Introduction to Launch Vehicle Analysis and Design

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Lecture - 30

Ballistic Reentry Solution

Hello and welcome. In the last lecture, we had introduced the concept of entry missions and had considered some aspects of the basic mechanism of entry or reentry missions. In this context, we had also mentioned that there are two possible broad categories of strategies which are commonly employed for entry missions, one with lift equal to zero called the ballistic entry.

And other one with a lift such that the lift to drag ratio is within a specified limited range. In this lecture, we will now look at the first of these strategies that is the ballistic entry trajectories. So let us begin.

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So, our discussion on ballistic entry trajectories is divided into two segments the first of which refers to the steep ballistic stick entry while the second one refers to a shallow or orbital ballistic trajectory. So let us first look at the basic idea of the steep ballistic trajectory.

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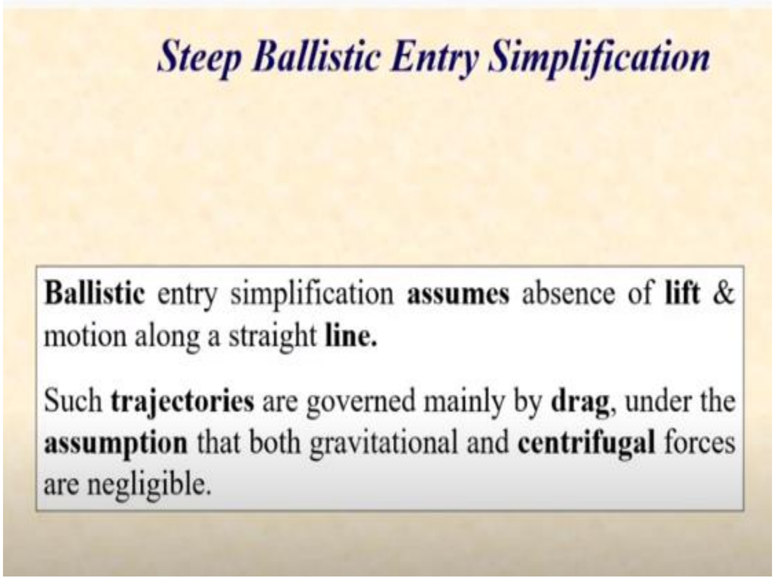

As we have already seen, the ballistic entry assumes that there is no lift and broadly the motion is along a straight line. As we immediately note, such trajectories are governed primarily by drag under the assumption that both gravitational and centrifugal forces are negligible. So, in addition to the neglect of lift we also neglect the centrifugal forces because the trajectory curvature is zero.

So, the radius of curvature is infinite and $\frac{V^2}{r}$ term goes to zero. And we also ignore gravitational acceleration as the drag is the primary force that is responsible for the trajectory parameters.

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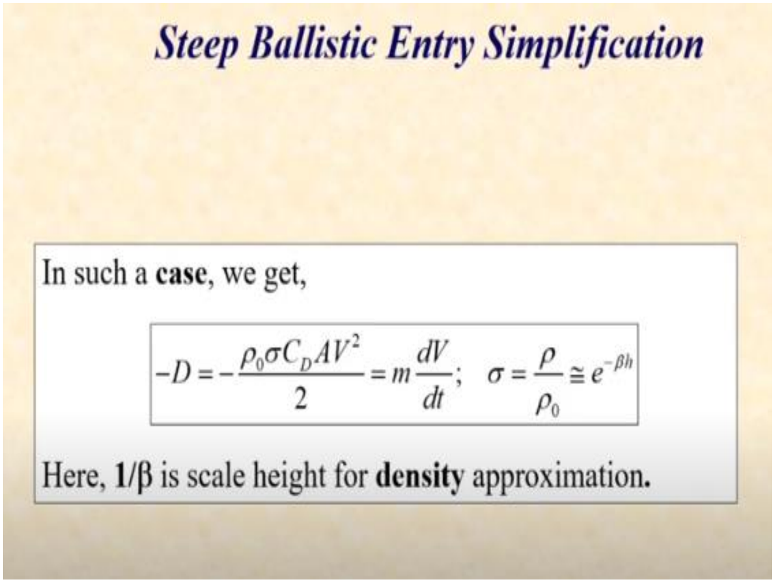

In such a case, we get the following system of equations. We just equate the drag which is defined as $\frac{\rho_0 \sigma C_D A V^2}{2}$, which is equal to $\frac{mdV}{dt}$ directly; a simple differential equation which is nonlinear as we have V^2 on the right-hand side. Further, we also have an altitude term to the parameter σ which represents the ratio of the density at an altitude h with respect to the sea level density ρ_0.

And is typically expressed as an exponentially decaying function with β as the parameter. It should be mentioned here that $\frac{1}{\beta}$ is called the scale height commonly employed for density approximation, while capturing the effect of atmosphere on the vehicles.

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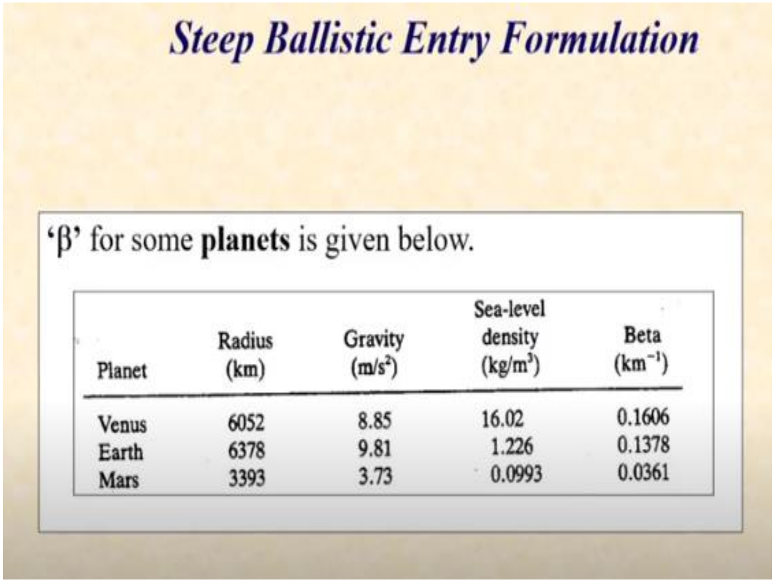

Planet	Radius (km)	Gravity (m/s^2)	Sea-level density (kg/m^3)	Beta (km^{-1})
Venus	6052	8.85	16.02	0.1606
Earth	6378	9.81	1.226	0.1378
Mars	3393	3.73	0.0993	0.0361

Given below are some values of β for various planets which have atmosphere. So, for example Venus the value is 0.1606, for earth it is 0.1378 and for Mars it is 0.0361 that indicates also the kind of atmosphere which is present on these planets.

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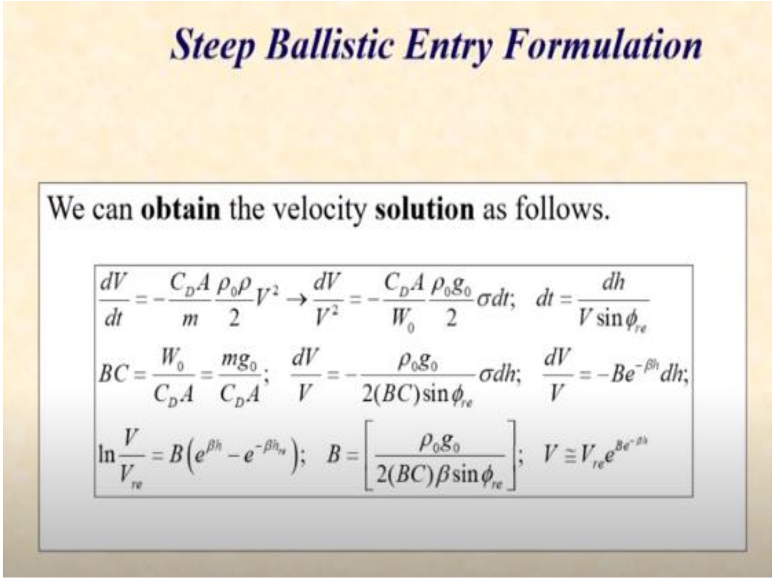

With this in place, it is not very difficult to solve the nonlinear differential equation in closed form and we can obtain the velocity solution as given below. So, we can write $\frac{dV}{dt}$ in this form. So, we can write on the left-hand side $\frac{dV}{V^2}$ in terms of the various constants such as drag coefficient, the frontal area A, W_0 which is the weight of the vehicle.

And before we write the final expression, there is an important definition that I introduced called the ballistic coefficient written as BC which is the ratio of the weight $\frac{W_0}{C_D}$ into area. Please note that C_D is the drag coefficient, A is the area which are both constants for a given vehicle. Similarly, we assume that there is no change in the weight of the vehicle as it reenters.

So, it is a constant for a given vehicle configuration. We try to then solve this differential equation in terms of the ballistic coefficient and the entry angle ϕ. And we can obtain a solution of velocity in terms of the velocity at the reentry and a parameter B which combines the various quantities which are constants and the exponential function of the density.

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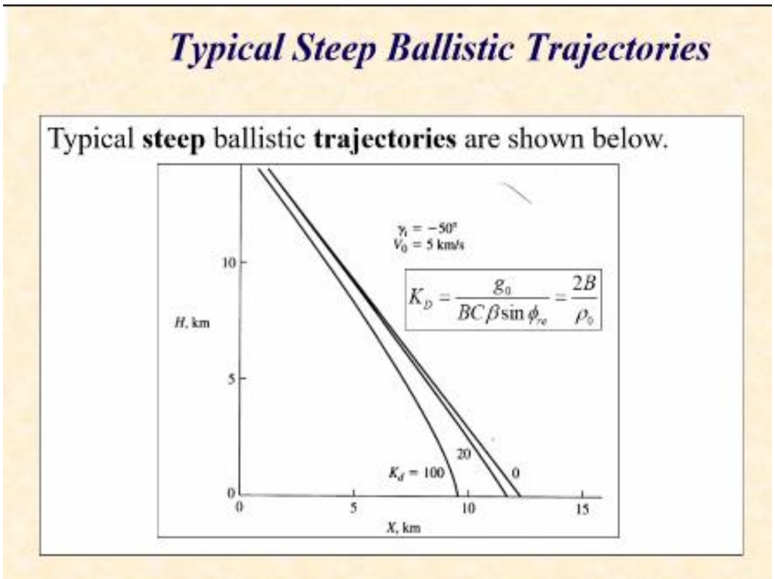

Let us now try to visualize these trajectories for different values of angles and the constant K_D which is the ratio of $\frac{2B}{\rho_0}$ and for an entry angle of $-50°$ which means that at the time of entry, the object makes an angle of $-50°$ with the local horizon and has a velocity of 5 km/s. Here we see the trajectories which are roughly from the height of 15 km down to the surface of the earth.

And we note that for a major part of the trajectory, it is merely a straight line. In fact, for $K_D = 0$, it is exactly a straight line. And as we can see from the expression of K_D, which contains ballistic coefficient that would happen in the case of a large value of ballistic coefficient and a large value of ballistic coefficient as we have seen earlier is directly possible with a small value of drag coefficient or a large weight.

So, which means that vehicles which either are heavy or have a small drag coefficient or a small frontal area will describe a straight-line trajectory and the equations that we have used will be reasonably applicable to give us the velocity solution. Of course, we also see from this picture that as K_D value increases that is the ballistic coefficient decreases.

Which indirectly means that either weight decreases or the drag coefficient increases, there is a little bit of curving of the trajectory as we reach the lower part of atmosphere, which is much denser. And as it is much denser, the effect of drag increases significantly indicating that some amount of curving or the trajectory is possible.

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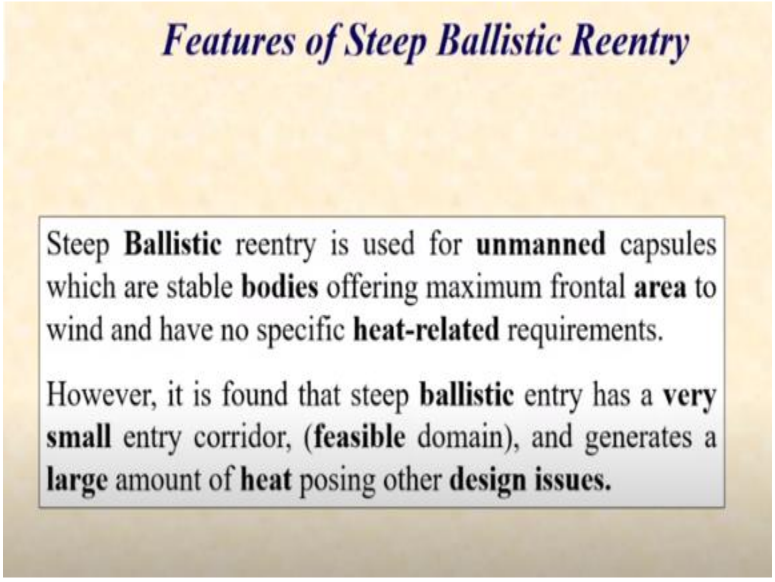

It is to be noted here that this kind of entry trajectory or entry mechanism is commonly used for unmanned capsules, which are designed as stable bluff bodies, which offer maximum frontal area to wind and generally have no specific heat related requirements so that we can generate a large amount of drag and dissipate the energy quickly in a shorter duration during a shorter span of the trajectory.

Of course, we find that the steep ballistic trajectory entry has a very small entry corridor which is typically what we call the feasible domain and obviously generates a large amount of heat. And even in the context of unmanned capsules can pose some design issues with regard to material, the strength, the integrity of the structure etc.

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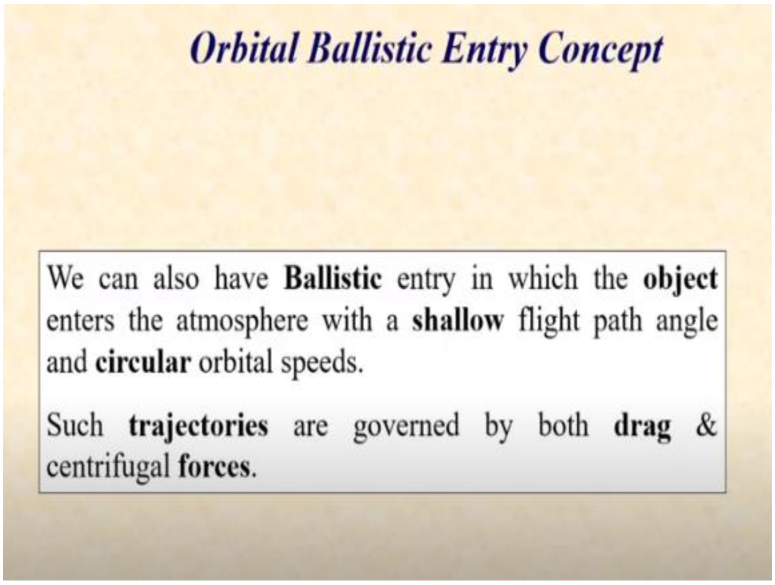

Let us now turn our attention to another form of ballistic entry in which instead of using a very large negative angle for entry, a shallow angle for trajectory is employed. And is typically also called shallow ballistic trajectory or orbital ballistic trajectory. Which means that the vehicle enters the earth's atmosphere on an orbit such that its velocity vector is nearly parallel to the local horizon with a very small angle with respect to the local horizon.

In such cases, the entry is typically close to orbital velocities, which we have seen to be can be of the order of 8000 to 11,000 m/s. Whereas in the steep ballistic trajectory, we make the entry with a smaller velocity of the order of about 4000 to 5000 m/s as we have seen in the example.

We will note that such trajectories are governed by both drag and centrifugal forces and they need to be included in our model description.

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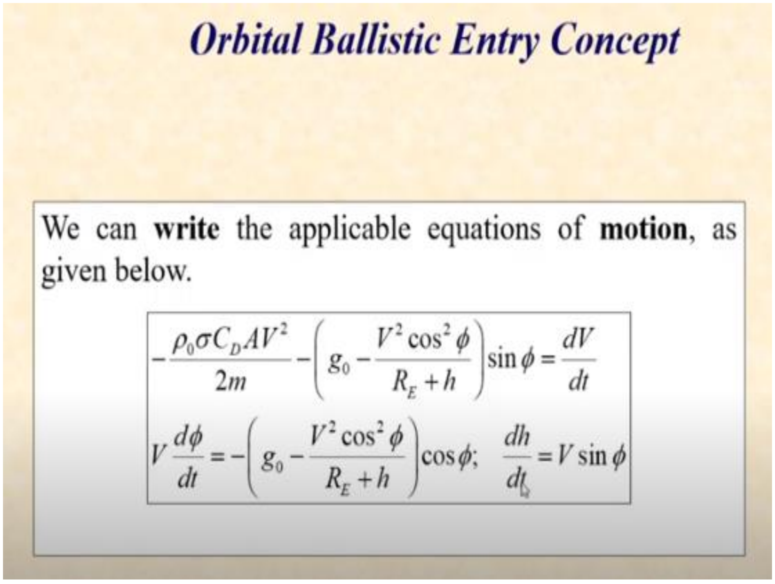

So, in this case, the model equations are that apart from the drag term, which is same as the steep ballistic trajectory, we include both the gravitational force g_0 and the centrifugal force term which is significant due to large velocities. And then of course, it is a two-dimensional motion, instead of the motion being along a straight line. So, we have the normal equilibrium also, $V\frac{d\phi}{dt}$. And then we have the kinematic relation $\frac{dh}{dt}$ as $V\sin\phi$.

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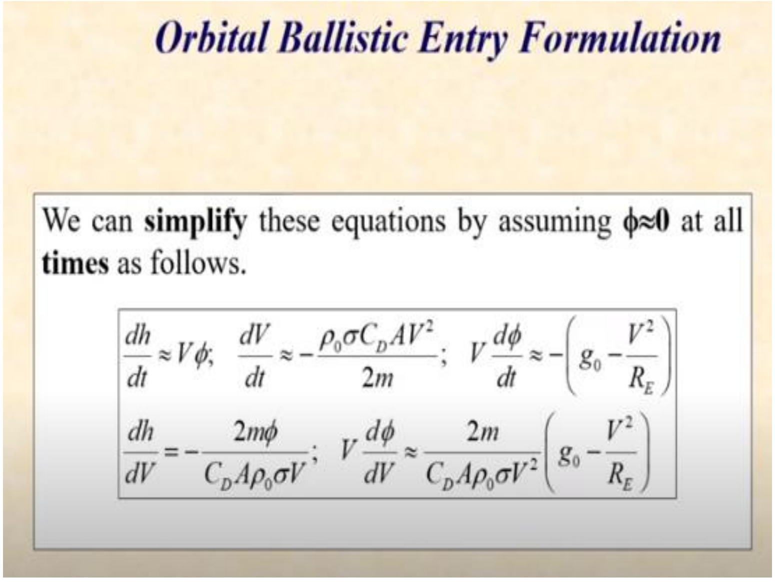

As ϕ in this case is very close to 0, we realize that $\cos\phi$ is very close to 1 and the $\sin\phi$ can be replaced with ϕ that makes the equations linear and to some extent simplify our solution process. So, the moment we do that, we get $\frac{dh}{dt}$ as $V\phi$, $\frac{dV}{dt}$ remains a nonlinear differential equation in V^2.

The normal equilibrium has $V\frac{d\phi}{dt}$. And with that, we get the following two differential equations, one for the altitude in terms of velocity and other one velocity in terms of the angle ϕ. While these equations are a little bit more complicated than the steep ballistic trajectory equations, it is still possible to carry out the solutions.

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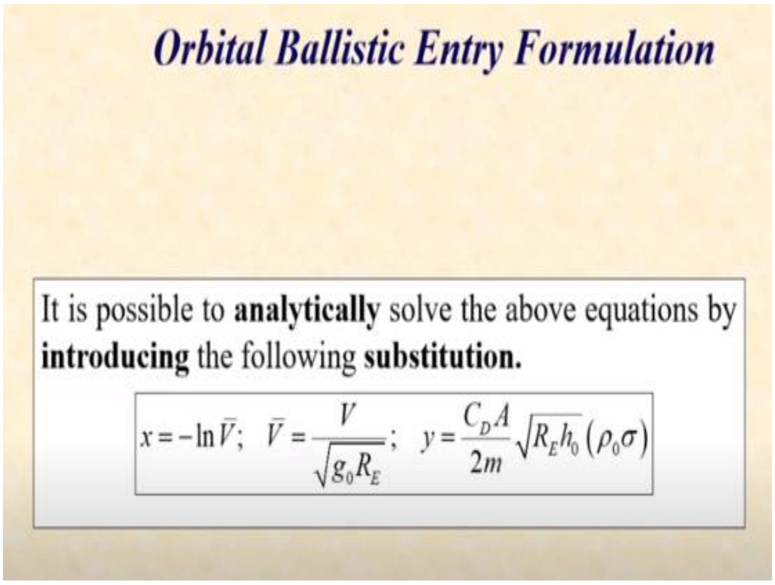

Analytically by introducing certain change of variable through substitution. So, we replace x as $-\ln \bar{V}$ where, $\bar{V}$ we defined as $\frac{V}{\sqrt{g_0 R_E}}$. Here let me make a mention that this normalization of V with respect to a parameter called $\sqrt{g_0 R_E}$ enables the values of V to be in a reasonable range between let us say 0 and 1 because $\sqrt{g_0 R_E}$ is nothing but the circular orbital velocity at a specific altitude that is surface of the earth.

Then we also introduce another parameter y in terms of the drag coefficient, the frontal area and the other parameters of the problem.

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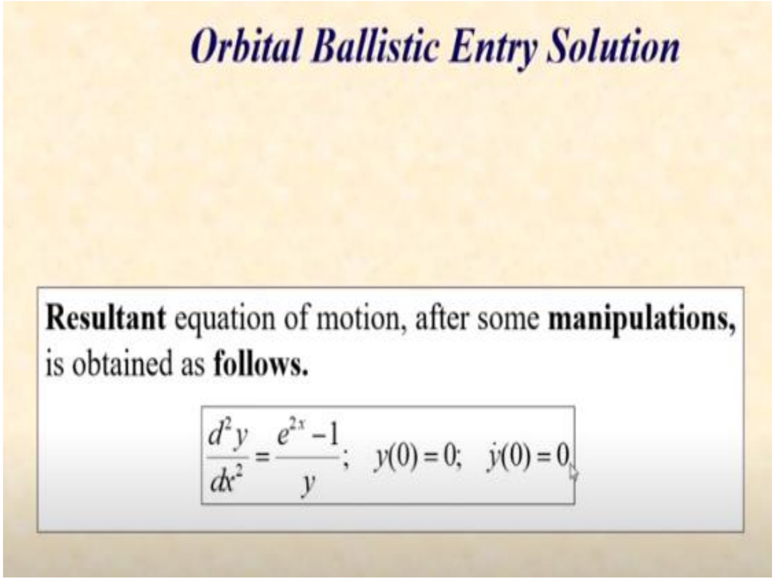

We can do a bit of algebraic manipulations after which the resultant equation of motion is obtained as follows. This is now a second order differential equation in y with respect to modified variable x and is in terms of an exponential function of x and y and as it is a second order differential equation, it will have two initial conditions or boundary values $y(0)$ is 0 and $\dot{y}(0)$ is 0.

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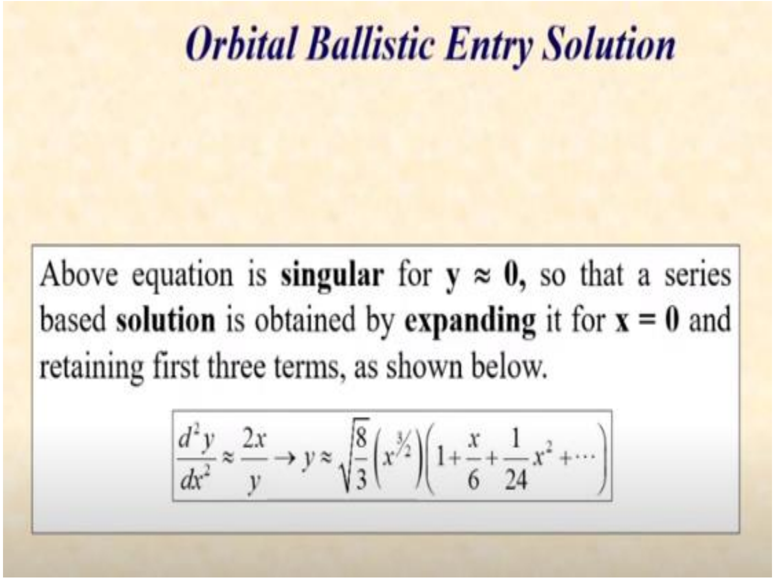

It is possible to solve this equation but it requires a little bit of interpretation. The above equation is found to be singular for $y = 0$. So, there are techniques in calculus, which aim to solve similar differential equations through what is called a perturbation technique wherein we expand the solution in the form of a series around that singular point and then generate the solution in the form of a power series.

In the present case, if we do this expansion, it can be shown that the resultant modified differential equation through this perturbation expansion can be given by $\frac{d^2y}{dx^2} = \frac{2x}{y}$. And this differential equation is not very difficult to solve in the closed form analytically for which we get a solution for y in the closed form. Here I have shown only the first three terms of this series.

But depending upon the requirement of specific context, one could include as many number of terms in this series as are required to achieve the desired accuracy.

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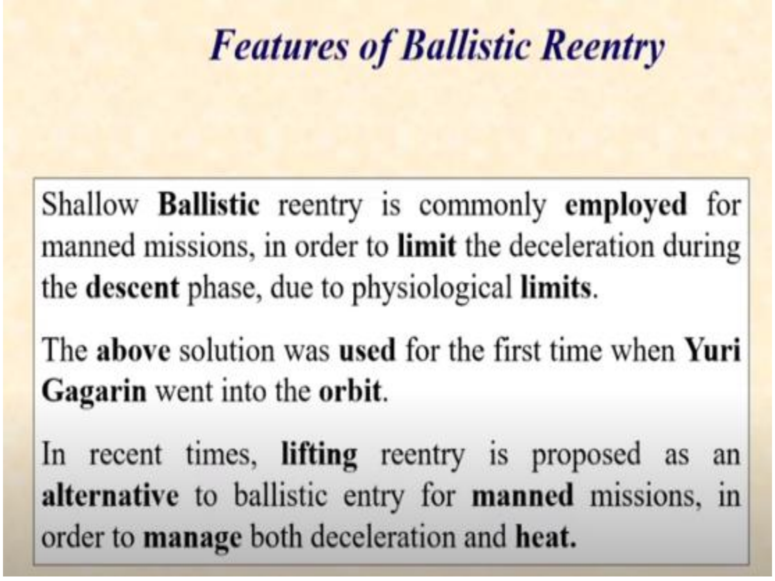

So, what are the broad features of such a ballistic reentry? The shallow ballistic reentry is commonly employed for manned missions as against the unmanned missions, which use steep ballistic entry. The idea is to limit the overall deceleration by limiting the drag that is generated in the descent phase. This is primarily because a large drag acting on the vehicle will result in very large amount of deceleration which may exceed the physiological limits of the humans which are part of the vehicle.

The above solution that we have seen was first used when Yuri Gagarin went into the orbit and returned to earth in his space capsule and the same space capsule was brought back through a shallow ballistic reentry so that the deceleration could be kept to manageable limits. Of course, this still has issues in terms of not having any explicit control over the acceleration.

And most of the time the acceleration is through designing of the configuration that fixes the value of your drag coefficient, the frontal area and other quantities. In recent times, this option has been replaced with a more flexible and viable option of lifting reentry in which the lift is used to manage the deceleration and also is able to manage the heat load which is not possible during the shallow ballistic trajectory.

If you want to do the heat management in the shallow ballistic trajectory, then the amount of time that it would require to remain in the orbit before it comes back to earth is going to be very large. So, the lifting reentry provides the additional flexibility of maintaining the deceleration and the heat load while performing entry for manned missions.

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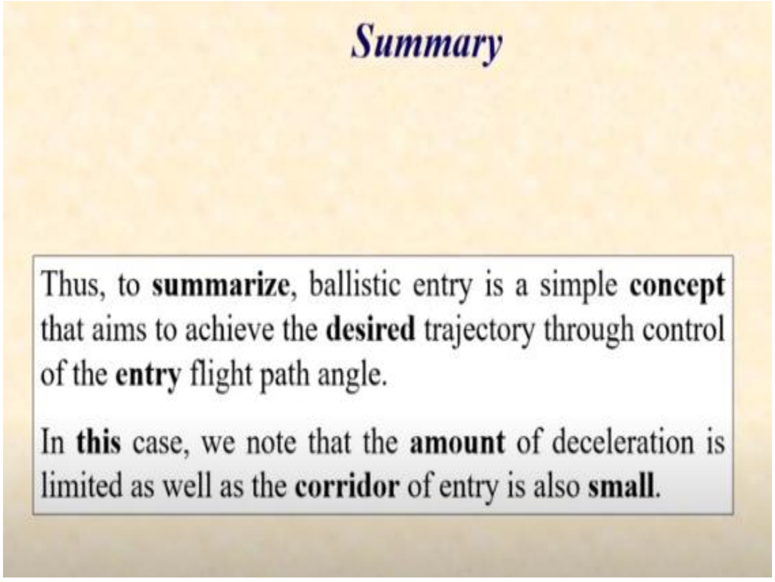

So, to summarize, the ballistic entry is a simple concept that aims to achieve the desired trajectory through control of entry flight path angle. In this case, we note that the amount of deceleration is limited as well as the corridor of entry is also quite small. Of course, we have seen that by making the entry angle shallow, it is possible to manage the heat load and the deceleration.

But then, it is possible that the time spent during the flight is likely to be significantly large. Hi, so in this lecture, we have seen some features of the ballistic entry trajectory through two scenarios, one with a steep entry angle and the other with a shallow entry angle, both having their own benefits as well as limitations.

We have also established that by using lift it is possible for us to manage both deceleration and the heat load in a more elegant manner. And this is what we will explore in the next lecture. So, bye. See you in the next lecture and thank you.