Introduction to Launch Vehicle Analysis and Design

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Lecture - 26
Jet Damping and Ballistic Missiles

Hello and welcome with the discussions on the trajectory and rocket configuration concluded. We now turn our attention to certain special effects that are present in rockets and in practical contexts, need to be taken into account during the design, analysis and operation. In this regard, we will first consider the concept of jet damping, which is present in most rockets.

And then we will introduce the basic ideas of ballistic missiles which have a reasonable amount of similarity with the rockets. So let us begin.

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So let us begin our discussion on the basic ideas of the jet damping phenomena.

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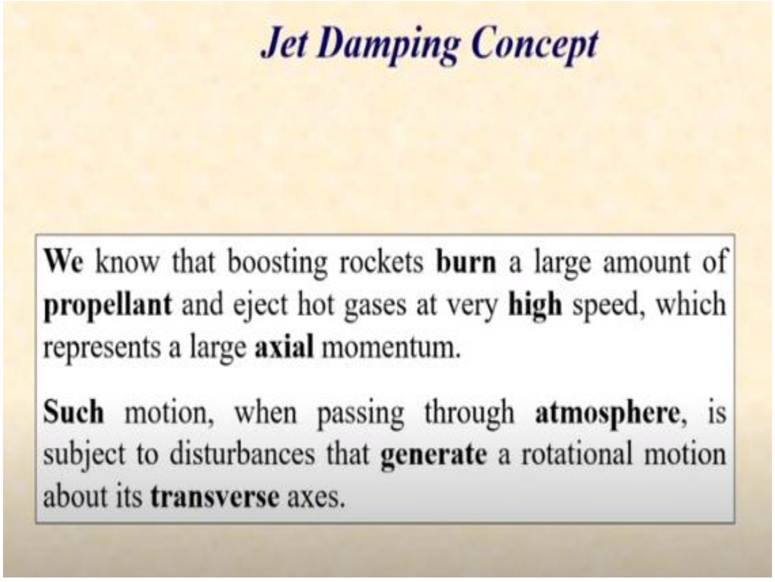

So let us start by recalling the fact that all the boosting rockets burn a large amount of propellant and eject hot gases at very high speed which represents a large axial momentum. Such motion when passing through atmosphere is subject to disturbances that generate a rotational motion of the vehicle about its transverse axes.

This is due to the fact that a transverse disturbance in the form of a wind or a wind shear would generate a transverse movement about the axis and under the action of such a moment the vehicle will rotate and acquire a rotational velocity.

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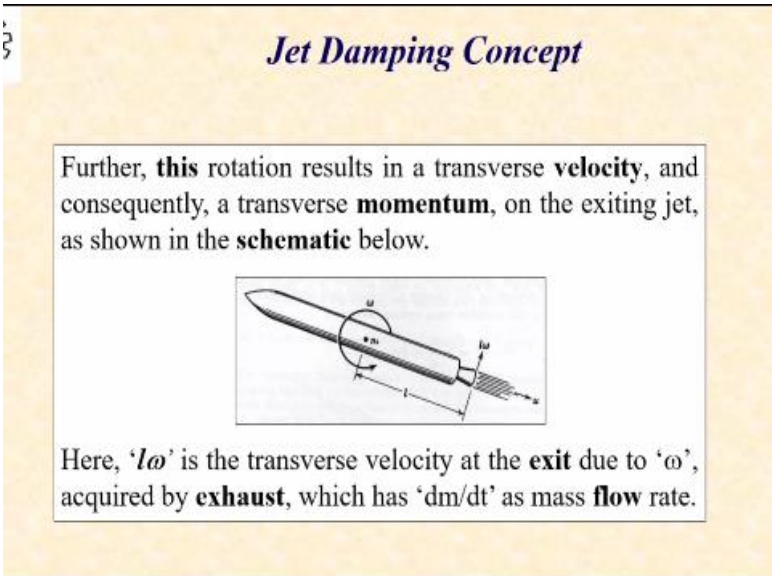

Now as vehicle rotates about its axis its front and back execute rectilinear motion and in the process, the jet which is exiting from the rear of the rocket acquires apart from the axial velocity due to burning of the propellant also a transverse velocity due to the angular velocity disturbance. The phenomenon is shown schematically below.

So here you see that there is a rocket which has acquired an angular velocity ω about its axis and under the action of this angular velocity ω which is anti-clockwise, the jet which is exiting at a distance l behind the center of mass of the launch vehicle acquires a rectilinear velocity $l\omega$ in the direction normal to the axial velocity u and represents the transverse momentum.

We see from this schematic that the transverse velocities $l\omega$ is acquired by the exiting mass flow rate $\frac{dm}{dt}$ which is due to the burning of the propellant.

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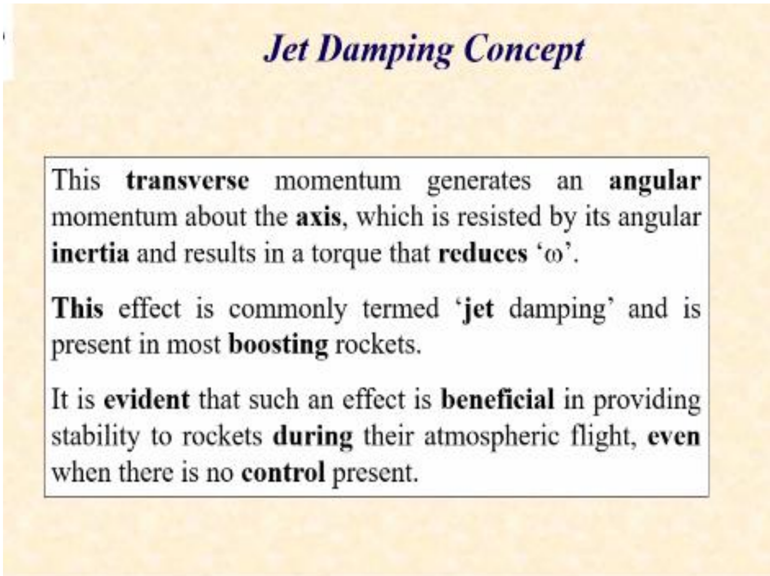

Now this transverse velocity has a transverse momentum and rate of change of that transverse momentum generates a transverse force which is nothing but $\dot{m}l\omega$. And now this force tries to generate a moment about the axis which is $\dot{m}l^2\omega$. Now this moment as we all know from our Newton's laws is resisted by its angular inertia such that there is a resistance to this motion which reduces ω as time progresses.

This effect is commonly termed jet damping as it aims to dampen out or reduce the amplitude of the angular velocity and is present in most boosting rockets. It is evident that such an effect is beneficial in providing stability to rockets during their atmospheric flight even when there is no control present.

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Jet Damping Formulation

Following is the formulation applicable in this **context**.

$$\frac{d}{dt}(I\omega) - \frac{dm}{dt}l^2\omega = 0; \quad I = mk^2 \rightarrow \text{Rocket Moment of Inertia}$$

$$I\frac{d\omega}{dt} + \omega\frac{dI}{dt} - \frac{dm}{dt}l^2\omega = 0 \rightarrow I\frac{d\omega}{dt} + \omega\frac{d(mk^2)}{dt} - \frac{dm}{dt}l^2\omega = 0$$

$$mk^2\frac{d\omega}{dt} + \omega k^2\frac{dm}{dt} + m\omega\frac{d(k^2)}{dt} - \frac{dm}{dt}l^2\omega = 0; \quad m\omega\frac{d(k^2)}{dt} \approx 0$$

$$m\frac{d\omega}{dt} + \omega\frac{dm}{dt} - \frac{dm}{dt}\frac{l^2}{k^2}\omega = 0 \rightarrow \frac{d\omega}{\omega} = \left(\frac{l^2}{k^2} - 1\right)\frac{dm}{m}$$

Following is a basic formulation applicable in this context. So let us start with the angular moment balance equation. So, the rate of change of angular momentum $I\omega$ is to be balanced by the torque which is applied and as this torque is in direction opposite to the inertia torque, we have this equation. And as there are no other external moments acting on it, the right-hand side is zero.

Here, we simplify the representation of the mass moment of inertia of the rocket by writing it as the mk^2 where, k is the standard radius of gyration. Now of course, we can expand the derivative and after a little bit of algebraic manipulation, which I would suggest that you verify and go through step by step we can show that the final differential equation which governs the motion of such a disturbed rocket about its axis can be given by this simplified differential equation.

Here, there is a small assumption which is made as mentioned here that the rate of change of radius of gyration as time progresses is small so that the corresponding term can be neglected. This also is reasonable as not only the rate of change of radius of gyration with time is small, it also is multiplying with ω which also is supposed to be a disturbance and a small quantity.

So, the product of two small quantities anyway is expected to be small compared to other quantities present in the differential equation. Let us now look at the solution of this differential equation, which is as given below.

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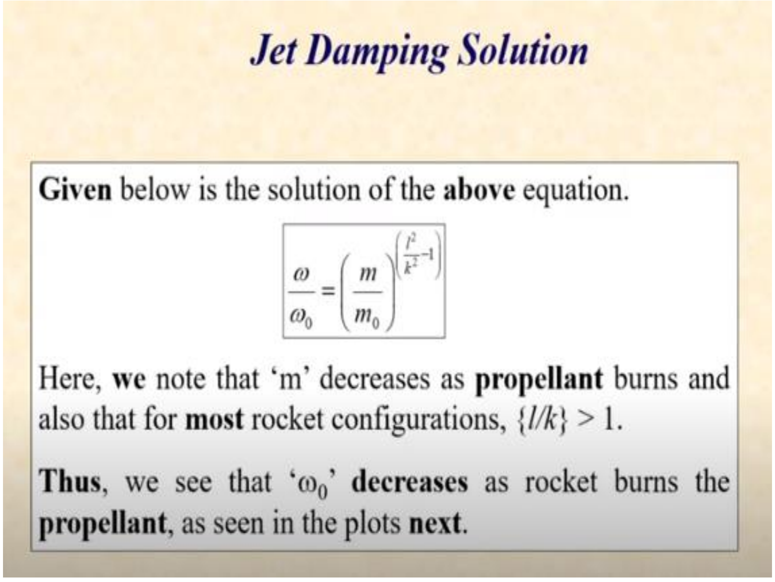

We can apply the simple formulae of integral calculus and we can show that $\frac{\omega}{\omega_0}$ where, ω_0 is the initial angular velocity acquired at time t equal to zero is nothing but $\frac{m}{m_0}$, where m_0 is the mass of vehicle at that instant of time raised to the power $\frac{l^2}{k^2} - 1$. And then we note couple of things with regard to the solution.

The first thing that we will note is that as time progresses, we burn propellant so that $\frac{m}{m_0}$ is a decreasing quantity as a function of time. Further, we find that for most practical rocket configurations, $\frac{l}{k}$ is a quantity which is always greater than 1, which obviously indicates that as $\frac{m}{m_0}$ decreases with time, $\frac{l^2}{k^2} - 1$ is a positive integer or a positive real number rather, $\frac{\omega}{\omega_0}$ will also continue to decrease as mass decreases.

The basic behavior of this equation is generically showed in the plots next.

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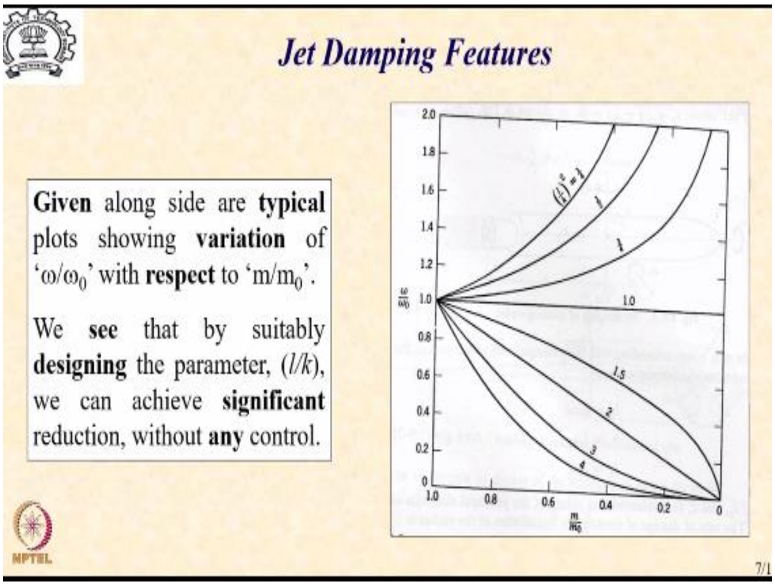

So let us see typical plots showing the variation of $\frac{\omega}{\omega_0}$ with respect to $\frac{m}{m_0}$. So now let us see the features of this solution. So let us first look at the degenerate case of $\frac{l}{k} = 1$. So, if $\frac{l}{k} = 1$, the exponent $\frac{l^2}{k^2} - 1$ goes to zero and which means that $\frac{\omega}{\omega_0} = 1$ or a constant. So, in such a scenario the acquired initial angular velocity will not decay.

Next, let us look at the cases of $\frac{l}{k} > 1$, like 1.5, 2, 3, 4 and we can see that as $\frac{m}{m_0}$ decreases continuously to 0 from 1 which is at t equal to 0. $\frac{\omega}{\omega_0}$ also continuously goes to a smaller value until it also becomes 0 as $\frac{m}{m_0}$ become 0. This indicates that an initial angular disturbance generating an angular velocity ω_0 will automatically decay to 0 just by the fact that your mass is continuously decreasing without having to take any other external action.

Of course, we also see that if $\frac{l}{k} < 1$, then this represents now an unstable configuration such that your disturbance will start growing as a function of time. As the mass decreases, the angular velocity will start growing. So now we realize that by suitably designing this parameter $\frac{l}{k}$, not only we can achieve stability of the disturbed motion, but we can also choose a specific rate of decay as desired from the trajectory and performance requirements.

Of course, you would also realize that the value of $\frac{l}{k}$ directly has an impact on the overall configuration design of the launch vehicle and typically, this fact is included in the overall design process, while looking at the freezing of the configuration of the rocket.

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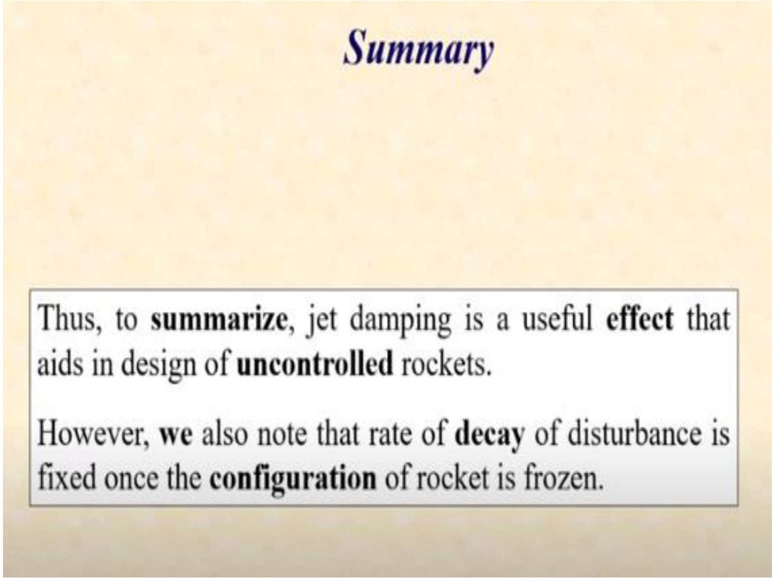

So, to summarize, jet damping is a useful effect that aids in design of uncontrolled rockets. However, we also note that rate of decay of disturbance is fixed once the configuration for rocket is frozen.

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Let us now take the next topic of special interest that is ballistic missile trajectories.

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Ballistic missiles are those class of space vehicles that generally resemble rockets from an external perspective but have different objectives and consequently the trajectory and it is important to look at this aspect just to understand the ideas of rockets that can be applied to ballistic missile solutions. So, one of the similarities which is there is that these objects are launched like rockets.

But instead of releasing a payload at a designated altitude, which is a spacecraft, these objects travel great distances over earth of the order of 5000 to 10,000 km and then they re-enter the earth's atmosphere and have an impact at a designated point on the surface of the earth.

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Of course, with this kind of a trajectory requirement, there are now differences in the way that missiles are configured as compared to rockets. As we have also seen, there are certain areas of similarity between the two so that whatever analysis that we have done so far with regard to rockets or launch vehicles, a part of it can be borrowed and applied to these objects.

Firstly, the ballistic missiles also burn large amount of propellant and pass-through atmosphere on a trajectory which is similar to that of rocket so that the relations for trajectory that we have developed so far for rocket motion through atmosphere from the launch can be applied and arrived at the solution for the trajectory parameters. Secondly, as these are expected to travel great distances, they also need large velocities.

So, there are going to be large amount of accelerations that these vehicles will experience and similar kind of burn profiles that we have looked at in the context of rockets could also be applicable in the context of ballistic missiles. There could also be multi-stage vehicles as rockets are commonly.

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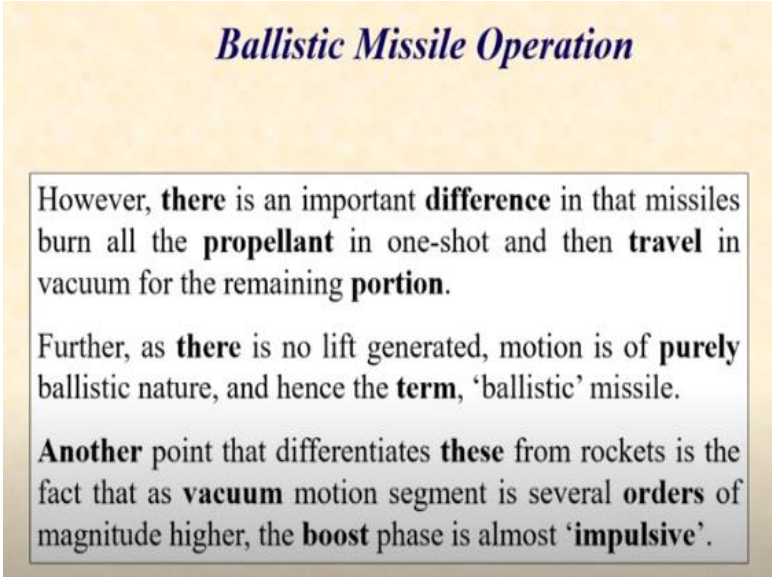

However, there is an important difference in that ballistic missiles burn all the propellant in one shot and then travel in vacuum for the remaining portion of trajectory without any thrust. Further as it is traveling in vacuum, there is no other transverse force like aerodynamic lift generated. So, the motion is purely ballistic in nature governed by the gravitational field of earth and hence the term ballistic missile.

Another point that differentiates these from rockets is the fact that as vacuum motion segment is several orders of magnitude higher, the boost phase is almost impulsive launch as compared to the rockets, where we try to avoid impulsive launch, which has a drawback of increasing the drag loss. In the present case, we can still assume the launch to be fairly impulsive and the motion in the gravitational field of earth can be modelled just using a simple initial velocity and kinematic relations.

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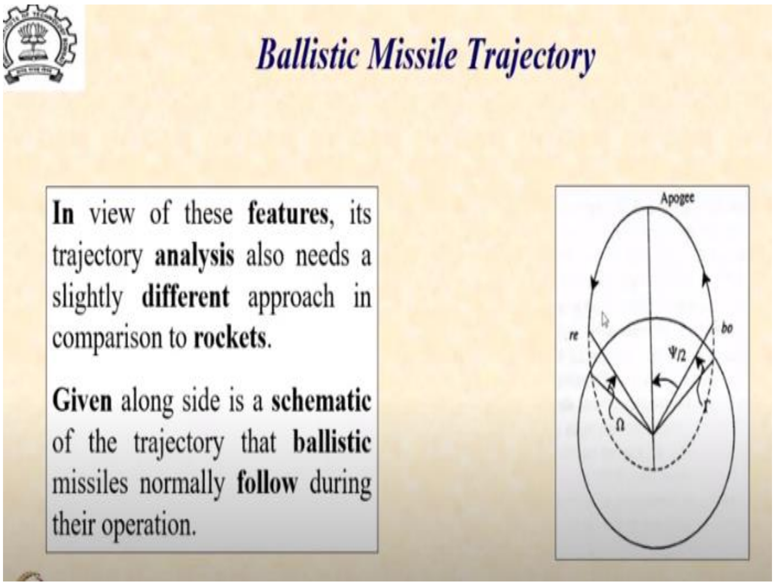

Let us look at these in a schematic that is shown alongside wherein we launch a missile and it is supposed to have an impact on the surface of the earth. So, in this we assume that there is an altitude point represented by burnout *bo* where the missile is given a velocity with which it starts traveling in the gravitational field of earth.

And from our projectile motion understanding, we know that in the absence of drag, it will reach a highest point above the surface of the earth which is commonly termed apogee of the, or maximum altitude point. And then the altitude will start decreasing and it will start turning back towards earth until it will reach a point which we call the reentry, which means it could re-enter the earth's atmosphere.

And with that, you will find that after some time it will have an impact on the surface of the earth as shown by the two small dotted line segments. You can clearly see that the segment of trajectory within the atmosphere is much smaller in length in comparison to the trajectory segment, which is outside the atmosphere which is the primary assumption of a ballistic missile.

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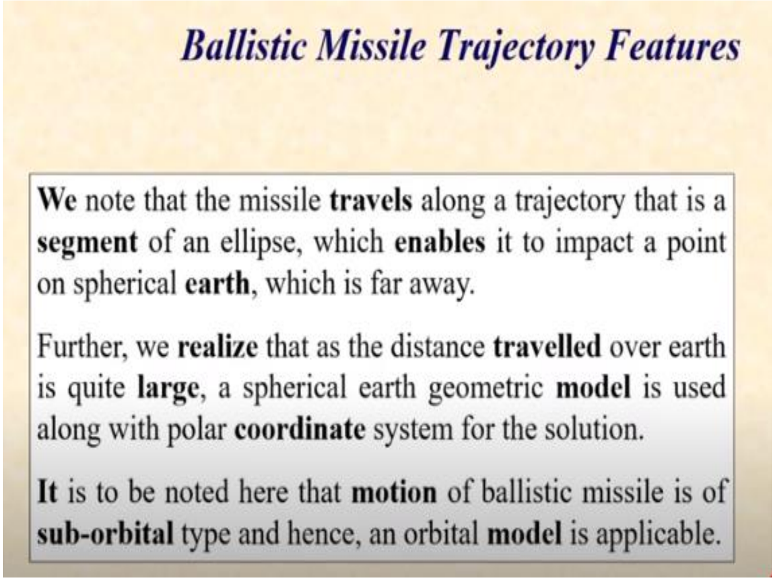

It is generally found that the best possible trajectory that such a missile can take is a segment of an ellipse which provides directly the impact point from geometric relations of the ellipse. Of course, we realize that as the distance traveled over earth is quite large, the flat earth the model that we commonly employ in the context of launch vehicles or rockets is not really applicable.

And we must use a spherical earth geometric model along with the polar coordinates to achieve the trajectory solution. Let me also mention here that the nature of the solution is somewhat similar to the solution that we commonly get for reusable launch vehicles as well as spacecraft orbits, which you will probably see in a different context. We are not covering these aspects here. But there is a similarity in the two scenarios.

The only point which I will mention is that the motion of ballistic missile in relation to a spacecraft or a reentry vehicle is suborbital type, so that an orbital motion model can also be applied to arrive at the trajectory solutions.

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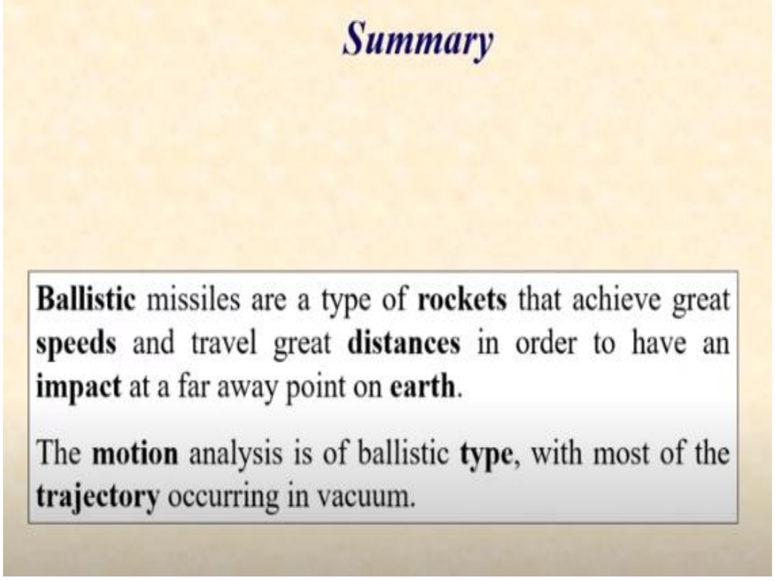

So, to summarize, ballistic missiles are a type of rockets that achieve great speeds and travel great distances in order to have an impact at a faraway point on earth. Further, we note that motion analysis is of ballistic type with most of the trajectory occurring in vacuum. Hi, so in this lecture segment, we have looked at two special concepts.

One, the concept of jet damping and the other the concept of ballistic missiles, which are connected to the trajectory and configuration design of rockets and provide important design inputs from a practical perspective.

In the next segment, we will now look at some special type of rocket configurations that are currently being developed for different kind of missions including futuristic missions to different planets, establishing colonies on faraway planets and we will see how such concepts can bring in value to our trajectory and configuration design of launch vehicles. So, bye, see you in the next lecture and thank you.