Introduction to Launch Vehicle Analysis and Design

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Lecture - 35
Multi-stage Rocket Concept

Hello and welcome to tutorial number 3. As I had mentioned in the previous tutorial number 2, in this tutorial we will solve a few problems concerning the multi-stage rocket configuration and related features. So let us begin.

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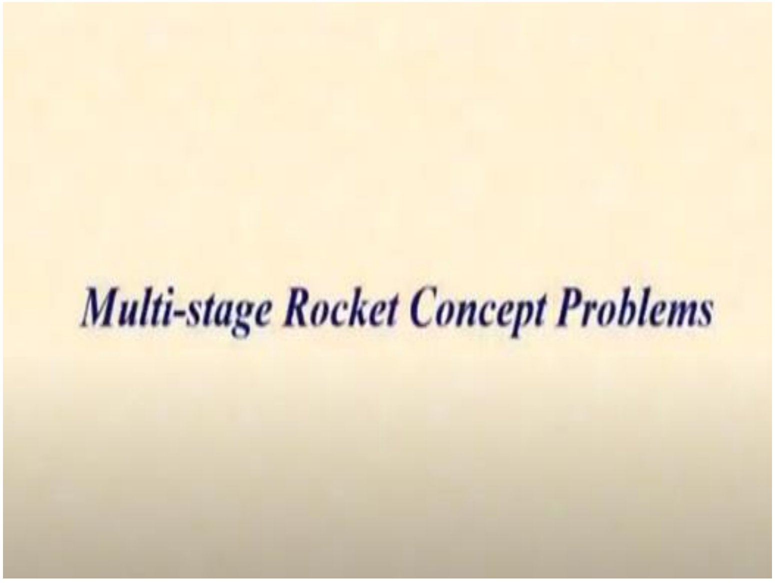

So, the problems that we are going to consider are related to the concept of multi-stage rocket as well as the basic configuration related solution processes.

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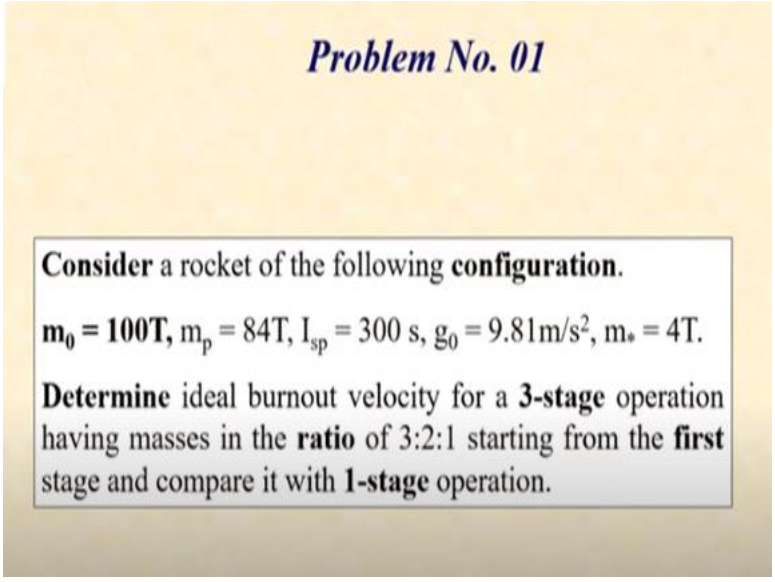

So let us start with problem number 1. Here let us consider a rocket of the following configuration. It has a liftoff mass of 100 tons, a propellant mass of 84 tons which has an I_{sp} of 300 seconds. We are going to assume sea level gravity conditions g_0 and can lift a payload of 4 tons.

Our task is to determine the ideal burnout velocity for a 3-stage operation of a very specific configuration in which the masses within the stages excluding the payload mass are in the ratio of 3:2:1. And the order of this ratio is first stage, second stage, and third stage. Which means, the first stage, second stage, and third stage mass are in the ratio of 3:2:1.

And then we have to compare this with 1-stage operation to see what benefit does a 3-stage operation brings in.

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Solution No. 01

The **solution** is as follows.

$$V_{b-ideal-1stage} = 9.81 \times 300 \times \ln\frac{100}{16} = 5393 m/s$$

Total mass of 3 stages: $48T$; $32T$; $16T$

Propellant mass of 3 stages: $42T$; $28T$; $14T$

$$V_{b-ideal-3stage} = 9.81 \times 300 \times \left[\ln\frac{100}{58} + \ln\frac{52}{24} + \ln\frac{20}{6}\right] = 7422 m/s$$

3-stage **operation** gives 38% higher **velocity**.

So let us look at the solution now. So let us first consider the ideal burnout velocity for the case of 1-stage operation. In this case, as we know all the 84 tons of propellant would be burnt in a single shot so that the ideal burnout velocity expression is $9.81 \times 300 \times \ln\frac{100}{16}$ which is what will be left at the end of the burnout which results in the burnout velocity of 5393 m/s.

Let us now move over to the 3-stage operation. It has been mentioned that the 3-stage operation is to be conducted with a configuration in which the stage masses are in the ratio of 3:2:1 excluding the payload mass. So let us first remove the payload mass from the total mass and what we are left with is 96 tons that is to be distributed between the three stages in the ratio of 3:2:1.

It is not very difficult to find out that if the mass is distributed in the following manner that is 48 tons in the first stage, 32 tons in the second stage, and 16 tons in the third stage we would have distributed the total 96 tons in the ratio of 3:2:1. Let us now do the same exercise for the propellant mass as well because the same ratio has been specified also for the propellant mass.

So, the 84 tons of propellant also can be distributed in the three stages as follows. That is first stage will carry 42 tons. The second stage will carry 28 tons and the third stage will carry 14 tons. You can see that all these add up to 84 tons and these are in the ratio of 3:2:1. Now with this, let us calculate the ideal burnout velocity for the 3-stage operation. So now, we get the same 9.81×300, but inside now we have three terms.

The first term corresponds to the burning of propellant in the first stage which is 42 tons. So, we start with 100 tons, burn the 42 tons and what we are left with at the end of first stage operation is 58 tons. Now when we move over from first stage to the second stage, the inert mass of first stage is what we are going to remove. And as we can see, the inert mass of the first stage is 6 tons. So, we remove 6 tons from 58.

So, we start the second stage operation with only 52 tons. And then we burn 28 tons of propellant and what we are left with is 24 tons at the end of the second stage. Once we finish the second stage operation, the 4 tons of the inert mass or the structural mass of the second stage will get removed.

And we will start the third stage operation with only 20 tons of which we will burn 14 tons of propellant so that at the end of the third stage operation, 6 tons will remain which will comprise 2 tons of the structural mass or the inert mass of the third stage and 4 tons of the payload. If we perform this operation, we get the velocity as 7422 m/s.

So, we find that from 1-stage operation to a 3-stage operation, where the masses are distributed in the ratio of 3:2:1, we get a 38% higher velocity indicating that a simple multi-staging can result in significant benefit in terms of the total mechanical energy that we can achieve from a given amount of propellant with a given I_{sp} value.

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Problem No. 02

A **3-stage** rocket has following **stage-wise** configuration and can launch a **payload** of 5T.

$$\begin{array}{l} \text{1-Stage: } \varepsilon_1 = 0.073, \quad \pi_1 = 0.191 \\ \text{2-Stage: } \varepsilon_2 = 0.102, \quad \pi_2 = 0.358 \\ \text{3-Stage: } \varepsilon_3 = 0.143, \quad \pi_3 = 0.737 \end{array}$$

Determine stage-wise mass **distribution** and the total lift-off **mass.**

Let us now move over to problem number 2. So, in this problem, we have a 3-stage rocket which has a specific payload and the structural ratio configuration and it is known that can launch a payload of 5 tons. The first stage structural ratio is 0.073, whereas its payload ratio is 0.191. For the second stage, the structural ratio is 0.102 while the payload ratio is 0.358. For the third stage, the structural ratio is 0.143 but the payload ratio is 0.737.

Our task is to determine the stage-wise mass distribution and arrive at the total liftoff mass.

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Solution No. 02

The **solutions** for 3rd and 2nd **stages** are as follows.

$$m_{s3} = \varepsilon_3 \left(\frac{1-\pi_3}{\pi_3} \right) m_* = 0.143 \times 5 \times 0.357 = 0.255T$$

$$m_{p3} = (1-\varepsilon_3) \left(\frac{1-\pi_3}{\pi_3} \right) m_* = 0.857 \times 5 \times 0.357 = 1.530T$$

$$m_{03} = m_{p3} + m_{s3} + m_* = 6.786T$$

$$m_{s2} = \varepsilon_2 \left(\frac{1-\pi_2}{\pi_2} \right) m_{03} = 0.102 \times 6.786 \times 1.793 = 1.241T$$

$$m_{p2} = (1-\varepsilon_2) \left(\frac{1-\pi_2}{\pi_2} \right) m_{03} = 0.898 \times 6.786 \times 1.793 = 10.93T$$

$$m_{02} = m_{p2} + m_{s2} + m_{03} = 18.953T$$

So, as we have seen in our lectures, the mass configuration of a rocket begins from the top. So, we start from the third stage first, then move over to the second stage, then move over to the first stage and then we get total liftoff mass. So let us first look at the solutions for third and the second stages as follows. So, for the third stage structural mass is driven by the third stage structural efficiency, third stage payload ratio and the payload mass.

So, when we perform this operation, we find that the structural mass of the third stage is 0.255 tons. Now we can get the propulsion mass for the third stage as the following expression, again a function of the payload mass. And it turns out to be 1.53 tons.

Now what we do is we generate m_{03} that is the mass at the beginning of the third stage operation, which is composed of the propellant mass of the third stage, the structural mass of the third stage and the payload which is going to be launched. If we sum up all these, the total mass that we get is 6.786 tons. And we immediately realize that this is going to become the payload for the second stage operation.

So, m_{03} will have the same character as m_* for the second stage operation. So, we just take the same expressions, replace third stage with the second stage and replace m_* with m_{03}. And if we do that, then we find that the mass of the structure of the second stage is 1.241 tons while the mass of propellant for the second stage is 10.93 tons.

Now similar to what we did for the third stage, we can now find out the starting mass for the second stage that is m_{02} which is a sum of propellant mass of second stage, the structural mass of the second stage and the payload for the second stage which is the starting mass for the third stage or m_{03}. If we sum all these the total mass for the second stage operation is 18.953 tons.

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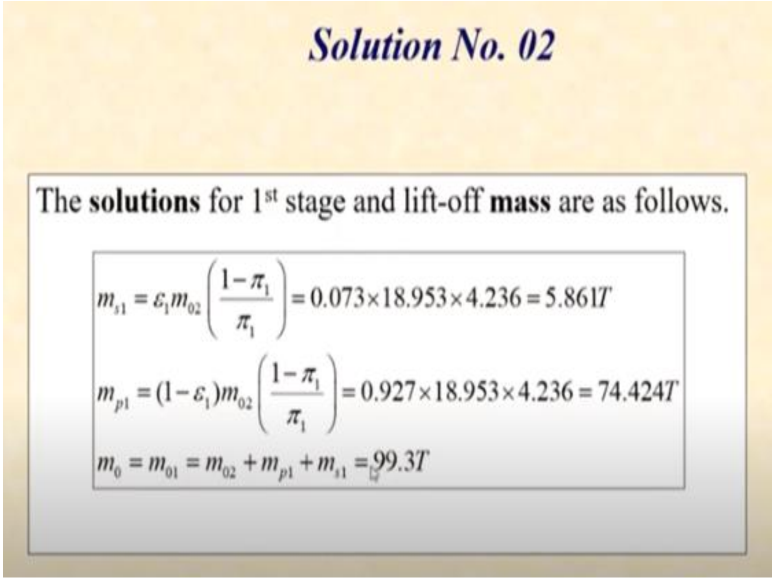

Now we repeat the same process for the first stage as follows. So, for the first stage, again we assume m_{02} to be the payload for the first stage and based on that, we get the structural mass of first stage as 5.861 tons and the propellant mass of the first stage as 74.424 tons.

Now the starting mass of the first stage, which is same as also the total liftoff mass is nothing but the sum of the propellant mass of first stage, the structural mass of the first stage and the payload for the first stage which is the starting mass for the second stage. If we sum up all this, what we get is 99.3 tons. Hi, so in this tutorial, we have looked at two problems.

The first problem is to arrive at the performance by converting a single stage rocket into a specific configuration of a 3-stage operation. And we have characterized the benefit that accrues to us when we go from a single-state operation to a multi-stage operation. In the second problem, for a given technological configuration in terms of structural ratios, and the payload ratios, we have used the payload capability to arrive at what kind of rocket would make such a mission possible.

As you would know, these represent fundamental solution techniques in the context of a multi-stage operation. With that, we come to the end of this tutorial. In the next tutorial, which is the last tutorial that we will have, we will look at the methodologies of arriving at an optimal configuration of the rocket with either velocity as the objective function or the mission payload ratio as the objective function.

We will also look at the approximate optimal solution procedure. And we will conclude the next tutorial with a problem on the tradeoff ratios to indicate the overall efficiency of the design, as well as its stretchability. So, bye, see you and thank you.