Introduction to Aircraft Design
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Lecture - 82
Tutorial on V-n Diagram of Transport Aircraft

Hello, let us have a look at how the V-n diagram is calculated for a long range transport aircraft.

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As is the usual in this course, we have taken Boeing 787 Dreamliner as our reference aircraft.

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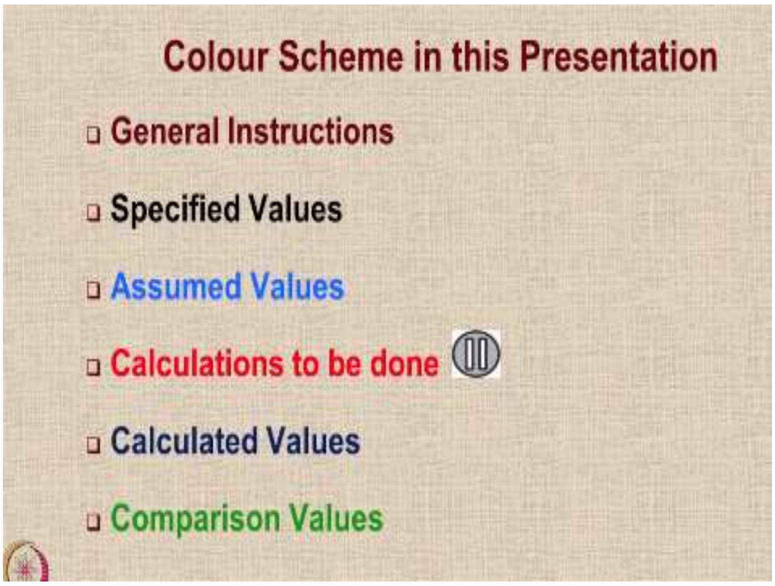

Let us look at the color scheme in this presentation which is also standard, you must be used to it by now. The general instructions are given in brown color. If there are any values which are specified in any standard document or any online source, which is taken as a reference, they will be shown in the black color. The values which are assumed or which are given in the database are in the light blue color.

Wherever we do you do some calculations, there is a hint that it is in red color and also there is a pause button after that. So I would request if you really want to learn to do these calculations properly. It is best that when you see the pause button you can pause the video, do the required calculations and then proceed further. The values which are calculated are going to be shown in the dark blue color.

And any comparison is going to be in the green color. But in this presentation of course we do not have anything to compare.

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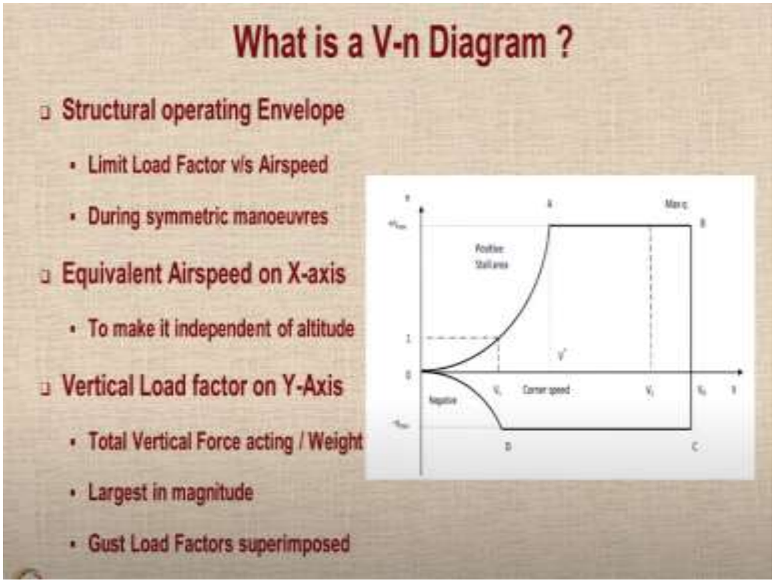

Let us first start with a quick recap about what is a V-n diagram. At this point, I suggest if you have not watched the video clips regarding V-n diagram, it is a good idea to first go and watch them and then do the tutorial. But those of you who have already watched the video clips here is a quick recap about a V-n diagram. It is basically a structural operating envelope of the aircraft. In this diagram, we plot the limit load factor versus the airspeed.

But this is applicable only for symmetric maneuvers in the xy plane. We use equivalent airspeed on x axis. The reason for that is very clear, because we want to use the same diagram for various altitudes. Usage of equivalent airspeed allows that to happen. On the y axis, we have the vertical load factor experienced by the aircraft, which is the total vertical force acting divided by the aircraft weight.

This is the largest in magnitude among the three load factors N_x, N_y, N_z. So this is actually N_z, but since we will talk only about one load factor, we are not going to use the subscript z. So when we say load factor N we actually mean the vertical load factor N_z. And on this particular diagram, the regulatory bodies have suggested that we should superimpose some gust load factors, okay.

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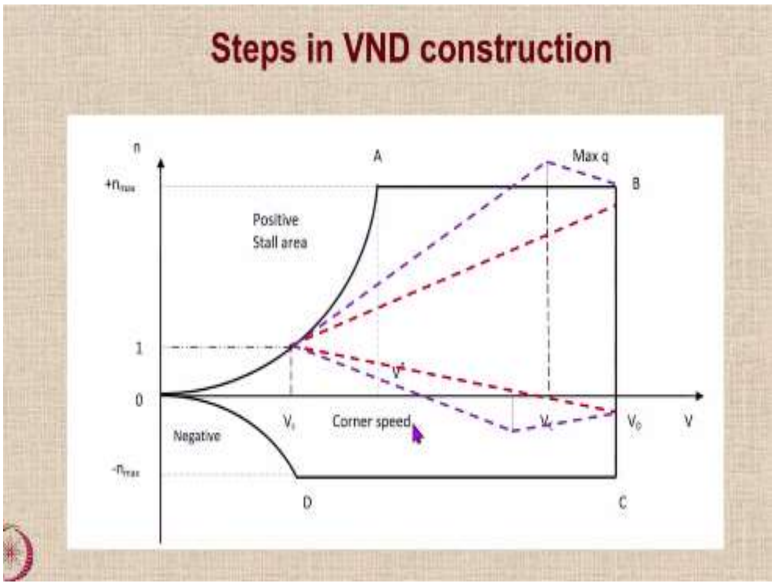

Let us look at the steps in the construction of a V-n diagram. So here is the V-n diagram. So first, we normally calculate the value of V_s which is the stall speed in level flight. It is also called as 1g stall speed. And while calculating this number you use load factor $N_z = 1$ and V equal to V_{stall}. And all along this line from O to A the C_L is actually equal to $C_{L_{max}}$ at various speeds, okay.

So therefore, this area on the left of this line is the positive stall area. So we cannot have sustained flight in this area because the aircraft is going to stall. So in a way, this gives you an operating limitation that you cannot operate for a long time on the left of the line OA. The next important point is the point D which corresponds to the negative load factor area usually when you have inverted flight.

So here what we do is we calculate the point D as the y axis value will be the most negative permitted value of the load factor, there is minus n max and the corresponding speed V depending on what is the lift coefficient maximum permissible in inverted flight. So just like we got the this point V_s for 1g level flight this is for you know maximum, this is for the maximum negative.

The next point is point A which is a very important point. It is called as the point of corner speed because the speed corresponding to this point is the corner speed. At this point there are two things which are applicable. The C_L is equal to $C_{L_{max}}$ because we are along this line, okay. And also the N_z is equal to positive N_z max because we are also on this line.

So this is the intersection of the positive maximum vertical load factor and the smallest speed because it corresponds to the highest value of C_L. This speed should be as low as possible okay because corresponding to this speed you have the tightest turn and the fastest turn rate. So this is called as the corner speed. This is obtained by the intersection between this line and the maximum permissible load factor line.

The next point of interest is the point corresponding the V-n diagram due to the cruise speed. So cruise speed is either specified or taken as a multiple multiplied fraction of the stall speed 1.25 or 1.5 okay. So here the load factor here is going to be the maximum value and the speed is going to be the maximum permitted cruise speed.

The next important point corresponds to the point where you have the maximum dynamic pressure. So this corresponds to the point which is having the design diving speed and also the N_z max. So this particular point gives the maximum loading on the aircraft. Therefore, flight on the right hand side of this line is not permitted from the structural safety considerations.

So corresponding to point B there is a point C on the negative load factor. This again corresponds to the most highest negative value of the dynamic pressure. So because it is at design diving speed and the maximum negative value of the load factor. And then on that we superimpose the gust lines. So first this is a gust line to do a positive gust starting from level flight. So that means n = 1.

And then as we will see very soon the delta N_z because of gust which is going to be the number above this line is proportional to V. So it will be a straight line up to V_c. Similarly, there will be another straight line up to the highest speed at which you are allowed to fly inverted. In this diagram or in this sketch it is shown that this speed is lower than the speed V_c for the positive flight but it could also be the same.

It depends on the specifications of the aircraft. And then up to design diving speed, we have been asked to also plot the delta N_z both in the positive direction and in the negative direction as shown here. And finally, we just join these lines from cruise to design diving speed both in the top of the graph and the bottom of the graph.

So therefore, if the dark lines shown earlier correspond to the limit maneuver V-n diagram and if these lines correspond to the limit gust diagram. So if you superimpose these two, this particular curve, the outermost curve is going to be the limit combined envelope.

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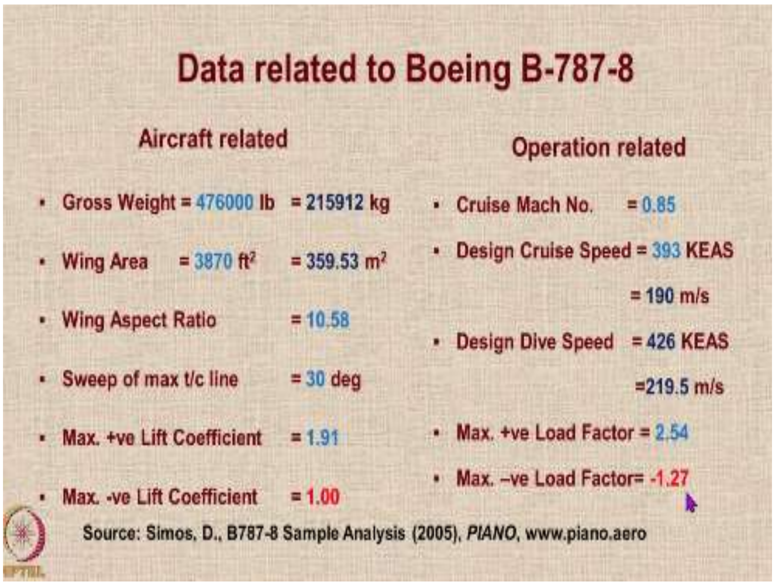

Now let us start doing the calculations for our aircraft Boeing 787-8. But before we proceed, we need to recall some of the important data and specifications of this aircraft. First, let us look at some aircraft related specifications. This aircraft is designed for a maximum gross weight of 476,000 pounds, which translates to 215912 kilograms, Wing area is 3870 square feet which translates to 359.53 square meters.

The wing aspect ratio is 10.58. The sweep of the maximum thickness line is 30 degrees. This information will be needed for calculating the value of the lift curve slope which is needed in the estimation of the additional load factor due to gust. Maximum positive lift coefficient is 1.91. And we do not have the value of maximum negative lift coefficient. So we will assume it to be 1.00. Remember this data is not available.

So we have just assumed it. As far as the operation related parameters are concerned, this aircraft has a cruise Mach number of 0.85. Its design cruise speed as specified in the document is 393 knots equivalent airspeed, which can be converted by multiplying by 0.51444 to get meters per second. If you want to do a more accurate conversion, you can multiply the knots with 1852 to convert it into meters, and then divide by 3600 to convert hours into seconds.

Similarly, the design diving speed is given as 426 knots EAS which converts to 219.5 meters per second. The maximum positive load factor that this aircraft is designed to sustain is specified in the sample analysis as 2.54 and we do not have any information regarding what is the maximum negative load factor.

So as specified by the airworthiness agencies, if the designer do not specify the maximum negative load factor, it can be assumed to be at least half of the maximum positive load factor. So please note that the maximum negative lift coefficient of 1 and maximum negative load factor of -1.27 are assumed values in this particular analysis.

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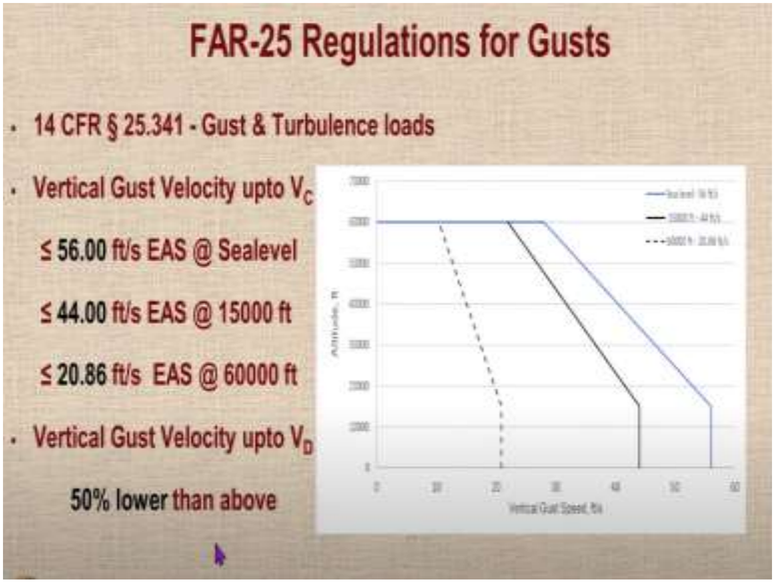

Let us look at the FAR-25 regulations for gusts. So these regulations are explained in detail in 14 CFR 25.341 titled Gust and Turbulence Loads. So there is a huge amount of information mentioned in that including things like how to look for changes in the gust velocity etc. We will take a very simple approximation and we will only look at specification of the vertical gust velocity up to the design cruise speed V_c.

So we need to take the gust speeds to be at least 56 feet per second equivalent airspeed at sea level. They can be reduced to 44 feet per second at the rate of 15,000 feet and at higher altitudes gusts are actually very weak. So therefore, the maximum velocity specified is only 20.86 feet per second equivalent airspeed. So in between, you can always interpolate to the value that you want.

As far as the specification of the gust velocity for the up to the design diving speed, it is considered to be just half of the above values. So this information can be graphically also indicated like this, where you can see that at sea level, we have the highest requirement of gust value gust speed of 56 feet per second which is supposed to be constant up to 15,000 feet. And then from there it is expected to reduce linearly, okay.

Then, if you look at the specifications at 15,000 feet, it is 44 up to 15,000 and then again it is reduced linearly. And if you look at the design diving speed, the value is 20.86 up to 15,000 feet. And then again it is to be linearly reduced. In our case, we are going to look mainly at the situation at sea level. So therefore, we are only going to be concerned with this velocity, 56 feet per second EAS at sea level.

And of course, the value up to design diving speed which will be half of this or 28 feet per second.

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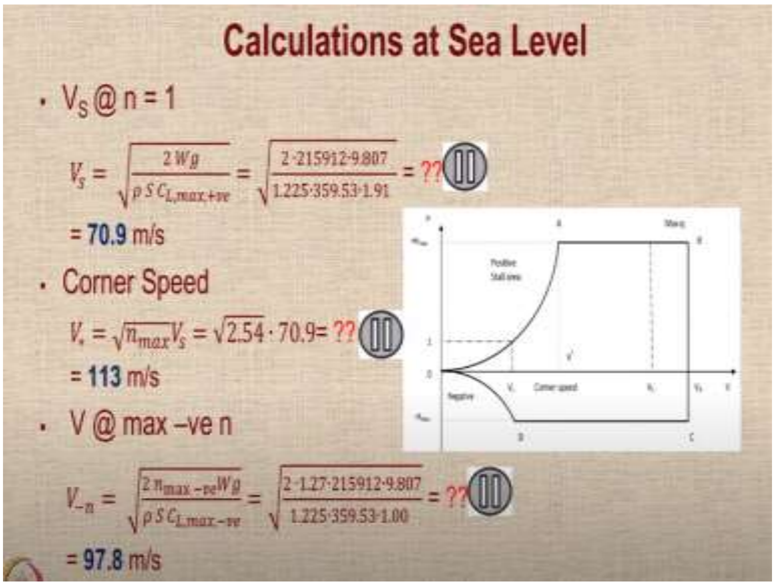

Let us look at the calculations at sea level. So this is our V-n diagram for a reference. First let us find out the value of stall speed at n = 1. That means the 1g level stall speed.

And we have to use $C_{L_{max}}$ positive which is 1.91. So at sea level conditions if you replace these, if you replace these symbols with the appropriate values, you should be able to get the estimate of the stall speed. At this stage, I would request you to pause the video and do this calculation. The value turns out to be 70 meters per second.

So the 1g level stall speed for this aircraft is 70 meters per 70.9 meters per second under sea level ISA conditions. Next we look at the corner speed. Remember for the corner speed, we both have C_L equal to $C_{L_{max}}$ positive and load factor n will be equal to this n max which is 2.54. So you can see that the value for corner speed will come from the same equation except W will be replaced by n_z max times W.

So therefore, what we can do is we can just assume it to be V* will be assumed to be or calculated. Please pause the video. Do this calculation. The value turns out to be 113 meters per second. Next, we look at what is the value of velocity when n is the maximum negative value.

That means we want to get the value of V corresponding to this particular point, okay. So at that point again the formula is same only we replace n with n max negative and we replace C_L with C_L max negative, okay. Be careful about the signs because we are going to have a square root. So because I know that in the numerator we have -1.27 the load factor and the denominator we have -1.

Therefore, I have just avoided putting the minus sign and I am getting the value directly. At this stage please pause the video and have a look at the calculation. So V at - n is going to be 97.8 meters per second. These three values please remember because we will need them in the future when we plot our V-n diagram.

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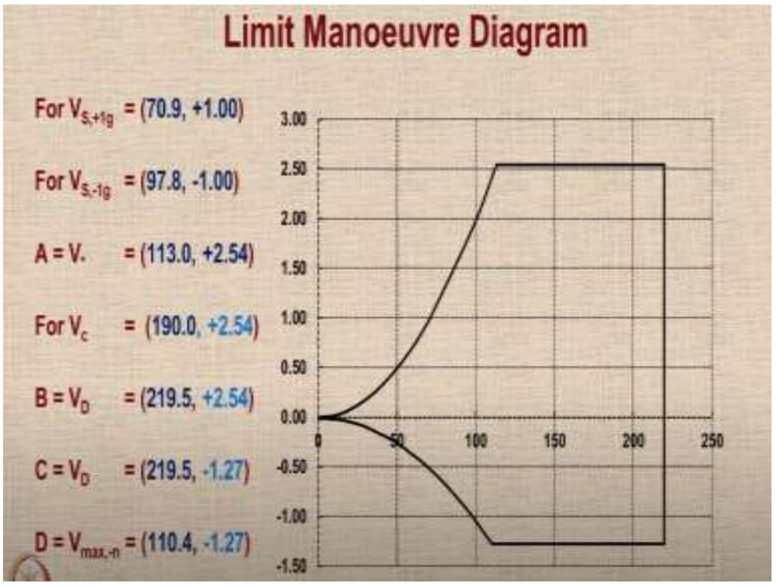

So this is our limit maneuver diagram or the limits imposed because of the maneuvers in the study at z plane okay. So the first number that we have already calculated is this particular point which corresponds to the stalling speed in 1g level flight. So n is equal to 1 and V as we just now calculate it is 70.9 meters per second. The next point is the point that corresponds to -1 the value of stalling speed at -1g.

So what is the -1g level flight stalling speed in inverted flight. So this one will be also along the same line and the only thing is n will be equal to -1. So for that there is a corresponding value of 97.8 meters per second. The next point is the corner speed, which we have just now calculated corresponding to maximum n_z 2.54 and also along this line. So the intersection of these two lines that is V* the corner speed.

The value is the coefficients of this particular point are going to be for the x axis it will be 113.0, that is the corner speed value and for the y axis it is going to be 2.54. Next point is the design cruising speed point. So this particular the contents of this point if you see here, they will correspond to cruising speed which is 190 and the value of n z here is maximum. So for V_c we have this particular point.

The next point is point B which corresponds to the maximum design diving speed and also the maximum load factor. So that would be 219.5 and +2.54. Corresponding to B there will be a similar point C which will have the same design diving speed value 219.5 but n_z is going to be now the maximum negative which is -1.27.

And finally, we have the point D where we have the value of velocity at which the load factor has the maximum negative value of -1.27. So you are on this particular line and on this line C_L will be maximum C_L permitted for inverted flight or in the negative condition maximum negative C_L and n_z will be the maximum negative permitted load factor.

So this point is analogous to the corner speed, but it is in inverted flight, okay. So this is the limit maneuver diagram which we have already plotted for this particular aircraft.

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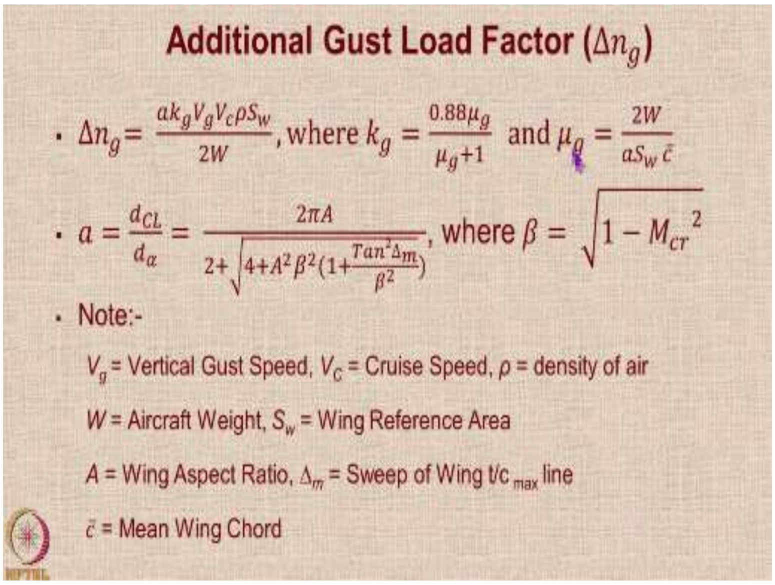

Let us now look at how to estimate the additional gust load factor and how additional limits get imposed in the V-n diagram due to these additional gust loads. For that recall from the previous video on V-n diagram, we have already shown there that the additional load factor due to a gust

$$\Delta n_g = \frac{a\, k_g V_g V_c \rho S_w}{2W}$$

Here k_g is the gust alleviation factor, So we notice that larger the wing loading W/S, smaller the lift curve slope and smaller the chord larger is the value of μ_g and then that adds that adds in the calculations here, okay.

So now the value of a that is needed here for the calculation of lift curve slope and also here for the calculation of μ_g is the $\frac{dC_l}{d\alpha}$ of the aircraft, which can be shown to be

$$a = \frac{dC_l}{d\alpha} = \frac{2\pi A}{2 + \sqrt{4 + A^2\beta^2\left(1 + \frac{tan^2\Lambda_m}{\beta^2}\right)}}$$

So where this Λ_m β A are aircraft related parameters. β is basically

$$\beta = \sqrt{1 - M_{cr}^2}$$

So in this case V_g here is the specified vertical gust speed which we just now saw. V_c is the cruise speed or the diving speed depending on which condition we calculate. ρ is the density of air which is used in the calculations here. Then W is the aircraft weight that is this W and S is the wing area. Many people use W/S directly in this particular calculation. So in that case S comes below and this comes 2 into W by S okay.

the value A here is the wing aspect ratio as I mentioned to you Λ_m is the sweep of the maximum thickness chord line. And $\bar{c}$ is the mean wing chord. So armed with this information, we will now first calculate the lift curve slope for this aircraft. Then using that we will calculate the value of μ_g. Once we know mu g we will calculate the value of k_g.

And once we know k_g we can put k_g here in the expression and get Δn_g. which means you already are flying at a load factor of 1. And either on that there is a vertical downward gust or a vertical upward gust. Depending on that we use either plus or minus.

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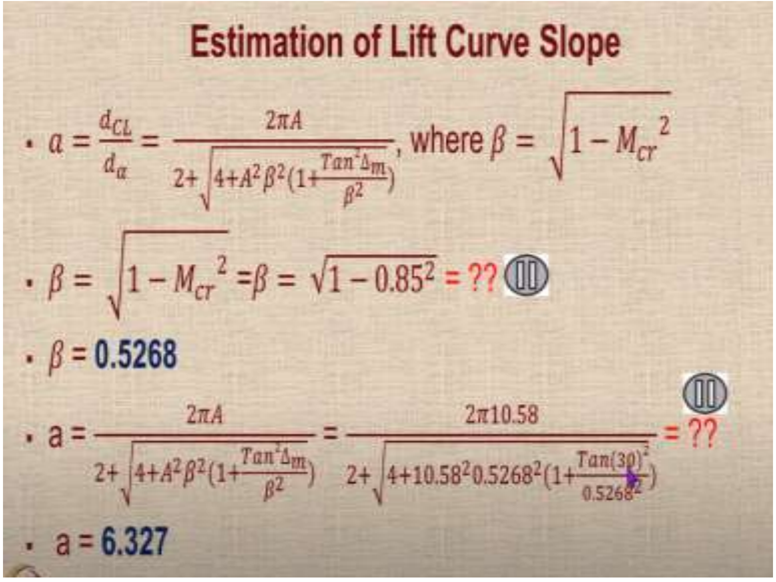

Let us estimate the lift curve slope of this aircraft. So the formula for that I have just reproduced from the previous slide. Now $\beta = \sqrt{1 - M_{cr}^2}$. So please calculate this number. It turns out to be 0.5268. So now that we know beta we can actually calculate the lift curve slope by putting all the numbers in this big formula.

So I would like you to pause the video and do this calculation and then compare with the value that I got. The value that I got is aa = 6.327. This is actually per degree because 30 is 30 degrees here, okay. Okay.

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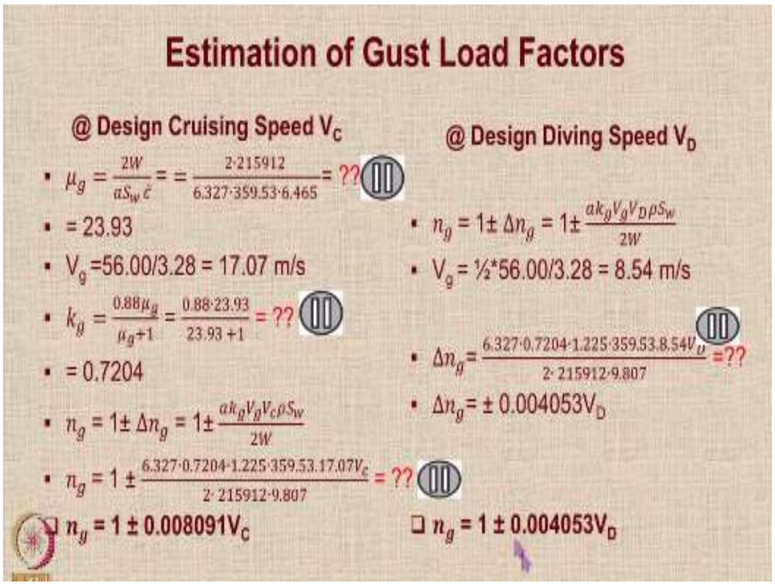

Now armed with the information let us now estimate the gust load factor. First we examine the gust load factors at the design cruising speed. So you know that μ_g is equal to $\frac{2W}{aS_w\bar{c}}$, a has just been calculated as 6.327. S_w is given 359.53. $\bar{c}$ is given 6.465. W is given 21592 kilograms. So just multiply by 2. So let us pause here and calculate the value. The value is 23.93.

Moving ahead, the specified value for the gust speed at sea level was 56 feet per second. You will divide by 3.28 to convert it to meters per second and k_g which will be used in the calculation for Δn_g. Once you know the value of μ, you can get the gust elevation factor k_g. Please calculate this value.

This number comes out to be 0.7204. So what it means is that a vertical gust acting on this aircraft, the sharp vertical gust will actually be reduced by a factor of around 28% to 0.7204. So the additional load factor created because of a vertical gust is going to be reduced by a factor of around 28% because of the value of k_g which is an aircraft specific parameter.

It depends upon the wing loading, it depends upon the lift curve slope and the span chord of the aircraft, okay. And what we have done here is we have just reproduced a 6.3272, k_g as 0.7204, V_g as 17.07, V_c remains as it is, ρ is 1.225 and S_w is 359.53.

And on the bottom we have replaced W by W into g. So just to take care of the units, okay. So please calculate the value of this whole expression. So it will be 1 plus a constant times V_c. That number turns out to be 0.008091. So additional gust would be this number times the cruise speed and that will add or subtract from 1 because it will act in the level flight either as a vertical upward gust in which case you will add it or a vertical downward goes in which you will subtract it.

Similarly, we have to apply the conditions for the design diving speed V_d. Remember, formula remains the same except that instead of V_c now we will have V_d in the numerator. And also we have to be careful that the value of the gusts speed V_g is also going to change. Because you know that V_g is specified as half of the value at the cruising speed. So the value of V_g will be half.

It will be 8.54 meters per second as compared to 17.07 meters per second. So substituting the values in the formula Δn_g equals 6.327 which is a. These are the other parameters like area okay weight n_g etc. I would like you to pause and calculate the value of Δn_g. The value comes to be 0.004053 times V_d okay. So therefore, n_g the gust load factor due to vertical and downward gust acting at the design diving speed would vary as the speed.

So from 0 to V_d with this particular formula 1 ± 0.004053. So notice that this n_g versus V will be a straight line and it will start from $V_c = 0$ it will become 1 and when V_c equal to whatever value is there for the design cruise speed it will add a number to that. So it will be a straight line with origin from the point 0, 1 okay. Similarly here.

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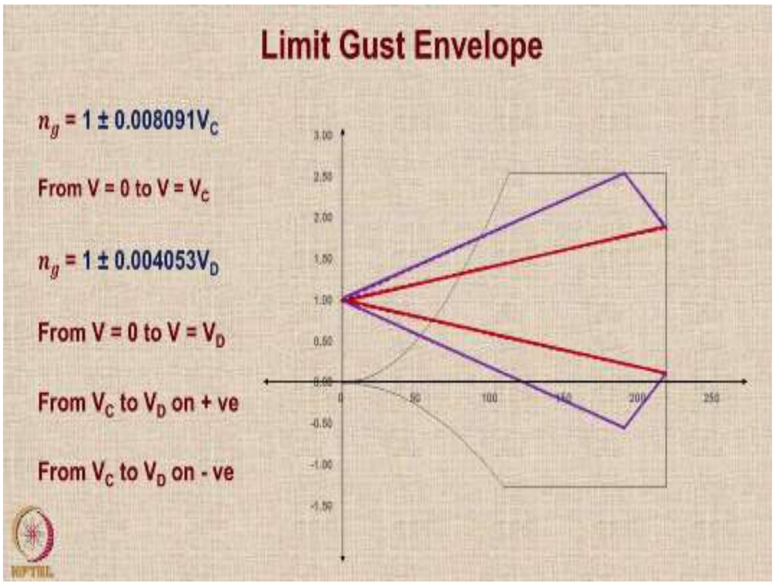

So let us look at the limit gust envelope. So the first equation is n_g equal to $1 \pm 0.008091V_c$. This is for the vertical gusts acting up to the cruising velocity when the gust speed is 56 feet per second. So from V equal to zero to V equal to V_c, we will just get two lines like this, where this point will correspond to you know it will correspond to 0, 1 level flight.

And as V changes as per this expression, the n_z value will be $1 + 0.008091V_c$. And this line will correspond to the value when you have V_c when you have from 0 to V_c when you have $1 - 0.008091$ times V_c. Similarly, if we look at the line for the additional load factor up to the design diving speed, so we notice that once again we get two lines, which start from the same point 0, 1.

One of them increases, the other of them decreases. This increases because this is equal to 1 plus a number, which is never going to become less. It is only going to only increase and this is going to be 1 minus a number okay. So that will depend upon the values. And you can see that even when you are at design diving speed, you still have some vertical load factor.

And then finally, what we do is, we just join these from straight lines from V_c to V_d on positive and from V_c to V_d on negative. So this is the limit gust envelope. That means, if you are in a if you are flying an aircraft, and if there is a gust acting on the aircraft, in level flight the aircraft can go into any of these areas. So the aircraft has to be strong enough to withstand the loads that come when the operating point is within this particular area.

So on this I have just superimposed the previous V-n diagram. So we see that there is not any problem because of gust, no additional material comes out. I mean, the gust envelope is almost completely inside the general maneuver envelope. So in this case, it turns out that the gusts are not imposing any additional limits. But that is not true always. That is not always true.

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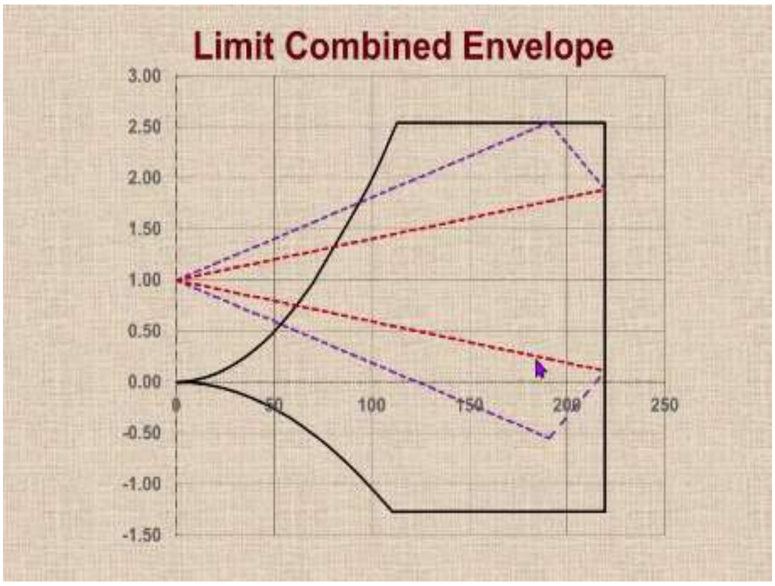

So here is our limit combined envelope. Here is our limit combined envelope in which the dotted lines represents the additional load factor imposed due to the vertical gusts which are known to be or taken to be 56 feet per second for up to design cruising speed and half of that up to design diving speed, both in the positive axis and in the negative axis.

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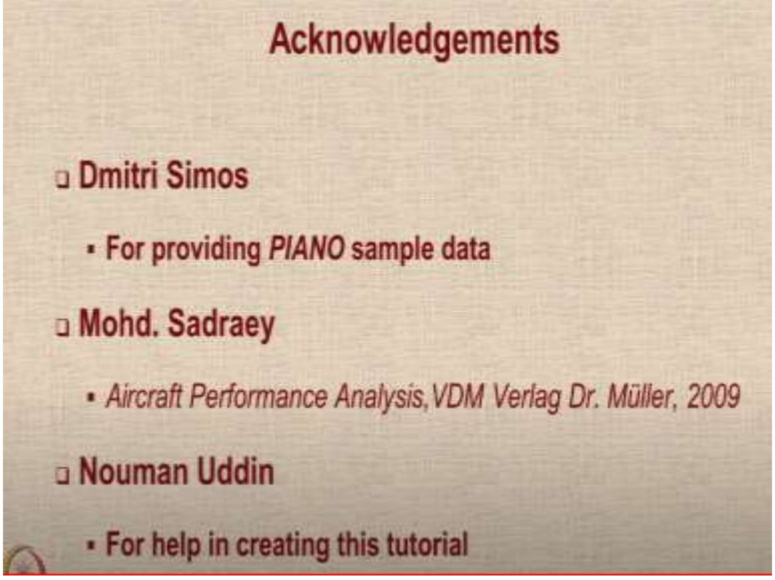

So thank you so much for your attention. I would like to acknowledge the contribution of the Dmitri Simos for providing the PIANO sample data which we have used in this calculation, Professor Mohd. Sadraey for his excellent textbook on Aircraft Performance Analysis, in which this particular V-n diagram calculation procedure is very nicely and lucidly explained.

And I have taken it from there and also some of the sketches that you have seen in this presentation. And last but not the least, I like to thank Nouman Uddin for help in creating this tutorial and plotting the V-n diagram. Thank you.