Introduction to Launch Vehicle Analysis and Design

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Lecture – 18
Multi-stage Problem Definition

Hello and welcome. In the last lecture, we had looked at the basic algebraic formulation for arriving at a mass configuration of a launch vehicle for given values of ε and π. We had also established a need for a formal procedure that would give us the values of π for the best possible rocket configuration. In this lecture, we will initiate the discussion on the various possibilities among which we will choose one of the possibilities for arriving at the best possible π values. So, let us begin.

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Let us begin by introducing the idea of an optimal design methodology.

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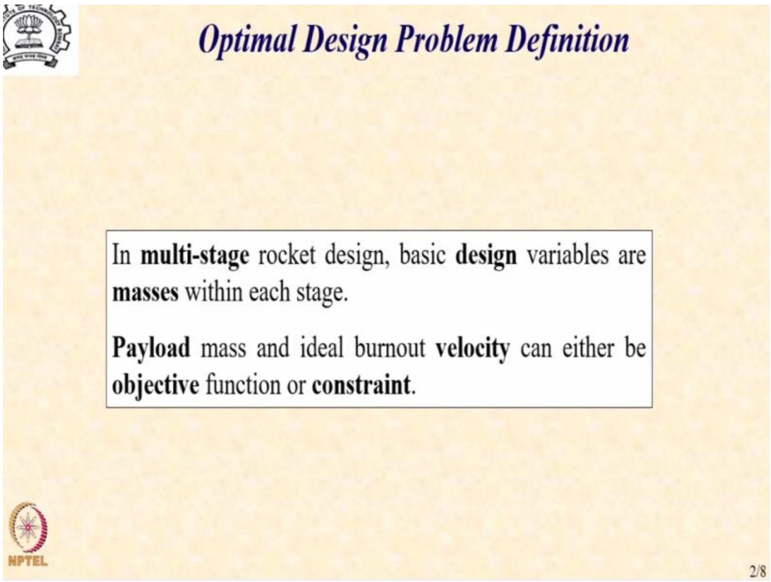

As we have already seen in multistage rocket design, basic design variables are masses within each stage. Further, we have also noted that there are two requirements from mission perspective that is the payload mass and the ideal burnout velocity and in a generic mechanism we can either use payload mass as a constraint. And try to maximize the ideal burnout velocity or we can put a constraint on the ideal burnout velocity.

And use payload mass as an objective function which is to be maximized. The implication of these are that in one case when we are putting payload mass as a constraint, we are saying that a particular spacecraft can be used for different missions for which you will require different velocities on the other hand, when we put the burnout velocity as a constraint and try to maximize the payload mass.

What we are indirectly saying is that we want to perform the same mission with different sized space craft and we will find that both these scenarios are possible to be handled in a reasonably structured and simple manner.

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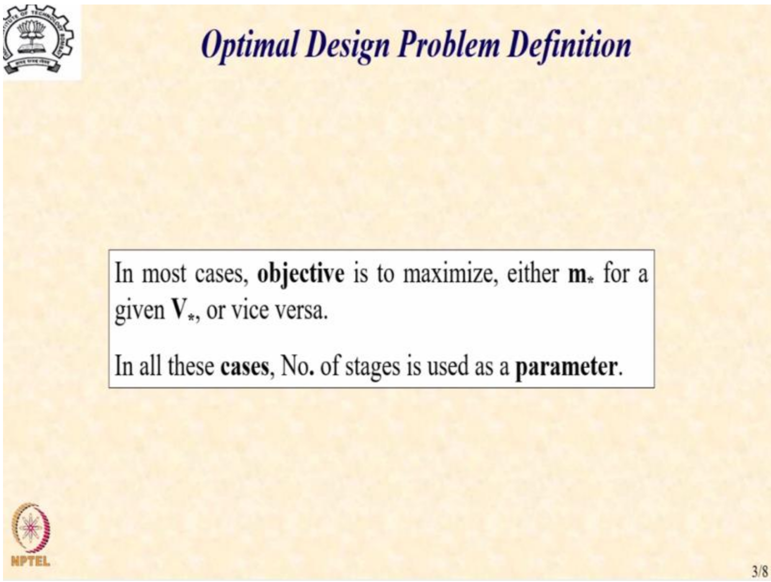

Now, we have already mentioned that we would either like to maximize the velocity for a given payload mass or maximize the payload mass for a given velocity. These are going to be our formal design objectives for designing a multistage rocket. Of course, it is important to take note of the fact that number of stages is strictly speaking an unknown and along with the stage wise mass parameters this can also be used as an unknown to arrive its value through the same optimization procedure.

However, there are a few issues that come up when we do that. So, the resulting optimization problem would now be a mixed integer real number optimization problem and these kind of optimization problems are not very easy to solve because you cannot really get the correct optimal solution because the N is going to get integer number. So, if you are getting a solution which is less than let us say 1.5 you might fix it to 1.

And then look at an optimal solution or if you are getting N as 3 or 3.5 then you may say okay, I will put it equal to 4. So, you realize that it is not a straight forward solution that we will have when we use N also as a design variable. So, what we do is that we take the next best course. We use the number of stages as a parameter. What essentially it means is that we generate a large number of optimal solutions for different values of N.

And then among those solutions we will pick one which is the more suitable for our mission objectives. There is another point that can be kept in mind which is that by and large most launch vehicles will be a minimum of two stages and possibly a maximum of five stages even if we use the strap-on or the 0^{th} stage. So, what it means is that you only have to do this exercise three or four times for different values of N between 2 to 5.

And we will have mapped the complete design space from which we can choose the configuration that is best suited. Lastly, we also need to realize that most space agencies will generally have technology to handle rockets of a certain number of stages even though most of them would have in their arsenal, rockets which will have different stages. Within a rocket family generally the number of stages would be not too many.

So, the overall exercise for using N as a parameter is still a fairly simple and workable exercise without getting into the hybrid integer real number optimization problem and for that reason this is the strategy which is most commonly employed.

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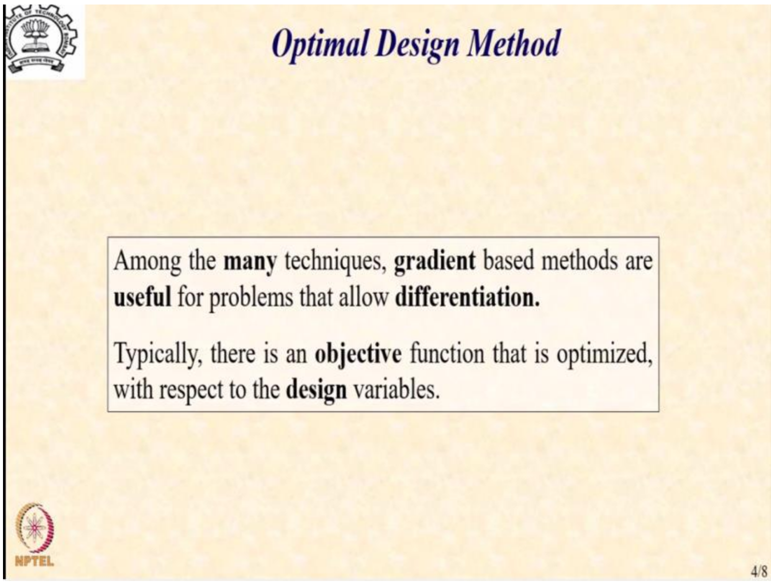

Let us now turn over to the actual optimal design methodology. There are many ways in which the optimization can be done. If you look at the methodology in the books in literature you will find linear programming, quadratic programming, nonlinear programming and host of other similar methodologies which provide best possible solution for a given problem statement.

In the present context, we will find that the gradient based techniques which is the simplest and the most intuitive and most commonly employed wherever applicable is going to be used particularly for those problems which allow differentiation. As you will find that the problem that you are going to formulate will permit derivatives it is possible for us to use the gradient based methods to arrive at an optimal solution.

In this methodology typically there is an objective function which is to be optimized that is either maximized or minimized with respect to the design variable.

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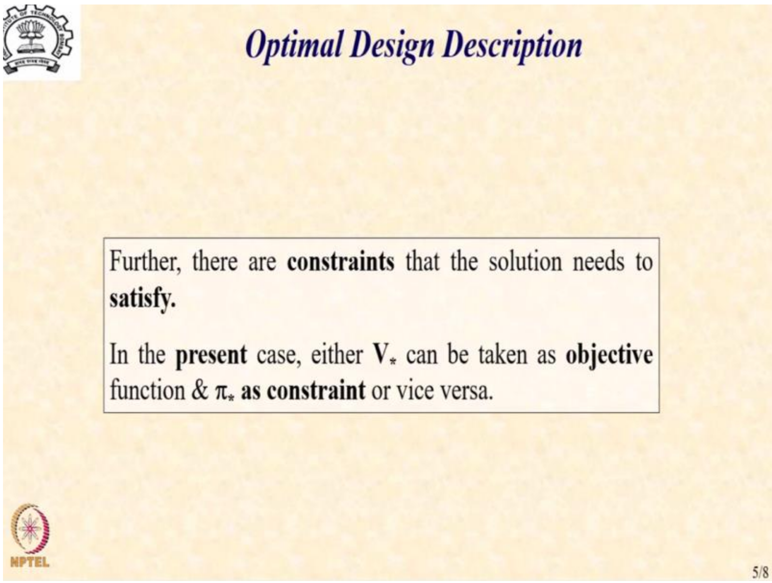

Further, there will generally be constraints that the solution needs to satisfy. The implication of constraints is that you are now defining the boundaries of your design space within which the best possible solution must be searched. So, the constraint essentially defines the boundaries within which you must look for a solution because we have stated two separate types of problems that we can solve.

We can also have the optimal design problem statement also in both these cases that is we can either take the burnout velocity as an objective function which is to be maximized and the mission payload fraction π^* as a constraint or we can reverse the process and we can choose π^* as the objective function that needs to be maximized and V_* as the constraint under which the solution must be obtained.

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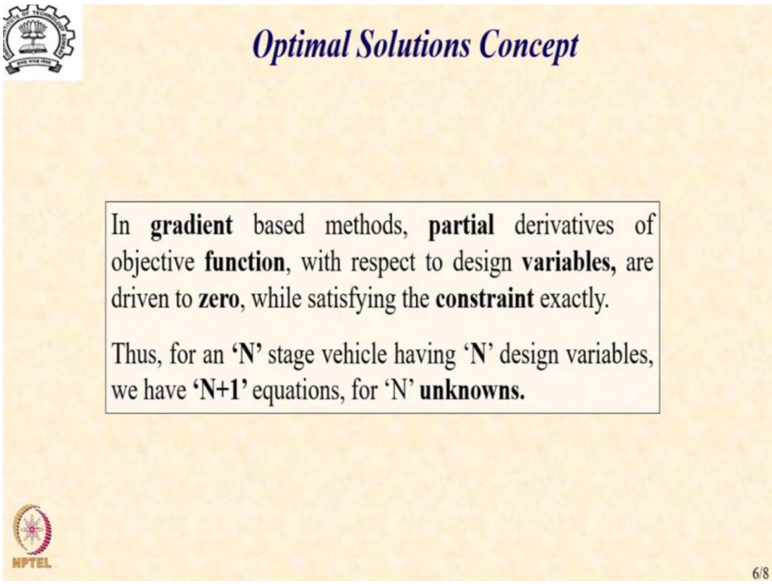

So, how does this gradient based method work? In gradient based methods, what we try and do is to generate the partial derivatives of the objective function with respect to all the design variables and from the basic understanding of calculus you would note that an optimal solution would be obtained when all the partial derivatives are zero at a single point. So, we now get as many numbers of equations as there are number of design variables which will be algebraic equations.

And when we solve those algebraic equations in terms of the design variables, we get a solution for a design variable which optimizes that is either maximizes or minimizes the objective function and all this must be done while satisfying the constraint exactly. So, if we have an N stage vehicle for each stage, we will have N values of the payload ratio or π's. So, we can treat them as N design variable which obviously means that we will get N partial derivatives or N algebraic equations.

Now, we also have a constraint either on V_* or on π^* that becomes our $(\mathrm{N}+1)^{th}$ equation. So, we are going to have $\mathrm{N}+1$ equations in which N equations are obtained from the partial derivative exercise and $(\mathrm{N}+1)^{th}$ equation is obtained from the constraint. And this system of $\mathrm{N}+1$ equations is an only N unknowns and you immediately realize that is something which needs to be now addressed before we can look at the solution itself.

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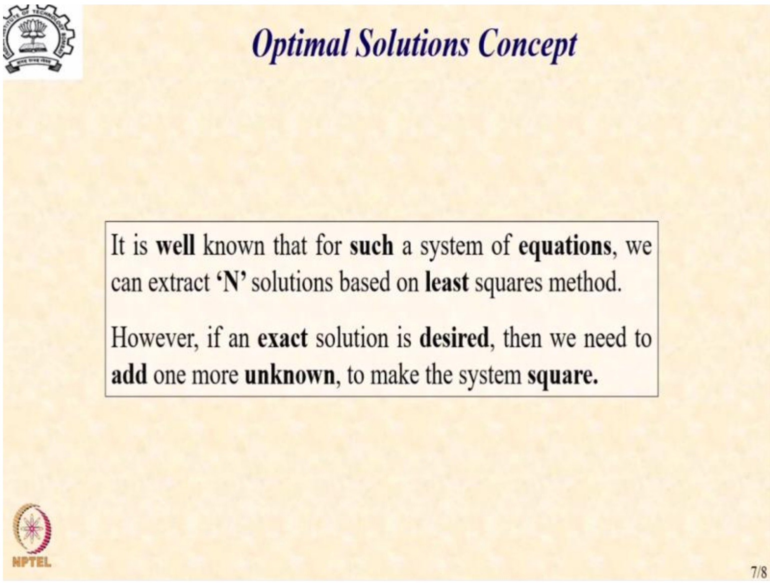

Now, let me recall something that you probably would be familiar with. If we have number of equations which are more than the number of unknowns in the context of linear algebra, we have a very well-established method of least squares which can extract the solution of N design variables using what is commonly called a pseudo inverse approach in which we use the set of $\mathrm{N}+1$ equations to arrive at an $N \times N$ nonsingular coefficient matrix whose inverse will give us the solution.

But what is the issue with this? The main issue with this is that this is not really an exact solution, what this is trying to do is to minimize the error between the exact solution and the solution given by the least square method and keeping that error to a minimum value using the quadratic formulation approach and that is why it is called the least squares method that the square of the error is a minimum and that is the condition under which the solution is obtained.

Of course, if we are not really looking at a very exact solution this works very well and is commonly employed in many cases. However, if we are looking at an exact solution then obviously this is not going to work. There are two possibilities that one can think of. One, either we can drop one equation and use the remaining N equations for N unknowns and solve for the N design variables.

This methodology can be a reasonable solution if the equation that you have dropped is not really of much consequence so that you will still get a reasonably good solution. However, if the equation that you have dropped is an important equation and has significant impact on the solution the chances are that you will get me an approximate solution which means you will get what is commonly called a suboptimal solution.

The other possibility is that can I add one more unknown so that we now have $N + 1$ unknowns for $N + 1$ equations that becomes our square system and we get an exact solution for all the $N + 1$ variables hoping that among that the N required design variable solution is also exact. So, for that we need to ensure the addition of these additional variables in a particular manner so that the solution that we get from such a formulation is exact.

We will first look at this aspect through a strategy and then of course we will also look at the other possibility of dropping one equation and still hoping for an exact solution. So, please note our objective is to get an exact optimal solution either through dropping one equation or through adding one additional variable.

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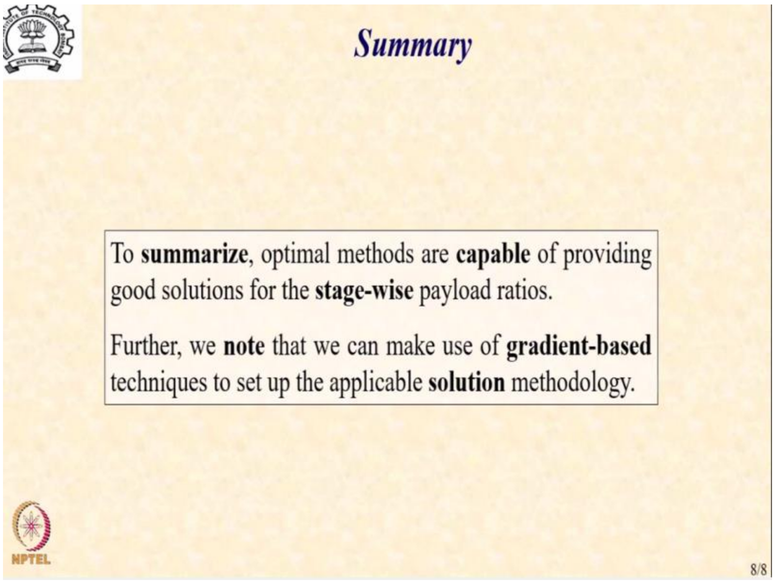

So, to summarize we have just noted that optimal methods are capable of providing good solutions for stage wise payload ratios. We have also noted that we can make use of gradient based techniques to setup the application solution methodology. So, we have seen that by appropriately choosing the objective function and the constraint it is possible for us to setup a formal optimization procedure based on the gradient methodology for arriving at a reasonably good optimal solution for the stage payload ratios.

We will look at the methodology in subsequent lectures. So, bye see you in the next lecture and thank you.