Introduction to Launch Vehicle Analysis and Design

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Lecture – 17
Multi-stage Solution Basics

Hello and welcome. In the last lecture, we had looked at the basic algebraic strategy to setup the problem of configuration design of a multistage rocket. And in the process, we defined important parameters for the stage as stage structural ratio and the stage payload ratio. In this lecture, we will now demonstrate those expressions through a simple example. And then we will also establish a reasoning for adopting a more rigorous approach for arriving at a launch vehicle mass configuration. So, let us begin.

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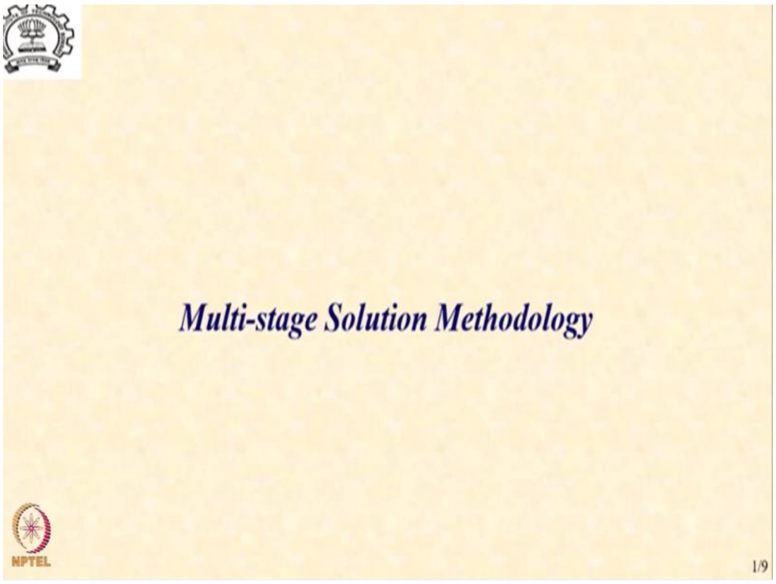

So, let us now mechanize the solution methodology based on the relations that we have derived in the previous lecture.

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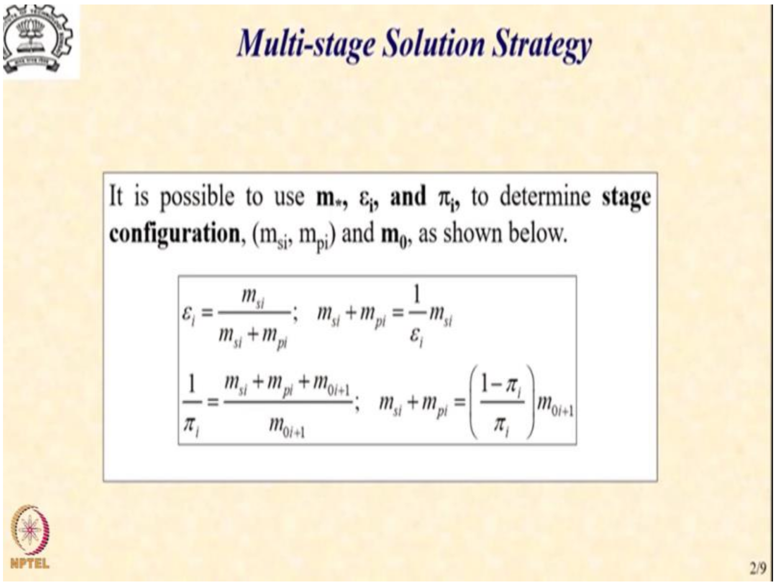

Let us make use of m^*, ε_i & π^* to determine the stage configuration which is meant by the m_{si} and m_{pi} for each stage as well as the total lift off mass in the following manner. So, let us first take the expression for ε_i which is written as $\frac{m_{si}}{m_{si}+m_{pi}}$. Now, let me do a little bit of algebraic jugglery and that gives me $m_{si} + m_{pi}$ as $\frac{1}{\varepsilon_i} m_{si}$.

Similarly, I take the expression for π_i invert it and write it as $\frac{1}{\pi_i}$ as $m_{si} + m_{pi} + m_{0i+1}$ which is nothing, but $\frac{m_{0i}}{m_{0i+1}}$. And I get another expression for $m_{si} + m_{pi}$ as $\left(\frac{1-\pi_i}{\pi_i}\right) m_{0i+1}$. So, now as you can see, I have two equations for m_{si} and m_{pi} as unknown in terms of the two known parameters ε_i and π_i. So, I am assuming that ε_i and π_i are going to be available to me.

And based on that I should be in a position to solve for m_{si} and m_{pi}. Of course, these are not very straight forward equation because if you see the second one m_{0i+1} appears in the second one. So, it cannot be directly solved and there is a strategy by which we will be in a position to solve these equations.

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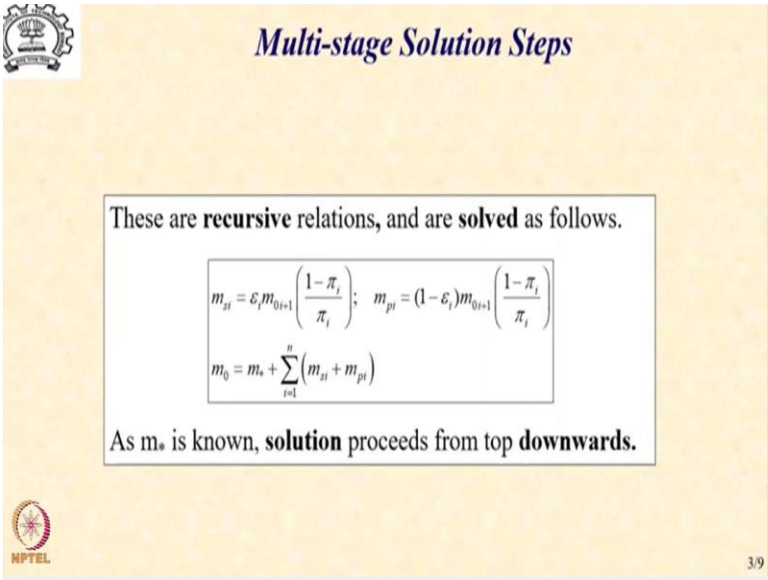

One thing that you might have realized that the relations that we have written are recursive which mean they're for each i. So, if I substitute i in the previous relation what I get is the relation in terms of $i + 1$. Indirectly, what it means is that my solution for i^{th} stage is going to be possible only if I am able to solve for $i + 1$ stage first. Now, let us extend this logic until we reach m^*.

And now we realize that m^* is something which is to be specified and it is generally available as a design requirement and now you realize that once m^* is specified you should be able to solve for n stage first that is you are starting now from the top. And once you solve for the n stage the n stage solution will drive the $n-1$ stage etcetera until you reach the last or the first stage.

And then when you add all this you are going to get the total rocket configuration. So, now this is the mechanization of the equations that we have just now seen. Of course, we can now use this to show that m_{si} for i^{th} stage is a function of m_{0i+1} and the π_i and the ε_i. Similarly, m_{pi} is directly driven by the same m_{0i+1} and now m_0 is going to be m^* plus sum of all these.

So, this is how I am going to actually solve the problem. You can see that if I have ε_i and π_i. I can solve for the mass configuration of the complete rocket including the lift off mass and because m^* is known as I have mentioned the solution proceeds from top downwards. So, this is how the design solution is proceeded for obtaining the overall rocket configuration.

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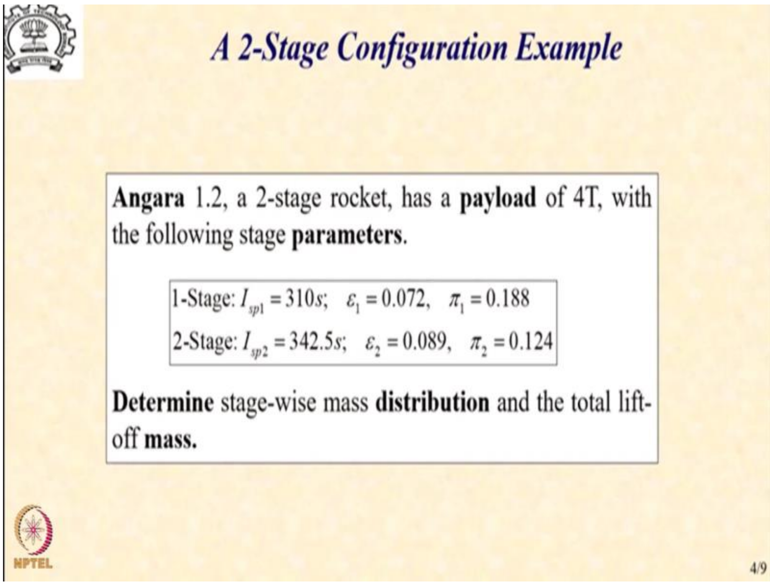

Let me demonstrate this through an example of a rocket which is in literature you can look it up called Angara 1.2 it is a two-stage rocket which is supposed to launch a 4-ton payload and has the following stage parameter as per our solution methodology. The first stage has the propellant with I_{sp} of 310s. Of course, for this exercise it is not really required, it has a structural ratio of 0.072 or what I call 7.2% structural mass.

And remaining 92.8% propulsion mass and a stage payload ratio of 0.188 which means the ratio between the mass above the first stage and the lift off mass of the first stage is 0.188. Similarly, for the second stage the I_{sp} is 342.5; ε_2 that is the stage structural ratio is 8.9% and the stage payload ratio is 0.124. With these numbers let us try and determine the first stage wise mass distribution.

And then get the total lift off mass and see if it matches with the actual lift off mass of this rocket which is recorded in literature.

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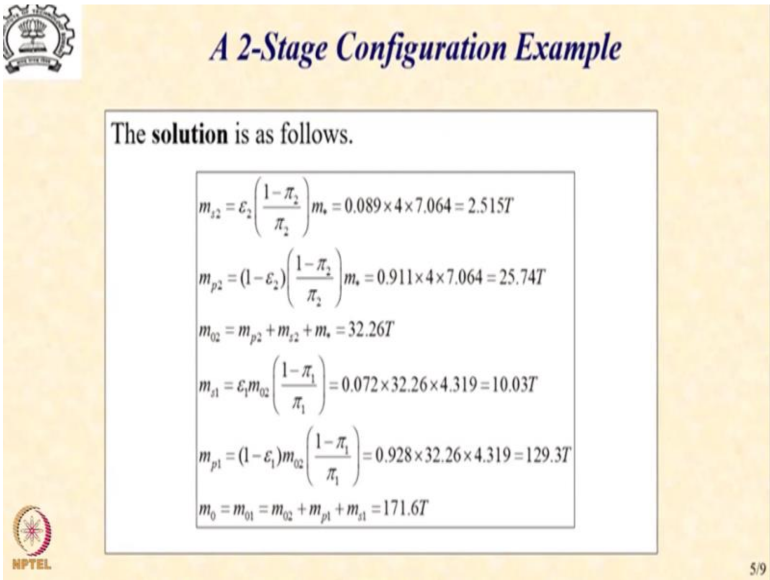

So, let us start the process let us go through it step by step. So, we start from the second stage because that is the last stage and m^* is known to us. So, I calculate m_{s2} as $\varepsilon_2 \times \frac{1-\pi_2}{\pi_2}$ into m^*. I perform this simple arithmetic and I get this structural mass of second stage is 2.51 tons. Similarly, I use the m_{p2} expression and I get the propulsion mass as 25.74 tons. Now my starting mass for the second stages $m_{p2} + m_{s2} + m^*$.

So, this is nothing but my m_{02} because this is going to drive the configuration of the first stage. With this, m_{02} now I immediately calculate m_{s1} which turns out to be 10 tons and m_{p1} which turns out to be 129 tons. I just add all these that is $m_{02} + m_{p1} + m_{s1}$ which is the m_{01} or the lift off mass it turns out to be 171.6 tons. This I will leave you to verify that the actual lift off mass or the gross lift off mass of Angara 1.2 is around 171 tons which means that the exercise that we have carried out essentially is feasible methodology for arriving at the stage wise distribution of masses for a given rocket which has ε_i's and π_i's specified.

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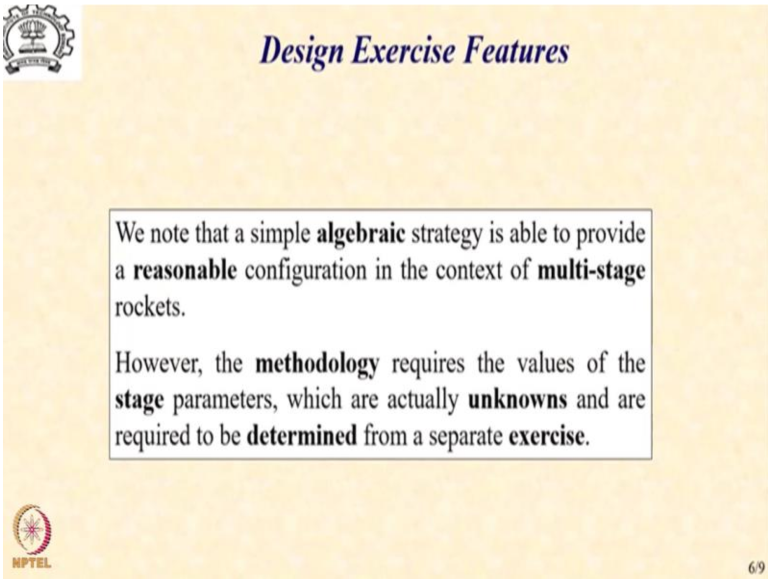

And we also note that the configuration is quite practical and realistic. The problem is that this requires the values of epsilons and π's. Now, we have already said that epsilons are known from structural technologies, but π's are the ones which we do not know and those are the ones which we are suppose to generate and only after we know the π_i's the methodology that I have demonstrated can be used to arrive at the actual mass configuration.

So, even before we can make use of these relations and the solution methodology, we have to talk about π_i's as the real unknowns which have to be determined from a different strategy.

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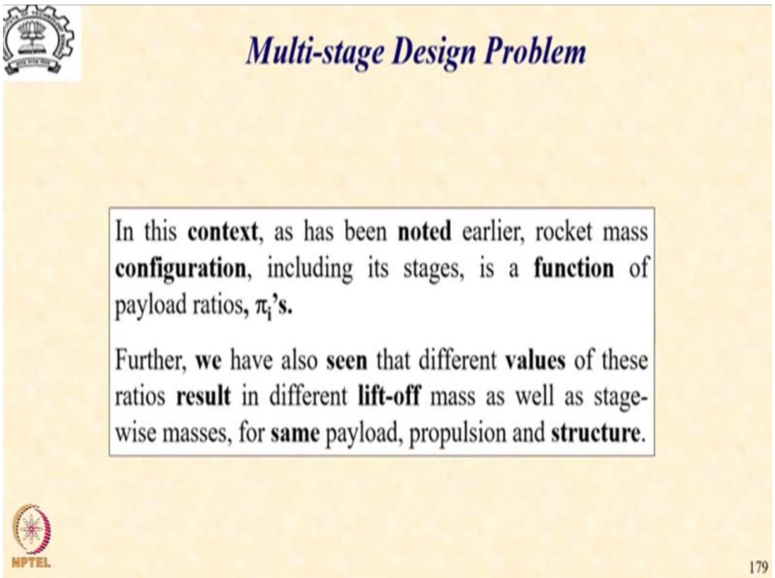

And how do we arrive at such a strategy and that will be nothing, but statement of our design problem. So, we have already noted earlier that the mass configuration through this problem is directly a function of π_i's. So, obviously different π_i's all of which satisfying the constraint that product of all $\pi's \Rightarrow \pi^*$ will generate different mass configurations. Question is among those many possibilities that we have which is the one which we should choose.

How do we ultimately get those π_i's which ultimately are compatible with the mission that we have in mind.

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And in this context, we need to realize that from a practical perspective we want a mission cost to be as small as possible. Now mission cost we normally define as amount of payload per unit lift off mass that we get and higher that value lower is the cost which means our mission is more cost effective if we are able to launch a higher payload with a smaller lift off mass.

There is another issue of safety and accuracy that we must do this exercise without affecting the overall safety of the vehicle as well as the accuracy of our orbital mission which is the primary objective. So, the design task in a nutshell is to choose among the many combination of π_i's which all satisfy the constraint and also result in the best rocket for a given mission. This is going to our overall objective in arriving at the configuration design of a multistage rocket.

Let me just make a mention that optimal techniques which are available in many forms are commonly employed to achieve this objective.

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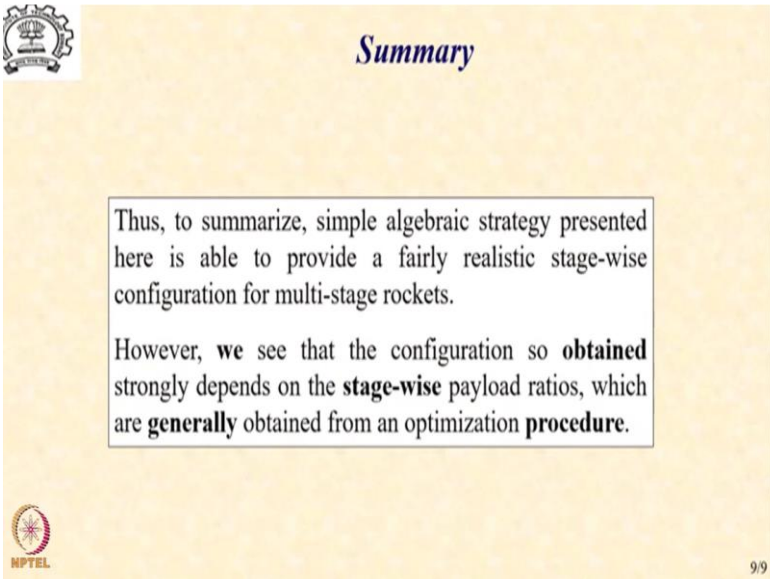

So, to summarize simple algebraic strategy presented here is able to provide a fairly realistic stage wise configuration for multistage rockets. However, we see that configuration so obtained strongly depends on the stage wise payload ratios which are generally going to be obtained from an optimization procedure. So, in this lecture we have seen the working of the formulation that we had given in the last lecture through a simple example of Angara 1.2.

And we found that the methodology is workable and give a realistic estimate of not only the lift off mass, but also the mass that is there in each of the stages. Of course, we have also noted the fact that this is subject to availability of values of ε_2 and π_2, ϵ_1, π_1 etcetera which have to be connected to the mission performance because they affect the mission performance itself.

Now, among the many possibilities we have kind of given a justification that we would need to use some optimization methodology for arriving at the values of π_i's that will give us the best possible rockets. We will look at some of these ideas in the next lecture. So, bye see you in the next lecture and thank you.