Introduction to Launch Vehicle Analysis and Design

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Lecture – 16
Multi-stage Configuration Basics

Hello and welcome. With the background that we have created in the last lecture with regard to the multi stage rocket configurations, let us begin our discussion on how to setup the problem that will give us a configuration for a multistage rocket along with the issues involved in arriving at the solution.

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So, let us begin our discussion on the multistage configuration of rockets with some ideas of how-to setup the problem.

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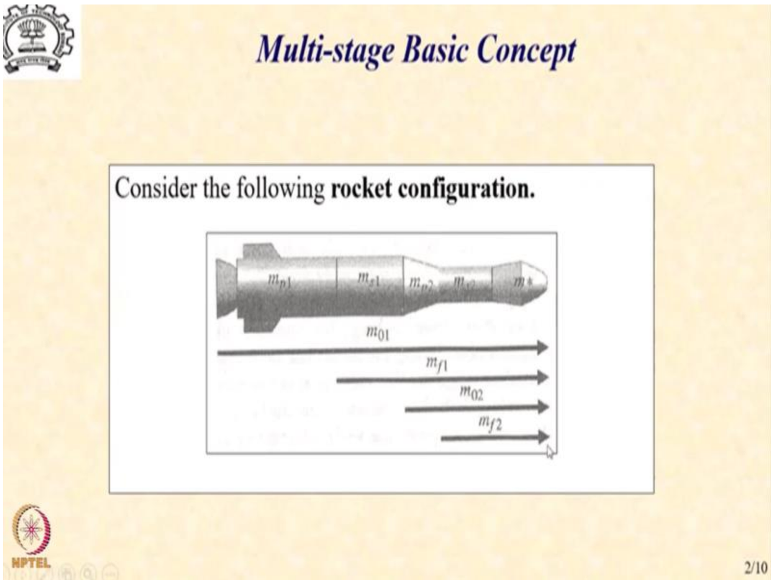

In this context, let us consider the following configuration that captures the spirit of a multistage rocket that we may have. In this picture, you note that the complete rocket has been divided into segments with the subscript 1 and 2 called the stages and the final payload module given a subscript m^*. Now, within each of the stages which is further subdivided into two parts, the propulsion and the remaining structure.

So, we have m_{p1} and m_{s1}. Similarly, we have m_{p2} and m_{s2}. Let us now introduce additional symbology that we are going to use in our solution procedure. So, we introduce m_{01} which is defined as the starting mass for stage 1 operation. In the present case, when the stage 1 starts operating it is almost like the complete rocket and in most cases, this will be same as your lift off mass.

In some special cases it may not so that we shall see later. Now as the state starts operating the propulsion gets burned until you finish all the propulsion and once you finish all the propellant what you are left with this m_{f1} which is the final mass for stage 1 operation. And that contains the residual empty shell or the inert mass of stage 1 and then of course the remaining part of the rocket which is the stage 2, the stage 3 etcetera.

And had the final payload stage called m star. Let us now come to m_{02} definition and this is where we implement the idea that we have discussed in the previous lecture, but before starting the second stage we get rid of the structural mass of the inert mass of the first stage m_{s1} so that we now have a smaller m_{02} compared to m_{f1}. So, m_{02} is nothing, but $m_{f1} - m_{s1}$.

And then of course the cycle continues that when second stage completes the operation it would have burnt the propellant m_{p2} and then what would be left will be m_{f2} and then of course the subsequent stages.

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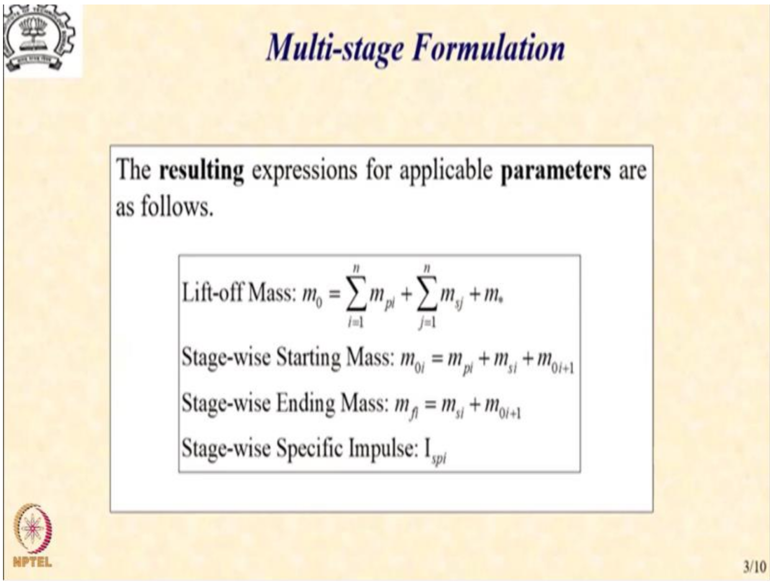

We can now write these m_{01}, m_{f1} etcetera in terms of the applicable parameters as shown here. So, we know that the lift off mass m_0 is going to be sum of propulsion mass for all the stages, the structural mass for all the stages and the payload stage mass. And now we introduce an important definition, the stage wise starting mass as the mass of that particular stage given by $m_{pi} + m_{si}$ and the mass of everything that is above it given by m_{0i+1}.

Similarly, stage wise ending mass would be $m_{0i} - m_{pi}$. So, you can see that in this expression of m_{fi}, m_{pi} does not appear and then of course we have the stage wise specific impulse I_{spi}.

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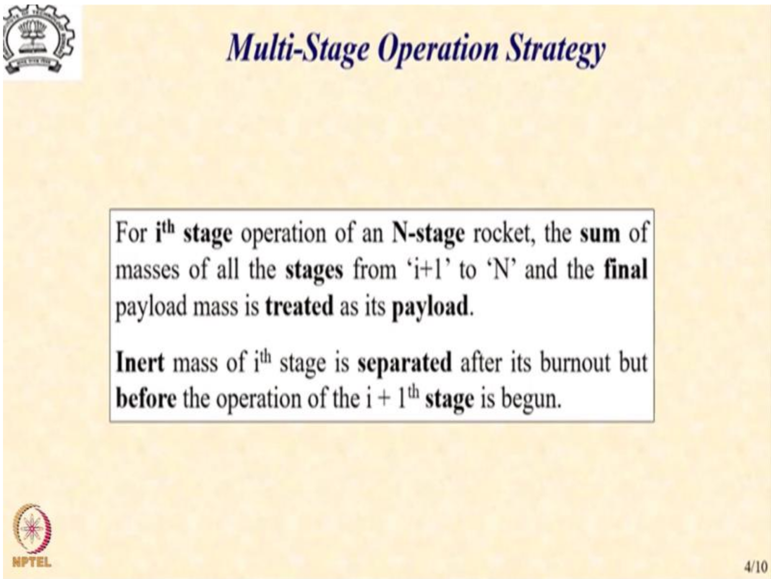

So, when we say that i^{th} stage of N stage rocket is operating then for that i^{th} stage whatever is above it is like a load on it which means the i^{th} stage is carrying everything starting from $i+1$ stage till the payload end and that becomes a kind of a payload for the i^{th} stage and that brings us to the definition of what we call the stage payload. So, m_{0i+1} becomes the payload for i^{th} stage because that is the amount of mass that the i^{th} stage will have to push along with its own mass of course and it's appropriate that we use the same interpretation.

And then of course we have already seen that the inert mass of the i^{th} stage is separated after its burnout before we start the $i+1^{th}$ stage and that becomes m_{02} which is the starting mass for the second stage for which m_{03} will become the payload and this cycle will continue.

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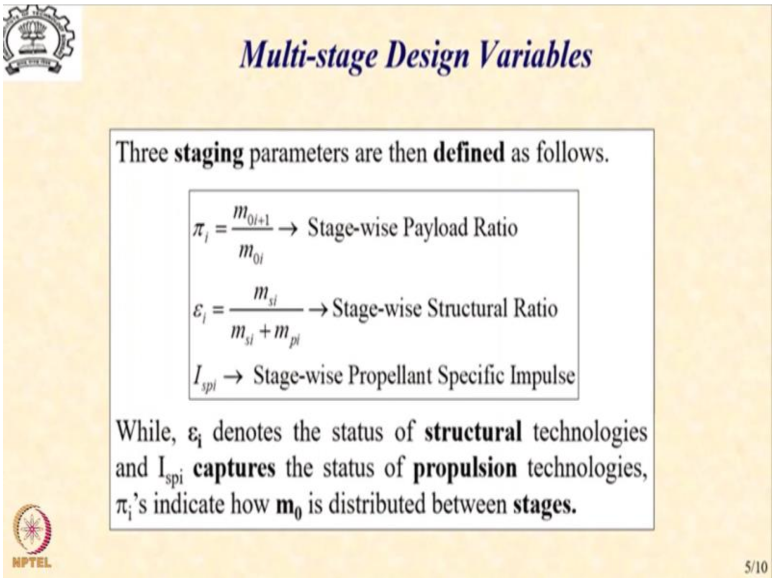

Based on this, let us now define the applicable design variables for a multistage configuration. So, let me introduce a parameter called π_i name stage wise payload ratio which is nothing, but the ratio of the payload of the i^{th} stage and the starting mass of the i^{th} stage. You may immediately connect this with the basic definition of the payload ratio that we have already used for the mission which is defined as $\frac{m^*}{m_0}$.

It is the same strategy that we are using here to say that for the rocket as a complete unit the payload ratio is $\frac{m^*}{m_0}$. So, in the same light for each stage the payload ratio is what it is pushing to its starting mass. We will find that it is an important parameter which will help us to setup the configuration design of multistage vehicles. We also introduce another ratio called the stage wise structural ratio.

And denoted by symbol ε_i which is nothing, but an indication of how much of the structure is used in a given stage. So, if the stage has 100 kg how much kg is the structure and the remaining will all be treated as the propellant. In some way, it also tells you how efficient is your stage that how much propellant you can carry for a given stage and lastly of course I_{spi} is as we have seen earlier.

I think it is also worth noting here that ε_i also indicate the status of the structural technologies. For example, whether we are talking about metallic structures, composite structures or even frame structure or Honeycomb structures the ε_i will be different for each of those. Similarly, I_{spi} captures the status of propulsion technologies which indicates whether you are talking about a solid propellant, liquid propellant, cryogenics or any other hybrid combination like air-breathing engines.

While the π_i's are the primary parameters which indicate how the lift off mass is distributed between different stages.

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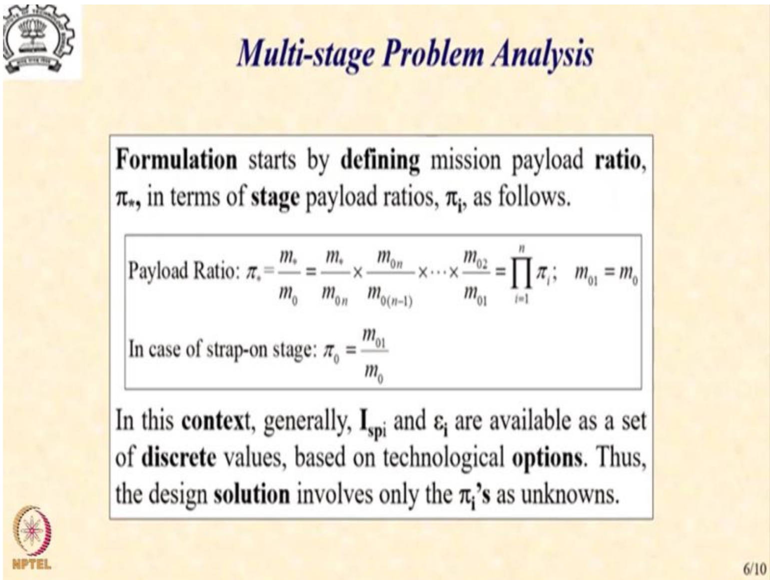

So, how do we formulate this problem? We formulate this problem by defining mission payload ratio π^* which is $\frac{m^*}{m_0}$ and there is a constraint that this particular ratio has to obey in terms of the stage payload ratios that we have defined in the following manner. So, we defined π^* as $\frac{m^*}{m_0}$ and then it is not very difficult to see that we can define a recursive process as shown through this multiplication until we arrive at m_{01}.

And as you can see each of these represents a stage payload ratio. For example, $\frac{m^*}{m_{0n}}$ is the payload ratio for N stage $\frac{m_{0n}}{m_{0n-1}}$ is the stage payload ratio for $n - 1$ stage etcetera such that finally we come to the last stage that this $\frac{m_{02}}{m_{01}}$ is the payload ratio for the first stage and this in a compact form we can represent as product of all the individual stage payload ratio and that becomes now the main constraint that this rocket must obey.

That the product of all the individual stages which is the actual mass configuration of the vehicle is not unconstraint. It cannot be done in an arbitrary manner. It has to be done in such a manner that their product is always equal to π^*. Of course, even within that constraint there are infinite possibilities of π_i's which will result in the same π^* and that will result in multiple design solutions for the same payload ratio specification.

And this is something that we will have to deal with when we are setting up the formal design process. Let me also make a mention of related issue which is going to be discussed in some detail later is that if there are strap on stages or what we call the boosters which in most rocket is used then there will be one more payload ratio which will get into this definition called π_0 which will be the payload ratio for the 0^{th} stage.

So, sometimes the booster stage or the strap on stage is also called the 0^{th} stage to differentiate from the first, second stage etc. We will talk a little bit more about it when we talk about the parallel stage. Now, let us understand how we are going to solve this problem and then we note that in most cases for most space agencies the propulsion and the structural technologies are generally available as a set of options that they have developed over a period of time.

And because of that the I_{spi} and ε_i's are available as a set of discrete values depending upon the number of technological options that a particular agency has. We will note that in such a case ε_i's and I_{sp} are not really directly designed per say, but are only selected from available set of discrete values in a given context and the only unknown for the design problem is the stage payload ratio π_i's under the constraint that their product must be equal to π^* and that will be our statement of the design problem.

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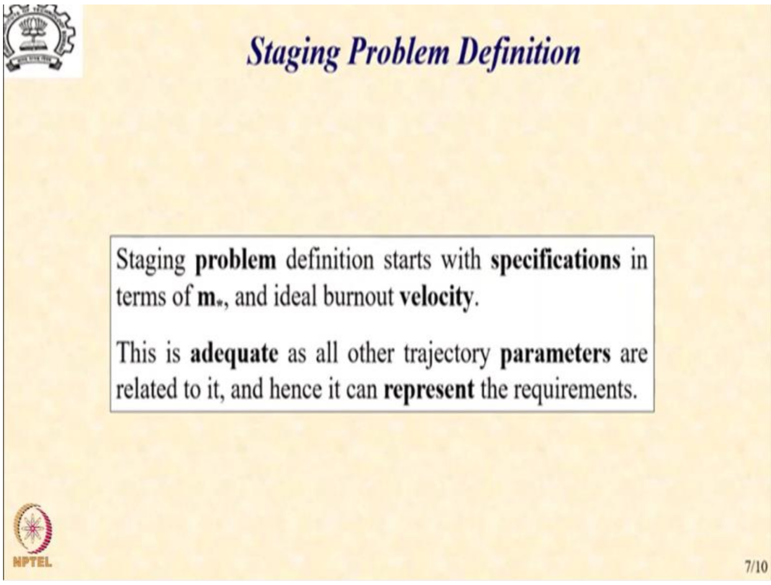

Apart from m^* or π^* as the primary driver for the design we have also seen that a rocket has a mission objective in terms of orbit and we have also established in our earlier lecture that the ideal burnout velocity is a good indicator of the nature of the mission that the rocket is required to perform. So, in that sense there are two design specifications that are commonly used to drive the design or the solution for π_i's which are m^* or π^* and the V^* the burnout velocity.

You will find that this is generally adequate for initial sizing and initial conceptual or even preliminary design of launch vehicles because most of the other parameters are now going to be related to these parameters. For example, if I know the ideal burnout velocity and if I know the mass configuration then by appropriately implementing the burn profile, I can find out what are going to be the actual velocity and altitude profiles and I am going to get the trajectory.

So, which means that once I get a configuration of rockets in terms of π_i's for a given V^* and π^* or m^* as the case maybe. We have a full rocket configuration and we also have the trajectory.

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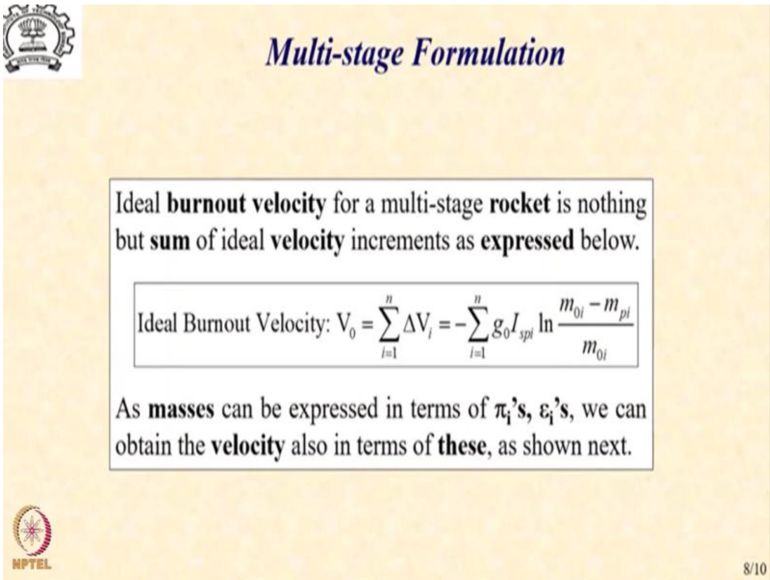

So, how do we set up this problem? The ideal burnout velocity for multistage rocket is nothing, but the sum of the velocity increments given by each of those stages. Each of those stages at an ideal ΔV and if I add all of these the total is nothing, but the ideal burnout velocity that I will get from this multistage rocket. So, I can express this in terms of the parameters that I have defined as follows.

So, we already know from our earlier discussion that the ideal burnout velocity from a higher stage is nothing, but $g_0 I_{spi} \ln \frac{m_0}{m_{pi}}$ which is nothing, but m_{fi} divided by m_{0i}. Of course, we just use the minus sign here I could easily invert the logarithm and the minus sign would disappear so it does not really matter. I going from 1 to n indicates that this velocity implement is summed up over n stages that gives us V_0 or V^* then both symbols are used interchangeably.

Now we have already defined the two mass parameters π_i's and ε_i's as non-dimensional parameters and as the ratio inside the ln is also non-dimensional. It is now appropriate that we introduce those two parameters as part of our velocity expression this is shown next.

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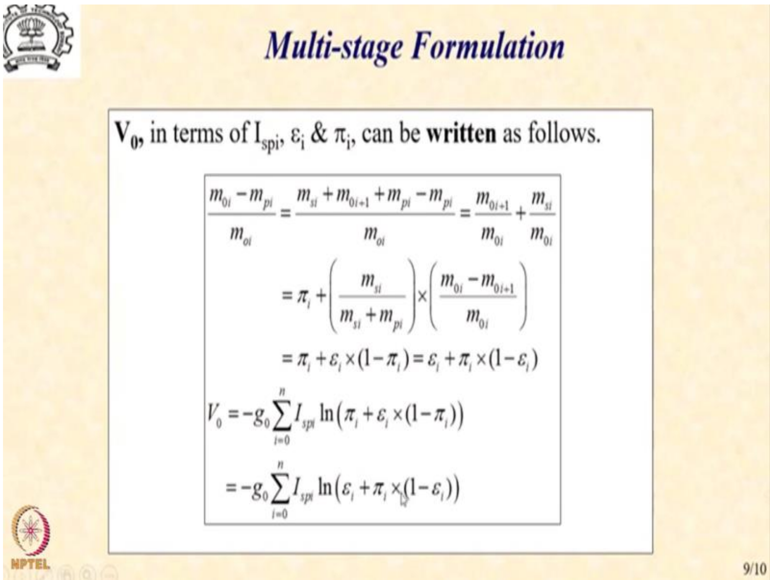

So, V_0 in terms of the three parameters $I_{spi}, \varepsilon_i, \& \, \pi_i$ can be written as follows. So, I am just showing you the long hand derivation of this. So, $\frac{m_0 - m_{pi}}{m_{0i}}$. So, m_{0i} is nothing, but $m_{si} + m_{pi} + m_{0i} + 1$, I know it from my definition. But now m_{pi} gets cancelled so what is left is $m_{si} + m_{0i} + 1$ divided by m_{0i} or I separate the two terms and the first term that is m_{0i+1}, m_{0i} is nothing, but my π_i by definition.

Similarly, the second term m_{si} and m_{0i}, I multiplied it by $m_{si} + m_{pi}$ and divided by the same. So, the first term if the division becomes my ε_i. In the second term which is $m_{si} + m_{pi}$ in the numerator I can rewrite the form as $m_{0i} - m_{0i+1}$. This is going to be which means the difference between the starting mass of the i th stage and the starting mass of $i + 1$ stage is nothing, but the mass of the i^{th} stage is $m_{si} + m_{pi}$.

The reason for this is quite obviously because I want to get this in terms of the parameters that I have defined. Again, I see that $\frac{m_{0pi}}{m_{0i}}$ is nothing, but 1 and $\frac{m_{0i+1}}{m_{0i}}$ is π_i and I get an expression called $\pi_i + \varepsilon_i \times (1 - \pi_i)$ and this can also be written as $\varepsilon_i + \pi_i \times (1 - \varepsilon_i)$. Substituting this back in our velocity expression we now have these two forms of the velocity expressions in terms of the non-dimensional parameters π_i and ε_i and the dimensional value $g_0 I_{sp}$; $g_0 \times I_{sp}$ is nothing, but the velocity units.

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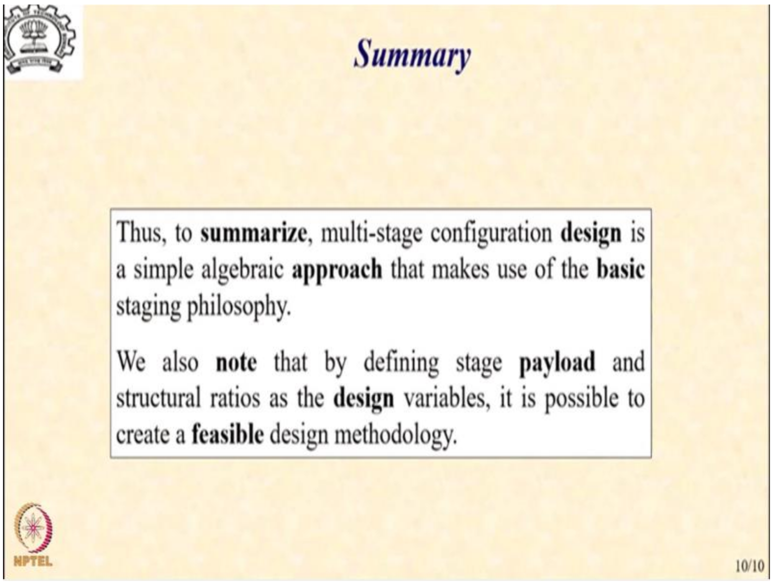

So, to summarize the multistage configuration design essentially is a simple algebraic methodology that makes use of the basic staging philosophy that we have discussed. The staging philosophy that we have discussed is typically called the serial or the series staging and, in some context, it is also called restricted staging. We also note that by defining stage payload and structural ratios it is possible to create a feasible design methodology.

We will look at the solution for these in the next lecture. So, in this lecture we have seen a simple algebraic formulation for a multistage rocket configuration based on the fundamental ideas of stage operations as has been discussed earlier. And we have noted that by defining stage payload ratios and the stage structural ratios it is possible for us to setup set of equations which when solved in an appropriate manner will generate a mass configuration that should meet the specification.

In the next lecture, we will look at the basic concept of solving these through a simple example and then understand the possibilities of arriving at the solution through different techniques. So, bye and see you in the next lecture. Thank you.