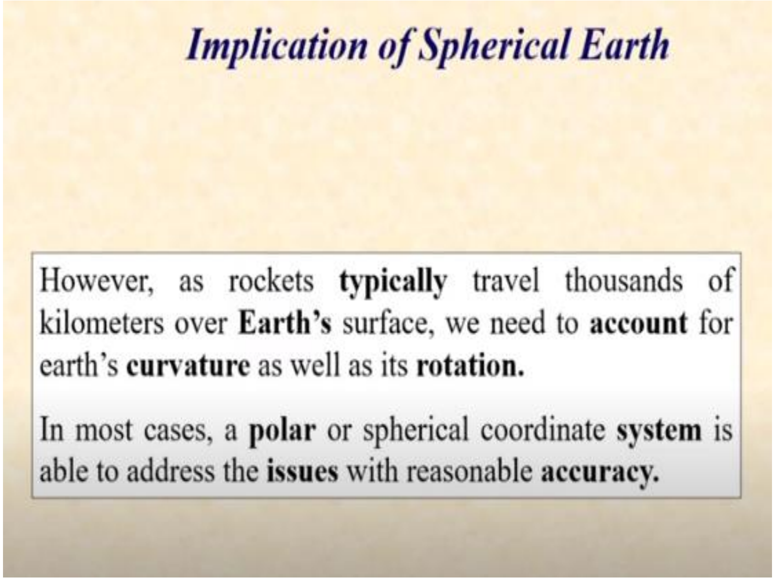

Of course, we also need to admit the possibility as we have also seen from our earlier schematic diagrams, that the trajectories over the complete ascent mission are going to be highly curvilinear in nature and going to move several thousands of kilometers over the surface of the earth. So obviously, this particular assumption of flat earth is not really going to be justifiable beyond a particular point.

And we will need to take into account the earth's curvature and of course maybe its rotation about its own axis. One way of getting around this difficulty is to then employ a polar or a spherical coordinate system which adequately addresses the issue of gravity vector direction, but introduces additional complexities in the mathematical model. So, we have to deal with those depending upon the context in which we are putting the model.

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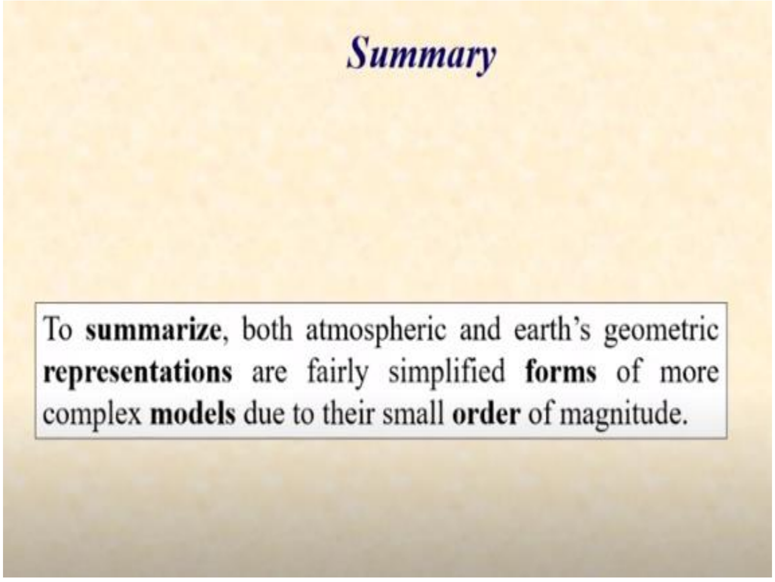

So, to summarize, both atmospheric and earth's geometric representation are fairly simplified forms of more complex models due to their small order of magnitude. Hi, so in this lecture, we have seen the implication of atmosphere on the modeling of drag that we need to make for our ascent mission. And we have also seen the implication of the curved surface of earth in terms of the coordinate system that is likely to become applicable.

With this we have assembled the right-hand side of our basic vector differential equation that we have seen earlier. And now, we are in a position to synthesize the full model and start working with the model to understand the ascent mission performance at various levels. So, we will do that starting from next lecture. So, bye, see you in the next lecture and thank you.