Introduction to Launch Vehicle Analysis and Design

Dr. Ashok Joshi
Department of Aerospace Engineering
Indian Institute of Technology-Bombay

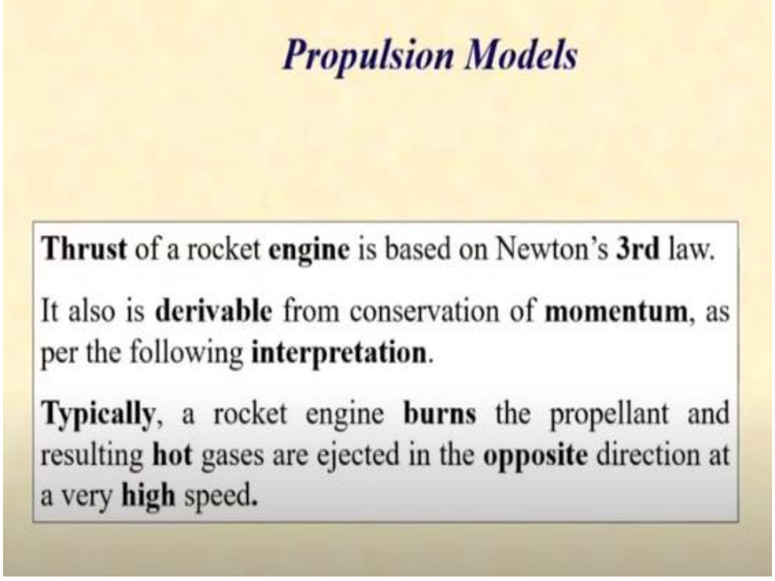
Lecture - 04

Force and Geometry Models -1

(Refer Slide Time: 00:16)

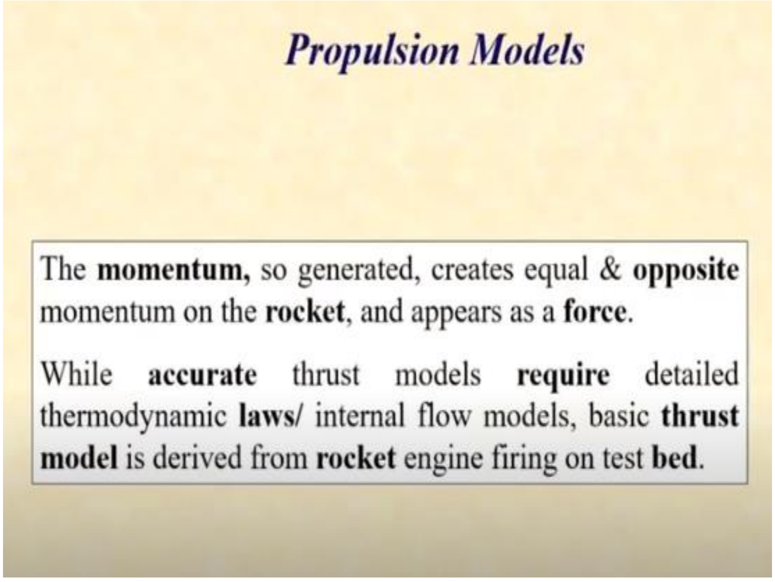
Hello and welcome to the first lecture on force models for ascent mission. As we have seen, there are three forces that we need to consider. That is propulsive, the gravity and the aerodynamic forces. So let us start our discussion with the propulsive force, which is also commonly called thrust.

(Refer Slide Time: 01:01)



We all know that thrust of a rocket engine is based on Newton's third law. That is action equal to reaction. Of course, we can also derive this from the conservation of momentum as per the following interpretation. So let us create a scenario which says that a rocket engine burns the propellant and resulting hot gases are ejected in the opposite direction at a very high speed.

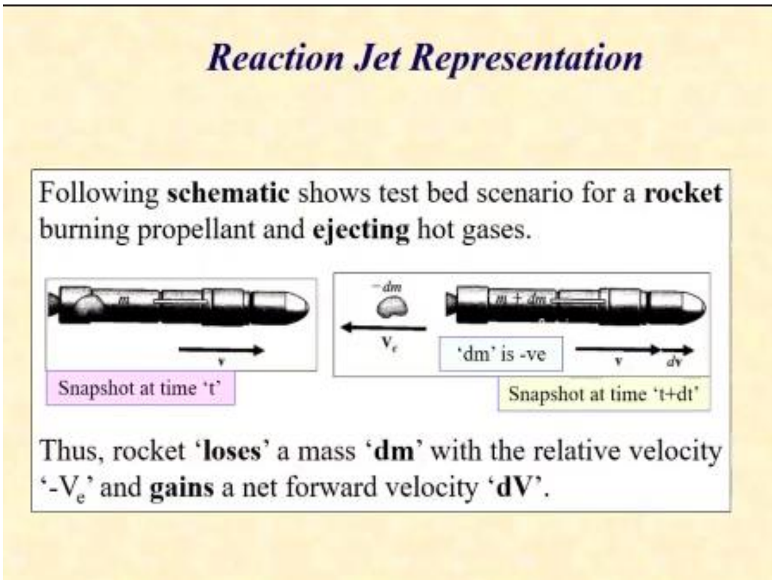
(Refer Slide Time: 01:56)



The momentum so generated creates equal and opposite momentum on the rocket so that the net momentum remains constant, but the momentum lost due to the exhaust gases appears in the form of momentum gained by the rocket or as a force on the rocket. Of course, we need to note here that accurate thrust models will require detailed thermodynamic laws, internal flow models etc.

However, for the ascent mission sizing and initial design a very basic thrust model can be derived from the concept of rocket engine firing on a test bed, which is a common configuration in which most of the rocket engines are tested before they are put on rocket.

(Refer Slide Time: 03:17)

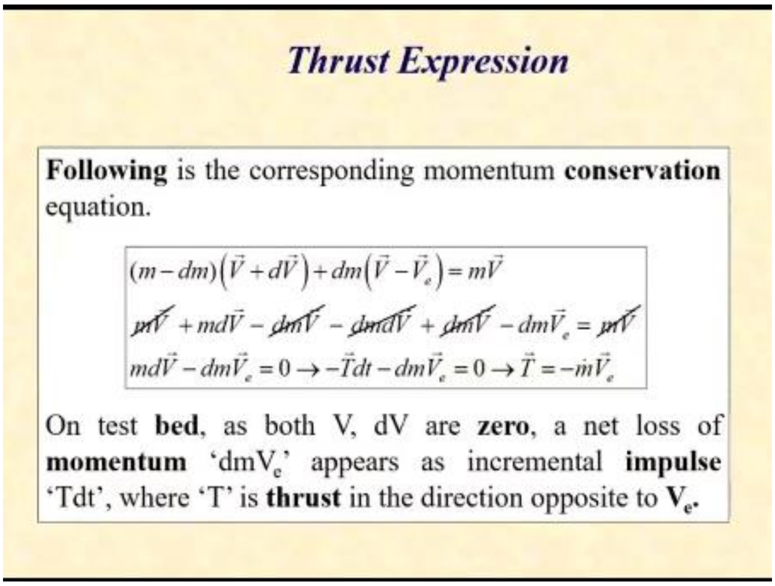


So let us look at this particular representation of a rocket engine firing. So, consider the following schematic in which a rocket is burning propellant and ejecting hot gases. Of course, in this case, we have introduced a velocity vector V just to make the derivation more general. But we could very easily make this velocity equal to zero which would represent the test bed configuration.

So, we take a rocket of mass m moving with a velocity v as shown in the picture. Let us assume that this is a snapshot at a time instant t. And then look at the snapshot at time $t + dt$. So, in that time interval dt the propellant has been burnt by a small amount represented by dm, which is ejected related to the rocket at a high-speed V_e and in the process the rocket velocity increases by an amount dV.

As the ejected mass and the rocket are considered to be part of the same control volume, we can say that the momentum within the control volume is conserved indirectly that means, that the momentum lost by the rocket through the exhaust gases must be balanced by the momentum gained by the rocket in the net sense and this can be stated as saying that rocket loses a mass dm with a relative velocity $-V_e$ and gains a net forward velocity dV.

(Refer Slide Time: 06:09)

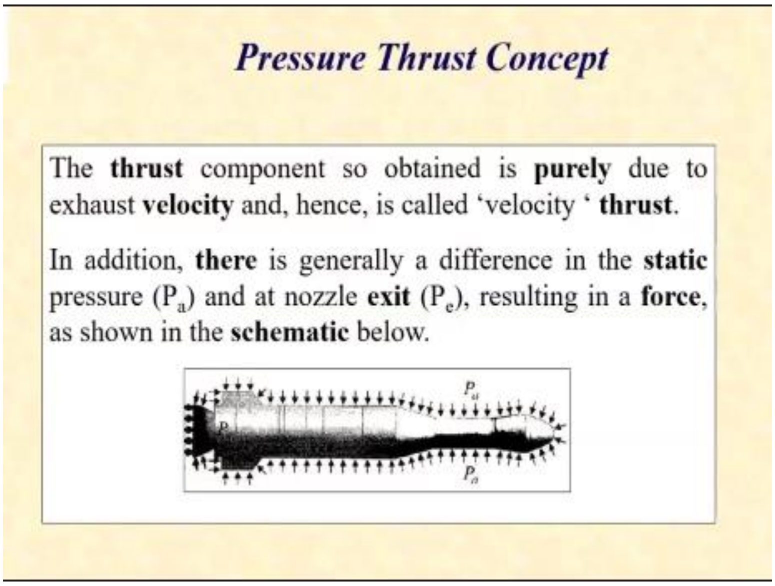


We can write the momentum balance as $(m-dm)(\vec{V}+d\vec{V})+dm(\vec{V}-\vec{V}_e)$ equal to the original momentum $m\vec{V}$. Of course, we just cancel the terms and ignore the higher order term $dmd\vec{V}$ with the net result that now we say that the net change in momentum is 0. That is $md\vec{V}$ the change in the momentum of the rocket is balanced by the momentum gained by the exhaust gases in the opposite direction that is $dm\vec{V}_e$.

Because this change in momentum results in a force in the forward direction, we hypothesize the presence of a force called thrust shown as T in this equation. And then we have an expression for thrust as $-\dot{m}\vec{V_e}$. The negative sign here indicates that $\dot{m}$ is a negative quantity. Because related to the rocket we are losing mass. So, rate of change of mass is negative.

Of course, on the test bed both V and dV are zero because the rocket does not move. The net loss of momentum dmV_e appears as incremental impulse which is like a force and on the test bed, this force is registered by support system as a reaction and typically, that reaction is measured as an indicator of the amount of thrust the rocket motor is generating.

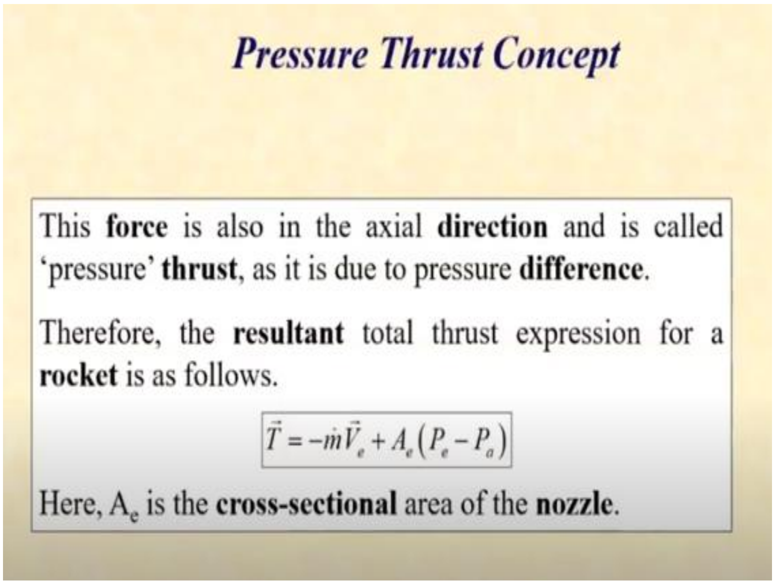
(Refer Slide Time: 08:18)



Of course, the thrust component so obtained is purely due to the exhaust velocity and also called the velocity thrust. In addition, there is generally a difference in the static pressure P_a in the atmosphere. And the nozzle exit static pressure P_e due to the internal flow characteristics of the exhaust gases and the design of the nozzle. This difference in the pressure also results in a force as shown in this schematic below.

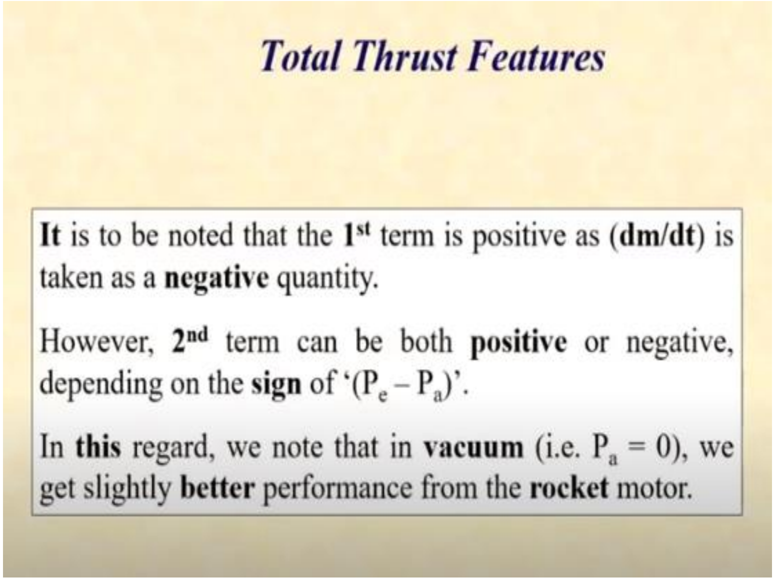
So, if you have a rocket in the atmosphere, the atmospheric pressured acts all along it and at all other points it balances each other so that there is no net force in those direction except the front and the back where the pressure at the front is different from the pressure at the back because of exhaust nozzle. And this difference in the pressure results in a net force which can be modelled as a simple differential pressure into area.

(Refer Slide Time: 09:51)



And the expression for that is as follows. That additional thrust is simply added to the expression for the basic thrust of velocity and we get two terms for thrust, the first one is the velocity thrust and second one is the pressure thrust. Here A_e is the cross-sectional area of the nozzle through which the exhaust gases exit.

(Refer Slide Time: 10:33)

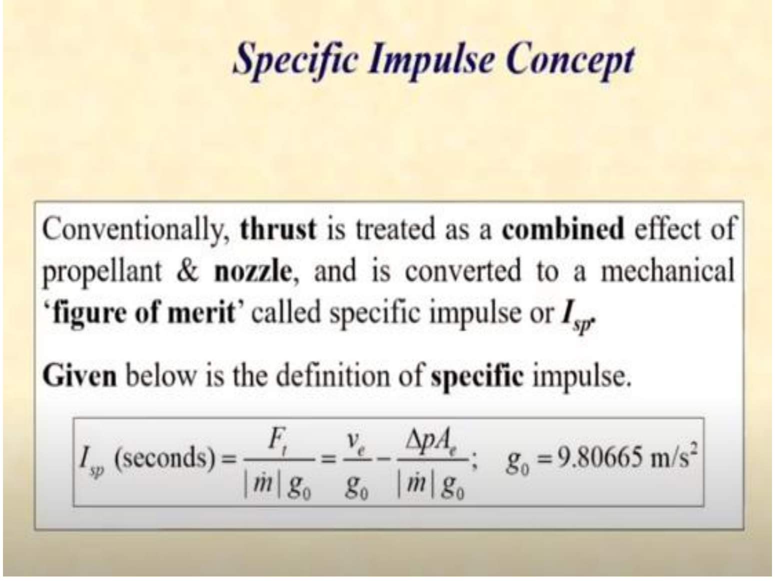


As I have mentioned, the first term is positive, because $\frac{dm}{dt}$ is negative. However, you will note that the second term now can be either positive or negative depending upon the sign of $P_e - P_a$. So, we know that if P_e is greater than P_a, you will get a positive value. On the other hand, if P_e is less than P_a you will get a negative value.

This has an important implication for such rocket nozzles, which are designed to fire with certain attributes at sea level where atmospheric pressure P_a is high. As the rocket moves to the atmosphere, the static pressure drops until it goes out of the atmosphere so that P_a becomes zero. And we find that if we reached the vacuum condition, where P_a is 0 while P_e will be a nonzero quantity.

There will be an additional positive thrust available at higher altitudes from the same rocket motor. So, the rocket motors that are designed with a particular thrust at sea level can actually give better performance at higher altitudes.

(Refer Slide Time: 12:28)

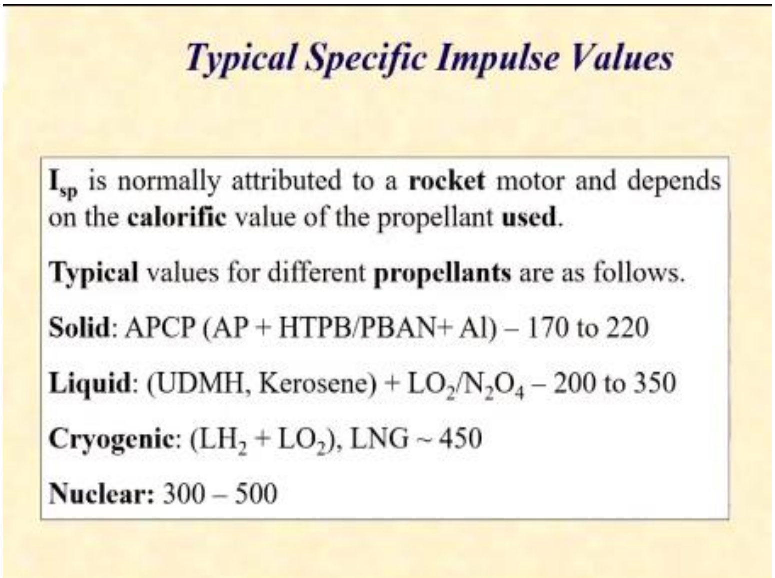


Let us now introduce the concept of specific impulse as a figure of merit for rocket motors. As we have already seen, thrust is a combined effect of propellant and nozzle because the propellant generates the temperature, pressure in the combustion chamber which is then expanded through the nozzle and then this combination generates both exit velocity V_e and exit pressure P_e which are part of our thrust definition.

So, this combination of propellant and nozzle is generally treated as a single unit called the rocket motor and is converted into a mechanical figure of merit which is called specific impulse or I_{sp}. The definition of I_{sp} is as given below. I_{sp} whose units are in seconds is the ratio of the net thrust or the force generated by the rocket motor divided by weight flow rate. So, $\dot{m}$ is the mass flow rate.

When we multiply this by g_0 the acceleration, this becomes weight flow rate. So, this is the thrust per unit weight flow rate. What it means is that per unit kilogram of fuel burnt in unit time what is the amount of thrust that one can generate and that becomes a parameter that characterizes a particular propellant rocket motor combination. In this definition, g_0 is an indicator of the acceleration due to gravity at sea level and a constant value is used as shown in this representation.

(Refer Slide Time: 15:05)



Of course, as I have mentioned, I_{sp} is normally attributed to a rocket motor which is a combination of the propellant and the nozzle and that is why they are also designed and fabricated together as it depends on the calorific value of the propellant and the nozzle shape. Of course, for different propellants we are going to get different values of these propellants.

For solid propellants typically what is used is an APCP, which is expanded as ammonium perchlorate composite propellant, which essentially a combination of ammonium perchlorate and which is an oxidizer. And then you have the fuel which is either HTPB or PBAN. HTPB represents hydroxyl terminated polybutadiene or PBAN which is polybutadiene aniline nitrile.

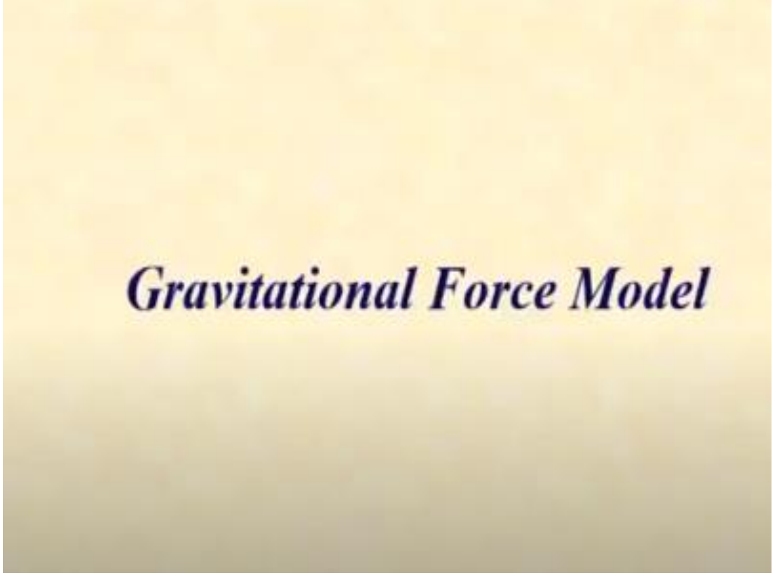
Both are hydrocarbons and represent the fuel. This combination is normally augmented with aluminum in the powder form in order to generate very high temperatures because aluminum powder once ignited at high temperature generates a large amount of heat. It is an exothermic reaction. And the I_{sp} for such propellants is in the range of 170 to 220 seconds.

When we use liquid propellants in rocket motors, there are two popular kinds which are commonly used as fuel that is either kerosene or UDMH, which expands as unsymmetric dimethylhydrazine, which is the hydrocarbon along with liquid oxygen as the oxidizer or N_2O_4 as the oxidizer. In these cases, we can get specific impulse between the range of 200 to 350 seconds.

When we come to the cryogenic engine, many of you would have heard that its liquid hydrogen and liquid oxygen is the most common cryogenic fuel which is used with hydrogen as the fuel and oxygen as the oxidizer. Sometimes, like natural gas is also used in place of hydrogen and typical I_{sp} which are possible with such engines are around 450 seconds.

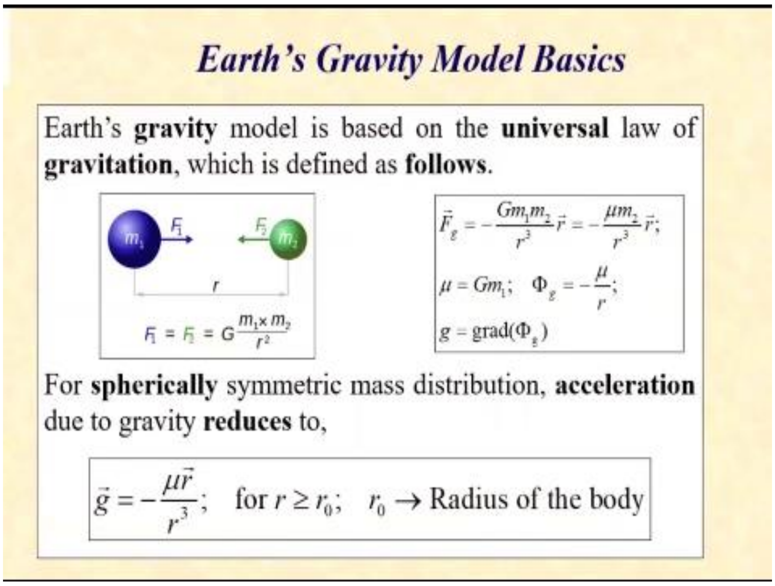
The nuclear propulsion which is also used in some applications of launch vehicles, rocket motors generate the I_{sp} in the range of 300 to 500.

(Refer Slide Time: 18:37)



With that discussion completed, let us now move over to the model for a gravitational force.

(Refer Slide Time: 18:49)



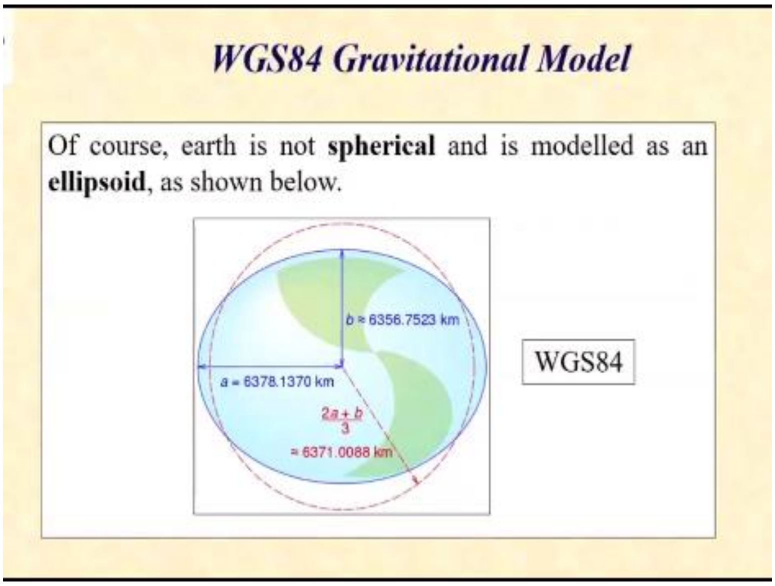
If we look at it from a very basic perspective, the earth's gravitational model is based on the universal law of gravitation, which is defined as follows. So, if we have two masses m_1 and m_2 placed at a distance of r, then they will generate gravitational forces F_1 and F_2 of attractive nature given by the expression $\frac{Gm_1m_2}{r^2}$, where G is the universal gravitational constant.

This law is universal and is applicable for all bodies, including earth and the launch vehicle. In the context of launch vehicle, what we do is that we rewrite this whole relation in a slightly different form to generate a force due to gravity on rocket as given by the above expression $-\frac{Gm_1m_2}{r^3}$.

And hypothesize a gravitational potential function, which is $-\frac{\mu}{r}$ such that the acceleration due to gravity g which is what we commonly used in most of our calculations is nothing but a gradient of the potential function $-\frac{\mu}{r}$.

Further for most applications, we can make use of a spherically symmetric mass distribution for earth such that acceleration due to gravity expression as seen above reduces to the following simple expression that is $\vec{g}$ as a vector quantity is $-\frac{\mu\vec{r}}{r^3}$, where $\vec{r}$ is the position vector from the center of earth to the center of mass of the launch vehicle. Of course, r_0 here is the radius of earth in the present case.

(Refer Slide Time: 21:22)

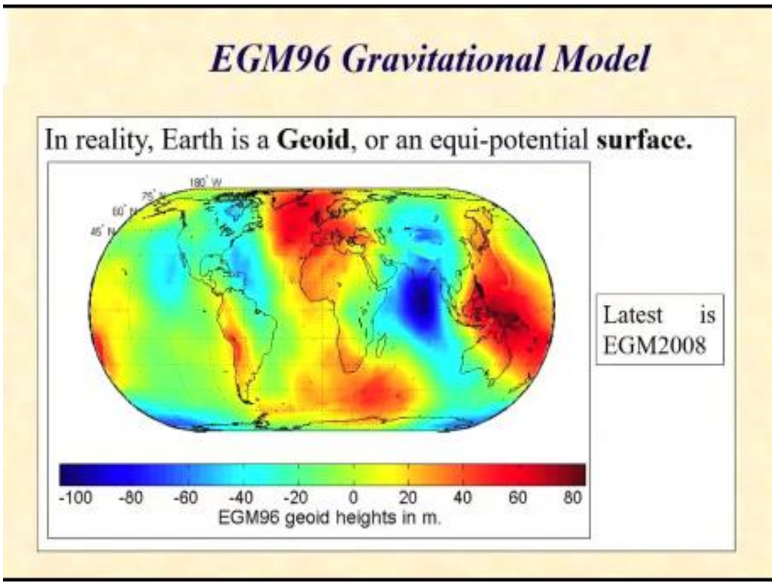


Now with this basic background on gravitational model, we now look at the realistic model of gravitation in the context of earth. The first thing that comes across is that earth is not spherical and is typically modelled as an ellipsoid as shown below. The blue lines show the ellipsoidal representation of earth geometry. So spherical approximation is the red line with an average radius of about 6371 kilometers.

While in the context of ellipsoidal model, the equatorial radius is 6378 kilometers, while the polar radius is 6356 kilometers so that there is a difference of roughly about 22 kilometers in the two radii in the ellipsoidal configuration. This particular representation has been adopted in defining the gravitational model using WGS84 standard which is used where WGS stands for World Geodetic System.

In 1984 this was first proposed and later was improved upon in 1995. So, you have a WGS95. I suggest that you can look at Wikipedia or any other internet resource to understand what are the implications of the modeling of the gravity using WGS84 and WGS95.

(Refer Slide Time: 23:24)

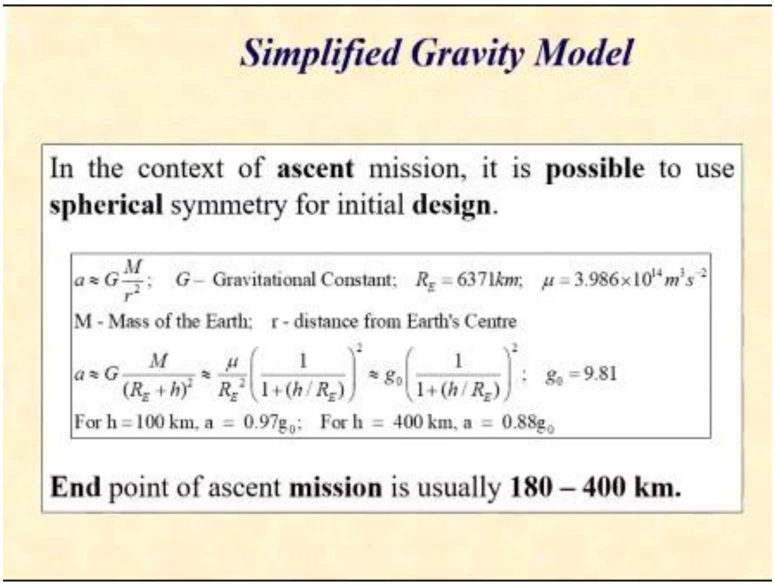


At the next level, there is a EGM96. EGM standing for Earth Gravitational Model evolved in 1996 where the ellipsoidal assumption is relaxed and earth is modeled as a geoid or an equipotential surface as shown in this picture, courtesy Wikipedia. So, it is the geoid or equipotential surface indicating that the gravitational potential on the surface is same everywhere.

So that depending upon its distance from the center the value of gravitational acceleration will be different at different locations given by the latitude and the longitudes. This is commonly modeled through a fairly complex mathematical representation which make use of zonal and tesseral harmonics, which we will not go into at the moment, but you must be aware of it.

The most recent of this kind of model has been given in 2008 under the name EGM2008.

(Refer Slide Time: 24:58)

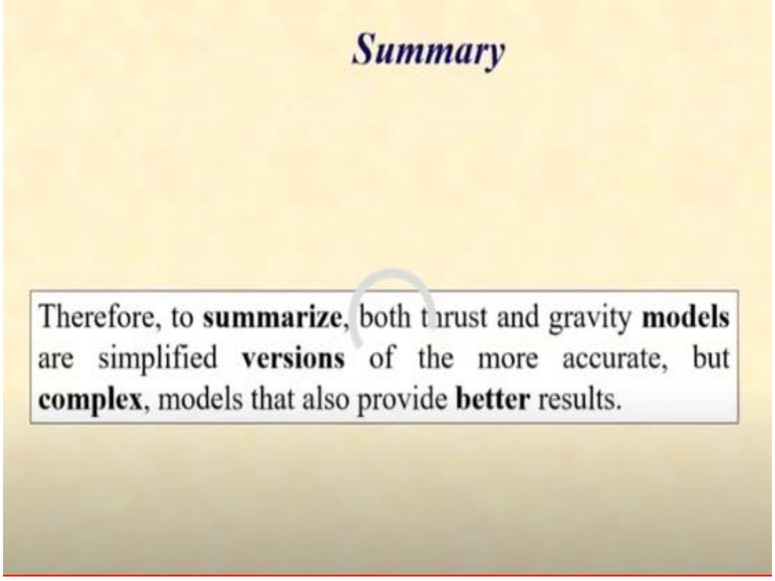


Of course, in the context of our ascent mission, for the initial sizing of the launch vehicle, for most such applications, we use the spherically symmetric model to start with. And if necessary, we can always make the corrections to improve the quality of result.

So, when we use this spherically symmetric model, we get a gravitational acceleration term, but we do a little bit of tweaking to this to show that we are not going to use a constant gravitational model, which means we are going to include the change in the gravitational acceleration due to change in the altitude.

This is necessary because you can see from the simple exercise that the gravitational acceleration of 9.81 at sea level or the surface of the earth reduces by 3% at an altitude of 100 kilometer and reduces by 12% at an altitude of 400 kilometers. As most of the ascent mission would end up between 250 to 400 kilometers, you will find that as you reach the terminal point, the value of gravity will be different from what you would assume at the start of the mission at the sea level.

(Refer Slide Time: 26:47)



So, to summarize, both thrust and gravity models are simplified versions of more accurate but complex models that also provide better results. But we have also seen that simplified models will help us to quickly size the vehicle and understand its performance, which then can be taken for more accurate analysis. So, with that, we close this lecture and we will next take up the aerodynamic and the geometric models. So, bye and see you in the next lecture and thank you.