Introduction to Launch Vehicle Analysis and Design

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Lecture - 25
Parallel Staging Benefits

Hello and welcome. So, in the last lecture, we had looked at the basic concept of parallel staging and what kind of configurations emerge when we employ this concept. In this lecture, we will explore the benefits and some basic formulation aspects of a parallel staging-based configuration and how it fits in with the staging solutions and the design aspects that we have discussed so far. So let us begin.

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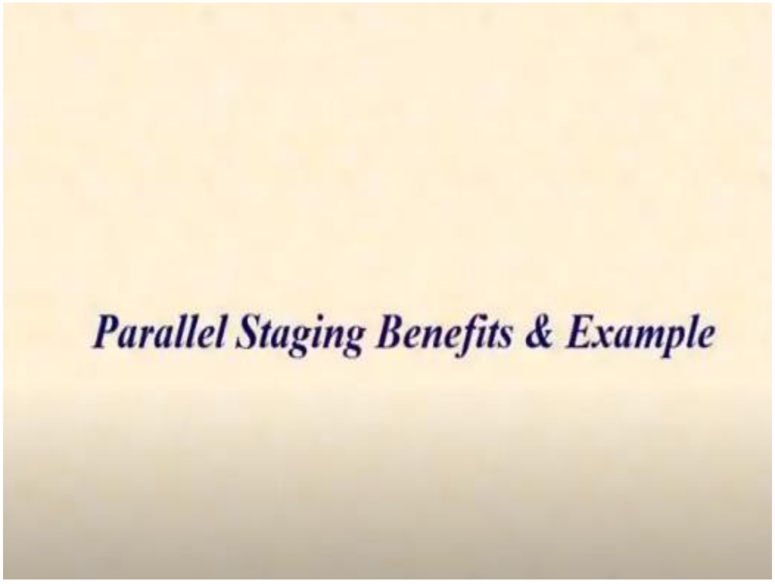

Let us look at first the benefits that accrue to us due to parallel staging.

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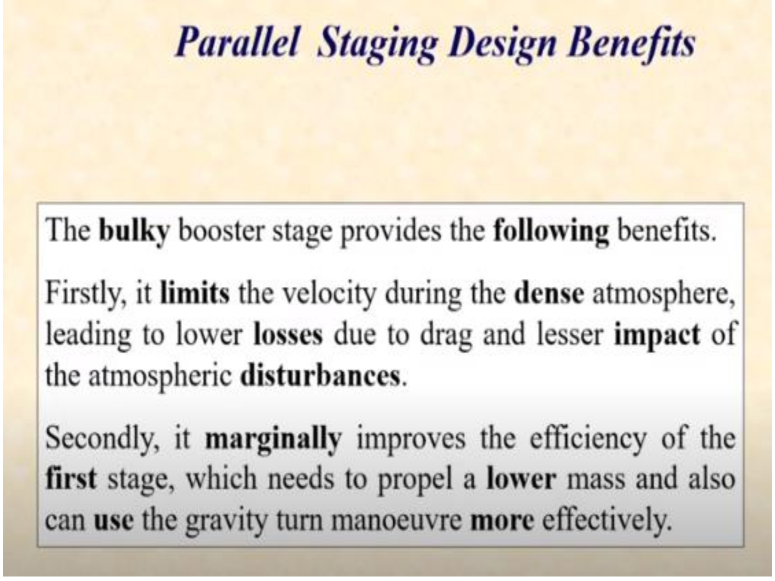

Now we have already noted that the booster stage will generally be a bit bulky and let us see what kind of benefits it brings in to our overall performance of a multi-stage rocket. The first thing that it does is to limit the velocity during the dense atmosphere and tries to reduce the drag losses which actually increased because of higher amount of base drag as the frontal area presented to the wind is larger because of parallel staging.

Secondly, we note that the efficiency of first stage improves marginally because it needs to propel a lower mass and more importantly, it can also make use of the gravity turn maneuver more effectively.

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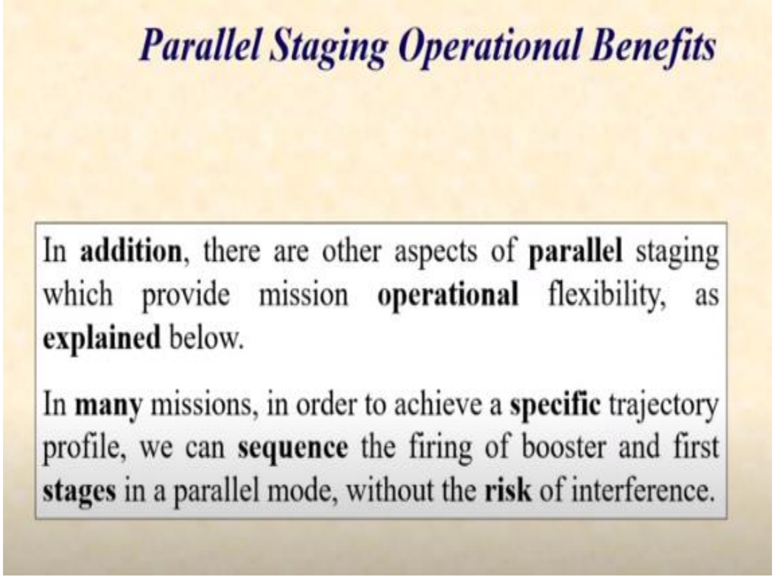

In addition, there are operational benefits of parallel staging as described below. The benefits are in the form of operational flexibility in operation of the zeroth stage in relation to the first stage and sometimes in conjunction with the first stage. In many missions, in order to achieve a specific trajectory profile, it is possible for us to sequence the firing of booster and the first stage in a parallel mode without the risk of interference.

Such an operation means that the booster stage and the first stage operate together in generating a huge amount of trust at the liftoff which is an important requirement particularly when we are designing heavy lift rockets that are currently the norm and SpaceX, heavy-lift Falcon 9 is an ideal example of such a configuration.

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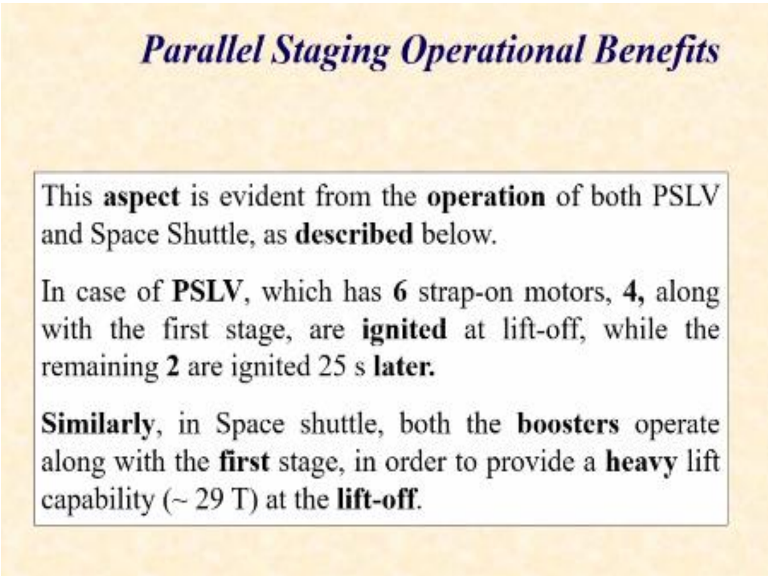

Let us explore this further through two examples of first PSLV and the Space Shuttle. So, in the case of PSLV, the normal configuration with boosters contains 6 strap-on motors of which at the lift-off only 4 are ignited along with the first stage. So, you use only four of them at the lift-off along with the first stage. So, both of them fire together indicating a heavier lift capability and the remaining two are ignited 25 seconds later.

You can clearly see that because the two boosters are held back while you will be able to lift cleanly, a larger mass enter lift-off. Your velocities will still be limited because you are not igniting all the boosters and the remaining two boosters are available once you have acquired a little altitude and that you are now entering a slightly less dense atmosphere.

Similarly, we find that in space shuttle both the boosters operate along with the first stage. Simultaneously and as, you have seen in the configuration, the boosters are the large size boosters, but they are lighter in comparison to the first stage so that they will finish the operation earlier to the first stage.

But the benefit of such an operation is that they provide a heavy lift capability at the lift-off as you have to realize that the Space Shuttle at the lift-off has a mass of 29 tons and that is the mass of the Space Shuttle itself. So obviously, you are going to require a very large thrust at the lift-off and such an operation of both the booster stage and the first stage firing together makes lot of sense.

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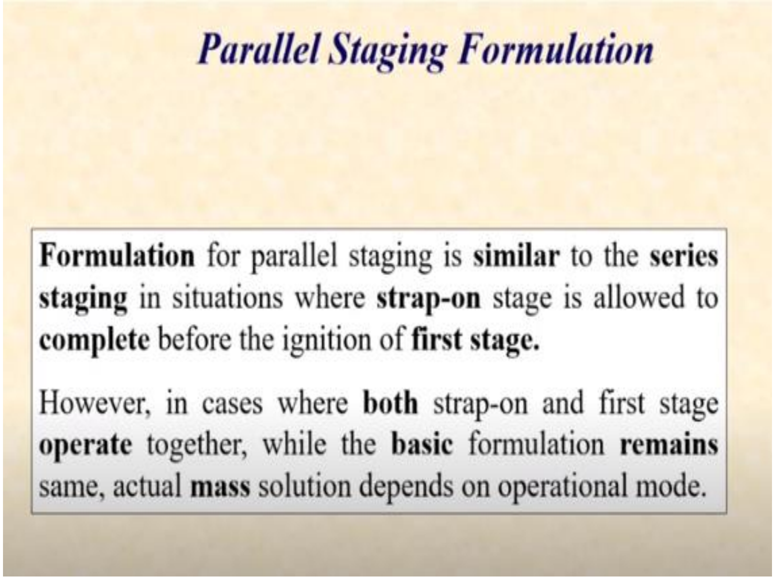

Let us now look at the formulational aspects of parallel staging configuration. So, at the outset, we note that the formulation for parallel staging is similar to series staging particularly in situations where strap-on stage is allowed to complete before the ignition of the first stage. So, if we finished all the step on stages before the first stage is started, then it is like any other serial rocket and the same relations that we have seen earlier hold good.

Of course, when both the strap-on stage and the first stage operate together, then there is a small change in the formulation that we need to look at. In fact, we need to also look at this aspect even for the strap-on stage because the strap-on stage is not really a single stage of rocket but a combination of multiple rockets firing together.

In fact, if you have seen the configurations, you will realize that the number of strap-on rockets which all of them fire either together or in a sequence is generally between four to nine, which means that we need to do a little bit of processing of the basic relations before this can be used as a single stage series operation concept.

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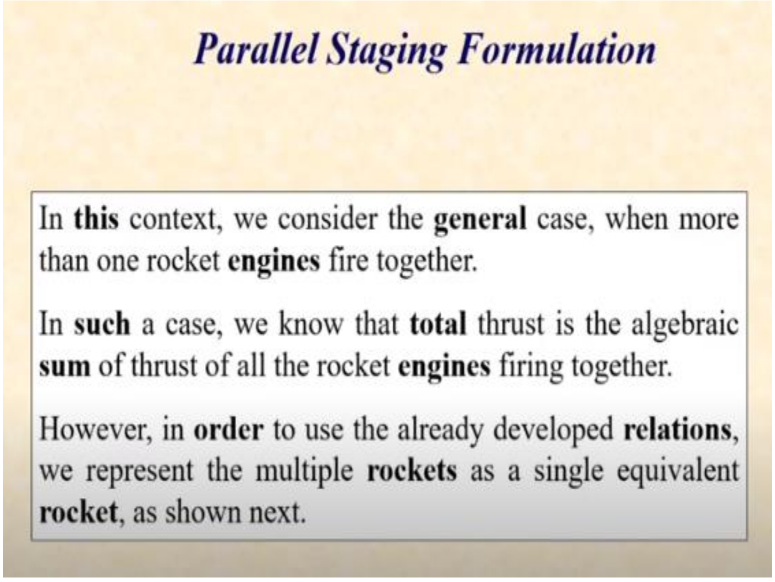

So let us consider a general case where there are n rockets firing together and let us assume that these n rockets could either be all of the zero stage or only some of the booster stage or all of the booster stage and the first stage together. It does not matter which of this case is actually operated as long as we assume that there is more than one rocket firing at the same time.

In such a case, we know the total thrust generated for this configuration is the algebraic sum of thrust of all rocket engines firing together. However, if we want to make use of the relations that we have already developed for trajectory calculations and multistage configuration design, then we need to represent the multiple rocket firing as a single equivalent rocket firing as has been shown next.

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Parallel Staging Formulation

Following are the applicable **equations** for a equivalent single rocket **stage**.

$$T_0 = \sum_{i=1}^{n} T_{0-i} = -g_0 \sum_{i=1}^{n} \dot{m}_{0-i} I_{sp0-i}; \quad \dot{m}_0 = \sum_{i=1}^{n} \dot{m}_{0-i}$$

$$T_0 = -g_0 \dot{m}_0 I_{sp0}; \quad I_{sp-0} = \frac{T_0}{g_0 \dot{m}_0} = \frac{\sum_{i=1}^{n} \dot{m}_{0-i} I_{sp0-i}}{\sum_{i=1}^{n} \dot{m}_{0-i}}$$

We see that I_{sp0} is now an **effective** mean I_{sp} of 0^{th}-stage.

So, following are the applicable equations for an equivalent single rocket stage for performance and design calculations that is derived from more than one rockets of different capabilities firing together. So let us start by saying that the total thrust of this particular stage still called the zeroth stage is sum of the thrust of rockets generating from all the rockets that are firing together, that is i from $1\ to\ n$.

Now we bring in our standard expression for the rocket thrust from our preparation formulation and we get the idea that this is going to be $-g_0$, $i = 1\ to\ n$, $\dot{m}$ that is the mass flow rate of each of those rockets into the specific impulse of each of those rockets firing together as zeroth stage. We also note that the total mass flow rate that is coming out of the zeroth stage would be sum of the mass flow rate of all these rockets.

So let us now hypothesize the presence of an effective I_{sp} of this single equivalent rocket called the I_{sp0} which is attached to the mass flow rate of all the rockets firing together and together generates the total thrust. Once we hypothesize the presence of I_{sp0}, we can now define this I_{sp0} as the ratio of the total thrust generated divided by the total weight flow rate that is $\dot{m}_0 g_0$ and is nothing but the ratio of the total thrust divided by the total mass flow rate.

And we see that this I_{sp0} is an effective mean I_{sp} of the zeroth stage. In this manner, we can express the case of a large number of dissimilar rockets firing together as a single rocket firing with an effective mass flow rate and an effective I_{sp}. You will immediately notice that once we have such a relation, we can use this directly for our trajectory calculations.

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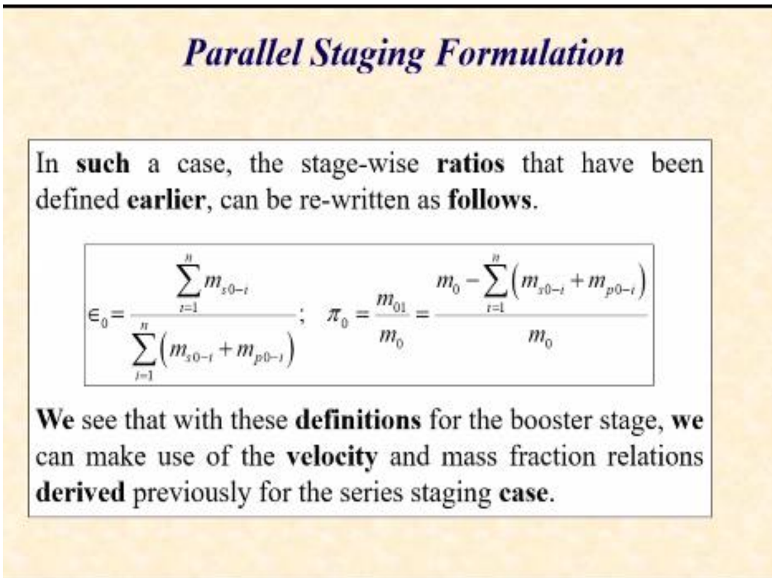

Next let us look at the changes that are likely to happen in the multistage design configuration, where we make use of the structural and the stage payload ratios for arriving at the multistage rocket configuration. So, these ratios in the context of many rockets firing together can be rewritten as shown below. So, the structural ratio of this zeroth single equivalent stage is the ratio of the sum of the structural mass of all the rockets firing together divided by the total mass of all these rockets firing together.

So, you realize that instead of a single mass value, if we replace this with the total mass of all the rockets firing together, we directly get the structural efficiency ratio ε_0 with regard to the stage payload ratio π_0. In the present case, it is now the ratio $\frac{m_{01}}{m_0}$, which can again be rewritten in terms of the $\frac{m_0 - \sum_{i=1}^{n}(m_{s0-i} + m_{p0-i})}{m_0}$.

In this manner, we can obtain the parameter ϵ and π_0, which can then be directly used in the design expressions for multistage rocket that we have seen for the optimal as well as the approximate staging solutions derived earlier. And with this, we can make use of the velocity and the mass fraction relations as they have been derived even for the parallel staging context.

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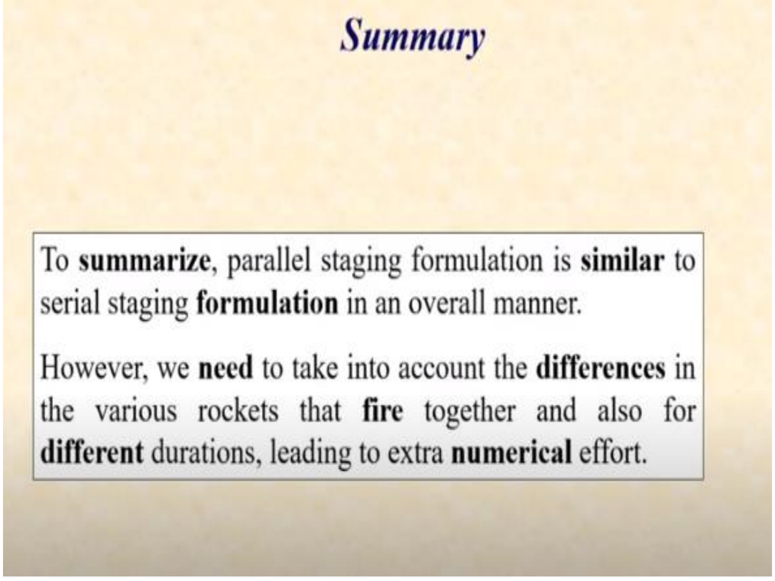

So, to summarize, the parallel staging formulation is similar to serial staging formulation in an overall manner. However, we need to take into account the differences in the various rockets that fire together and create a single equivalent rocket stage configuration in terms of its total mass, the structural ratio, the payload fraction and the effective mass flow rate as well as the effective I_{sp} for that single stage.

And you realize that this is going to involve a little bit extra numerical effort just to arrive at this information after which we can make use of already developed relation to design a rocket, which is a combination of both series and parallel staging.

Hi, so in this lecture, we have seen that in a reasonably simple manner, we can include the effect of multiple rockets firing together in a parallel staging configuration to arrive at a single equivalent rocket stage, which then can be used in the formulation and the solution that we have generated for series staging. And in that manner, we can arrive at both the trajectory and the rocket configuration solution for both series staging and the combined staging where you have both series and parallel staging.

With this, we conclude our discussion on the multistage rocket that we started some time back. We are now in a position to look at some of the special topics and the special features of launch vehicles that are also present. And in the next few lectures, we will look at the ideas that are currently being practiced by many space agencies such as air-breathing rockets, single-stage-to-orbit concepts, ballistic missiles as rockets.

So, we will do all that in the next couple of lectures. Bye. See you in the next lecture and thank you.